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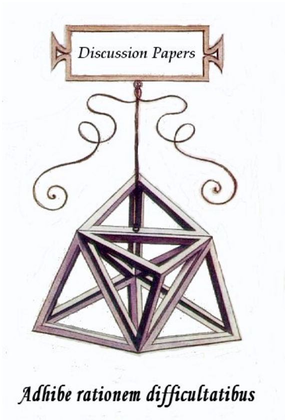

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Hiring a manager or not? When asymmetric equilibria arise under outsourcing to a rival

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Discussion Paper
n. 220

Luciano Fanti and Marcella Scrimitore

Hiring a manager or not? When asymmetric equilibria arise under outsourcing to a rival

Abstract

This paper reconsiders the issue of the endogenous choice of delegation in a market in which a vertically integrated producer (VIP) sells an input to a downstream competitor. The choice of whether to hire a manager or not is made at a preplay stage of a game developed by assuming that, within managerial firms, owners provide their managers with incentives affecting both the VIP's decision regarding the input price and retail competition. Our findings rule out that both the symmetric choices of being managerial or entrepreneurial can be implemented in equilibrium when firms compete à la Cournot, which contrasts with previous literature. The paper brings into focus the role of product differentiation in delivering asymmetric equilibria as solutions of the endogenous delegation game.

JEL codes: D43, L13, L21

Keywords: Managerial delegation, duopoly, vertically integrated firm.

1 Introduction

This paper deals with the issue of whether firms choose to be managerial or entrepreneurial or not, namely to hire a manager or not, in a duopoly. Using insights from strategic delegation theory, we assume that managerial behavior implies delegation of firm control from stockholders to managers who are instructed, for strategic purposes, to deviate to some extent from profit maximization.[1] We furthermore assume that entrepreneurial firms are strict profit-seeking. Such a firm attitude is strategically decided in a vertical structure where retail competition involves two firms offering differentiated products and a monopolist input supplier is integrated with a downstream competitor.[2] In this framework, we investigate the mechanism through which delegation to managers interacts with the incentives driving the choices of the input price and the retail variable made by the vertical integrated producer (VIP), concerned with both enhancing the demand of inputs from the rival and favouring its affiliate's downstream competitive advantage, and the decisions of the independent rival seeking for its retailing success.[3]

Dealing with managerial incentives in a vertical chain with one integrated firm and one independent firm is new to the literature. More generally, strategic delegation has received scarce attention in the literature on vertical relationships, despite the growing attention of researchers to competition issues in vertically related industries (Miklós-Thal et al., 2010; Villas-Boas, 2007) and the focus on the role of managerial discretion on firm behavior, dating back to Williamson (1963). The only exception, to the best of our knowledge, is the work by Park (2002),[4] who finds that the presence of an upstream monopoly in a market with two managerial downstream firms leads managers to be instructed to maximize profits, i.e., no discretion is given to managers regardless of whether firms compete downstream in quantities or prices. By letting the choice of the input wholesale price follow the choice of the managerial incentives, it is shown

[1] Starting from the idea that firm management has different objectives than stockholders, this literature shows how firms can benefit from such a separation of ownership from control for strategic reasons. By assuming that managers can be concerned with maximizing sales (Vickers, 1985), revenues (Fershtman and Judd, 1987; Sklivas, 1987), market shares (Jansen at al., 2007) or relative performance (Miller and Pazgal, 2001), rather than with profits, they are assigned by stockholders an incentive compatible contract based on a combination of profits and the manager's variable of interest. In these contexts, managerial discretion, i.e., a deviation from strict profit goals, arises at equilibrium and represents the means through which firms credibly commit to a more profitable course of action.

[2] A number of works have investigated the nature of firm interactions in vertical structures in which an upstream firm with market power provides a key input to one or more independent retailers (Arya et al., 2008a; Chen et al., 2011; Kabiraj and Sinha, 2016) or the former is integrated with one retailer and supplies a downstream competitor (Arya et al., 2008b; Moresi and Schwartz, 2017). See the same literature for related examples, which include network industries where bottleneck segments with natural monopoly characteristics are widespread.

[3] In order to focus on such a mechanism, we abstract from assuming that delegation is costly, as it is instead in Basu (1995) and Mukherjee (2001).

[4] It is also worth mentioning the work by Moner-Colonques et al. (2004) where delegation of sales to independent retailers in a multi-product context is interpreted as strategic delegation, but is not associated with the use of an incentive scheme.

how in this work the incentive contract offered to each manager is appropriately designed to account for two opposite effects, which balance out at equilibrium: that of exploiting the advantage on the downstream market and that of extracting some of the monopolist's rent. While the latter effect is caused by separation between the upstream and the downstream units, we conjecture that vertical integration of the upstream monopolist with a retailer may lead delegation to differently impact the aforementioned incentives of the two firms. We follow this perspective in the present paper, thus investigating whether managerial incentives, by affecting both the choice of the wholesale price by the VIP and retail competition, alter the choice of both firms to delegate or not with respect to the standard contexts of competition between independent firms.[5] In the standard scenarios of Cournot and Bertrand competition, independent firms are shown to choose to delegate market decisions to managers, this choice representing a dominant strategy for both firms in a duopoly, which yields a prisoner's dilemma equilibrium under Cournot and a Pareto-improving equilibrium under Bertrand (see Sklivas, 1987, among others).[6] The question of whether firms choose to behave as managerial or profit maximizer is approached from another perspective in Fanti and Scrimitore (2017), a companion to the present paper, where we assume the same vertical structure and managerial incentives affecting only downstream competition.[7] Under such a sequence of firms' choices, symmetric delegation is shown to arise as an equilibrium under both Cournot and Bertrand competition. This result is driven by the fact that managerial delegation, by shaping only firms' downstream incentives, allows the VIP and the independent firm to manipulate interactions on the retail market, the former to enhance the demand of inputs from the rival and commit to a higher input price, the latter to gain a market competitive advantage.[8] Such a mechanism of delegation turns out to be preferable to profit maximization for each firm, regardless of the rival's action and the degree of product differentiation, leading the symmetric choice to hire a manager to be an equilibrium in dominant strategies.[9]

The analysis carried out in the present paper is relevant in showing how the possibility for each firm to manipulate through managerial incentives both the

[5] In order to focus on such a mechanism, we abstract from assuming that delegation is costly, as it is instead in Basu (1995) and Mukherjee (2001).

