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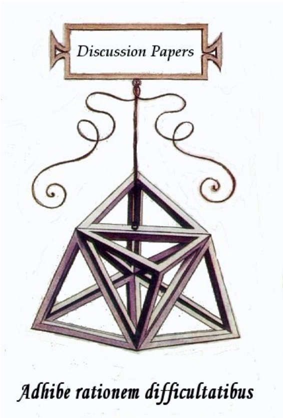

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How Reliable Are the Geographical Spatial Weights Matrices?

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Discussion Paper
n. 198

Davide Fiaschi* and Angela Parenti†

How reliable are the geographical spatial weights matrices?

Abstract

This paper shows how it is possible to estimate the interconnections between regions by a connectedness matrix recently proposed by Diebold and Yılmaz (2014), and discusses how the connectedness matrix is strictly related to the spatial weights matrix used in spatial econometrics. An empirical application using growth rate volatility of per capita GDP of 199 European NUTS2 regions (EU15) over the period 1981-2008 illustrates how our estimated connectedness matrix is not compatible with the most popular geographical weights matrices used in literature.

Classificazione JEL: C23; R11; R12; O52
Keywords: First-order Contiguity, Distance-based Matrix, Connectedness Matrix, European Regions, Network

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I. Introduction

In spatial econometric literature, "the matrix is the fundamental tool used to model the spatial interdependence between regions. More precisely, each region is connected to a set of neighboring regions by means of a *spatial pattern* introduced *exogenously* as a spatial weight matrix W" (Le Gallo et al., 2003, p.110).

The traditional specification of the spatial weights matrix relies on the geographical relation between observations, implying that areal units are neighbors when they share a common border (first-order contiguity), or the distance between their centroids is within a distance cut-off value (distance-based contiguity). As pointed out by Anselin and Bera (1998), other specifications of the spatial weights matrix are possible as, for example, weights reflecting whether or not two individuals belong to the same social network, or based on some "economic" distance. Although these specifications are desired, the resulting spatial process must satisfy necessary regularity conditions. "For example, this requires constraints on the extent on the range of interaction and/or the degree of heterogeneity implied by the weights matrices" (Anselin and Bera, 1998, p. 244).

Moreover, "in the standard estimation and testing approaches, the weights matrix is taken to be *exogenous*" (Anselin and Bera, 1998, p. 244). Therefore, the W matrix represents the *a priori* assumption about interaction strength between regions. However, in many cases considerable attention should be given to specifying the matrix W to represent as far as possible economic links (see Corrado and Fingleton, 2012).

In a companion paper (see Fiaschi and Parenti, 2013) we show how is possible to estimate the interdependence between European regions by a *connectedness matrix*, which is the result of a general variance decomposition analysis on the residuals of a VAR model. The connectedness matrix has the advantage to be immediately interpretable as a network, allowing for the use of network connectedness measures to understand the interdependence among regions (see Diebold and Yılmaz, 2014).

The aim of the paper is to discuss how our (contemporaneous) connectedness matrix is strictly related to the spatial matrix W, and to compare the network deriving from connectedness matrix with that deriving from a spatial model that can display mixed dynamics in both space and time. An empirical application using growth rate volatility of per capita GDP of 199 European NUTS2 regions (EU15) over the period 1981-2008 is used to illustrate our analysis.

The paper it is organized as it follows: Section II. explains the methodology to estimate the connectedness matrix. Section III. traces a comparison between our connectedness matrix and the spatial weights matrix. Section IV. contains the empirical application to EU regions. Section V. concludes.

II. The Methodology

The estimate of connectedness matrix follows the methodology described in Fiaschi and Parenti (2013). To sum up: firstly, a panel of growth rate volatilities (GRV) of per capita GDP for a sample of regions is estimated; then, the panel is used to perform a general variance decomposition analysis (GVD hereafter) on the residuals from a VAR in order to estimated the so-called *connectedness matrix*. The procedure is largely inspired by Diebold and Yılmaz (2014), with the additional difficulty arising in the estimate of VAR, that the number of observations for each region is generally lower than the number of regions, i.e. we face a typically high-dimensional problem (see Hastie et al., 2008). To overcome this problem a Bayesian Model Averaging is used.

