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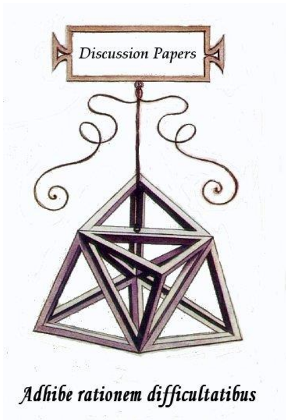

Thomas Renström and Luca Spataro

Optimal taxation, environment quality, socially responsible firms and investors

Discussion Paper n. 232

2018

Discussion Paper n. 232, presentato: **Maggio 2018**

Indirizzo dell'Autore:
Luca Spataro
Dipartimento di Economia e Management, via Ridolfi 10, 56100 PISA – Italy
Email: luca.spataro@unipi.it

Thomas Renström
Durham University Business School, Durham (UK).
Email: t.i.renstrom@durham.ac.uk

Si prega di citare così:
Thomas Renström and Luca Spataro (2018), "Optimal taxation, environment quality, socially responsible firms and investors ", Discussion Papers del Dipartimento di Economia e Management – Università di Pisa, n. 232 (http://www.ec.unipi.it/ricerca/discussion-papers.html).

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Abstract

We characterize the optimal pollution-, capital- and labour-tax structure in a continuous-time growth model in the presence of pollution (resulting from production), both in the first- and second-best, allowing investors to be driven by social responsibility objectives. The social responsibility objective takes the form of warm-glow, as in Andreoni (1990) and Dam (2011), inducing firms to reduce pollution through increased abatement activity. Among the results, the first best pollution tax is still positive under warm-glow, the second-best pollution tax displays the additivity property, and we show the circumstances under which the Chamley-Judd zero capital-income tax result does not hold.

Keywords: Socially responsible investment, corporate social responsibility, environmental quality, optimal taxation, pollution

JEL: D21, D53, G11, H21, H23, M14, Q58.

1. Introduction

The issue of environmental quality has been increasingly debated in the last decades. Many international summits (from Kyoto in 1997 to Paris 2015), the diffusion of credit rating agencies, shareholder activism, mobilization of NGOs and social media prove this growth of environmental concerns (Ballestrero et al. 2015). Socially responsible investment (SRI) has been argued to be a possible instrument to improve environmental quality through a market mechanism.

According to Eurosif, SRI "is a long-term oriented investment approach, which integrates ESG [i.e. Environmental, Social, Governance] factors in the research, analysis and selection process of securities within an investment portfolio. It combines fundamental analysis and engagement with an evaluation of ESG factors in order to better capture long term returns for investors, and to benefit society by influencing the behaviour of companies." (Eurosif 2016, p. 9). Hence, SRI is a process of identifying and investing in companies that meet certain standards of Corporate Social Responsibility (CSR) through such activities and strategies as positive or negative screening, shareholder advocacy, impact and community investing (for more details see GSIA, 2016).

As a matter of fact, according to the latest Global Sustainable Investment Alliance report (GSIA, 2016) global SRI assets amounted to $22.89 trillion at the start of 2016 (an increase of 25% since 2014), and represented 26% of total assets managed in the world, with $8.72 trillion in total assets managed in the US (10% in 2001), $12.04 trillion in Europe, $6.7 billion in Canada, $515.7 billion in Australia and New Zealand, $52.1 billion in Asia.

In light of this recent trend, several scholars have started analysing the phenomenon from an economic perspective. However, the economic literature is still embryonic and results are mixed.

On one hand, Dam (2011) argues that SRI creates a role for the stock market to deal with intergenerational environmental externalities, consisting in the fact that short-lived individuals fail to account for the long-term effects of pollution. The author shows that, although socially responsible investors are short-lived, the forward-looking nature of stock prices, reflecting the warm-glow motive, can help to mitigate the conflict between current and future generations.

Dam and Scholtens (2015) develop a model that links SRI and CSR, showing that responsible firms have higher returns on assets, although the overall effect on stock market returns depends on the relative strength of supply and demand side effects.

On the other hand, Dam and Heijdra (2011) analyse the effects of SRI and public abatement on environmental quality and the economy in a growth model where investors feel partly responsible for environmental pollution when holding firm equity (due to a warm-glow mechanism in their preferences as in Andreoni 1990), thus requiring a premium on the return on equity. In this scenario, the authors show that SRI behaviour by households partially offsets the positive effects on environmental quality of public abatement policies.

Finally, according to Vanwalleghem (2017), SRI may have a mixed effect on firms' incentives to remove negative externalities. Whereas SRI screening incentivizes the removal of externalities (as predicted by Heinkel et al. 2001 and confirmed by the empirical work of Hong and Kacperczyk 2009), SRI trading can disincentivise it when traders disagree on the externality removal's cash flow effects.

In this paper we ask how optimal pollution taxes, as well as capital and labour taxes are affected by socially responsible objectives of investors, both in the first- and second-best. If socially responsible investors manage to induce firms to reduce pollution, are still pollution taxes needed? If so, what is the structure of those taxes?

We specify a continuous-time growth model, where pollution is a by-product of production, but firms can engage in abatement, reducing net pollution. We model investors' social-responsibility objective through a warm-glow mechanism as in Andreoni (1990) and Dam (2011). Through investors' portfolio choice, firms are induced to engage in socially responsible activities (abatement). To the best of our knowledge, the analysis of the optimal tax structure in such a framework has not been done yet. By allowing for different specifications of the warm-glow function, we show the circumstances under which the well-established zero capital-income tax result, Chamley (1986) and Judd (1985) can be violated.

The paper is organized as follows: in section 2 we specify the model and characterize the decentralized equilibrium; in section 3 we present the Ramsey problem of optimal taxation and provide the solutions; in section 4 we discuss the results and section 5 concludes.