[6] Most of literature of strategic delegation has assumed firms to be managerial with respect to product market decisions. In some contributions, however, being managerial implies also delegation of decisions regarding technology transfer to a rival (Clark and Michalsen, 2010), firm locations in Hotelling models (Barcena-Ruiz and Casado-Izaga, 2005; Matsumura and Matsushima, 2012), R&D investments (Kopel and Riegler, 2006).

[7] This also implies that each owner keeps control of the wholesale price, irrespective of the nature - managerial or not - of the firm.

[8] An increase of the demand of inputs accruing to the VIP is obtained, under delegation, by inducing higher aggressiveness of its downstream rival with respect to the profit-maximizing case, for either Cournot or Bertrand competition. Notice that such gains for the integrated firm have been also highlighted by Moresi and Schwartz (2017) in a very recent study. Conversely, a higher competitive advantage is achieved by the independent retailer through higher (lower) aggressiveness in Cournot (Bertrand).

[9] The firm's choice to delegate is also shown to benefit (hurt) the rival under Cournot (Bertrand), which makes symmetric delegation more profitable for both firms in Cournot and causes a prisoner dilemma in Bertrand than under no-delegation.

VIP's choice of the wholesale price and retail competition leads a rich plethora of asymmetric equilibria - with one firm choosing to be entrepreneurial and the other to be managerial - to appear under Cournot, in dependence of the degree of product differentiation. The latter is also shown to determine uniqueness or multiplicity of the equilibria. More particularly, the advantages from delegation in a Cournot scenario are associated with the incentive for the independent retailer to be charged a lower wholesale price by reducing its demand of inputs, and the VIP's incentive to induce a higher demand of inputs by reducing the upstream price. The incentives towards delegation of both firms are clearly consistent with a reduction of their aggressiveness on the product market, which turns out to be profitable, regardless of the degree of product differentiation, when the rival does not delegate. Conversely, when the rival is managerial, behaving as profit-maximizer is the VIP's optimal reaction under sufficiently high product differentiation, which enhances the profitability of being more aggressive on the downstream market instead of favouring a more limited demand of inputs from the rival. Likewise, when the VIP is managerial, the higher likelihood of foreclosure occurring under sufficiently low product differentiation pushes the independent firm towards profit maximization, which implies its higher aggressiveness on the retail market. Accordingly, we observe a set of asymmetric equilibria over the interval of the product differentiation parameter, at the extremes of which a unique equilibrium arises, while multiple equilibria occur at moderate values.

It is worth noting how the preference for profit-maximizing behavior by one firm at the Cournot equilibria is in contrast with most of the previous studies in literature of strategic delegation. Indeed, most of the works concerned with the endogenous determination of firm behavior in a duopoly have pointed out the occurrence of a symmetric equilibrium with delegation (e.g., Lambertini, 2000; Krakel, 2004) or multiple equilibria (Chirco and Scrimitore, 2013), while that of asymmetric equilibria is rather uncommon, being associated to the presence of a moderate hiring cost (Basu, 1995) or social welfare maximization by a public firm (White, 2001).

The contribution of this paper to existing literature is twofold. First, it brings insights to the literature of vertical relationships by drawing attention on how delegation can negatively affect an integrated monopolist's ability to set a higher input price in a vertical chain,[10] leading to circumstances under which, in contrast to Moresi and Schwartz (2017), it prefers not to induce higher aggressiveness by the rival. Second, this study contributes to the literature of strategic delegation showing that the result highlighted in most of previous research that firms symmetrically choose to delegate at equilibrium does not generalize to a market with the upstream monopolist integrated with a downstream competitor.

The bulk of this paper deals with the analysis of the model under Cournot competition, which is developed in Section 2. Moreover, the baseline model has been extended to Bertrand competition, with the main results sketched in

[10] See Schelling (1960) for arguments underlying the benefits from commitment in strategic dynamics.

Section 3. Finally, Section 4 summarizes the findings and concludes.

2 The model

As in Fanti and Scrimitore (2017), which is also in line with Arya et al. (2008b), consider firm 1 as a vertically integrated producer (VIP) and firm 2 as an independent retailer which compete in a Cournot fashion on a downstream market, with the former supplying the latter with an essential input over which it exerts monopoly power. The two firms offer differentiated products and face the following inverse demand function used by Dixit (1979) and Singh and Vives (1984):

$$p_i = a - \gamma q_j - q_i$$

where p_i and q_i respectively denotes firm i's retail price and retail output (with $i = 1, 2$ and $j \neq i$) and the parameter $\gamma \in (0, 1)$ measures substitutability between the two products, with independent products at $\gamma = 0$ and homogeneous products at $\gamma = 1$.

We assume that each retailer has a one-to-one technology implying that one unit of output is produced by one unit of input, while downstream production costs imply c_i $(i = 1, 2)$ as marginal costs, with $c_1 \leq c_2$, and zero fixed costs.[11] Without any loss of generality, we also assume that firm 1's cost of input production is normalized to zero and that firm 2 procures the input at a price z, which allows us to write firm 1's and firm 2's profits as follows:

$$\pi_1 = zq_2 + (p_1 - c_1)\, q_1 \tag{1}$$

$$\pi_2 = (p_2 - z - c_2)\, q_2 \tag{2}$$

with z also representing firm 1's upstream margin, and $p_1 - c_1$ and $p_2 - z - c_2$ the downstream margin of firm 1 and firm 2, respectively.

