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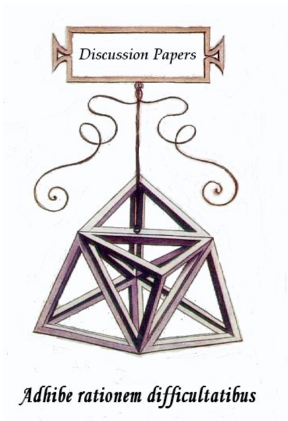

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The endogeneous choice of delegation in a duopoly with outsourcing to the rival

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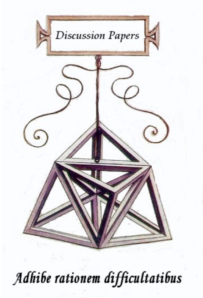

Luciano Fanti and Marcella Scrimitore

The endogeneous choice of delegation in a duopoly with outsourcng to the rival

Abstract

In a market in which a vertically integrated producer (VIP) also supplies an essential input to a retail rival, we explore the role of managerial delegation when it shapes downstream firms' incentives and determine the endogenous choice of delegation under both Cournot and Bertrand. The equilibrium choice of acting as a managerial firm, which is a standard result in literature of strategic delegation, is shown to be robust to the presence of a VIP in both the quantity competition and the price competition framework, regardless of the degree of product differentiation. The paper, however, highlights the different motives pushing the integrated firm and the independent retailer towards delegation, which also revert the standard result that delegation causes a prisoner's dilemma-type equilibrium under Cournot and a more profitable outcome under Bertrand. This result sheds new light on the role and implications of the managerial delegation in the real-world market structures.

JEL codes: D43, L13, L21

Keywords: Strategic delegation, outsourcing, Cournot competition, Bertrand competition, vertical integration

1 Introduction

Starting from the pioneering works by Vickers (1985), Fersthman (1985), Fersthman and Judd (1987) and Sklivas (1987) (VFJS hereafter), the need for managerial compensation on the basis of sales' maximization, stemming from separation of ownership from management in large companies (Berle and Means, 1932) and widely recognized in corporate governance literature (e.g., Baumol, 1958), has been reconsidered in a strategic context. The VFJS approach draws on the use of incentive contracts designed by firms' owners to manipulate their managers' behavior on the product market and attain a strategic advantage. Their main findings are that owners find optimal to distort their managers from profit maximization in order to commit to a more (less) aggressive behavior respectively under quantity (price) competition. Literature on strategic delegation has also extended the above basic models to explicitly model the endogenous choice of delegation in a duopoly, showing that the choice of hiring a manager emerges as the Nash equilibrium solution of both the Cournot game and the Bertrand game (e.g., Basu, 1995, Lambertini, 2017).[1] While in the former game firms face a prisoner-dilemma situation due to higher competition induced by the equilibrium quantities exceeding the profit-maximizing level,[2] in the latter the cooperative effect of delegation on equilibrium prices leads them to enjoy higher profits with respect to no-delegation. Such a result is also sustained by the fact that unilateral delegation induces a Stackelberg outcome which raises the delegating firm's profits and reduces those of the rival under Cournot, while it results in higher profits for both firms under Bertrand.

The above works hinge on the assumption that delegation to managers occurs within an integrated firm sourcing inputs and selling output, while it has never been assumed in a vertical structure in which an independent retailer buys an input rather than makes it. More generally, strategic delegation has received less attention in the literature on firms' vertical relationships, with the exception of Park (2002) and Moner-Colonques et al. (2004). The objective of this paper is to revisit strategic delegation of market decisions to managers in both a Cournot and a Bertrand duopoly in which one retailer is integrated with a manufacturer providing a key input to a downstream rival. This assumption is common in the Industrial Organization literature which focuses on the profit and welfare consequences of vertical integration (Riordan, 1998; Kuhn

[1] Most recent literature on strategic delegation has extended the VFJS framework to allow for R&D investments (Zhang and Zhang, 1997, Kopel and Riegler, 2006), firms' unionization (Fanti and Meccheri, 2013), endogenous timing (Lambertini, 2000; Fanti, 2017), mergers (Gonzalez-Maestre and Lopez-Cunat, 2001), to quote a few examples. See Lambertini (2017) for a comprehensive overview of strategic delegation in oligopoly games.

[2] For instance, as regards the Cournot competition, this is clearly resumed in the words of Berr (2011, p. 251): "Unfortunately, this [i.e. the use of managerial delegation] results in lower payoffs for both owners than in a standard Cournot game, and a prisoner's dilemma situation emerges, i.e. although both owners would benefit by abstaining from the use of incentives and, hence, play a normal Cournot game, they will not." Instead, in this paper we will show that both owners would benefit from the use of incentives in a standard Cournot game.

and Vives, 1999; Chen, 2001; Arya, 2008; Moresi and Schwartz, 2017).[3] Outsourcing to a vertically integrated rival also fits many real-world cases. For example, in the telecommunications and in the railway industries, vertically integrated incumbent operators routinely supply key inputs (e.g., telephone loops and fixed railway network) to retail competitors. Also the electronic industry is a typical real-world example of "giant" firms purchasing key inputs from direct competitor: as reported by Chen (2010, p. 302) "in 1980s, IBM outsourced the micro-processor for its PC to Intel and the operating system to Microsoft". Moreover, as also noted by Arya et al. (2008, pp. 1-2), other firms such as soft-drink producers, cereal manufacturers, and gasoline refiners have long supplied key inputs both to their downstream affiliates and to retail competitors.

Within the above framework, this paper examines the strategic choice to hire a sales-interested manager or not. This choice is made by firms' owners at a preplay stage of a Cournot and a Bertrand game. At the second stage, the vertically integrated producer (VIP) charges the independent retailer a wholesale price. At the third stage, each owner decides upon the degree of discretion to include in a managerial contract (if any), while quantity or price competition with managers acting as decision-makers takes place at the last stage of the game.[4]

The results of the paper are as follows. We find that a Nash equilibrium with symmetric delegation is the equilibrium choice also in the presence of a integrated firm . Such a presence, however, reverses the payoff sequence popularized by the established literature leading unilateral delegation by each firm to also benefit the rival in the Cournot model and to harm the rival in the Bertrand model, which causes a Pareto-improving equilibrium arising in the former and a prisoner-dilemma in the latter, contrasting with the standard result with independent firms. Such a result is driven by the VIP's incentive to manipulate, under delegation, downstream interactions and induce a greater demand of inputs from the rival at a sufficiently high wholesale price, both in Cournot and Bertrand. The latter ensures sufficiently high profitability for the VIP in both scenarios in which the greater demand of input is achieved at the cost of a relatively low production on its direct channel in Cournot (due to strategic substitutability), and through relatively low retail prices in Bertrand (due to strategic complementarity). Delegation by the independent firm, however, by causing its higher (lower) aggressiveness under Cournot (Bertrand), benefits (harms) the VIP through a higher (lower) demand of inputs. Based on the above considerations, unilateral delegation turns out to be profitable for one firm regardless of the mode of competition, and to also benefit the rival only under Cournot where firms' interactions lead firms' objectives to coincide. Strategic substitutability of delegation (Bulow et al., 1985), by allowing the VIP (independent firm) to

[3] See Wu (1992) for an analysis concerning the strategic impact of vertical integration in oligopolistic markets. See also Lafontaine and Slade (2007) for empirical evidence on the consequences of vertical mergers, and Abiru (1988) for some implications for anti-trust laws.