2. The model setup

In this section, we specify the benchmark model. The model contains H identical households and J identical firms. We assume that an infinitely lived consumer-investor in each period is endowed with a unit of time that can be allocated either to leisure or to work. Moreover, individuals are endowed with an instantaneous utility function $u(c(t), l(t), p(t), Q(t))$, where $c(t)$, is consumption for that individual at period t, $l(t)$ is labour supply, $p(t)$ is an index of the responsibility that the individual feels for the pollution caused by firms that it holds shares in (*warm-glow*) and $Q(t)$, is the environmental quality. This utility is assumed to be increasing in $c(t)$ and $Q(t)$, decreasing in $l(t)$ and $p(t)$ and strictly concave. Hence, in each period an individual chooses consumption, labour supply and saving allocation.

As for firms, we assume perfectly competitive markets and constant return to scale technology. As a consequence, we can retain the "standard" second-best framework, in the sense that there are no rents.

Finally, we assume the government finances an exogenous stream of per-capita expenditure g by issuing debt (which is the only clean asset in the market) and levying taxes. To retain the second-best, we levy taxes on the choices made by the families, i.e. savings, labour supply and by firms (pollution). Consequently, we introduce a capital-income tax, a labour income tax and a tax on pollution.

2.1. Households

The lifetime utility function of an individual household, at period 0, is:

$$U(0) = \int_0^\infty e^{-\rho t} u(c(t), l(t), p(t), Q(t))dt \tag{1}$$

with $u_c, u_Q > 0, u_l, u_p < 0, u_{cc}, u_{ll}, u_{pp}, u_{QQ} < 0$ and $\rho > 0$ the intertemporal discount rate. Population size, H, is assumed to be constant. In line with Dam and Scholtens (2015), the warm-glow $p(t)$ is assumed to be a function of the individual's portfolio invested in polluting firms:

$$p(t) \equiv \sum_{j=1}^{J} \frac{e^j(t)}{\bar{E}^j} \bar{p}^j(t) \tag{2}$$

where $e^j(t)$ is the number of shares of firm j owned by the individual, $\bar{E}^j$ is number of total shares of firm j, assumed to be constant, $\bar{p}^j(t)$ is the "pollution content" of firm j as perceived by the individual. We allow the latter to be a function of other potentially relevant variables:

$$\bar{p}^j(t) \equiv \bar{p}(x^j(t); X(t), F(t), Q(t)) \tag{3}$$

where $x^j(t)$ is the flow of pollution produced by the jth firm, $X(t) \equiv \sum_{j=1}^{J} x^j(t)$ the aggregate flow of pollution, $F(t)$ is the aggregate gross production of the homogenous good, $Q(t)$ is the environmental quality. We assume that $\bar{p}^j$ is linear in x^j (as any non-linearity can be captured by u). Notice that $x^j(t)$ is controlled by the firm j, i.e. each firm can affect its "rating" through its decision, while aggregate variables are taken as given by each firm. We will discuss possible formulations of $\bar{p}^j$ later in section 4.

At each instant of time t, individual's wealth is

$$a(t) \equiv b(t) + \sum_{j=1}^{J} e^j(t)\, P_e^j(t) \tag{4}$$

where $b(t)$ is per-capita public debt, $P_e^j(t)$ the stock market price of shares. By defining $\omega^j(t) \equiv \frac{e^j(t) P_e^j(t)}{a(t)}$ as the portfolio share invested in firm j, and $V^j(t) \equiv \bar{E}^j P_e^j(t)$ as the stock market value of firm j, we have that

$$p(t) \equiv \sum_{j=1}^{J} \frac{\omega^j(t) a(t)}{V^j(t)} \bar{p}^j(t) \tag{5}$$

and the individual budget constraint reads as[1]:

$$\dot{a}(t) = \sum_{j=1}^{J} \omega^j(t) \bar{r}_e^j(t)\, a(t) + \left[1 - \sum_{j=1}^{J} \omega^j(t)\right] \bar{r}(t) a(t) + \bar{w}(t) l(t) - c(t) - z(t) \tag{6}$$

[1] We follow Merton (1971).

where $\bar{r_e^j} \equiv r_e^j\left(1-\tau^a(t)\right)$ is net-of-tax return on share j, $\bar{r}(t) \equiv r(t)\left(1-\tau^a(t)\right)$ is net-of-tax interest rate on public debt, $\bar{w}(t) \equiv w(t)\left(1-\tau^l(t)\right)$ is the net-of-tax wage, $z(t)$ a lump sum tax (which we will set to zero in the second-best analysis) and $\tau^a(t), \tau^l(t)$ are the tax rates on capital income and labour income, respectively.

Returns on shares of firm j are:

$$r_e^j(t) \equiv \frac{\dot{V}^j(t)}{V^j(t)} e^j(t) + \frac{d^j(t)}{V^j(t)} \tag{7}$$

where $\frac{d^j(t)}{V^j(t)}$ is the dividend payout ratio and $d^j(t)$ total dividend payments by firm j.

The individual's problem is to maximize (1) w.r.t. $c(t), l(t), \omega^j(t)$ subject to (4) and (6). The associated current value Hamiltonian is:

$$\Lambda(t) = u(t) + q(t)\dot{a}(t) \tag{8}$$

with $q(t)$ the shadow price of wealth. FOCs yield:

$$u_c(t) - q(t) = 0 \tag{9}$$

$$u_l(t) + q(t)\bar{w}(t) = 0 \tag{10}$$

$$u_p(t)\frac{\bar{p}^j(t)}{V^j(t)}a(t) + q(t)a(t)[1-\tau^a(t)][r_e^j(t) - r(t)] = 0 \tag{11}$$

$$q(t)[[1-\tau^a(t)][\sum_{j=1}^{J}\omega^j(t)r_e^j+\left(1-\sum_{j=1}^{J}\omega^j(t)\right)r(t)]+u_p(t)\sum_{j=1}^{J}\frac{\omega^j(t)\bar{p}^j(t)}{V^j(t)}=\rho q(t)-\dot{q}(t) \tag{12}$$