II.A. The Connectedness Matrix

Following McConnell and Perez-Quiros, 2000 and Fiaschi and Lavezzi, 2011 the basic idea to build a panel of GRV is that the dynamics of growth rate of per capita GDP can be well-approximated by an autoregressive process of order p (denoted by AR (p)):

$$\gamma_{jt} = \mu_j + \phi_1 \gamma_{j,t-1} + ... + \phi_p \gamma_{j,t-p} + \epsilon_{jt}, \tag{1}$$

where ϵ_{jt} is assumed to be normally distributed. Given that ϵ_{jt} follows a normal distribution, an unbiased estimator of the standard deviation of ϵ_{jt}, σ^{ϵ}_{jt}, is given by:

$$\hat{\sigma}^{\epsilon}_{jt} = \sqrt{\frac{\pi}{2}} |\hat{\epsilon}_{jt}|. \tag{2}$$

From Eq. (2) we derive the unbiased estimator of the standard deviation of the growth rate of per capita GDP, σ^{γ}_{jt}. For example, if the growth rate follows an AR(1) process (see Hamilton (1994), p. 53), the standard deviation of the growth rate is given by:

$$\hat{\sigma}^{\gamma}_{jt} = \frac{\hat{\sigma}^{\epsilon}_{jt}}{\sqrt{1-\phi_1^2}} = \frac{\sqrt{\frac{\pi}{2}} |\hat{\epsilon}_{jt}|}{\sqrt{1-\phi_1^2}}. \tag{3}$$

This method is easily extended to higher-order AR models (see Hamilton, 1994, pp. 58-59).

Once the panel of GRV has been estimated, we follow Diebold and Yılmaz (2014) in the use of a vector-autoregressive (VAR) model to represent the process governing the GRV of regions, and estimate the GVD which allows to measure the population *connectedness*, i.e. assessing the share of forecast error variance in a region due to shocks arising *elsewhere*.

The use of VAR implicitly implies that relationships across units of observations are essentially linear, and that the contemporaneous relationships are well represented by pairwise correlations (i.e., the variance-covariance matrix). Moreover, the use of GVD is subject to some restrictive assumptions, the most notable is the Gaussian distribution of shocks.[1]

Assume that a VAR of order p is a good approximating model of the process governing the GRV of regions:[2]

$$\mathbf{x}_t = \mathbf{c} + \sum_{i=1}^{p} \mathbf{\Phi}_i \mathbf{x}_{t-i} + \epsilon_t, \quad t = 1, ..., T, \tag{4}$$

where $\mathbf{c}$ is a $N \times 1$ vector of constants, $\mathbf{x}_t = (x_{1t}, ..., x_{Nt})'$ is a $N \times 1$ vector of jointly determined dependent variables, $\mathbf{\Phi}_i,\ i = 1, ..., N$ is the $N \times N$ coefficients matrix and ϵ_t is an error term such that $E(\epsilon_t) = \mathbf{0}$, $E(\epsilon_t \epsilon_t') = \Sigma\ \forall t$, where $\mathbf{\Sigma} = \{\sigma_{ij},\ j = 1, ..., N\}$ is an $N \times N$ positive definite matrix, and $E(\epsilon_t \epsilon_{t'}') = \mathbf{0}$ for all $t \neq t'$.

Assuming also that all roots of $|\mathbf{I}_N - \sum_{i=1}^{p} \mathbf{\Phi}_i z^i| = 0$ fall inside the unit circle, that is $\mathbf{x}_t$ is covariance-stationary (see Pesaran and Shin, 1998), Eq. (4) can be rewritten as the infinite moving average representation:

$$\mathbf{x}_t = \mu + \sum_{i=0}^{\infty} \mathbf{\Theta}_i \epsilon_{t-i}, \quad t = 1, ..., T, \tag{5}$$

where $\mu = (\mathbf{I}_N - \mathbf{\Phi}_1 - \cdots - \mathbf{\Phi}_p)^{-1}\mathbf{c}$ is the mean of the process, and the $N \times N$ coefficient matrices $\mathbf{\Theta}_i$ can be obtained as $\mathbf{\Theta}_i = \mathbf{\Phi}_1 \mathbf{\Theta}_{i-1} + ... + \mathbf{\Phi}_p \mathbf{\Theta}_{i-p},\ \ i = 1, 2, ...$ with $\mathbf{\Theta}_0 = \mathbf{I}_N$ and $\mathbf{\Theta}_i = 0$ for $i < 0$.

To measure the effect of shocks at a given point in time on the expected future values of variables in a dynamical system, Koop et al. (1996) advance the *generalized impulse response function*. In particular the *scaled* generalized impulse response function of x_t at horizon H is given by:

$$\psi_j^g(H) = \sigma_{jj}^{-\frac{1}{2}} \mathbf{\Theta}_H \mathbf{\Sigma} \mathbf{e}_j, \tag{6}$$

where $\mathbf{e}_j$ is the selection vector (a vector of all zeros with 1 in the j-th element), which measures the effect of one standard error shock to the j-th unit of observations at time t on expected values of $\mathbf{x}$ at time $t + H$.