We model the owner-manager relationship within a delegating firm by assuming that owners have profit maximization as their objective and that managers are concerned with profits and sales so that, according to Vickers (1985), the objective function of firm i's manager is the following:

$$u_i = \pi_i + \lambda_i q_i \tag{3}$$

where the weight attached to the volume of sales λ_i $(i = 1, 2)$ is chosen by the profit-maximizing owner to determine her manager's incentives.

As above already mentioned, the model is based on a multistage game played by managerial or entrepreneurial firms which runs as follows. In a preplay stage of the game, firms' owners decide on whether to hire a manager or not, i.e. to be entrepreneurial or managerial where E stands for entrepreneurial, while M

[11] Indeed, as standard in this literature (see also Arya et al., 2008b), we assume that the VIP is at least as efficient as its downstream rival.

stands for managerial. At the second stage, the owner of each managerial firm (if any) optimally chooses the degree of discretion to give her manager, while at the third stage the VIP's owner or manager, according to the choice made at the previous stage(s), sets the upstream input price. At the final stage firms' owners, directly or through their managers, engage in Cournot competition. The game is solved by backward induction.

In Subsections 2.1-2.3 we derive the equilibrium outcomes of the subgames that follow all the possible choices regarding E or M. Such choices made at equilibrium at the preplay stage of the game are then investigated in Subsection 2.4, which identifies the subgame perfect Nash equilibria (SPNE) of the game.

2.1 Symmetric delegation (MM)

Let us develop and solve the subgame under the assumption that both firms' stockholders delegate control over their assets to managers. Given the objective function of firm i's manager ($i = 1, 2$) in (3), the following maximization problem is solved at the market stage:

$$\max_{q_i} u_i = \pi_i + \lambda_i q_i$$

which yields the following reaction functions, respectively for firm 1 and firm 2:

$$\begin{aligned} q_1 &= \frac{a - c_1 - \gamma q_2 + \lambda_1}{2} \\ q_2 &= \frac{a - c_2 - \gamma q_1 + \lambda_2 - z}{2} \end{aligned}$$

The solution of the system given by the two reaction functions gives the optimal quantities as functions of the input price z and the incentive parameters λ_1 and λ_2:

$$q_1 = \frac{2(a - c_1) - \gamma(a - c_2) + \gamma(z - \lambda_2) + 2\lambda_1}{4 - \gamma^2} \tag{4}$$

$$q_2 = \frac{2(a - c_2) - \gamma(a - c_1) - \gamma\lambda_1 - 2(z - \lambda_2)}{4 - \gamma^2} \tag{5}$$

Notice that $\frac{\partial q_1}{\partial z} > 0$ and $\frac{\partial q_2}{\partial z} < 0$ (with $i = 1, 2$ and $j \neq i$) and, moreover, $\frac{\partial q_i}{\partial \lambda_i} > 0$ and $\frac{\partial q_i}{\partial \lambda_j} < 0$, which reveals that a reduction of the wholesale price z lowers q_1 and raises q_2, while a higher λ_i raises q_i, translating into higher firm i's downstream aggressiveness, and lowers q_j.

At the previous stage of the game, the manager of the integrated firm still maximizes (3), with $i = 1$ in this case, after incorporating the optimal quantities in (4) and (5). This allows us to identify the optimal wholesale price firm 1 charges the rival:

$$z^{MM} = \frac{4\left(2-\gamma^2\right)(a-c_2)+\gamma^3(a-c_1)+\gamma^3\lambda_1+4\lambda_2\left(2-\gamma^2\right)}{2\left(8-3\gamma^2\right)}$$

We find that $\frac{\partial z^{MM}}{\partial \lambda_1} > 0$ and $\frac{\partial z^{MM}}{\partial \lambda_2} > 0$, namely, the wholesale price increases both in λ_1 and λ_2. This reflects the incentive of both firm 1 and firm 2 to reduce z by limiting downstream aggressiveness, the former with the aim to enhance the demand of inputs q_2 consistently with a reduction of q_1, the latter with the objective of gaining an advantage on the retail market.

The analysis of the delegation stage of the game confirms the existence of the above incentives. At this stage, firm i's owner ($i = 1, 2$) maximizes with respect to λ_i the profits obtained after substituting z^{MM} in (1) and (2), respectively for firm 1 and firm 2, thus choosing the optimal extent of delegation to assign her own manager. Accordingly, we get the following reaction functions:

$$\begin{aligned}
\lambda_1 &= -\frac{2\gamma\left(a-c_2-(a-c_1)\gamma+\lambda_2\right)}{8-3\gamma^2} \\
\lambda_2 &= -\frac{\left(4-3\gamma^2\right)\left(\left(a-c_2-(a-c_1)\gamma\right)-\gamma\lambda_1\right)}{6\left(2-\gamma^2\right)}
\end{aligned}$$

Focusing on firms' reaction functions at the delegation stage, we easily prove that $\frac{\partial \lambda_1}{\partial \lambda_2} < 0$ and $\frac{\partial \lambda_2}{\partial \lambda_1} > 0$, which implies respectively strategic substitutability and strategic complementarity of delegation (Bulow et al., 1985). Such properties identify firm 1's decreasing best reply function and firm 2's increasing best reply function, which can be interpreted as follows. Strategic substitutability for firm 1 captures the fact that the marginal profit this firm gains by increasing λ_1 is decreasing in λ_2. That is, suppose a reduction of λ_2: on the one hand it induces an increase of q_1 through a contraction of q_2, on the other hand it reduces z causing q_1 (q_2) to decrease (increase), with a positive net effect on firm 1's marginal profit following an increase of λ_1, which limits a further reduction of z and positively impacts q_1. By contrast, as long as a reduction of λ_1 occurs, thus favoring an increase of q_2 through both a contraction of q_1 and a reduction of z, firm 2 will respond by reducing λ_2 to further reduce z and indirectly expand q_2.