Note that eq. (9) and eq. (11) provide:

$$\frac{u_p(t)}{u_c(t)}\frac{\bar{p}^j(t)}{[1-\tau^a(t)]}+V^j(t)[r_e^j(t)-r(t)]=0 \tag{13}$$

Equations (9)-(10) provide the usual optimality conditions for consumption and labour supply; (13) is the optimal portfolio choice condition. Notice that the return on assets in production is greater than the return on government bonds and the difference is proportional to the pollution content by the firm, thus there is a pollution premium, compensation to the household for holding "dirty assets". Exploiting (7), (13) becomes:

$$\frac{u_p(t)}{u_c(t)}\frac{\bar{p}^j(t)}{[1-\tau^a(t)]}+\dot{V}^j(t)+d^j(t)-r(t)V^j(t)=0 \tag{14}$$

Finally, pre-multiplying (11) by $\omega^j(t)$ and summing from j=1 to J and using (12) we have:

$$q(t)[1-\tau^a(t)]r(t)=\rho q(t)-\dot{q}(t). \tag{15}$$

2.2. Firms

We assume that each firm runs its business in a perfectly competitive market, endowed with constant-returns-to-scale production technology that uses capital and labour inputs to produce a homogenous good. We shall also assume that each firm's technologies are the same. Hence, it will be possible to aggregate the firms to obtain a representative firm. The production function for firm j is:

$$y^j(t) = f^j\big(k^j(t), l^j(t)\big) \tag{16}$$

with $k^j(t)$ physical capital input and $l^j(t)$ labour input, respectively.

We follow Copeland and Taylor (1994) by assuming that, at any time t, every unit of output generates ε units of pollution as a joint product of output and that pollution can be reduced by abatement activity of the firm, $\alpha(t)$. The latter is supposed to be carried out through a CRS technology which is an increasing function of the total scale of firm activity $f(t)$ and of the firm's efforts at abatement, $f^{\alpha}(t)$. If abatement at level $\alpha(t)$ removes $\varepsilon \cdot \alpha(t)$ units of pollution, we have that total emissions (pollution) $x(t)$ by firm j is equal to:

$$x^j(t) = \varepsilon \cdot f^j(t) - \varepsilon \cdot \alpha\big(f^j(t), f^{\alpha j}(t)\big) \tag{17}$$

Defining $\Psi^j(t) \equiv \frac{f^{\alpha j}(t)}{f^j(t)}$ as the fraction of output devoted to abatement activity and exploiting CRS, we get:

$$\frac{x^j(t)}{f^j(t)} = \varepsilon \cdot \left[1 - \alpha\big(1, \Psi^j(t)\big)\right] = \varepsilon \cdot \left[1 - \alpha\big(\Psi^j(t)\big)\right] \tag{18}$$

with α increasing in Ψ^j and, thus, eq. (18) gives $\Psi^j(t) = \Psi\left(\frac{x^j(t}{f^j(t)}\right)$. Gross operating profits of the firms are:

$$\pi^j(t) \equiv \left[1 - \Psi^j\left(\frac{x^j(t}{f^j(t)}\right)\right] f^j\big(k^j(t), l^j(t)\big) - w(t)l^j(t) - \tau^x(t)x^j(t) \tag{19}$$

where $\tau^x(t)$ is the tax on pollution at time t. Given that we assume that the number of shares remains constant and that we abstract from corporate bonds, new investments, $i^j(t)$, can only by financed via retained earnings, $Re(t)$, i.e. $\pi^j(t) = d^j(t) + Re^j(t)$.That is, by exploiting the capital accumulation identity:

$$\dot{k}^j(t) = i^j(t) - \delta k^j(t) \tag{20}$$

with δ the (constant) instantaneous depreciation rate, we get:

$$\dot{k}^j(t) = \pi^j(t) - d^j(t) - \delta k^j(t) \tag{21}$$

and, exploiting (19), (21) becomes:

$$\dot{k}^j(t) = \left[1 - \Psi^j\left(\frac{x^j(t}{f^j(t)}\right)\right] f^j\big(k^j(t), l^j(t)\big) - w(t)l^j(t) - \tau^x(t)x^j(t) - d^j(t) - \delta k^j(t) \tag{22}$$

Now, integrating (14) we get:

$$V^j(0) = \int_0^\infty e^{-\int_0^t r(s)ds} \left[d^j(t) + \frac{u_p(t)}{u_c(t)} \frac{\bar{p}^j(t)}{[1-\tau^a(t)]}\right] dt \tag{23}$$

which provides the value of the firm at time 0. Substituting for $d^j(t)$ from (22), (23) reads as:

$$V^j(0) = \int_0^\infty e^{-\int_0^t r(s)ds} \left\{\left[1 - \Psi^j\left(\frac{x^j(t}{f^j(t)}\right)\right] f^j(k^j(t), l^j(t)) - w(t)l^j(t) - \tau^x(t)x^j(t) - \delta k^j(t) + \frac{u_p(t)}{u_c(t)} \frac{\bar{p}^j(t)}{[1-\tau^a(t)]} - \dot{k}^j(t)\right\} dt \tag{24}$$

Given the assumption of perfect competition, the firm hires labour, $l^j(t)$, on the spot market and remunerates it according to its marginal productivity. In fact, FOCs on (24) w.r.t. $l^j(t)$ and $x^j(t)$ yield, respectively:

$$\left[1-\Psi^j\left(\frac{x^j(t}{f^j(t)}\right)+\Psi^{j\prime}\left(\frac{x^j(t)}{f^j(t)}\right)\frac{x^j(t)}{f^j(t)}\right]f_l^j(t)-w(t)=0 \tag{25}$$

$$\frac{u_p(t)}{u_c(t)}\frac{1}{[1-\tau^a(t)]}\frac{\partial \bar{p}^j(t)}{\partial x^j(t)}-\Psi^{j\prime}\left(\frac{x^j(t)}{f^j(t)}\right)-\tau^x(t)=0 \tag{26}$$