[4] In this work we assume that managerial contracts are publicly observable. For a discussion on the observability and commitment problems in delegation models, see Kockesen and Ok (2004) and literature therein.

further reduce (enhance) its downstream aggressiveness with respect to unilateral delegation, implies that the equilibrium with symmetric delegation is more profitable for both firms with respect to no-delegation. By contrast, the conflict between opposite objectives arising in Bertrand lets unilateral delegation be profit-detrimental for the rival, which also causes a prisoner dilemma under symmetric delegation, due to strategic complementarity of delegation.

This paper contributes to the literature in several ways. First, in contrast to the existing literature, a market structure rather relevant in the real-world is considered. Second, the implications of the outsourcing between rival firms on the effects of the managerial delegation, so far not explored, are analysed. Third, it is shown a profit-enhancing role of the unilateral delegation for both rival firms, which has not been evidenced in the established literature.[5]

The reminder of the paper is organized as follows. Section 2 develops the model under Cournot competition and under Bertrand competition, discussing the main results. Section 3 draws some conclusions.

2 The model

We assume that firm 1 is a vertically integrated producer (VIP) that is the sole producer of an essential input supplied to its downstream unit and to its downstream independent competitor, firm 2. Assumptions on firms' technology are that each retailer can convert one unit of input into one unit of output at no cost, and sustains output production costs implying constant marginal costs, equal to c_1 and c_2 respectively for firm 1 and firm 2 (with $c_1 \leq c_2$),[6] and zero fixed costs. Without loss of generality, firm 1's cost of input production is normalized to zero.

We introduce the following demand for differentiated product faced by firm i ($i = 1, 2$), as in Dixit (1979) and Singh and Vives (1984):

$$p_i = a - \gamma q_j - q_i$$

where q_i and p_i are respectively firm i's retail output and retail price, while the parameter $\gamma \in (0, 1)$ captures imperfect substitutability.

Therefore, firm's profits are given by:

$$\pi_1 = z q_2 + (p_1 - c_1)\, q_1 \tag{1}$$

$$\pi_2 = (p_2 - z - c_2)\, q_2 \tag{2}$$

[5] One partial exception is the result by Fanti and Meccheri (2017), according to which at the equilibrium of the standard Cournot game "the more efficient firm may obtain higher profits provided that the degree of cost asymmetry between firms is sufficiently large" (p. 279). However such a result concerns only one firm and holds only under appropriate conditions, while the present one always applies to both rival firms.

[6] Indeed, as standard in this literature (see also Arya et al., 2008), we assume that the VIP is at least as efficient as its downstream rival.

Indeed, the VIP's profit π_1 is the sum of the wholesale profit gained from supplying the input to firm 2 at a unit input price z and the profit accruing from its own retail sales, while π_2 reflects the firm 2's retail profits.

The model is described by a multi-stage game solved in the usual backward fashion. We assume that at stage 0 (i.e., a pre-play stage) firm i's owner chooses whether to maximize profits, thus acting as an entrepreneurial firm, or to hire a sales-interested manager who is offered the following compensation contract:

$$u_i = \pi_i + \lambda_i q_i \tag{3}$$

where parameter λ_i $(i = 1, 2)$ represents the weight attached to the volume of sales and is optimally chosen by firm i's owner on a profit-maximizing basis (Vickers, 1985).[7] The higher (lower) the assigned λ_i, the more (less) aggressive is the manager's behavior on the product market, with $\lambda_i = 0$ capturing pure profit maximization and $\lambda_i > 0$ $(\lambda_i < 0)$ implying more (less) aggressiveness than under profit maximization, i.e., managers are allowed to care about (penalized for) sales. At stage 1, firm 1's owner sets the input price charged to firm 2, while at the stage 2 she chooses the compensation scheme, i.e., the optimal λ_i to assign her manager, if any. The last stage of the game identifies product market competition in which managers or owners, depending on the choice made at stage 0, simultaneously decide upon the optimal level of market variable, price or quantity.

According to the choice at stage 0, the following configurations may arise, which lead to the four market subgames described in the following subsections:

- (MM), where 'M' stands for 'managerial': each firm delegates retail choices (output or prices) to a manager;

- (EE), where 'E' stands for 'entrepreneurial': both firms are profit-maximizers,

- (ME): firm 1 hires a manager and firm 2 does not (thus behaving as a profit-maximizer);

- (EM): firm 2 hires a manager and firm 1 does not (thus behaving as a profit-maximizer).

The solutions of the subgames in the hypothesis of quantity competition are derived in Subsection 2.1, which also addresses the question regarding the firm's structure, managerial or entrepreneurial, endogenously determined at stage 0, while Subsection 2.2 solves the game under the hypothesis of price competition.

2.1 The quantity competition case

2.1.1 Symmetric behavior

Symmetric delegation (MM)

[7] In our model, using the managerial objective function introduced by Vickers (1985) is formally equivalent to using that defined by Fershtman and Judd (1987) as a linear combination of firm's profits and revenues.

Given the above-mentioned timing of the game and given the objective function in (3), under symmetric delegation the two managers solve the following maximization problems:

$$
\begin{aligned}
\max_{q_1} u_1 &= \pi_1 + \lambda_1 q_1 \\
\max_{q_2} u_2 &= \pi_2 + \lambda_2 q_2
\end{aligned}
$$

yielding the following reaction functions, respectively for firm 1 and firm 2:

$$
\begin{aligned}
q_1 &= \frac{a - c_1 - \gamma q_2 + \lambda_1}{2} \\
q_2 &= \frac{a - c_2 - \gamma q_1 + \lambda_2 - z}{2}
\end{aligned}
$$

which exhibit strategic substitutability. Solving the system of the two reaction functions, we obtain the optimal quantities as functions of the input price z and the incentive parameters λ_1 and λ_2:

$$
q_1 = \frac{2(a - c_1) - \gamma(a - c_2) + \gamma(z - \lambda_2) + 2\lambda_1}{4 - \gamma^2} \tag{4}
$$

$$
q_2 = \frac{2(a - c_2) - \gamma(a - c_1) - \gamma\lambda_1 - 2(z - \lambda_2)}{4 - \gamma^2} \tag{5}
$$

At the second stage of the game, i.e., the delegation stage, owner i $(i = 1, 2)$ maximizes with respect to λ_i her own profits obtained after substituting (4) and (5) respectively in (1) and (2), thus choosing the optimal extent of delegation to assign each manager. We get the following reaction functions:

$$
\begin{aligned}
\lambda_1 &= \frac{\gamma\left(2\gamma(a - c_1) - \gamma^2(a - c_2) - 2z\left(2 - \gamma^2\right) - \gamma^2\lambda_2\right)}{4(2 - \gamma^2)} \\
\lambda_2 &= \frac{\gamma^2\left(2(a - c_2) - \gamma(a - c_1) - 2z - \gamma\lambda_1\right)}{4(2 - \gamma^2)}
\end{aligned}
$$

Notice that $\frac{\partial \lambda_1}{\partial \lambda_2} < 0$ and $\frac{\partial \lambda_2}{\partial \lambda_1} < 0$, which implies strategic substitutability of delegation.