From the above generalized impulses, Pesaran and Shin (1998) derive the generalized (i.e., order-invariant) forecast error variance decomposition, defined as the proportion of the H-step ahead forecast error variance of variable i which is accounted for by innovations in variable j. Then, for $H = 1, 2, ...$, H-step GVD matrix $\mathbf{D}^{gH} = [d_{ij}^{gH}]$ has entries:[3]

$$d_{ij}^{gH} = \frac{\sigma_{jj}^{-1} \sum_{h=0}^{H-1} (e_i' \mathbf{\Theta}_h \mathbf{\Sigma} e_j)^2}{\sum_{h=0}^{H-1} (e_i' \mathbf{\Theta}_h \mathbf{\Sigma} \mathbf{\Theta}_h' e_i)} \tag{7}$$

[1] Alternatively, the use of Cholesky-factor identification is sensitive to ordering of the units of observations.

[2] Notation refers to Pesaran and Shin (1998).

[3] Notice that $H = 1$ actually corresponds to the contemporaneous connectedness.

where $\mathbf{\Theta}_h$ is the coefficient matrix of the h-lagged shock vector in the MA representation of the *non-orthogonalized* VAR, $\mathbf{\Sigma}$ is the covariance matrix of the shock in the *non-orthogonalized* VAR, and σ_{ii} its diagonal.

As in Diebold and Yılmaz (2014) we normalize the GVD matrix by row in order to have unity sums of forecast error variance contribution (remember that the shocks are not necessarily orthogonal in GVD, therefore their sum is not equal to 1 in the standard decomposition). Therefore, the *connectedness matrix* has entries as:

$$\tilde{d}_{ij}^{gH} = \frac{d_{ij}^{gH}}{\sum_{j=1}^{N} d_{ij}^{gH}}. \tag{8}$$

	x_1	...	x_N	From Others
x_1	$\tilde{d}_{11}^{gH}$	...	$\tilde{d}_{1N}^{gH}$	$\sum_{j=1}^{N} \tilde{d}_{1j}^{gH}$, $j \neq 1$
$\vdots$	$\vdots$		$\vdots$	$\vdots$
x_N	$\tilde{d}_{N1}^{gH}$	...	$\tilde{d}_{NN}^{gH}$	$\sum_{j=1}^{N} \tilde{d}_{Nj}^{gH}$, $j \neq N$
To Others	$\sum_{i=1}^{N} d_{i1}^{H}$, $i \neq 1$	...	$\sum_{i=1}^{N} \tilde{d}_{iN}^{gH}$, $i \neq 1$	$\frac{1}{N} \sum i, j = 1^N \tilde{d}_{ij}^{gH}$, $i \neq j$

Table 1: Connectedness Matrix derived from the GVD Matrix.

In particular, $\tilde{d}_{ij}^{gH}$ is the fraction of region's i H-step forecast error variance due to shocks in region j. The *cross*-variance decomposition, that is the off-diagonal elements (i.e., $i \neq j$), measure the *pairwise directional connectedness*; in general, $\tilde{d}_{ij}^{gH} \neq \tilde{d}_{ji}^{gH}$), i.e. GVD matrix is not symmetric. On the other hand, the diagonal elements (*own connectedness*) measure the fraction of region's i H-step forecast error variance due to shocks arising in the same region (i.e. idiosyncratic shocks).

The connectedness matrix is conditioned to the predictive horizon H, which is in turn related to the concept of *dynamic* connectedness. In particular, GVD 1-step ahead represents the *contemporaneous* connectedness. As the predictive horizon H increases there is more possibility for connectedness to appear. In this sense, we can distinguish between short-run and long-run connectedness.

The typical dimensions of datasets used in cross-country and cross-region analysis are such that the number of countries/regions N is much higher than the length of time series T, i.e. we generally face a high-dimensional problem with $N \gg T$. Firstly, this suggests to maintain the order of VAR at the minimum level equal to 1, i.e. GRV of each region at time t will depend on a constant, on its lagged GRV at time $t-1$, and on the GRV of all other regions at time $t-1$. Secondly, since the total number of parameters to be estimated equal to $K = N + 1$, i.e. all lagged GRV of regions plus constant, is higher than the number of observations T, the VAR(1) cannot be estimated by standard OLS. We overcome this problem by using a Bayesian Model Averaging approach (see Fiaschi and Parenti, 2013, for technical details).