The system of such reaction functions gives the following solutions of the delegation stage:

$$\begin{aligned}
\lambda_1^{MM} &= -\frac{\gamma\left(8-3\gamma^2\right)\left(a-c_2-(a-c_1)\gamma\right)}{2\left(3\gamma^4-19\gamma^2+24\right)} \\
\lambda_2^{MM} &= -\frac{\left(4-3\gamma^2\right)\left(8-\gamma^2\right)\left(a-c_2-(a-c_1)\gamma\right)}{4\left(3\gamma^4-19\gamma^2+24\right)}
\end{aligned}$$

At the SPNE, the equilibrium wholesale price is:

$$z^{MM} = \frac{\left(4-\gamma^2\right)\left(8-5\gamma^2\right)(a-c_2)+\gamma\left(2-\gamma^2\right)\left(8-\gamma^2\right)(a-c_1)}{4\left(3\gamma^4-19\gamma^2+24\right)}$$

while the retail output for the two firms:[12]

$$\begin{aligned} q_1^{MM} &= \frac{\left(8-\gamma^2\right)\left(3-\gamma^2\right)(a-c_1)-2\gamma\left(4-\gamma^2\right)(a-c_2)}{2\left(3\gamma^4-19\gamma^2+24\right)} \\ q_2^{MM} &= \frac{\left(8-\gamma^2\right)\left(a-c_2-(a-c_1)\gamma\right)}{2\left(3\gamma^4-19\gamma^2+24\right)} \end{aligned}$$

The expressions of the equilibrium profits are included in Appendix A (eqts A1-A2).

Remark 1 The analysis of the equilibrium choices made at the delegation stage reveals that $\lambda_1^{MM} \leq 0$ and $\lambda_2^{MM} \leq 0$ under the non-foreclosure condition (i.e., $\gamma \leq \frac{a-c_2}{a-c_1}$), which implies that both managers are penalized for sales at the market stage, i.e., both firms put a lower weight on sales than under profit maximization (i.e., $\lambda_i = 0$). By behaving less aggressively on the product market, both firms succeed in reducing z ($z^{MM} \leq z^{EE}$), being firm 1 oriented to enhance the demand of inputs relative to its own output and firm 2 to gain from an output expansion.

This reveals the mechanism through which the sequence of firm decisions assumed in this paper alters the role played by delegation with respect to Fanti and Scrimitore (2017). In the latter delegation, by affecting only retail competition, represents a credible commitment for the integrated (the independent) firm to behave less (more) aggressively downstream - λ_1 and λ_2 respectively increases and decreases compared to the profit-maximizing case - which allows the VIP to benefit from charging a higher input price on the increased demand of inputs, thus exploiting its upstream monopoly power, and the independent firm to gain a competitive advantage downstream. Conversely, in the current scenario we show how lower λ_1 and λ_2 than in the profit-maximizing case, by allowing for a reduction of z, is the means through which firm 1 gets a higher demand of inputs and firm 2 achieves higher market competitiveness.

2.2 Symmetric no-delegation (EE)

In this subsection we briefly resume the equilibrium outcome when both firms are entrepreneurial, which can be also borrowed from Fanti and Scrimitore

[12] The non-foreclosure condition for firm 2 (i.e., $q_2 \geq 0$) holds when $\gamma \leq \frac{a-c_2}{a-c_1}$. We assume it is always satisfied throughout the paper, since it applies in all the other subgames.

(2017) or Arya et al. (2008). The same equilibrium outcome can be also recovered by placing $\lambda_1 = 0$ and $\lambda_2 = 0$ in (4) and (5).

$$z^{EE} = \frac{4\left(2-\gamma^2\right)(a-c_2)+\gamma^3(a-c_1)}{2\left(8-3\gamma^2\right)}$$

$$q_1^{EE} = \frac{\left(8-\gamma^2\right)(a-c_1)-2\gamma(a-c_2)}{2\left(8-3\gamma^2\right)}$$
$$q_2^{EE} = \frac{2\left(a-c_2-\gamma(a-c_1)\right)}{8-3\gamma^2}$$

The equilibrium profits in this EE framework are included in Appendix A (eqts A3-A4).

2.3 Unilateral delegation

2.3.1 Unilateral delegation by firm 1 (ME)

When only firm 1 delegates the output choice to a manager and firm 2 is a non-delegating (profit-maximizing) firm, the solutions of the quantity stage of the game are the following:

$$q_1 = \frac{2(a-c_1)-\gamma(a-c_2)+\gamma z+2\lambda_1}{4-\gamma^2} \quad (6)$$
$$q_2 = \frac{2(a-c_2)-\gamma(a-c_1)-\gamma\lambda_1-2z}{4-\gamma^2} \quad (7)$$

At the previous stage of the game, maximization of $U_1 = \pi_1 + \lambda_1 q_1$ with respect to z, after substituting in it the optimal quantities in (6) and (7) allows us to get the optimal z as a function of λ_1**:**

$$z^{ME} = \frac{4\left(2-\gamma^2\right)(a-c_2)+\gamma^3\left((a-c_1)+\lambda_1\right)}{2\left(8-3\gamma^2\right)}$$

It can be easily checked that $\frac{\partial z^{ME}}{\partial \lambda_1} > 0$.