We get the following solutions of the delegation stage:

$$
\lambda_1 = \frac{\gamma\left(\gamma\left(4 - \gamma^2\right)(a - c_1) - 2\gamma^2(a - c_2) - 2z\left(4 - 3\gamma^2\right)\right)}{(\gamma^4 - 12\gamma^2 + 16)} \tag{6}
$$

$$
\lambda_2 = \frac{\gamma^2\left(\left(4 - \gamma^2\right)(a - c_2) - 2\gamma(a - c_1) - 2z\left(2 - \gamma^2\right)\right)}{(\gamma^4 - 12\gamma^2 + 16)} \tag{7}
$$

In this regard, also notice that $\frac{\partial \lambda_1}{\partial z} = -\frac{2\gamma\left(4-3\gamma^2\right)}{(4-\gamma(2+\gamma))(4+\gamma(2-\gamma))} \leq 0$ and $\frac{\partial \lambda_2}{\partial z} = -\frac{2\gamma^2\left(2-\gamma^2\right)}{(4-\gamma(2+\gamma))(4+\gamma(2-\gamma))} \leq 0$.

At the first stage of the game, by maximizing firm 1's profits in (1) calculated at the optimal quantities and the optimal delegation parameters with respect to z, we solve for the equilibrium wholesale price charged to firm 2 in this setting :

$$z^{MM} = \frac{16\left(2-\gamma^2\right)\left(4-3\gamma^2\right)(a-c_2)+\gamma^7(a-c_1)}{2\left(128+48\gamma^4+\gamma^6-160\gamma^2\right)}$$

The equilibrium incentive parameters are:

$$\begin{aligned}
\lambda_1^{MM} &= -\frac{2\gamma\left(4-\gamma^2+2\gamma\right)\left(4-\gamma^2-2\gamma\right)(a-c_2-(a-c_1)\gamma)}{128+48\gamma^4+\gamma^6-160\gamma^2} \\
\lambda_2^{MM} &= \frac{\gamma^2\left(16-\gamma^4-8\gamma^2\right)(a-c_2-(a-c_1)\gamma)}{128+48\gamma^4+\gamma^6-160\gamma^2}
\end{aligned}$$

The quantities at the subgame perfect Nash equilibrium (SPNE) are therefore:

$$\begin{aligned}
q_1^{MM} &= \frac{\left(128+\gamma^6+8\gamma^4-96\gamma^2\right)(a-c_1)-8\gamma\left(8-5\gamma^2\right)(a-c_2)}{2\left(48\gamma^4+128+\gamma^6-160\gamma^2\right)} \\
q_2^{MM} &= \frac{2\left(16-\gamma^4-8\gamma^2\right)(a-c_2-\gamma(a-c_1))}{128+48\gamma^4+\gamma^6-160\gamma^2}
\end{aligned}$$

which reveals that a non-foreclosure condition for firm 2 (i.e., $q_2 \geq 0$) applies when $\gamma \leq \frac{a-c_2}{a-c_1}$. Under such a condition, we find $\lambda_1^{MM} \leq 0$ and $\lambda_2^{MM} \geq 0$, for any γ, which shows that at equilibrium firm 1's manager is penalized for sales (Fershtman and Judd, 1987, p. 938), while firm 2's manager is allowed to consider sales to some extent.

Finally, we calculate the equilibrium profits:

$$\begin{aligned}
\pi_1^{MM} &= \frac{a^2\left(4-\gamma^2-2\gamma\right)\left(48-\gamma^4+2\gamma^3-24\gamma^2-8\gamma\right)+c_1^2\left(128+\gamma^6+16\gamma^4-96\gamma^2\right)}{4(128+48\gamma^4+\gamma^6-160\gamma^2)} - \\
&\quad \frac{2c_1 a\left(128+32\gamma^3+\gamma^6-96\gamma^2+16\gamma^4-64\gamma\right)+32c_2\left(2-\gamma^2\right)(2a(1-\gamma)-c_2+2c_1\gamma)}{4(128+48\gamma^4+\gamma^6-160\gamma^2)} \\
\pi_2^{MM} &= \frac{2\left(2-\gamma^2\right)\left(16-\gamma^4-8\gamma^2\right)^2(a(1-\gamma)-c_2+c_1\gamma)^2}{(128+48\gamma^4+\gamma^6-160\gamma^2)^2}
\end{aligned}$$

Symmetric no-delegation (EE)

We solve the game under the EE configuration in which at the market stage the profit-maximizer owners directly make their output choices, by placing $\lambda_1 = 0$ and $\lambda_2 = 0$ in (4) and (5). By running the above model under such conditions, we recover the solutions identified by Arya et al. (2008), which are as follows:

$$z^{EE} = \frac{4\left(2-\gamma^2\right)(a-c_2)+\gamma^3(a-c_1)}{2\left(8-3\gamma^2\right)}$$

$$\begin{aligned} q_1^{EE} &= \frac{\left(8-\gamma^2\right)(a-c_1)-2\gamma(a-c_2)}{2\left(8-3\gamma^2\right)} \\ q_2^{EE} &= \frac{2\left(a-c_2-\gamma(a-c_1)\right)}{8-3\gamma^2} \end{aligned}$$

$$\begin{aligned} \pi_1^{EE} &= \frac{\left(8+\gamma^2\right)(a-c_1)^2+4(a-c_2)^2-8\gamma(a-c_1)(a-c_2)}{4(8-3\gamma^2)} \\ \pi_2^{EE} &= \frac{4((a-c_2)-\gamma(a-c_1))^2}{(8-3\gamma^2)^2} \end{aligned}$$

The comparison between the two settings of symmetric delegation and no-delegation under quantity competition allows us to introduce the following remark.

Remark 1 Symmetric delegation under quantity competition implies that the VIP behaves at the market stage less aggressively and the independent retailer more aggressively than under no-delegation. This result relies on the higher incentive of the VIP to gain from a higher demand of inputs from the rival upstream than from downstream competition, and the incentive of firm 2 to achieve a competitive advantage through an output expansion induced by delegation to a more aggressive manager

2.1.2 Unilateral delegation

We now derive the SPNE of the game in the two frameworks which assume that only one of the two firms delegates output decisions to a manager.