II.B. A Network Interpretation of Connectedness Matrix

The proposed methodology has a straightforward interpretation in terms of network and of percolation of shocks through it. As stated by Diebold and Yılmaz (2014, p.123) "[...] *variance decompositions are networks.* More precisely, the variance decomposition matrix D, which defines our connectedness table, and all associated connectedness measure, is a network adjacency matrix A. Hence network connectedness measures can be used in conjunction with variance decompositions to understand connectedness among components". Specifically, GVD defines a weighted, directed network.

For the sake of simplicity consider the case with three regions and a representation by VAR(1), whose variance-covariance matrix $\mathbf{\Sigma}$ is given by:

$$\mathbf{\Sigma} = \begin{bmatrix} \sigma_{11} & \sigma_{12} & 0 \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ 0 & \sigma_{32} & \sigma_{33} \end{bmatrix}, \tag{9}$$

from which the GVD matrix at $H = 1$:

$$\mathbf{D}^{g1} = \begin{bmatrix} 1 & \frac{\sigma_{12}}{\sigma_{22}} & 0 \\ \frac{\sigma_{21}}{\sigma_{11}} & 1 & \frac{\sigma_{23}}{\sigma_{33}} \\ 0 & \frac{\sigma_{32}}{\sigma_{22}} & 1 \end{bmatrix}. \tag{10}$$

In case of VAR(1) $\mathbf{\Theta}_0 = \mathbf{I}_N$, $\mathbf{\Theta}_1 = \mathbf{\Phi}$, $\mathbf{\Theta}_2 = \mathbf{\Phi}^2$, ..., where $\mathbf{\Phi}$ is the coefficient matrix of VAR.

The network representation related to $\mathbf{D}^{g1}$ in Eq. (10) is reported in Fig. (1). The structure of *contemporaneous* network fully reflects the shape of $\mathbf{\Sigma}$ both in terms of existence of links and in terms of their strength. However, differently from $\mathbf{\Sigma}$, $\mathbf{D}^{g1}$ is not symmetric, i.e. the contemporaneous network is both *weighted* and *directional*. The row standardization has not an impact on the analysis in the case we are interested in only the existence of links between two regions. In a more complete analysis of percolation of shocks trough network this normalization is however not neutral and the use of the original values of GVD matrix is the best option.

Assuming that the coefficient matrix $\mathbf{\Theta}_1$ of the *1*-lagged shock vector in the MA representation of the VAR(1) is given by:

$$\mathbf{\Theta}_1 = \mathbf{\Phi} = \begin{bmatrix} \phi_{11} & 0 & \phi_{13} \\ 0 & 0 & 0 \\ \phi_{31} & 0 & \phi_{33} \end{bmatrix} \tag{11}$$

GVD matrix at $H = 2$ is given by:

$$\mathbf{D}^{g2} = \begin{bmatrix} \frac{\sigma_{11}+\phi_{11}^2\sigma_{11}}{\sigma_{11}+\phi_{11}^2\sigma_{11}+\phi_{13}^2\sigma_{33}} & \frac{\sigma_{12}}{\sigma_{22}}\left[1+\left(\phi_{11}+\phi_{13}\frac{\sigma_{32}}{\sigma_{12}}\right)^2\right] & \frac{\phi_{13}^2\sigma_{33}}{\phi_{33}\phi_{13}\sigma_{33}+\phi_{31}\phi_{11}\sigma_{11}} \\ \frac{\sigma_{21}}{\sigma_{11}} & 1 & \frac{\sigma_{23}}{\sigma_{33}} \\ \frac{\phi_{31}^2\sigma_{11}}{\phi_{11}\phi_{31}\sigma_{11}+\phi_{13}\phi_{33}\sigma_{33}} & \frac{\sigma_{32}}{\sigma_{22}}\left[1+\left(\phi_{33}+\phi_{31}\frac{\sigma_{12}}{\sigma_{32}}\right)^2\right] & \frac{\sigma_{33}+\phi_{33}^2\sigma_{33}}{\sigma_{33}+\phi_{33}^2\sigma_{33}+\phi_{31}^2\sigma_{11}} \end{bmatrix} \tag{12}$$

The network representation related to $\mathbf{D}^{g2}$ in Eq. (12) is reported in Fig. (2). The structure of network appears crucially affected by $\mathbf{\Phi}$ both in terms of the emergence of new links and in terms of their strength.