Profit maximization by firm 1 with respect to λ_1 gives the following solution:

$$\lambda_1^{ME} = -\frac{2\gamma\left(a-c_2-(a-c_1)\gamma\right)}{8-3\gamma^2}$$

while the other equilibrium variables are:

$$z^{ME} = \frac{2\left(4-\gamma^2\right)\left(8-5\gamma^2\right)(a-c_2)+\gamma^3\left(8-\gamma^2\right)(a-c_1)}{2\left(8-3\gamma^2\right)^2}$$

$$
\begin{aligned}
q_1^{ME} &= \frac{\left(8-\gamma^2\right)^2 (a-c_1) - 8\gamma\left(4-\gamma^2\right)(a-c_2)}{2\left(8-3\gamma^2\right)^2} \\
q_2^{ME} &= \frac{2\left(8-\gamma^2\right)(a-c_2-\gamma(a-c_1))}{\left(8-3\gamma^2\right)^2}
\end{aligned}
$$

Notice that $\lambda_1^{ME} \leq 0$ under the non-foreclosure condition, i.e., when $\gamma \leq \frac{a-c_2}{a-c_1}$.

The equilibrium profits in this ME framework are included in Appendix A (eqts A5-A6).

Remark 2 Unilateral delegation by the VIP alters as follows the trade-off between the VIP's incentive to gain a competitive advantage by behaving aggressively on the retail market and its incentive to favor a higher demand of inputs by reducing the wholesale price. We get $\lambda_1^{ME} \leq 0$, i.e., firm 1's manager receives an overcompensation for profits, which reveals that unilateral delegation pushes towards a greater incentive to compete less vigorously downstream, thus enhancing the demand of inputs from the rival through a z's reduction with respect to the no-delegation case ($z^{ME} \leq z^{EE}$). Moreover, we find that the weight put on sales in the incentive contract is lower under unilateral delegation than under symmetric delegation case (namely, $\lambda_1^{ME} \leq \lambda_1^{MM} \leq 0$ regardless of γ),[13] in this showing the role of delegation by firm 2 in inducing the rival to compete downstream more aggressively due to strategic substitutability of delegation, further lowering z through a reduction of its demand of inputs ($z^{MM} \leq z^{ME}$).

Remark 3 It is worth considering the pattern of the optimal delegation weight over the interval $\gamma \in (0,1)$. We find that both λ_1^{MM} and λ_1^{ME} are U-shaped, which can be explained as follows. As long as γ is sufficiently low and limits product rivalry on the downstream market, increasing product substitutability leads the VIP's incentive to induce a higher demand of inputs through delegation to dominate its incentive to behave more aggressively on the retail market. This determines a decreasing weight put on sales by the VIP's owners (starting from zero at $\gamma \to 0$) The opposite occurs when γ is high enough and foreclosure of firm 2 becomes more likely, case in which the latter incentive dominates the former, thus causing the optimal weight on sales to increase in γ and approach zero when $\gamma \to \frac{a-c_2}{a-c_1}$.

2.3.2 Unilateral delegation by firm 2 (EM)

Now we proceed to the analysis of the game with firm 2 behaving as a managerial firm and firm 1 is an entrepreneurial firm, the solutions of the quantity stage of the game are the following:

[13] Indeed, we find: $\lambda_1^{MM} - \lambda_1^{ME} = \frac{\gamma\left(4-3\gamma^2\right)\left(8-\gamma^2\right)(a-c_2-\gamma(a-c_1))}{2\left(3\gamma^4-19\gamma^2+24\right)\left(8-3\gamma^2\right)} > 0$.

$$q_1 = \frac{2(a-c_1) - \gamma(a-c_2) + \gamma(z-\lambda_2)}{4-\gamma^2} \quad (8)$$

$$q_2 = \frac{2(a-c_2) - \gamma(a-c_1) - 2(z-\lambda_2)}{4-\gamma^2} \quad (9)$$

At the third stage of the game, firm 2's owner maximizes with respect to z her own profits calculated at the quantities in (8) and (9), which gives the optimal wholesale price z as a function of λ_2**:**

$$z^{ME} = \frac{4\left(2-\gamma^2\right)(a-c_2+\lambda_2) + \gamma^3(a-c_1)}{2(8-3\gamma^2)}$$

It can be easily checked that $\frac{\partial z}{\partial \lambda_2} > 0$.

Finally, at the second stage of the game the managerial firm maximizes $U_2 = \pi_2 + \lambda_2 q_2$ with respect to λ_2, which is at equilibrium:

$$\lambda_2^{EM} = -\frac{\left(4-3\gamma^2\right)(a-c_2-\gamma(a-c_1))}{6(2-\gamma^2)}$$

We then obtain the following market variables calculated at λ_2^{EM}:

$$z^{EM} = \frac{2(a-c_2) + \gamma(a-c_1)}{6}$$

$$q_1^{EM} = \frac{2\left(3-\gamma^2\right)(a-c_1) - \gamma(a-c_2)}{6(2-\gamma^2)}$$

$$q_2^{EM} = \frac{a-c_2-\gamma(a-c_1)}{3(2-\gamma^2)}$$

Notice that $\lambda_2^{EM} \leq 0$, under the non-foreclosure condition, i.e., when $\gamma \leq \frac{a-c_2}{a-c_1}$.

The equilibrium profits in this EM case are included in Appendix A (eqts A7-A8).