The optimality condition for $k^j(t)$, $\frac{dV^j(0)}{dk^j(t)}=\frac{d}{dt}\frac{dV^j(0)}{dk^j(t)}$, classical calculus of variation, gives:

$$\int_0^\infty e^{-\int_0^t r(s)ds}\left\{\left[1-\Psi^j\left(\frac{x^j(t)}{f^j(t)}\right)+\Psi^{j\prime}\left(\frac{x^j(t)}{f^j(t)}\right)\frac{x^j(t)}{f^j(t)}\right]f_k^j(t)-\delta\right\}dt=\frac{d}{dt}\int_0^\infty e^{-\int_0^t r(s)ds}\,dt \Rightarrow$$

$$\left[1-\Psi^j\left(\frac{x^j(t)}{f^j(t)}\right)+\Psi^{j\prime}\left(\frac{x^j(t)}{f^j(t)}\right)\frac{x^j(t)}{f^j(t)}\right]f_k^j(t)-\delta=r(t) \tag{27}$$

Finally, it can be shown that, by plugging (25)-(27) into (24) and exploiting CRS in $f^j(t)$, then $maxV^j(0)\equiv\bar{E}^jP_e{}^j(0)=k^j(0)$.

3. The Ramsey problem

We now solve the optimal tax problem (Ramsey problem). In doing so, we adopt the primal approach, consisting of the maximization of a direct social welfare function through the choice of quantities (i.e. allocations; see Atkinson and Stiglitz 1972). For this purpose, we must restrict the set of allocations among which the government can choose to those that can be decentralized as a competitive equilibrium. We now provide the constraints that must be imposed on the government's problem in order to comply with this requirement.

In our framework there is an implementability constraint associated with the individual's intertemporal choice plan. More precisely this constraint is the individual budget constraint with prices substituted for by using the individual's first order conditions, which yields (see Appendix A.1):

$$a(0)q(0) = \int_0^\infty e^{-\rho t}\left[u_p(t)p(t) + u_l(t)l(t) + u_c(t)c(t) + u_c(t)z(t)\right]dt \tag{28}$$

Finally there are two feasibility constraints, one requires that, under the assumption that firms are equal, private and public consumption plus investment be equal to aggregate output, i.e[2].

$$\dot{K}(t) = \left[1 - \Psi\left(\frac{X(t)}{F(t)}\right)\right]F\big(K(t), L(t)\big) - c(t)H - \delta K(t) - G(t). \tag{29}$$

with $L(t) = l(t)H$. The other one is given by the dynamics of environmental quality which we assume, as in Dam and Heijdra (2011):

$$\dot{Q}(t) = -\mu Q(t) - \eta X(t) + \phi \tag{30}$$

Notice that in equilibrium $\omega^j(t) \equiv \frac{V^j(t)}{H \cdot a(t)}$; then, by (4)

[2] In fact aggregating over firms we get:
$\sum_{j=1}^{J}\left[1 - \Psi^j\left(\frac{x^j(t)}{f^j(t)}\right)\right]f^j\left(k^j(t), l^j(t)\right) = \sum_{j=1}^{J}\left[1 - \Psi\left(\frac{x(t)}{f(t)}\right)\right]f\left(k(t), l(t)\right) = \left[1 - \Psi\left(\frac{X(t)}{F(t)}\right)\right]F\left(K(t), L(t)\right),$
with $L(t) = l(t)H$.

$$p(t) \equiv \sum_{j=1}^{J} \frac{\omega^j(t)a(t)}{V^j(t)} \bar{p}^j(t) = \sum_{j=1}^{J} \frac{\bar{p}^j(t)}{H} = \frac{\bar{p}(X(t);F(t),X(t),Q(t))}{H}$$

where the last equality follows from $\bar{p}^j(t)$ being linear in $x^j(t)$.

Suppose that the tax programme is chosen in period 0, hence the problem of the policymaker is to maximize (1) subject to eq. (28) and, $\forall t \geq 0$, (29) and (30). The current value Hamiltonian is:

$$\Lambda(t) = Hu\left(c(t), l(t), \frac{\bar{p}(t)}{H}, Q(t)\right) + \lambda H\left[u_p(t)p(t) + u_l(t)l(t) + u_c(t)c(t) + u_c(t)z(t)\right] +$$
$$q^k(t)\left\{\left[1 - \Psi\left(\frac{X(t)}{F(t)}\right)\right] F(K(t), l(t)H) - c(t)H - \delta K(t) - G(t)\right\} + q^Q(t)\{-\mu Q(t) - \eta X(t) + \phi\} \quad (31)$$

where λ is the multiplier associated with the implementability constraint[3] and $q^k(t)$ and $q^Q(t)$ are the co-states associated with the other constraints and where we have made use of the equilibrium condition $L(t) = l(t)H$.