Unilateral delegation by firm 1 (ME)

When only firm 1 delegates the output choice to a manager and firm 2 is a non-delegating (profit-maximizing) firm, the solutions of the quantity stage of the game are obtained posing $\lambda_2 = 0$ in (4) and (5). Running the model under this assumption, we obtain the following solutions:

$$z^{ME} = \frac{a-c_2}{2}$$

$$\lambda_1^{ME} = -\gamma \frac{a - c_2 - (a - c_1)\gamma}{2(2-\gamma^2)}$$

$$q_1^{ME} = \frac{a(2-\gamma) - 2c_1 + \gamma c_2}{2(2-\gamma^2)}$$
$$q_2^{ME} = \frac{a(1-\gamma) - c_2 + \gamma c_1}{2(2-\gamma^2)}$$

Notice that $\lambda_1^{ME} \leq 0$ under the non-foreclosure condition, i.e., when $\gamma \leq \frac{a-c_2}{a-c_1}$.[8]

The equilibrium profits are:

$$\pi_1^{ME} = \frac{a^2(3-2\gamma) - 2c_2 a(1-\gamma) - 2ac_1(2-\gamma) + 2c_1^2 + c_2^2 - 2c_2\gamma c_1}{4(2-\gamma^2)}$$
$$\pi_2^{ME} = \frac{(a(1-\gamma) + \gamma c_1 - c_2)^2}{4(2-\gamma^2)^2}$$

Remark 2 Unilateral delegation under quantity competition alters the trade-off between the VIP's incentive to behave aggressively downstream and the incentive to exploit a higher demand of inputs upstream as follows. We find $\lambda_1^{MM} \leq \lambda_1^{ME} \leq 0$, regardless of γ, which reveals that firm 1 assigns lower discretion to its manager under unilateral delegation than under no-delegation, thus competing less aggressively downstream and exploiting to a greater extent the profit margin on a higher demand of inputs induced by strategic substitutability of quantities. Indeed, the manager receives an overcompensation for profits, as under symmetric delegation. The latter, moreover, implies a further reduction of firm 1's aggressiveness, due to greater aggressiveness of the managerial rival and strategic substitutability of delegation.

Unilateral delegation by firm 2 (EM)

When firm 2 is assumed to delegate market discretion to a manager and firm 1 is a profit-maximizing firm, the solutions of the last stage of the game are obtained by assessing (4) and (5) at $\lambda_1 = 0$. Given that, the solutions of this subgame are as follows:

$$z^{EM} = \frac{2\left(4-3\gamma^2\right)(a-c_2) + \gamma^3(a-c_1)}{2(8-5\gamma^2)}$$

[8] It can be easily checked that the solution of the delegation stage of the game, which yields firm 1's incentive parameter as a function of the wholesale price z, is $\lambda_1 = \gamma\left(2\gamma(a-c_1) - \gamma^2(a-c_2) - 2z\left(2-\gamma^2\right)\right)/\left(4\left(2-\gamma^2\right)\right)$ and that $\partial\lambda_1/\partial z < 0$ for any γ in the considered interval.

$$\lambda_2^{EM} = \frac{\gamma^2 (a - c_2 - \gamma (a - c_1))}{8 - 5\gamma^2}$$

$$\begin{aligned} q_1^{EM} &= \frac{a (2+\gamma)(4-3\gamma) + 2\gamma c_2 - c_1 \left(8 - 3\gamma^2\right)}{2(8-5\gamma^2)} \\ q_2^{EM} &= \frac{2 (a - c_2 - \gamma (a - c_1))}{8 - 5\gamma^2} \end{aligned}$$

Notice that $\lambda_2^{EM} \geq 0$, under the non-foreclosure condition, i.e., when $\gamma \leq \frac{a-c_2}{a-c_1}$.[9]

The equilibrium profits are:

$$\begin{aligned} \pi_1^{EM} &= \frac{4\left(3a^2+c_2^2+2c_1^2\right)-8c_2 a(1-\gamma)-8c_1 a(2-\gamma)-\gamma^2(a-c_1)^2-8\gamma\left(a^2+c_2 c_1\right)}{4(8-5\gamma^2)} \\ \pi_2^{EM} &= \frac{2\left(2-\gamma^2\right)(a(1-\gamma)+\gamma c_1-c_2)^2}{(8-5\gamma^2)^2} \end{aligned}$$

Remark 3 According to a standard result in the literature on managerial incentives under quantity competition, unilateral delegation by the independent retailer allows it to commit to a more aggressive behavior than under no-delegation, namely to instruct its manager to put a positive bonus on sales. Indeed, we get $0 \leq \lambda_2^{EM} \leq \lambda_2^{MM}$, regardless of γ. This also implies that symmetric delegation induces firm 2 to compete more aggressively than under unilateral delegation, due to lower aggressiveness of the managerial VIP and strategic substitutability of delegation.

2.1.3 The Cournot delegation game

In this section the endogenous choice of whether to act as a managerial firm or not is identified with the equilibrium of the delegation game described in the following matrix.

Figure 1
The pay-off matrix of the Cournot delegation game

1/2	M	E
M	π_1^{MM}; π_2^{MM}	π_1^{ME}; π_2^{ME}
E	π_1^{EM}; π_2^{EM}	π_1^{EE}; π_2^{EE}

[9] It can be easily checked that the solution of the delegation stage of the game, which yields firm 2's incentive parameter as a function of the wholesale price z, is $\lambda_2 = \gamma^2 (2(a-c_2) - (a-c_1)\gamma - 2z) / \left(4\left(2-\gamma^2\right)\right)$ and that $\partial\lambda_2/\partial z < 0$ for any γ.

We can state the following proposition:

Proposition 1. Managerial delegation emerges as the endogenous choice made by both firms in the Cournot game of delegation, regardless of the degree of product differentiation γ. Indeed, it represents a dominant strategy for each firm, which leads the symmetric choice (MM) to arise as the unique subgame perfect Nash equilibrium and to Pareto-dominate the symmetric outcome with no-delegation.

*Proof***.**

Let us consider the following profit differentials:

- $\pi_1^{ME} - \pi_1^{EE} = \frac{\gamma^2(a(1-\gamma)+\gamma c_1 - c_2)^2}{4(2-\gamma^2)(8-3\gamma^2)} \geq 0$
- $\pi_1^{MM} - \pi_1^{EM} = \frac{\gamma^2\left(16-\gamma^4-8\gamma^2\right)(a(1-\gamma)+\gamma c_1 - c_2)^2}{(128+48\gamma^4+\gamma^6-160\gamma^2)(8-5\gamma^2)} \geq 0$
- $\pi_2^{EM} - \pi_2^{EE} = \frac{2\gamma^4(4-3\gamma)(4+3\gamma)(a(1-\gamma)+\gamma c_1 - c_2)^2}{(8-5\gamma^2)^2(8-3\gamma^2)^2} \geq 0$
- $\pi_2^{MM} - \pi_2^{ME} = \frac{\gamma^4\left(1024+128\gamma^4-1280\gamma^2+320\gamma^6-81\gamma^8-8\gamma^{10}\right)(a(1-\gamma)-c_2+c_1\gamma)^2}{4(128+48\gamma^4+\gamma^6-160\gamma^2)^2(2-\gamma^2)^2} \geq 0$