Remark 4 Unilateral delegation by the independent firm allows it to gain a competitive advantage on the retail market by reducing the wholesale price through a reduction of its demand of inputs. Accordingly, we observe lower aggressiveness on the final market with respect to the no-delegation case, with firm 2's manager instructed to put a negative bonus on sales, i.e., $\lambda_2^{EM} \leq 0$, and a reduced input price ($z^{EM} \leq z^{EE}$). Moreover, we find that the weight put on sales in the incentive contract is higher under unilateral delegation than under symmetric delegation case (namely, $\lambda_2^{MM} \leq \lambda_2^{EM} \leq 0$ regardless of γ),[14] in this showing the role of delegation by firm 1 in inducing the rival to compete downstream less aggressively due to strategic complementarity of delegation, which further reduces z ($z^{MM} \leq z^{EM}$).

Remark 5 Also in this case we assess the pattern of the optimal delegation weight over the interval $\gamma \in (0,1)$. We find that both λ_2^{MM} and λ_2^{EM} are monotonically increasing in γ, starting from the lowest value equal to $-1/3\,(a-c_2)$ when $\gamma \to 0$, and approaching zero when $\gamma \to \frac{a-c_2}{a-c_1}$. This monotonicity property proves the existence of the incentive for the independent firm to succeed on the downstream market by reducing λ_2, and thus the wholesale price z. Such an incentive is maximum, and leads to the lowest weight put on sales, when product rivalry is absent (i.e., $\gamma \to 0$) and the cost of limiting λ_2 is low compared to the advantage of reducing z with respect to the profit-maximizing case. Such a cost progressively increases, weakening the incentive to reduce λ_2, when γ increases and makes firm 2's foreclosure more likely, up to $\gamma = \frac{a-c_2}{a-c_1}$ at which the incentive to delegate vanishes and firm 2 optimally chooses to maximize profits.

2.4 The delegation game

The equilibrium of the game described by the following matrix, the payoffs of which are the equilibrium profits of the above subgames, identifies the optimal choice by firm 1 and firm 2 in the strategy space $S = \{M, E\}$, i.e., the endogenous choice of whether to act as a managerial firm or an entrepreneurial firm.

Figure 1
The pay-off matrix of the delegation game

1/2	M	E
M	$\pi_1^{MM};\ \pi_2^{MM}$	$\pi_1^{ME};\ \pi_2^{ME}$
E	$\pi_1^{EM};\ \pi_2^{EM}$	$\pi_1^{EE};\ \pi_2^{EE}$

where the mathematical expressions of π_1^{MM}, π_1^{EE}, π_1^{ME}, π_1^{EM}, π_2^{MM}, π_2^{EE}, π_2^{ME} and π_2^{EM} are given in Appendix A (eqts. A1–A8).

We can now state the following proposition:

Proposition 1. The subgame perfect Nash equilibrium (SPNE) emerging in a scenario in which a vertical integrated producer (firm 1) and an independent

[14] Indeed, we find: $\lambda_2^{MM} - \lambda_2^{EM} = -\frac{\gamma^2(4-3\gamma^2)(8-3\gamma^2)(a-c_2-\gamma(a-c_1))}{12(3\gamma^4-19\gamma^2+24)(2-\gamma^2)} \leq 0$.

firm (firm 2) choose whether to hire a manager or not depends on the degree of product differentiation. Firm 2 has (M) as a dominant strategy in the interval $\gamma \in (0, 0.832)$ in which (EM) arises as a unique equilibrium due to the VIP's incentive to choose the strategy that is opposite to the rival's, while firm 1 has a dominant strategy (M) in the interval $\gamma \in (0.838, 1)$ where (ME) is the unique equilibrium, as the independent firm optimally chooses the strategy that is opposite to the rival's. In the interval $\gamma \in (0.832, 0.838)$, each firm plays M (E) when the rival plays E (M), this implying that both the asymmetric configurations (EM) and (ME) arise as SPNE in this interval.

Proof.

Let us consider the following profit differentials:

- $\pi_1^{ME} - \pi_1^{EE} = \frac{a^2\gamma^2(1-\gamma)^2}{(8-3\gamma^2)^2} > 0$, for any γ.
- $\pi_1^{EM} - \pi_1^{MM} = \frac{(1-\gamma)^2 a^2\gamma^2\left(8-3\gamma^2\right)\left(48w-9\gamma^6+60\gamma^4-106\gamma^2\right)}{72(2-\gamma^2)^2(3\gamma^4-19\gamma^2+24)^2} > 0 \Rightarrow 0 < \gamma < 0.838$

The above inequalities prove that (M) is a dominant strategy for firm 1 when $\gamma \in (0.838, 1)$ and that, as long as $\gamma \in (0, 0.838)$ and firm 2 plays E (M), the strategy M (E) is the best firm 1's reply.

- $\pi_2^{EM} - \pi_2^{EE} = \frac{a^2(1-\gamma)^2\left(4-3\gamma^2\right)^2}{6(2-\gamma^2)(8-3\gamma^2)^2} > 0$, for any γ.
- $\pi_2^{MM} - \pi_2^{ME} = \frac{a^2\left(4-3\gamma^2\right)(1-\gamma)^2\left(8-\gamma^2\right)^2\left(81\gamma^8-822\gamma^6+2872\gamma^4-3840\gamma^2+1536\right)}{8(3\gamma^4-19\gamma^2+24)^2(8-3\gamma^2)^4} > 0 \Rightarrow 0 < \gamma < 0.832$

The above inequalities prove that (M) is a dominant strategy for firm 2 when $(0, 0.832)$ and that, as long as $\gamma \in (0.832, 1)$ and firm 1 plays E (M), the strategy M (E) is the best firm 2's reply.[15]

We can now move on to compare all the equilibrium variables on the upstream and the downstream market in the four subgames, which allows us to interpret the above proposition.