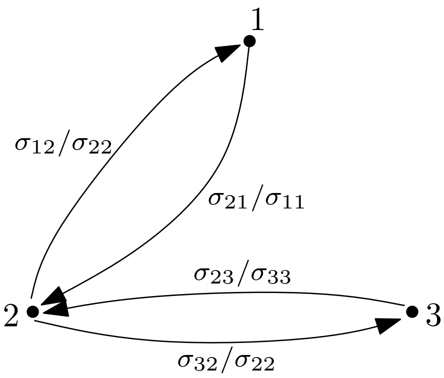

Figure 1: Network representation of GVD matrix at horizon $H = 1$.

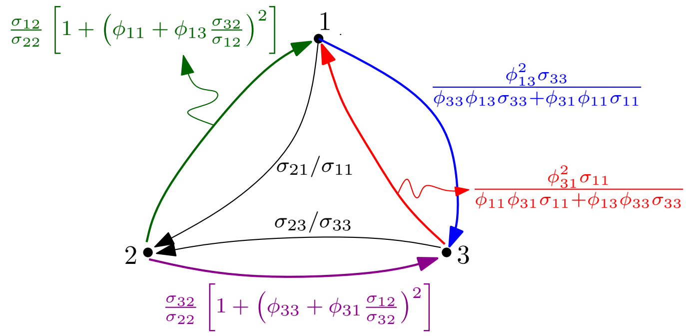

Figure 2: Network representation of GVD matrix at horizon $H = 2$.

In particular, new links appear connecting Regions 1 and 3 through the VAR coefficients ϕ_{13} and ϕ_{31}.

VAR coefficients also drive the extent of persistence of shocks over time; for example, d_{11}^{g2} depends on ϕ_{11} (the effect of autoregressive component of Region 1) and ϕ_{13} (the shocks received from Region 3); coefficients appear to have a power proportional to time horizon (i.e. for $H = 2$), that is shocks have an exponential decay. It is straightforward to show that a longer time horizon increases the strength of links with an exponential decay (for example d_{11}^{g3} includes terms like ϕ_{11}^4 and ϕ_{13}^2).

Region 2, missing any lag with itself and with the other regions in the VAR, displays a network partially independent of the time horizon considered. In particular, the connectedness from Region 2 to other regions are affected through the contemporaneous covariances σ_{12}, σ_{22}, and σ_{32}, while the connectedness from other regions to Region 2 are not affected by H.

Relaxing the assumption of VAR(1), for example in favor of VAR(2), increases both the percolation of shocks through network and their persistence, but the qualitative results remain the same.

III. Connectedness Matrix versus Spatial Weigths Matrix

As discussed in the introduction the main goal of the paper is to get some insights on the shape of spatial weights matrix $\mathbf{W}$, which in spatial literature measures the spatial dependence across different regions.

$\mathbf{W}$ is generally taken as exogenous in spatial literature, and it is specified or in term of geographic contiguity or in terms of geographical distance (see Anselin, 2001). Corrado and Fingleton (2012) formulate three main critiques to current literature: i) the values in the cells of $\mathbf{W}$ comprise an explicit hypothesis about the strength of interlocation connection", in particular, "*a priori* assumption about interaction strength"; ii) "Typically, isotropy is assumed, so that only distance between j and h is relevant, not the direction j to h"; iii) "The potential for dynamic $\mathbf{W}$ matrices poses some problems for estimation, given the assertion that $\mathbf{W}$ is necessarily a fixed entity. While this may not be such an issue for cross-section approaches, [...], with the extension of spatial econometrics to include panel data modelling it may be the case that $\mathbf{W}$ is evolving."

To discuss how our contemporaneous connectedness matrix $\mathbf{D}^{g1}$ is strictly related to $\mathbf{W}$ assume that the data generating process of GRV of N regions y follows:

$$\mathbf{y}_t = (\mathbf{I}_N - \rho\mathbf{W})^{-1}\left[\mu_N + \mathbf{X}_t\beta + \mathbf{u}_t\right] = (\mathbf{I}_N - \rho\mathbf{W})^{-1}\left[\mu_N + \mathbf{X}_t\beta\right] + \mathbf{v}_t \qquad (13)$$

where $\boldsymbol{\mu}_N$ is the vector of fixed effects of length N, $\mathbf{y}_t$ is a vector of length N, $\mathbf{X}_t$ is a matrix of dimensions $(N \times k)$, $\boldsymbol{\beta}$ is a vector of coefficients of length k, and $\mathbf{u}_t$ is the vector of error component of length N, and $\mathbf{v}_t$ is the vector of spatially filtered error component of length N.[4] The error component $\mathbf{u}_t$ is specified as:

$$\mathbf{u}_t = \lambda\mathbf{W}\mathbf{u}_t + \psi\mathbf{W}\mathbf{u}_{t-1} + \delta\mathbf{u}_{t-1} + \epsilon_t, \qquad (14)$$

where ϵ_t is the vector of innovations, with $E\left[\epsilon_t,\right] = 0$, $E\left[\epsilon_t\epsilon_t',\right] = \sigma_\epsilon^2 I_N$, and $E\left[\epsilon_t\epsilon_{t'}',\right] = 0$ for each $t' \neq t$. Eq. (14) reflect the possibility that $\mathbf{u}_t$ can display mixed dynamics in both space and time. We follow the literature assuming that $\mathbf{W}$ is the same for the spatially lagged dependent variable and the errors.