The above inequalities prove that M is a dominant strategy for both firms. Also, consider the following inequalities:

- $\pi_1^{MM} - \pi_1^{EE} = \frac{\gamma^2\left(48-24\gamma^2-\gamma^4\right)(a(1-\gamma)-c_2+c_1\gamma)^2}{(128+48\gamma^4+\gamma^6-160\gamma^2)(8-3\gamma^2)} \geq 0$
- $\pi_2^{MM} - \pi_2^{EE} =$
$\frac{2\gamma^2(a(1-\gamma)-c_2+c_1\gamma)^2\left(8192-13\,824\gamma^2+7936\gamma^4-1984\gamma^6+416\gamma^8-80\gamma^{10}-9\gamma^{12}\right)}{(128+48\gamma^4+\gamma^6-160\gamma^2)^2(8-3\gamma^2)^2} \geq 0$

which prove that the equilibrium (MM) Pareto-dominates (EE).

In order to capture the forces shaping firms' incentives to act as a managerial firm, we compare the market variables in the four subgames.

- $z^{EM} \leq z^{EE} \leq z^{MM} \leq z^{ME}$
- $q_1^{MM} \leq q_1^{ME} \leq q_1^{EM} \leq q_1^{EE}$
- $q_2^{EE} \leq q_2^{ME} \leq q_2^{EM} \leq q_2^{MM}$
- $\pi_1^{EE} \leq \pi_1^{ME} \leq \pi_1^{EM} \leq \pi_1^{MM}$
- $\pi_2^{EE} \leq \pi_2^{EM} \leq \pi_2^{ME} \leq \pi_2^{MM}$

The above inequalities, which hold regardless of γ, demonstrate the existence of an incentive for firm 1, when it unilaterally delegates, to gain from the upstream market by limiting downstream aggressiveness. This lets a higher demand of input accrue from the rival due to strategic substitutability of quantities and a relatively high wholesale price charged ($\partial\lambda_1/\partial z < 0$). A lower q_1 (higher q_2) is then observed in (ME) than under (EE) . With respect to (ME) , moreover, symmetric delegation (MM) further reduces firm 1's aggressiveness through higher firm 2's aggressiveness and due to strategic substitutability of delegation, which limits its ability to set a higher input price. This causes z, as well as $q_1(q_2)$, to be lower (higher) in (MM) than in (ME). Conversely, firm 2's output is higher in (EM) than in (EE) due to its greater aggressiveness under unilateral delegation. The Stackelberg leadership firm 2 gets under such circumstances also induces firm 1, which is unable to affect downstream interactions since it chooses the optimal output by taking as given the rival's output, to produce less than under (EE) and exploit firm 2's greater demand of inputs through a lower z ($\partial\lambda_2/\partial z < 0$). Symmetric delegation (MM) enhances firm 2's aggressiveness with respect to (EM) : in this case, indeed, also firm 1 delegates, becoming less aggressive downstream and setting a higher z upstream, which makes the rival more aggressive due to strategic substitutability of delegation. As a result, $q_1(q_2)$ will be lower (higher) in (MM) than in (EM).

The above effects also explain the reasons for (MM) to arise as an equilibrium under quantity competition. Indeed, acting as a managerial firm is a dominant strategy for firm 1 since it allows it to pursue the objective of gaining from a higher input margin rather than competing aggressively downstream, independently of the rival's strategy. Similarly, it is a dominant strategy for firm 2 which succeeds, thanks to delegation, in gaining higher profits through an output expansion. Consequently, a deviation from (MM) is never profitable for any firm. Indeed, for firm 1 it would imply a higher output and a reduced profit margin on the upstream market under (EM), while for firm 2 it would imply both a lower output and a higher wholesale price charged under (ME). In such a framework of quantity competition, therefore, unilateral delegation not only represents a mechanism through which firm 1 (firm 2) gains higher profits on the upstream (downstream) market, but also a strategy allowing the rival to benefit from a competitive advantage downstream (a higher profit margin upstream). This leads (MM) to Pareto-dominate (EE).

2.2 The price competition case

In the price competition framework, we consider the following direct demand function faced by firm i ($i = 1, 2$):

$$q_i = \frac{a(1-\gamma) - p_i + \gamma p_j}{(1-\gamma^2)}$$

We keep the assumptions on technology and managerial contracts of the previous section and solve the subgames under the same market configurations as above.

2.2.1 Symmetric behavior

Symmetric delegation (MM)

Given the manager's objective function in (3), symmetric delegation under price competition implies the following maximization problems:

$$\begin{aligned} \max_{p_1} u_1 &= \pi_1 + \lambda_1 q_1 \\ \max_{p_2} u_2 &= \pi_2 + \lambda_2 q_2 \end{aligned}$$

We obtain the following reaction functions:

$$p_1 = \frac{a(1-\gamma) + c_1 + \gamma p_2 - \lambda_1 + z\gamma}{2} \tag{8}$$

$$p_2 = \frac{a(1-\gamma) + c_2 + \gamma p_1 - \lambda_2 + z}{2} \tag{9}$$

which exhibit strategic complementarity, and the following solutions of the price stage:

$$\begin{aligned} p_1 &= \frac{a(2+\gamma)(1-\gamma) + \gamma(c_2 - \lambda_2) + 2(c_1 - \lambda_1) + 3z\gamma}{4-\gamma^2} \\ p_2 &= \frac{a(2+\gamma)(1-\gamma) + \gamma(c_1 - \lambda_1) + 2(c_2 - \lambda_2) + z\left(2+\gamma^2\right)}{4-\gamma^2} \end{aligned}$$

At the delegation stage, profit maximization by each owner gives the following reaction functions:

$$\begin{aligned} \lambda_1 &= \frac{\gamma\left(\gamma^2(a-c_2) - \gamma\left(2-\gamma^2\right)(a-c_1) + \gamma^2\lambda_2 + 4z\left(1-\gamma^2\right)\right)}{4\left(2-\gamma^2\right)} \\ \lambda_2 &= \frac{\gamma^2\left(\gamma(a-c_1) - \left(2-\gamma^2\right)(a-c_2) + \gamma\lambda_1 + 2z\left(1-\gamma^2\right)\right)}{4\left(2-\gamma^2\right)} \end{aligned}$$

Notice that $\frac{\partial \lambda_1}{\partial \lambda_2} > 0$ and $\frac{\partial \lambda_2}{\partial \lambda_1} > 0$, i.e., the reaction functions exhibit strategic complementarity.