Preliminarily, notice that, by eq. (3), we allow the warm-glow to be a function of other variables, say S_i, so that the partial derivative of the Hamiltonian with respect to any such variable will be (omitting time subscripts):

$$\frac{d\Lambda}{dS_i} = \frac{\partial\Lambda}{\partial S_i} + \frac{\partial\Lambda}{\partial p}\frac{\partial p}{\partial S_i},$$

with

$$\frac{\partial\Lambda}{\partial p} = Hu_p(1 + \lambda\Delta_p) \quad (32)$$

[3] It is possible to show that λ is positive if the constraint binds (as $\left.\frac{\partial\Lambda}{\partial z}\right|_{z=0} = \lambda H u_c > 0$). For this reason, it is usually interpreted as a measure of the deadweight loss brought about by distortionary taxation. We will set $z = 0$ in the second-best analysis.

and $\Delta_p \equiv 1+\frac{u_{pp}p}{u_p}+\frac{u_{lp}l}{u_p}+\frac{u_{cp}c}{u_p}$, referred to as the "general equilibrium elasticity" of the warm-glow. The first order conditions of the Ramsey problem are (omitting time subscripts):

$$\frac{\partial\Lambda}{\partial c}=0 \Longrightarrow u_c(1+\lambda\Delta_c)=q^k \tag{33}$$

$$\frac{\partial\Lambda}{\partial l}=0 \Longrightarrow u_p\left(1+\lambda\Delta_p\right)\frac{\partial\bar{p}}{\partial F}F_L+u_l(1+\lambda\Delta_l)+q^k\left[1-\Psi\left(\frac{X}{F}\right)+\Psi'\left(\frac{X}{F}\right)\frac{X}{F}\right]F_L=0 \tag{34}$$

$$\frac{\partial\Lambda}{\partial K}: u_p\left(1+\lambda\Delta_p\right)\frac{\partial\bar{p}}{\partial F}F_K+q^k\left\{\left[1-\Psi\left(\frac{X}{F}\right)+\Psi'\left(\frac{X}{F}\right)\frac{X}{F}\right]F_K-\delta\right\}=\rho q^k-\dot{q}^k \tag{35}$$

$$\frac{\partial\Lambda}{\partial X}=0 \Longrightarrow u_p\left(1+\lambda\Delta_p\right)\frac{\partial\bar{p}}{\partial X}-q^k\Psi'\left(\frac{X}{F}\right)-\eta q^Q=0 \tag{36}$$

$$\frac{\partial\Lambda}{\partial Q}: u_p\left(1+\lambda\Delta_p\right)\frac{\partial\bar{p}}{\partial Q}+Hu_Q\left(1+\lambda\Delta_Q\right)=(\rho+\mu)q^Q-\dot{q}^Q \tag{37}$$

with $\Delta_c \equiv 1+\frac{u_{cc}c}{u_c}+\frac{u_{lc}l}{u_c}+\frac{u_{pc}p}{u_c}$, $\Delta_l \equiv 1+\frac{u_{ll}l}{u_l}+\frac{u_{lc}c}{u_l}+\frac{u_{pl}p}{u_l}$, usually referred to as the "general equilibrium elasticities" of consumption and labour, respectively and $\Delta_Q \equiv \frac{u_{pQ}p}{u_Q}+\frac{u_{lQ}l}{u_Q}+\frac{u_{cQ}c}{u_Q}$. By dividing (34) by (33), exploiting (25) (recognizing that $f_l^j(t)=F_L$) and the equilibrium condition stemming from (9) and (10) (i.e. $u_c\bar{w}=-u_l$) we get:

$$\frac{\tau^l}{1-\tau^l}=\lambda\frac{(\Delta_l-\Delta_c)}{(1+\lambda\Delta_c)}+\frac{u_p}{u_l}\frac{\left(1+\lambda\Delta_p\right)}{(1+\lambda\Delta_c)}\frac{\partial\bar{p}}{\partial F}F_L \tag{38}$$

which provides the implicit expression for the labour-income tax. As for the capital income tax, at steady state, $\dot{q}^k = 0$ and $\dot{q} = 0$; hence, by equating (15) and (35) and exploiting (27) (recognizing that $f_k^j(t) = F_K$) if follows that

$$-\frac{u_p}{u_c}\frac{(1+\lambda\Delta_p)}{(1+\lambda\Delta_c)}\frac{\partial\bar{p}}{\partial F}F_K = \rho\frac{\tau^a}{1-\tau^a} \tag{39}$$

As for the Pigouvian tax on pollution *X*, in steady state $\dot{q}^Q = 0$. By substituting for q^Q from (34) into (35) and exploiting (33) one gets:

$$\frac{u_p}{u_c}\frac{(1+\lambda\Delta_p)}{(1+\lambda\Delta_c)}\left[\frac{\eta}{\rho+\mu}\frac{\partial\bar{p}}{\partial Q} - \frac{\partial\bar{p}}{\partial X}\right] + \frac{\eta}{\rho+\mu}H\frac{u_Q}{u_c}\frac{(1+\lambda\Delta_Q)}{(1+\lambda\Delta_c)} = -\Psi'\left(\frac{X}{F}\right) \tag{40}$$

Next, substituting (40) into (26) and rearranging terms we can provide the following decomposition of τ^x:

$$\tau^x = \tau_{FB}^x + \tau_{FB}^x(p) + \tau_{SB}^x + \tau_{SB}^x(p) \tag{41}$$

with

$$\tau_{FB}^x = \frac{\eta}{\rho+\mu}H\frac{u_Q}{u_c}$$

$$\tau_{FB}^x(p) = \frac{u_p}{u_c}\left(\frac{\eta}{\rho+\mu}\frac{\partial\bar{p}}{\partial Q} - \frac{u_p}{u_c}\frac{\partial\bar{p}}{\partial X}\frac{\partial\bar{p}}{\partial F}\frac{F_K}{\rho}\right)$$

$$\tau_{SB}^x = \frac{\eta}{\rho+\mu}H\frac{u_Q}{u_c}\lambda\frac{(\Delta_Q-\Delta_c)}{(1+\lambda\Delta_c)}$$

$$\tau_{SB}^x(p) = \frac{u_p}{u_c}\lambda\frac{(\Delta_p-\Delta_c)}{(1+\lambda\Delta_c)}\left(\frac{\eta}{\rho+\mu}\frac{\partial\bar{p}}{\partial Q} - \frac{\partial\bar{p}}{\partial X} - \frac{u_p}{u_c}\frac{\partial\bar{p}}{\partial X}\frac{\partial\bar{p}}{\partial F}\frac{F_K}{\rho}\right)$$

where $\tau_{FB}^{x}, \tau_{SB}^{x}$ are the first-best and second-best tax components in a framework without the warm-glow component ($u_p = 0$) and $\tau_{FB}^{x}(p) + \tau_{SB}^{x}(p)$ are the first-best and second-best components of the pollution tax that add to the previous ones in the presence of warm-glow. Notice that the optimal tax on pollution displays the "additive property" first obtained by Sandmo (1975) in the context of taxation of consumption goods with externalities.