From Eq. (14) we derive:[5]

$$\mathbf{u}_t = (\mathbf{I}_N - \lambda\mathbf{W})^{-1}\left\{\sum_{i=0}^{\infty}\delta^i\left[\left(\mathbf{I}_N + \left(\frac{\psi}{\delta}\right)\mathbf{W}\right)(\mathbf{I}_N - \lambda\mathbf{W})^{-1}\right]^i \epsilon_{t-i}\right\}$$

[4]See Elhorst (2013) for a general introduction to spatial panel models.
[5]We are assuming that the first-order spatial autoregressive process is ergodic.

from which we get the variance-covariance matrix of $\mathbf{u}_t$, $\mathbf{U}$, for all t, i.e.:

$$\begin{aligned}\mathbf{U} = E\left[\mathbf{u}_t\mathbf{u}_t'\right] &= \sigma_\epsilon^2\left(\mathbf{I}_N - \lambda\mathbf{W}\right)^{-1} \times \\ &\times \left[\mathbf{I}_N - \delta^2\left(\mathbf{I}_N - \lambda\mathbf{W}'\right)^{-1}\left(\mathbf{I}_N + \left(\frac{\psi}{\delta}\right)\mathbf{W}'\right)\left(\mathbf{I}_N + \left(\frac{\psi}{\delta}\right)\mathbf{W}\right)\left(\mathbf{I}_N - \lambda\mathbf{W}\right)^{-1}\right]^{-1} \times \\ &\times \left(\mathbf{I}_N - \lambda\mathbf{W}'\right)^{-1} \end{aligned} \quad (15)$$

Therefore, the variance-covariance matrix of $\mathbf{v}_t$, $\mathbf{V}$, for all t is given by:

$$\mathbf{V} = E\left[\mathbf{v}_t\mathbf{v}_t'\right] = \left(\mathbf{I}_N - \rho\mathbf{W}\right)^{-1}\mathbf{U}\left(\mathbf{I}_N - \rho\mathbf{W}'\right)^{-1}. \quad (16)$$

Assuming that the VAR representation well approximates the dynamics of $\mathbf{y}_t$, a possible estimation of $\mathbf{V}$ is given by the variance-covariance matrix of the VAR(1) model, i.e. $\hat{\mathbf{\Sigma}}$.

The approximation of stochastic process of $\mathbf{y}_t$ through a VAR representation allows to overcome the *incidental parameters* problem discussed in Anselin (2002). In our model the total number of parameters to be estimate is equal to $N^2 - N + 5$ (all no-zero elements of $\mathbf{W}$ plus λ, δ, ψ, ρ, and σ_ϵ^2) and the number of observations are equal to N^2 (the elements of $\mathbf{V}$); under the assumption $N > 5$ it is therefore possible to estimate the elements of $\mathbf{W}$ as well as the other parameters of Eq. (16). But the estimate of $\mathbf{W}$ from $\hat{\mathbf{\Sigma}}$ becomes very unreliable already for small N: for our sample of $N = 199$ observations, the total number of parameters o estimate is equal to 39407 against a number of observations equal to 39601.

However, a comparison between Eqq. (7) and (16) makes clear that $\mathbf{D}^{g1}$ and $\mathbf{W}$ are calculated on the same information set, i.e. $\hat{\mathbf{\Sigma}}$.

Moreover, the comparison highlights how spatial panels whose observations refer to variable with different timing (e.g. panel of annual observations versus panel with five-year average observations) should include different spatial matrix reflecting the different degrees of interconnectedness (five-year average observations are likely to have a higher level of interconnectedness). A similar argument is made in network literature (see, e.g., Newman, 2009).