- $q_1^{ME} \leq q_1^{MM} \leq q_1^{EE} \leq q_1^{EM}$
- $q_2^{EM} \leq q_2^{MM} \leq q_2^{EE} \leq q_2^{ME}$
- $z^{MM} \leq z^{EM} \leq z^{ME} \leq z^{EE}$

Looking at the firms' individual quantities, we can capture the effects of delegation on firms' incentives. Clearly, in the unilateral delegation scenarios (ME) and (EM), we observe that q_1^{ME} and q_2^{EM} provide the minimum values

[15] The above inequalities also prove the existence of the following asymmetric SPNE when products are complements, i.e., over the interval $\gamma \in (-1, 0)$: ME (EM) emerges as a unique equilibrium when $\gamma \in (-1, -0.838)$ $(\gamma \in (-0.832, 0))$, while both ME and EM arise as multiple equilibria when $\gamma \in (-0.838, -0.832)$.

and q_1^{EM} and q_2^{ME} the maximum values, which confirms the incentive of the VIP to let the rival procure the largest amount of inputs by reducing its own output downstream and the incentive of the independent firm to limit its demand of inputs to reduce the wholesale price, thus favouring the rival. In such circumstances, the downward pressure exerted by both firms on the wholesale price with respect to the profit-maximizing case results from the inequalities $z^{ME} \leq z^{EE}$ and $z^{EM} \leq z^{EE}$. Instead, under symmetric delegation (MM) firm 1 is induced to compete downstream more aggressively than under (ME), due to strategic substitutability of delegation, which causes $q_1^{ME} \leq q_1^{MM}$; however it charges the rival with a lower input price ($z^{MM} \leq z^{ME}$) due to the role played under symmetric delegation by firm 2, which reduces z by lowering its demand of inputs ($q_2^{MM} \leq q_2^{ME}$). Conversely, in (MM) firm 2 competes downstream less vigorously than under (EM), due to strategic complementarity of delegation. However, the fact that firm 1 is induced to push towards a higher demand of inputs when it delegates, causes $q_2^{EM} \leq q_2^{MM}$, with the lower downstream aggressiveness by both firms causing a further reduction of the wholesale price ($z^{MM} \leq z^{EM}$).

The above considerations explain why it is optimal for each firm to choose the strategy opposite to that of its competitor in the different intervals of γ identified in Proposition 1. Indeed, firms have an interest to avoid market configurations in which they are both entrepreneurial or are both managerial. While deviation from (EE) is always profitable for each firm regardless of the degree of product differentiation, as pointed out in Remarks 2 and 4, the degree of product differentiation plays a role in leading firm 1 or firm 2 to deviate from symmetric delegation (MM). Indeed, when product differentiation is sufficiently high (i.e., when $0 < \gamma < 0.838$) and reduces firm 1's advantage from delegation as compared to profit-maximization (markets are less interconnected, which limits firm 1's ability to induce higher q_2). In this case firm 1 finds profitable to deviate from (MM) to (EM) . Moreover, higher profitability of firm 2's unilateral delegation than no-delegation leads (EM) to be an equilibrium in the same interval. Likewise, we prove that when product differentiation is sufficiently low (i.e., when $0.832 < \gamma < 1$), and makes firm 2's foreclosure more likely, the advantages of firm 2 from delegation, namely the gains from limiting the demand of inputs to reduce z , are low compared to profit-maximization shrinks (markets are more interconnected, which makes firm 2's foreclosure more likely). In this case firm 2 finds profitable to deviate from (MM) to (ME). Moreover, higher profitability of firm 1's unilateral delegation than no-delegation leads (ME) to be an equilibrium in this interval. The same argument explains why (EM) and (ME) coexist as Nash equilibria in the interval $0.832 < \gamma < 0.838$.

3 An extension to the price competition case

In this section we present the main results obtained by extending the above model to price competition.[16] The equilibrium outcomes of the Bertrand model

[16] Detailed calculations are available from the authors upon request.

run in the four frameworks of symmetric and unilateral delegation allow us to define the following rankings:

- $p_1^{ME} \leq p_1^{MM} \leq p_1^{EM} \leq p_1^{EE}$
- $p_2^{ME} \leq p_2^{EE} \leq p_2^{MM} \leq p_2^{EM}$
- $z^{EM} \leq z^{MM} \leq z^{EE} \leq z^{ME}$

which basically prove that the mechanism leading to the asymmetric equilibria highlighted in Proposition 1 does not apply in the Bertrand model. In this setting, indeed, the objective pursued through delegation of enhancing the demand of inputs for the integrated firm and that of achieving market success for the independent retailer are consistent with the aim of keeping the input price higher and lower, respectively for the two firms. The VIP's decision on the latter is actually affected by the choice of the incentives schemes at the previous stage, however that strategic choice turns out to enable both firm to credibly commit to the most desirable course of action, independently of the degree product differentiation. Despite the different sequence of firms' decisions, the mechanism at work is the same as that highlighted in Fanti and Scrimitore (2017). This causes delegation to a manager, as in the companion paper, to be the optimal strategy chosen at the equilibrium by each firm.[17]

The above discussion allows us to state the following proposition.

Proposition 2. The symmetric SPNE (MM), with both firms acting as managerial firms, always emerges at equilibrium, irrespective of the degree of product substitutability.

Proof.