Table 1 summarizes our findings on τ^{x}.

Table 1: Summary of the results on the Pigouvian tax τ^{x}

	τ^{x}	
	No warm-glow (u_p =0)	With warm-glow ($u_p < 0$)
First-Best	τ_{FB}^{x}	$\tau_{FB}^{x} + \tau_{FB}^{x}(p)$
Second-Best	$\tau_{FB}^{x} + \tau_{SB}^{x}$	$\tau_{FB}^{x} + \tau_{FB}^{x}(p) + \tau_{SB}^{x} + \tau_{SB}^{x}(p)$

4. Discussion of the results

In this section we analyse and comment on our obtained results on the optimal structure of taxes. We first present the optimal tax structure at the first-best, then we focus on the second-best. While our results are quite general, in order to better characterize their economic content we can focus on some specifications of the perceived pollution content function $\bar{p}^{j}(t)$ and of the utility function:

Assumption H1: *The warm-glow perceived pollution content function* $\bar{p}^{j}(t)$ *assumes one of the following forms:*

$$\bar{p}^{j}(t) = \gamma \cdot x^{j}(t) \qquad \text{(S.1)}$$

$$\bar{p}^{j}(t) = \gamma \cdot \frac{x^{j}(t)}{Q(t)} \qquad \text{(S.2)}$$

$$\bar{p}^j(t) = \gamma \cdot \frac{x^j(t)}{F(t)} \tag{S.3}$$

$$\bar{p}^j(t) = \gamma \cdot \frac{x^j(t)}{X(t)} \tag{S.4}$$

with $\gamma > 0$.

The first and the second specification for $\bar{p}^j(t)$ are in line with the existing literature (e.g. Dam 2011 and Dam and Heijdra 2011), while the others, to the best of our knowledge, are new and are meant to represent a situation in which the pollution content of a firms, as perceived by the individual for warm-glow purposes, is relative to either aggregate economic activity (gross GDP) (when $\bar{p}^j(t) = \gamma \cdot \frac{x^j(t)}{F(t)}$), or aggregate total pollution ($\bar{p}^j(t) = \gamma \cdot \frac{x^j(t)}{X(t)}$).

The second assumption is concerned with the shape of the utility function, on which we introduce the restriction of partial additivity.

Assumption H2: *The instantaneous utility function assumes the following form*:

$$u(c, l, p, Q) = v(c, l) + h(p, Q)$$

i.e. additive and separable in (c, l) *and* (p, Q).

We now provide the following Lemma:

Lemma 1: Under H2, if leisure is non-inferior, then:

L.1) $\Delta_l - \Delta_c > 0$

L.2) $\Delta_p - \Delta_c > 0$

L.3) $\left(1+\lambda\Delta_p\right)>0$

Moreover, if also $\frac{u_{pQ}p}{u_Q}\geq 1$, then

L.4) $\Delta_Q-\Delta_c>0$.

Proof. See Appendix A.2. □

As a comment on point L.4) of Lemma 1, notice that the assumption of $\frac{u_{pQ}p}{u_Q}\geq 1$ implies that negative of $p(t)$ and $Q(t)$ are substitutes, i.e. marginal value of the environmental stock is falling with the cleanliness of the portfolio held by the household. Under the above assumptions, we can now provide the following Proposition characterizing the first-best tax structure.

Proposition 1. *At the steady state, the first-best tax structure is the following:*

a) $\tau^x>0$;

b) $\tau^a,\tau^l=0$ *under specifications (S.1), (S.2) and (S.4),*

c) $\tau^a,\tau^l<0$ *under specification (S.3).*

Proof. The result on τ^x descends from eq. (41), whereby at the first-best $\tau^x=\tau^x_{FB}+\tau^x_{FB}(p)$, with $\tau^x_{FB}>0$ for all specifications of $\bar{p}^j(t)$ and $\tau^x_{FB}(p)>0$ for specifications (S.2) and (S.3), (zero otherwise[4]). As for τ^a,τ^l, the results descend from the fact that, at the first-best, $\lambda=0$ and, that, under specifications (S.1), (S.2) and (S.4), $\frac{\partial\bar{p}}{\partial F}=0$, so that the results sub b) of zero taxes follow from mere observation of (38) and (39). Under specification (S.3), $\frac{\partial\bar{p}}{\partial F}<0$;

[4] Notice that, under specification (S.4), the equilibrium value of the warm-glow function is $p(t)=\sum_{j=1}^{J}\frac{\bar{p}^j(t)}{H}=\gamma\cdot\frac{1}{H}$, so that the term $\frac{\partial\bar{p}}{\partial x}$ appearing in the expression for $\tau^x_{FB}(p)$ is zero.

moreover, recall that $u_c > 0, u_l, u_p < 0$. Hence, by observation of (38) and (39), $sign(\tau^a) = sign(\tau^l)$. Finally, by (33), $(1 + \lambda\Delta_c) > 0$ and by Lemma 1, sub L3), $(1 + \lambda\Delta_p) > 0$, so that $\tau^a, \tau^l < 0$.

□

Under formulation (S.1) and (S.4) there is no correction for warm-glow in the pollution tax (i.e. $\tau_{FB}^{x}(p) = 0$) and we get standard Pigou tax, while the add-on correction is present under the (S.2) and (S.3) specifications. Furthermore, both taxes on capital and labour income are either zero or, in case the warm-glow depends on the scale of economic activity (S.3), negative (i.e. both inputs should be subsidized to reduce the individual's perceived damage caused by firms).