IV. Empirical Application

Our sample consists of a panel of GRV of per capita GDP of 199 European NUTS2 regions belong to EU15 over the period 1981-2008.[6]

In order to compare our connectedness matrix with the spatial weights matrices mostly used in the spatial econometric literature, we construct a network derived from the contemporaneous connectedness matrix $\mathbf{D}^{g1}$ (that is the GVD

[6]See Fiaschi and Parenti (2013) for details on the sample, sources of data and estimation of the GRV. All the calculations are made using R (R Core Team, 2014). Codes and data are available on author's web page `http://dse.ec.unipi.it/~fiaschi/`.

at time horizon H=1)[7] and two networks derived from the GVD of the variance-covariance matrix of a spatial model as the one in Eq. (16) of Section III.. In particular, we assume two different spatial weights matrices, i.e. a first-order contiguity matrix, $\mathbf{W}_{cont}$, and a distance based matrix with cut-off, $\mathbf{W}_{invDistQ1}$, (both row-standardized) whose weights are given by:

$$w_{cont}(i,j) = \begin{cases} 1 & \text{if } i \text{ and } j \text{ share a border} \\ 0 & \text{otherwise} \end{cases},$$

and

$$w_{invDistQ1}(i,j) = \begin{cases} dist_{ij}^{-2} & \text{if } dist_{ij} < 370 \text{ miles} \\ 0 & \text{otherwise} \end{cases};$$

we have calibrated the parameters of the model as ρ=0.32, λ=0.42, ϕ=0.32, ψ=0 and σ_ϵ^2=1 to get networks which are similar to our contemporaneous network in terms of mean degrees (see Table 2).[8]

	n	m	c	S	l	d	C	r
$\mathbf{D}^{g1}$	199	1188	**5.97**	1	3.3	6	0.15	0.83
$\mathbf{W}_{cont}$	199	1082	**5.44**	18	2.47	16	0.56	0.94
$\mathbf{W}_{invDistQ1}$	199	1237	**6.22**	5	4.05	21	0.63	0.97

Table 2: Characteristics of the networks derived from the GVD matrix with H=1, $\mathbf{D}^{g1}$, $\mathbf{W}_{cont}$ and $\mathbf{W}_{invDistQ1}$. n is the number of vertices (the number of regions), m the number of edges (the number of no-zero links), c the mean degree (i.e. m/n), S the fraction of vertices in the largest (weakly connected) component, l the mean geodesic distance (any two no-connected links are excluded by calculation), d the diameter of network (the length of the longest finite geodesic path), C the average clustering coefficient (based on transitivity in weak form), and r the assortative coefficient (see Newman, 2009).

To analyze our connectedness matrix as an unweighed and direct network links, i.e. pairwise directional connectedness, with a strength greater than 2.12% will be set equal to 1, while all the others equal to 0. In other words, we have assigned a value of 1 to the ij element of adjacent matrix if the fraction of region's

[7]In Fiaschi and Parenti (2013) we extensively discuss the connectedness matrix estimated at different time horizons (contemporaneous H1, 5-year ahead H5, 10-year ahead H10 and 20-year ahead H20).

[8]Table 2 reports some basic statistics of the networks. We follow the notation in Newman (2009) labelling by n the number of vertices (the number of regions), m the number of edges (the number of no-zero links), c the mean degree (i.e. m/n), S the fraction of vertices in the largest (weakly connected) component, l the mean geodesic distance (any two no-connected links are excluded by calculation), d the diameter of network (the length of the longest finite geodesic path), C the average clustering coefficient (based on transitivity in weak form), and r the assortative coefficient.

i 1-year ahead forecast error variance due to shocks in region j, d^1_{ij}, is higher than 2.12%, which corresponds to a significance level equal to 2.5% under the null hypothesis of no percolation of shocks from region j to region i (see Fiaschi and Parenti, 2013 for details).[9]

Figures (3)-(5) report the Kamadakawai network for the adjacent matrix derived from D^{g1}, W_{cont} and $W_{invDistQ1}$ respectively. In all the figures the colours of the vertices are the same for regions belonging to the same country.

[9] In the estimation of the unweighed and direct networks derived from the two spatial matrices we have used the same level of significance on the pairwise directional connectedness equal to 2.12%.

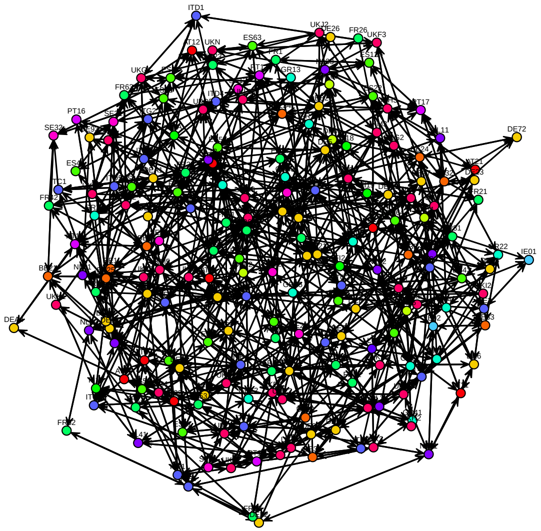

Figure 3: Kamadakawai network with threshold on the share of GVD equal to 2.12% for the adjacent matrix derived from $\mathbf{D}^{g1}$.