The solutions of the delegation stage are the following:

$$\begin{aligned} \lambda_1 &= -\frac{\gamma\left(a\gamma(1-\gamma)\left(4-\gamma^2+2\gamma\right) - c_1\gamma\left(4-3\gamma^2\right) + c_2\gamma^2\left(2-\gamma^2\right) - 2z\left(1-\gamma^2\right)(2-\gamma)(2+\gamma)\right)}{(4-\gamma^2+2\gamma)(4-\gamma^2-2\gamma)} \\ \lambda_2 &= -\frac{\gamma^2\left(a(1-\gamma)\left(4-\gamma^2+2\gamma\right) + c_2\left(3\gamma^2-4\right) + c_1\gamma\left(2-\gamma^2\right) - 4z\left(1-\gamma^2\right)\right)}{(4-\gamma^2+2\gamma)(4-\gamma^2-2\gamma)} \end{aligned}$$

Notice that $\frac{\partial\lambda_1}{\partial z} = \frac{2\gamma(1+\gamma)(1-\gamma)(2-\gamma)(2+\gamma)}{(4-\gamma^2-2\gamma)(4-\gamma^2+2\gamma)} > 0$ and $\frac{\partial\lambda_2}{\partial z} = \frac{4\gamma^2(1-\gamma)(1+\gamma)}{(4-\gamma^2-2\gamma)(4-\gamma^2+2\gamma)} > 0$.

Finally, at the first stage of the game, by maximizing firm 1's profits in (1) calculated at the optimal prices and the optimal delegation parameters with respect to z, we solve for the equilibrium wholesale price charged to firm 2 in this setting :

$$z^{MM} = \frac{\gamma^7 (a - c_1) + 8 \left(4 - \gamma^2\right) \left(2 - \gamma^2\right)^2 (a - c_2)}{2 \left(128 + 64\gamma^4 - 7\gamma^6 - 160\gamma^2\right)}$$

The delegation parameters at the SPNE are:

$$\begin{aligned}
\lambda_1^{MM} &= \frac{\gamma \left(2 - \gamma^2\right) \left(4 - \gamma^2 + 2\gamma\right) \left(4 - \gamma^2 - 2\gamma\right) (a - c_2 - (a - c_1)\gamma)}{128 + 64\gamma^4 - 7\gamma^6 - 160\gamma^2} \\
\lambda_2^{MM} &= -\frac{\gamma^2 \left(5\gamma^4 + 16 \left(1 - \gamma^2\right)\right) (a - c_2 - (a - c_1)\gamma)}{128 + 64\gamma^4 - 7\gamma^6 - 160\gamma^2}
\end{aligned}$$

while equilibrium prices are:

$$\begin{aligned}
p_1^{MM} &= \frac{a\left(4-\gamma^2+2\gamma\right)\left(3\gamma^4+10\gamma^3-24\gamma^2-16\gamma+32\right)-4c_2\gamma^3\left(2-\gamma^2\right)+c_1\left(128-11\gamma^6+72\gamma^4-160\gamma^2\right)}{2(128+64\gamma^4-7\gamma^6-160\gamma^2)} \\
p_2^{MM} &= \frac{4a\left(48-2\gamma^6+21\gamma^4-56\gamma^2\right)+2c_2\left(4-\gamma^2\right)\left(2-\gamma^2\right)\left(4-3\gamma^2\right)-\gamma\left(64\left(1-\gamma^2\right)+\gamma^4\left(20-\gamma^2\right)\right)(a-c_1)}{2(128+64\gamma^4-7\gamma^6-160\gamma^2)}
\end{aligned}$$

A non-foreclosure condition $\gamma \leq \frac{a-c_2}{a-c_1}$ also applies under price competition. Under such a condition, we find $\lambda_1^{MM} \geq 0$ and $\lambda_2^{MM} \leq 0$, for any γ, which shows that at equilibrium firm 1's manager is instructed to care about sales, to some extent, while firm 2's manager is penalized for sales.

Finally, we calculate the equilibrium profits:

$$\begin{aligned}
\pi_1^{MM} &= \frac{a^2\left(192-320\gamma^2-31\gamma^6+176\gamma^4\right)-(a-c_1)^2\gamma^8-8c_2\left(2-\gamma^2\right)^3(2a-c_2)}{4(1-\gamma)(1+\gamma)(128+64\gamma^4-7\gamma^6-160\gamma^2)} - \\
&\quad \frac{16\gamma\left(2-\gamma^2\right)^3(a-c_2)(a-c_1)+c_1(2a-c_1)\left(128-23\gamma^6+128\gamma^4-224\gamma^2\right)}{4(1-\gamma)(1+\gamma)(128+64\gamma^4-7\gamma^6-160\gamma^2)} \\
\pi_2^{MM} &= \frac{2\left(2-\gamma^2\right)\left(16+5\gamma^4-16\gamma^2\right)^2(a(1-\gamma)-c_2+c_1\gamma)^2}{(1-\gamma)(1+\gamma)(128+64\gamma^4-7\gamma^6-160\gamma^2)^2}
\end{aligned}$$

Symmetric no-delegation (EE)

We solve the game under the EE configuration in which at the market stage the profit-maximizer owners directly make their output choices, by placing $\lambda_1 = 0$ and $\lambda_2 = 0$ in (8) and (9). By running the above model under such conditions, we obtain the following equilibrium market variables:

$$z^{EE} = \frac{8\left(a-c_2\right)+\gamma^3\left(a-c_1\right)}{2\left(8+\gamma^2\right)}$$

$$\begin{aligned}
p_1^{EE} &= \frac{8\left(a+c_1\right)+2\gamma\left(a-c_2\right)-\gamma^2\left(a-3c_1\right)}{2\left(8+\gamma^2\right)} \\
p_2^{EE} &= \frac{2a\gamma^2+4\left(3a+c_2\right)-\gamma\left(4+\gamma^2\right)\left(a-c_1\right)}{2\left(8+\gamma^2\right)}
\end{aligned}$$

$$\begin{aligned}
\pi_1^{EE} &= \frac{a^2(1-\gamma)(2+\gamma)\left(\gamma^2-\gamma+6\right)+4c_2(c_2-2a(1-\gamma))}{4(1-\gamma^2)(8+\gamma^2)} + \\
&\quad \frac{2c_1a(\gamma-1)\left(\gamma^3+\gamma^2+4\gamma+8\right)-8c_2\gamma c_1+c_1^2\left(-\gamma^4-3\gamma^2+8\right)}{4(1-\gamma^2)(8+\gamma^2)} \\
\pi_2^{EE} &= \frac{(a(1-\gamma)+\gamma c_1-c_2)^2\left(2+\gamma^2\right)^2}{(1-\gamma^2)(8+\gamma^2)^2}
\end{aligned}$$

The comparison between the two settings of symmetric delegation and no-delegation under price competition allows us to introduce the following remark.

Remark 4 Symmetric delegation under price competition implies that the VIP behaves at the market stage more aggressively and the independent retailer less aggressively than under no-delegation. This result relies on the higher incentive of the VIP to gain from a higher demand of inputs from the rival upstream than from downstream competition, and the incentive of firm 2 to relax downstream competition through delegation to a less aggressive manager.