The reason is that in formulation (S.3), each firm realises that its individual pollution affects the perceived pollution content, but does not take into account the effect on aggregate production. If the firm could increase aggregate production, it would do so in order to reduce the perceived pollution content and lower the pollution premium (to lower the cost of capital). In the first best, this needs to be corrected for. Thus, capital and labour are subsidised to increase aggregate production. However, the correction to increase aggregate production, to lower the perceived pollution content, will imply that the abatement incentive for the firm is lowered. Therefore, the new Pigou tax needs to contain the extra (positive) component.

In formulation (S.2), while firms realise the consequence of pollution on its own perceived pollution content, they do not realise that they (in the aggregate) affect the state of the environment (environmental quality). If they did realise they affected the aggregate, they would have an incentive to lower pollution at each date to increase Q (again in order to lower the cost of capital). This needs to be corrected for in the first best, with an extra (positive) component added to the Pigou tax.

Let us now turn to the second-best tax structure, which we characterize through the following Proposition:

Proposition 2. *At the steady state, the second-best tax structure is the following:*

I) As for τ^x*:*

I.A) its sign is ambiguous;

I.B) Under Lemma 1, sub L.4) $\tau^x > 0$*;*

II) As for τ^a:

II.A) $\tau^a = 0$ *under specifications (S.1), (S.2) and (S.4),*

II.B) $\tau^a < 0$ *under specification (S.3).*

III) As for τ^a:

III.A) $\tau^l > 0$ *under specifications (S.1), (S.2) and (S.4),*

III.B) Its sign is ambiguous under specification (S.3).

Proof: As for τ^x, in the Proof of Proposition 1 we already showed that $\tau^x_{FB}, \tau^x_{FB}(p) > 0$. The sign of τ^x_{SB} is in general ambiguous, in that depends on the sign of $\left(\Delta_Q - \Delta_c\right)$. By Lemma 1, sub L.4), the latter difference is positive, so that in this case $\tau^x_{SB} > 0$. Finally, under Lemma 1, sub L.2), $\tau^x_{SB}(p) > 0$. As for τ^a, the argument presented in Proposition 1 applies. Finally, as for τ^l, under specifications (S.1), (S.2) and (S.4), the term $\frac{\partial \bar{p}}{\partial F}$ in eq. (38) is zero, while $(\Delta_l - \Delta_c) > 0$ by Lemma 1, sub L.1), so that $\tau^l > 0$. Under specification (S.3), $\frac{\partial \bar{p}}{\partial F} < 0$, so that the sign of τ^l is ambiguous. □

As a final comment on our results, we notice that under complete additive separability, $\Delta_Q = 0$ and $sign\left(\Delta_Q - \Delta_c\right) = sign(-1 - \frac{u_{cc}c}{u_c})$. For example, in case of log utility the latter is equal to zero, so that $\tau_{SB}^{x} = 0$ and $\tau^{x} > 0$. Finally, the ambiguity of the sign of the labour income tax in specification (S.3) stems from the fact that the second-best component would make it optimal for the policymaker to levy positive taxes on labour income, while the first-best component does exert an opposite effect. We can summarize our results through the following Table.

Table 2: Summary of the results on first-best and second best-tax structure

<table>
<tr><td></td><td colspan="2">τ^x</td><td colspan="2">τ^a</td><td colspan="2">τ^l</td></tr>
<tr><td></td><td colspan="2"></td><td>Specifications (S.1), (S.2), (S.4)</td><td>Specification (S.3)</td><td>Specifications (S.1), (S.2), (S.4)</td><td>Specification (S.3)</td></tr>
<tr><td>**First-Best**</td><td colspan="2">$\tau^x > 0$</td><td rowspan="2">$\tau^a = 0$</td><td rowspan="2">$\tau^a < 0$</td><td>$\tau^l = 0$</td><td>$\tau^l < 0$</td></tr>
<tr><td>**Second -Best**</td><td>τ^x ambiguous</td><td>Under Lemma 1, sub L.4), or completely additive separable utility with $\frac{u_{cc}c}{u_c} \geq 1, \tau^x > 0$</td><td>$\tau^l > 0$</td><td>$\tau^l$ ambiguous</td></tr>
</table>

4. Conclusions

In this paper we characterize optimal taxes in a continuous-time growth model in the presence of pollution as a joint product of production. We explicitly allow investors to engage in socially responsible investments through a *warm-glow* mechanism as in Andreoni (1990) and Dam (2011) and firms to engage in corporate socially responsible activities through pollution abatement activity.

We show that the first-best tax structure consists in positive taxation of pollution and either zero or negative taxation of production-factor incomes (negative taxation arising in case the perceived pollution content of firms is negatively related to the total scale of economic activity).

As for the second-best structure, we show that the pollution tax has the additivity property, consisting of the first-best component, plus the first best-warm-glow component, plus a second-best component, plus the second-best warm-glow component. Leisure being non-inferior is sufficient for the add-on components to be positive or zero, apart from the second-best warm-glow component, which can take on any sign.

While the first-best tax rule for the capital income tax also holds in the second-best, it emerges that in general, sufficient for the labour income tax to be positive is that leisure is non inferior (though its sign can be ambiguous, if the perceived pollution content of the firm depends on gross GDP).