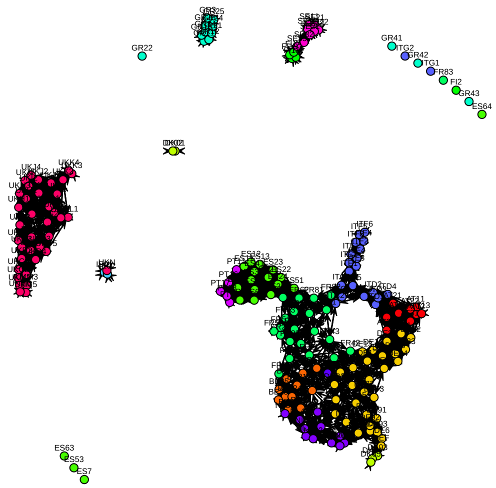

Figure 4: Kamadakawai network with threshold on the share of GVD equal to 2.12% for the adjacent matrix derived from the spatial model with $\mathbf{W}_{cont}$.

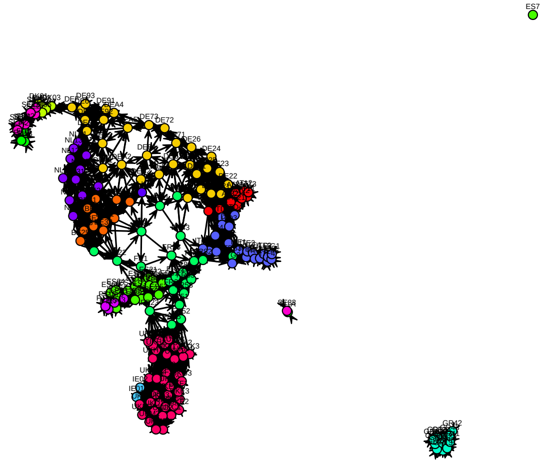

Figure 5: Kamadakawai network with threshold on the share of GVD equal to 2.12% for the adjacent matrix derived from the spatial model with $\mathbf{W}_{invDistQ1}$.

The network for the adjacent matrix derived from the contemporaneous connectedness matrix $\mathbf{D}^{g1}$, is very different from $\mathbf{W}_{cont}$ and $\mathbf{W}_{invDistQ1}$. In particular, although the number of mean degrees, i.e. the mean number of links, is very similar across the three networks (we choose the parameters in Eq. 16 to match this characteristic), the network derived from the GVD matrix with H=1 shows no evidence of specific geographical pattern.[10] The opposite holds for the networks derived from the GVD matrix with $\mathbf{W}_{cont}$ and $\mathbf{W}_{invDistQ1}$, which obviously impose a geographical structure through the exogenous definition of $\mathbf{W}$.

Hence, the assumption that spatial interaction between regions is represented by a geographical weights matrix can lead to a misspecification of the spatial interdependence structure. The effects of such a misspecification are studied by Florax and Rey (1995), who show that both over and under-specification of the geographical weights matrix increases the mean square errors for spatial econometric models. However, no systematic exploration has been conducted so far; the intuition is that misspecification of the spatial weights matrix could lead to a substantial bias in the estimate.

V. Concluding Remarks

The estimate of the connectedness matrix for EU regions proposed in the paper has allowed to highlight how the most popular spatial weights matrices used in literature are very far from the true spatial weights matrix (if any).

This paper would represent a first step in the development of a methodology to estimate a spatial weights matrix which explicitly takes into account the critiques advanced by Corrado and Fingleton (2012). The next step should be the definition of a methodology that, starting from the estimated connectedness matrix, allows to estimate the associated spatial weights matrix. The biggest obstacle appears the high number of matrix elements to estimate, which calls for some non-standard econometrics techniques and /or for imposing some regularity conditions on the shape of the spatial weights matrix.

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[10] In Fiaschi and Parenti (2013) we show that a clear pattern of core-periphery exists but not defined in geographical terms, and that most of the regions belonging to Belgium, Finland and Sweden tend to be more connected to the rest of Europe than to regions belonging to their country, while for most of the regions of Denmark, Greece and Italy the opposite holds.

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