2.2.2 Unilateral delegation

Unilateral delegation by firm 1 (ME)

When only firm 1 delegates the output choice to a manager and firm 2 is a non-delegating (profit-maximizing) firm, the solutions of the quantity stage of the game are obtained posing $\lambda_2 = 0$ in (8) and (9). Running the model under this assumption, we obtain the following equilibrium market variables:

$$z^{ME} = \frac{a - c_2}{2}$$

$$\lambda_1^{ME} = \frac{\gamma\left(a - c_2 - \gamma\left(a - c_1\right)\right)}{4}$$

$$\begin{aligned} p_1^{ME} &= \frac{a + c_1}{2} \\ p_2^{ME} &= \frac{a\left(3 - \gamma\right) + \gamma c_1 + c_2}{4} \end{aligned}$$

Notice that $\lambda_1^{ME} \geq 0$ under the non-foreclosure condition, i.e., when $\gamma \leq \frac{a-c_2}{a-c_1}$.[10]

The equilibrium profits are:

$$\begin{aligned} \pi_1^{ME} &= \frac{a^2(3+\gamma)(1-\gamma)+c_2^2-2c_2(a-\gamma(a-c_1))+c_1^2\left(2-\gamma^2\right)-2c_1 a(2+\gamma)(1-\gamma)}{8(1-\gamma^2)} \\ \pi_2^{ME} &= \frac{(a(1-\gamma)+\gamma c_1-c_2)^2}{16(1-\gamma^2)} \end{aligned}$$

Remark 5 Unilateral delegation under price competition alters the trade-off between the VIP's incentive to behave aggressively downstream and the incentive to exploit a higher demand of inputs upstream as follows. We find $0 \leq \lambda_1^{MM} \leq \lambda_1^{ME}$, regardless of γ, which reveals that firm 1 assigns greater discretion to its manager under unilateral delegation than under no-delegation, thus competing more aggressively downstream and exploiting to a greater extent the profit margin on a higher demand of inputs induced by strategic complementarity of prices. In this case, as in the symmetric delegation case, the manager is instructed to put a positive weight on the volume of sales. Symmetric delegation, however, implies a reduction of firm 1's aggressiveness with respect to unilateral delegation, due to lower aggressiveness of the managerial rival and strategic complementarity of delegation.

Unilateral delegation by firm 2 (EM)

When firm 2 is assumed to delegate market discretion to a manager and firm 1 is a profit-maximizing firm, the solutions of the last stage of the game are

[10] It can be easily checked that the solution of the delegation stage of the game, yielding firm 1's incentive parameter as a function of the wholesale price z, is $\lambda_1 = \gamma\left(\left(1-\gamma\right)\left(4z\left(1+\gamma\right) - a\gamma\left(2+\gamma\right)\right) + \gamma c_1\left(2-\gamma^2\right) - \gamma^2 c_2\right) / \left(4\left(2-\gamma^2\right)\right)$ and that $\partial\lambda_1/\partial z > 0$.

obtained by assessing (8) and (9) at $\lambda_1 = 0$. By solving the subsequent stages, we obtain the following market variables at the subgame perfect equilibrium:

$$z^{EM} = \frac{\left(4-\gamma^2\right)\left(2-\gamma^2\right)(a-c_2)+(a-c_1)\gamma^3}{2\left(\gamma^4-5\gamma^2+8\right)}$$

$$\lambda_2^{EM} = -\frac{\gamma^2\left(a-c_2-\gamma\left(a-c_1\right)\right)}{\gamma^4-5\gamma^2+8}$$

$$\begin{aligned} p_1^{EM} &= \frac{2a\left(4+\gamma^4+\gamma\right)-\gamma^2 a\left(7+\gamma\right)-\gamma c_2\left(2-\gamma^2\right)+c_1\left(8-3\gamma^2\right)}{2\left(\gamma^4-5\gamma^2+8\right)} \\ p_1^{EM} &= \frac{2a\left(4+\gamma^4+\gamma\right)-\gamma^2 a\left(7+\gamma\right)-\gamma c_2\left(2-\gamma^2\right)+c_1\left(8-3\gamma^2\right)}{2\left(\gamma^4-5\gamma^2+8\right)} \end{aligned}$$

Notice that $\lambda_2^{EM} \leq 0$, under the non-foreclosure condition, i.e., when $\gamma \leq \frac{a-c_2}{a-c_1}$.[11]

$$\begin{aligned} \pi_1^{EM} &= \frac{a^2(1-\gamma)\left(2\gamma^4-\gamma^3-9\gamma^2+4\gamma+12\right)+c_1^2\left(8+2\gamma^4-9\gamma^2\right)}{4(1-\gamma^2)(8-5\gamma^2+\gamma^4)} + \\ &\quad \frac{2c_1 a(\gamma-1)\left(\gamma^4-\gamma^3-5\gamma^2+4\gamma+8\right)+c_2\left(2-\gamma^2\right)^2(2\gamma(a-c_1)+c_2-2a)}{4(1-\gamma^2)(8-5\gamma^2+\gamma^4)} \\ \pi_2^{EM} &= \frac{2\left(2-\gamma^2\right)(a(1-\gamma)+\gamma c_1-c_2)^2}{(1-\gamma^2)(8-5\gamma^2+\gamma^4)^2} \end{aligned}$$

Remark 6 According to a standard result in the literature on managerial incentives under price competition, unilateral delegation by the independent retailer allows it to commit to a less aggressive behavior than under no-delegation, namely to instruct its manager to put a negative weight on sales. Indeed, we get $\lambda_2^{EM} \leq \lambda_2^{MM} \leq 0$, regardless of γ. This also implies that symmetric delegation induces firm 2 to compete more aggressively than under unilateral delegation, due to higher aggressiveness of the managerial VIP and strategic complementarity of delegation.

[11] It can be easily checked that the solution of the delegation stage of the game, which yields firm 2's incentive parameter as a function of the wholesale price z, is $\lambda_2 = \gamma^2\left(a\left(2+\gamma\right)\left(\gamma-1\right)+c_2\left(2-\gamma^2\right)-\gamma c_1+2z\left(1-\gamma^2\right)\right)/\left(4\left(2-\gamma^2\right)\right)$ and that $\partial\lambda_2/\partial z > 0$.

2.2.3 The Bertrand delegation game

In this section the endogenous firm choice of whether to act as a managerial firm or not is the equilibrium of the following delegation game.

Figure 2
The pay-off matrix of the Bertrand delegation game

1/2	M	E
M	$\pi_1^{MM};\ \pi_2^{MM}$	$\pi_1^{ME};\ \pi_2^{ME}$
E	$\pi_1^{EM};\ \pi_2^{EM}$	$\pi_1^{EE};\ \pi_2^{EE}$

We can state the following proposition:

Proposition 2. Managerial delegation emerges as the endogenous choice made by both firms in the Bertrand game of delegation, regardless of the degree of product differentiation γ. Therefore, (MM) is the unique subgame perfect Nash equilibrium in dominant strategies, which turns out to be a prisoner-dilemma-type equilibrium.