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Appendix A.1. The implementability constraint

Time derivative of $q(t)a(t)$ is

$$\frac{d}{dt}[q(t)a(t)] = \dot{q}(t)a(t) + q(t)\dot{a}(t) \quad \text{(A.1.1)}$$

Exploiting (6) and (15), (A.1.1) reads as:

$$\frac{d}{dt}[q(t)a(t)] = q(t)\{\overline{w}(t)l(t) - c(t) - z(t)\} - u_p(t)\sum_{j=1}^{J}\frac{\omega^j(t)\bar{p}^j(t)}{V^j(t)}a(t) + a(t)\rho q(t) \quad \text{(A.1.2)}$$

By substituting for $q(t)$ from (9) and (10) and exploiting (4) it follows:

$$\frac{d}{dt}[q(t)a(t)] - \rho q(t)a(t) = -u_p(t)p(t) - u_c(t)c(t) - u_c(t)z(t) - u_l(t)l(t) \tag{A.1.3}$$

Multiplying both sides by $e^{-\rho t}$ (A.1.3) can be written as:

$$\frac{d}{dt}[q(t)a(t)e^{-\rho t}] = -e^{-\rho t}[u_p(t)p(t) + u_c(t)c(t) + u_c(t)z(t) + u_l(t)l(t)] \tag{A.1.4}$$

and integrating it follows that:

$$a(0)q(0) = \int_0^\infty e^{-\rho t}\left[u_p(t)p(t) + u_l(t)l(t) + u_c(t)c(t) + u_c(t)z(t)\right]dt \tag{A.1.5}$$

which is eq. (28) in the text.

Appendix A.2. Proof of Lemma 1

q is the marginal shadow value of assets and is inversely related to assets. We take as normality of, say c, the case when $\frac{dc}{dq} < 0$, as it corresponds to $\frac{dc}{da} > 0$, the "income effect" keeping prices fixed. Normality of leisure is when labour increases in q, that is $\frac{dl}{dq} > 0$.

Differentiating (9) and (10) for partially separable utility (assumption H2) we have:

$$\begin{bmatrix} u_{cc} & u_{cl} \\ u_{lc} & u_{ll} \end{bmatrix} \begin{bmatrix} \frac{\partial c}{\partial q} \\ \frac{\partial l}{\partial q} \end{bmatrix} = \begin{bmatrix} 1 \\ -\overline{w} \end{bmatrix} = \begin{bmatrix} 1 \\ \frac{u_l}{u_c} \end{bmatrix} \tag{A.2.1}$$

Cramer's rule provides the following:

$$\frac{\partial l}{\partial q} = \frac{1}{u_l}\frac{\frac{u_{cc}}{u_c}-\frac{u_{lc}}{u_l}}{u_{cc}u_{ll}-u_{cl}u_{lc}} \tag{A.2.2}$$

Concavity of u implies

$$u_{cc}u_{ll} - u_{cl}u_{lc} > 0 \tag{A.2.3}$$

Then, since $u_l < 0$, $sign\left(\frac{\partial l}{\partial q}\right) = sign\left(-\frac{u_{cc}}{u_c} + \frac{u_{lc}}{u_l}\right)$, that is, leisure is non-inferior if $-\frac{u_{cc}}{u_c} + \frac{u_{lc}}{u_l} \geq 0$.

From definition of Δ_c and Δ_l

$$\Delta_c - \Delta_l = \left(\frac{u_{lc}}{u_l} - \frac{u_{cc}}{u_c}\right)c + \left(\frac{u_{ll}}{u_l} - \frac{u_{lc}}{u_c}\right)l. \tag{A.2.4}$$

From eq. (6) in steady state

$$c = \bar{w}l + \bar{R}a = -\frac{u_l l}{u_c} + \bar{R}a \tag{A.2.5}$$

where $\bar{R}$ is after-tax return on assets. Substituting (A.2.5) into (A.2.4) and rearranging we have:

$$\Delta_c - \Delta_l = \frac{u_l}{u_c^2}l\left[u_{cc} - 2\frac{u_c}{u_l}u_{lc} + \left(\frac{u_c}{u_l}\right)^2\right] + \left(\frac{u_{lc}}{u_l} - \frac{u_{cc}}{u_c}\right)\bar{R}a\,. \tag{A.2.6}$$

The term in squared brackets is a quadratic form of the Hessian of u and is negative under concavity. Since $u_l < 0$, we have $\Delta_c - \Delta_l > 0$ if $-\frac{u_{cc}}{u_c} + \frac{u_{lc}}{u_l} \geq 0$, i.e. if leisure is non-inferior.

From definition of Δ_p and Δ_c,

$$\Delta_p - \Delta_c = \frac{u_{pp}}{u_p}p - \frac{u_{cc}}{u_c}c - \frac{u_{lc}}{u_l}l. \tag{A.2.7}$$

Using (A.2.5), (A.2.7) can be written as

$$\Delta_p - \Delta_c = \frac{u_{pp}}{u_p}p + \frac{u_l}{u_c}\left(\frac{u_{cc}}{u_c} - \frac{u_{lc}}{u_l}\right)l - \frac{u_{cc}}{u_c}\bar{R}a\,. \tag{A.2.8}$$

Then, $-\frac{u_{cc}}{u_c} + \frac{u_{lc}}{u_l} \geq 0 \Rightarrow \Delta_p - \Delta_c > 0$, since $\frac{u_{pp}}{u_p}p > 0$.

Next, from definition of Δ_Q and using the steps above, we have

$$\Delta_Q - \Delta_c = \frac{u_{pQ}}{u_Q} p - 1 + \frac{u_l}{u_c}\left(\frac{u_{cc}}{u_c} - \frac{u_{lc}}{u_l}\right) l - \frac{u_{cc}}{u_c} \bar{R}a \,. \tag{A.2.9}$$

Non-inferiority of leisure implies $\Delta_Q - \Delta_c > \frac{u_{pQ}}{u_Q} p - 1$.

If complete additive separability, $\Delta_Q = 0$, then from the definition of Δ_c we have $\Delta_Q - \Delta_c = -\frac{u_{cc}}{u_c} c - 1$.

Finally, from (33) $(1 + \lambda\Delta_c) > 0$, next $(1 + \lambda\Delta_c) = (1 + \lambda\Delta_p) - \lambda(\Delta_p - \Delta_c)$. So, $\Delta_p - \Delta_c > 0$ implies $(1 + \lambda\Delta_p) > 0$.

Email della redazione: lfanti@ec.unipi.it