By considering the following profit differentials:

- $\pi_1^{ME} - \pi_1^{EE} = \frac{a^2\gamma^2(1-\gamma)}{(1+\gamma)(\gamma^2-\gamma^4+8)(8+\gamma^2)} > 0$, for any γ.
- $\pi_1^{EM} - \pi_1^{MM} = -\frac{a^2\gamma^2(1-\gamma)\left(8+\gamma^2\right)\left(11\gamma^6-88\gamma^4-28\gamma^2+672\right)}{144(1+\gamma)(2\gamma^2-5\gamma^4+48)^2} < 0$, for any γ.
- $\pi_2^{EM} - \pi_2^{EE} = \frac{a^2(1-\gamma)(2+\gamma)^2\left(2+\gamma^2\right)(2-\gamma)^2}{24(1+\gamma)(8+\gamma^2)^2} > 0$, for any γ.
- $\pi_2^{MM} - \pi_2^{ME} = \frac{a^2(1-\gamma)\left(2+\gamma^2\right)\left(4-\gamma^2\right)\left(8-\gamma^4-\gamma^2\right)^2\left(768-3\gamma^{10}-4\gamma^8-7\gamma^6-4\gamma^4+384\gamma^2\right)}{2(1+\gamma)(2\gamma^2-5\gamma^4+48)^2(8+\gamma^2)^2(8+\gamma^2-\gamma^4)^2} >$ 0, for any γ.

which prove that both firm 1 and firm 2 have (M) as a dominant strategy:

[17] Such an equilibrium turn out to be also profit-detrimental with respect to the no-delegation case, as in Fanti and Scrimitore (2017).

4 Concluding remarks

This paper identifies the driving forces behind the endogenous choice to hire a manager or not made by an independent firm and an integrated firm in a vertical structure. Firms' incentives towards delegation have been analyzed in a scenario in which the incentive schemes designed for managers affect both the integrated firm's decision upon the optimal wholesale price and downstream competition. We have shown that the possibility to manipulate through delegation firms' decisions at both stages of the vertical chain alters the equilibrium choice with respect to previous studies. In particular, we have shown that symmetric delegation is never an equilibrium in the Cournot model in which, conversely, there exists a set of (unique o multiple) asymmetric equilibria, with one firm choosing to be managerial and the other one to be entrepreneurial, depending on the degree of product differentiation.

An intuitive explanation of the above results is as follows. Delegation turns out to be the strategic device leading each firm to limit its aggressiveness on the retailing market: indeed, by reducing its own output, the integrated firm sustains a higher demand of inputs from the rival at the cost of a wholesale price reduction, while the independent firm induces a wholesale price's reduction by reducing its demand of inputs. As long as delegation is unilateral, this yields higher profits accruing to the delegating firm (whatever firm) with respect to a scenario without delegation, regardless of the degree of product differentiation. Moreover, the managerial attitude of the independent firm alters the integrated firm's trade-off between a higher demand of inputs and higher success on the retailing market, pushing towards the former when it also delegates. Being entrepreneurial turns out to be the VIP's optimal response to the rival's choice of delegating when sufficiently high product differentiation reduces the advantages of inducing a greater demand of inputs through delegation. Likewise, under managerial delegation by the integrated firm, which implies a higher output of the independent firm, the latter chooses to be entrepreneurial, provided that product differentiation is sufficiently low. In such circumstances in which its foreclosure is more likely, the advantages from exploiting market competitiveness by behaving as an entrepreneurial firm dominates those gained by reducing the input price through delegation. It follows from the above argument that, in the baseline Cournot model, while EM (ME) turns out to be the equilibrium as long as product differentiation is high (low) enough, EM and ME coexist as equilibria for intermediate values of the product differentiation parameter.

Finally, by extending the baseline model to Bertrand competition, this paper has proved the existence of a symmetric equilibrium with delegation regardless of any degree of product differentiation, thus pointing out the gains each firm gets unambiguously from delegation when competes with respect to prices.

Future research should aim at verifying whether the effects highlighted in this study extend to the case in which hiring a manager is costly, also assessing the impact of this assumption of the equilibrium profile.

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Appendix A

$$\pi_1^{MM} = \frac{9A^2\gamma^8-2\gamma\left(8-3\gamma^2\right)^2\left(4-\gamma^2\right)AB}{8(3\gamma^4-19\gamma^2+24)^2} - \frac{9\left(17a^2-16Cc_1-Dc_2\right)\gamma^6-2\left(419a^2-377Cc_1-42Dc_2\right)\gamma^4}{8(3\gamma^4-19\gamma^2+24)^2} + -\frac{32\left(57a^2-49Cc_1-8Dc_2\right)\gamma^2-1408a^2+1152Cc_1+256Dc_2}{8(3\gamma^4-19\gamma^2+24)^2} \quad \text{(A1)}$$

$$\pi_2^{MM} = \frac{3\left(2-\gamma^2\right)\left(8-\gamma^2\right)^2(B-A\gamma)^2}{8(3\gamma^4-19\gamma^2+24)^2} \quad \text{(A2)}$$

$$\pi_1^{EE} = \frac{A^2\left(8+\gamma^2\right)+4B^2-8\gamma AB}{4(8-3\gamma^2)} \quad \text{(A3)}$$

$$\pi_2^{EE} = \frac{4(B-\gamma A)^2}{(8-3\gamma^2)^2} \quad \text{(A4)}$$

$$\pi_1^{ME} = \frac{A^2\gamma^4-16\gamma AB\left(4-\gamma^2\right)-8\left(3a^2-2Cc_1-Dc_2\right)\gamma^2+96a^2-64Cc_1-32Dc_2}{4(8-3\gamma^2)^2} \quad \text{(A5)}$$

$$\pi_2^{ME} = \frac{4\left(8-\gamma^2\right)^2(B-\gamma A)^2}{(8-3\gamma^2)^4} \quad \text{(A6)}$$

$$\pi_1^{EM} = \frac{6\gamma^4A^2-2\gamma\left(8-3\gamma^2\right)AB+\left(28Cc_1+3Dc_2-31a^2\right)\gamma^2+44a^2-36Cc_1-8Dc_2}{36(2-\gamma^2)^2} \quad \text{(A7)}$$

$$\pi_2^{EM} = \frac{(B-A\gamma)^2}{6(2-\gamma^2)} \quad \text{(A8)}$$

where
$A = a - c_1$
$B = a - c_2$
$C = 2a - c_1$
$D = 2a - c_2$