Proof.

Let us consider the following profit differentials:

- $\pi_1^{ME} - \pi_1^{EE} = \frac{\gamma^2(a-c_2-\gamma(a-c_1))^2}{8(1-\gamma^2)(8+\gamma^2)} \geq 0$
- $\pi_1^{MM} - \pi_1^{EM} = \frac{\gamma^2\left(2-\gamma^2\right)^2\left(16-\gamma^4-8\gamma^2\right)(a-c_2-\gamma(a-c_1))^2}{4(1-\gamma^2)(\gamma^4-5\gamma^2+8)(128+64\gamma^4-7\gamma^6-160\gamma^2)} \geq 0$
- $\pi_2^{EM} - \pi_2^{EE} = \frac{\gamma^4\left(64-46\gamma^2-5\gamma^4+6\gamma^6-\gamma^8\right)(a-c_2-\gamma(a-c_1))^2}{(1-\gamma^2)(\gamma^4-5\gamma^2+8)^2(8+\gamma^2)^2} \geq 0$
- $\pi_2^{MM} - \pi_2^{ME} = \frac{\gamma^4\left(1024-49\gamma^8+96\gamma^6+384\gamma^4-1280\gamma^2\right)(a(1-\gamma)-c_2+c_1\gamma)^2}{16(1-\gamma^2)(128+64\gamma^4-7\gamma^6-160\gamma^2)^2} \geq 0$

The above inequalities prove that M is a dominant strategy for both firms. Also, consider the following inequalities:

- $\pi_1^{MM} - \pi_1^{EE} = -\frac{\gamma^2\left(16-8\gamma^2-3\gamma^4+2\gamma^6\right)(a(1-\gamma)+\gamma c_1-c_2)^2}{(1-\gamma^2)(8+\gamma^2)(128+64\gamma^4-7\gamma^6-160\gamma^2)} \leq 0$
- $\pi_2^{MM} - \pi_2^{EE} =$
$\frac{-\gamma^2(a(1-\gamma)+\gamma c_1-c_2)^2\left(49\gamma^{14}-650\gamma^{12}+3328\gamma^{10}-2560\gamma^8-27776\gamma^6+91\,648\gamma^4-111616\gamma^2+49152\right)}{(1-\gamma^2)(8+\gamma^2)^2(128+64\gamma^4-7\gamma^6-160\gamma^2)^2}$
≤ 0

which prove that the equilibrium (MM) is a prisoner dilemma.

Let us compare the market variables in the four subgames in order to capture the forces leading (MM) to arise as an equilibrium.

- $z^{EM} \leq z^{EE} \leq z^{MM} \leq z^{ME}$
- $p_1^{ME} \leq p_1^{MM} \leq p_1^{EE} \leq p_1^{EM}$
- $p_2^{ME} \leq p_2^{EE} \leq p_2^{MM} \leq p_2^{EM}$
- $\pi_1^{EM} \leq \pi_1^{MM} \leq \pi_1^{EE} \leq \pi_1^{ME}$
- $\pi_2^{ME} \leq \pi_2^{MM} \leq \pi_2^{EE} \leq \pi_2^{EM}$

The above inequalities, holding regardless of γ, demonstrate the existence of an incentive for firm 1 under unilateral delegation to compete more aggressively downstream by setting the lowest retail price, which lets a higher demand of input accrue from the rival due to strategic complementarity of prices. This allows firm 1 to set the highest wholesale price ($\partial\lambda_1/\partial z > 0$). With respect to (ME), symmetric delegation, by reducing firm 1's aggressiveness (firm 2 also delegates in this case, becoming less aggressive and thus limiting the rival's aggressiveness due to strategic complementarity of delegation), restricts its ability to set a higher wholesale price which is indeed lower in (MM) than in (ME). The same forces lead both retail prices to be higher in (MM) than in (ME). Conversely, unilateral delegation by firm 2 leads to its highest retail price due the incentives towards its lowest aggressiveness. The latter, moreover, by softening downstream competition (i.e., both retail prices are higher under (EM) than under (EE), leads firm 1 to optimally react to the reduced demand of inputs by setting the lowest wholesale price ($\partial\lambda_2/\partial z > 0$). That is, due to the impossibility for the non-delegating firm 1 to affect downstream interactions through an appropriate choice of the retail price, which is conversely chosen by taking the rival's price as given, it strategically induces a higher demand of input through a lower wholesale price as compared to (EE). With respect to (EM), finally, symmetric delegation, by enhancing firm 2's aggressiveness due to greater firm 1's aggressiveness, positively affects firm 1's ability to set a higher input price under (MM). The same forces lead both retail prices to be lower in (MM) than in (EM).

We are now able to discuss the properties of the equilibrium (MM) under price competition. Indeed, to delegate to a manager is a dominant strategy for firm 1 since it allows it to gain a higher profit margin upstream rather than competing for a downstream competitive advantage, independently of the rival's strategy. It is also a dominant strategy for firm 2 which succeeds, thanks to delegation, in gaining higher profits by relaxing downstream competition. Therefore, a deviation from (MM) is not profitable for neither firm 1, which would face a reduced profit margin upstream and a higher retail price under (EM), nor firm 2 which would set a lower retail price and would be charged a higher wholesale price under (ME) Despite unilateral delegation represents a profit-enhancing mechanism through which firm 1 and firm 2 exploit higher profitability respectively on the wholesale and on the retail market, it hurts the rival by weakening firm 2's competitive position downstream and reducing firm 1's upstream profit margin. This leads (MM) to emerge as a prisoner dilemma

3 Concluding remarks

In this paper we have highlighted the role of a vertically integrated producer (VIP), which supplies a key input to a downstream rival, in defining the firms' strategic incentives towards delegation. Indeed, literature on managerial incentives has focused on the profit-enhancing character of delegation as a means for independent firms to credibly commit to a more or less aggressive conduct respectively under Cournot and Bertrand, which weakens the rival but leads to a more competitive market outcome in the former and softens overall market competition in the latter. As a result, a prisoner-dilemma-type equilibrium arises under Cournot and a more profitable equilibrium outcome arises under Bertrand. Conversely, in our scenario we have shown that delegation represents the equilibrium choice made by both the independent firm and the integrated firm since it works as a mechanism through which the former exploits the competitive advantage on the product market, while the latter orients downstream interactions to fully exploit its market power on the upstream market. In such a context, managerial delegation turns out to be beneficial to the rival under strategic substitutability of quantities and to be detrimental under strategic complementarity of prices, which results in higher profits for both firms under Cournot and causes a prisoner dilemma under Bertrand. Our findings rely on the assumption that the integrated firm sets the wholesale input price prior to the optimal choice of the managerial contract(s). The analysis of the decision concerning the optimal firm structure when delegation also affects the upstream monopolist's choice of the wholesale price, as well as the analysis under different assumptions on the vertical structure of the industry (e.g., assuming competition on the upstream market or vertical separation between the upstream and downstream units), are left to future research.

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