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Partial decentralisation and inter-governmental electoral competition in local public good provision

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Abstract

Over the last few years, the literature on partial decentralisation has largely grown, pointing out that one of its effects is a reduction in accountability because voters are imperfectly informed about each government contribution. However, the possibility for politicians to directly manipulate this asymmetry in information has not been addressed yet. This paper provides a simple model in which two levels of government are involved in the provision of a local public good with the local government that can decide to spend its budget either on the provision of the public good or in spending that influences the information of the voters in its favour. A central result is that the conflict of interest that arises among the levels of government reduces the spending in the public good at both levels, while it generates a wasteful spending to pander to voters.

Keywords: partial decentralisation; party alignment; accountability; intergovernmental transfers

JEL: D72; H72; H77
1 Introduction

In the last decades decentralisation reforms in public good provision and management have been implemented all over across countries, both in developed countries and developing ones, aiming to increase overall government accountability, in complete agreement with the famous decentralisation theorem (Oates, 1972).

Yet the empirical evidence is far from conclusive about the positive results of decentralisation reforms implemented across countries. On the contrary, results of many different studies support the opposite outcome. An example coming from the history of the United States can be found in Wallis, Fishback, and Kantor (2006); they show how, during the New Deal, the direct intervention of the Federal Government in substituting local government in the management of welfare programs dramatically reduced the corruption that at the time affected those programs. A similar result can be found in Fan, Lin, and Treisman (2009) who, exploiting a cross-national dataset, show that in countries with higher tiers of government the reported corruption is consistently higher.

Summarising these contradictory results, Treisman (2007) notices that “Empirical studies have found almost no solid, general result about the consequences of decentralisation. Decentralising government in a particular place and time is very much a leap in the dark”.

One of the most common explanations for this lack of results is that decentralisation reforms are often incomplete. In some cases, even after the provision of a public good has been formally decentralised, the central government de facto remains deeply involved in its provision. In other cases even if the decisions about public good provision are really decentralised, the central government provides a large share of funds. However, in spite of the different characteristics of each case, the common result is a weakening of political accountability, which typically requires the possibility for voters to easily assign a policy outcome to a given incumbent official, thus undermining the very reason for which decentralisation was implemented in the first place.

In the economic literature, these situations fall under the concept of partial decentralisation, defined by Devarajan, Stuti, and Shah (2009) as an “attempt at decentralisation that has not led to citizens’ being able to hold local governments accountable for budgetary allocations and their outcomes”. The contribution of this paper is to develop a model taking into account the possibility that, despite the fact that they cannot observe the single contribution of each single government tier to the provision of a public good, voters still have an opinion about it, and since they condition their electoral choice on that opinion politicians will have the incentive to influence it.

Two tiers of government, central and local, are involved in the provision of a public good. The local government is directly involved since it is the actual provider, while the central government is indirectly involved since it provides resources. Incumbents are rent-seeking politicians with career concerns. Voters form an opinion on the relative contribution of each tier onto which they base their electoral choices for

\footnote{For example, in its World Development Report (World Bank, 2003) the World Bank has proposed decentralisation reforms as an effective way to reduce corruption in developing countries.}

\footnote{The first situation is more common in developing countries, where the quality of institutions is overall lower, then boundaries between different tiers are less clear. The second situation is more common in developed unitary States going through a process of decentralisation, like Italy.}
both incumbents. The local incumbent can divert part of the budget to spend on communication aiming to increase the perceived relative contribution of the local government at the expense of the central incumbent.

The main feature of this model is that, since the credit assigned for the provision of the public good is a zero sum game, there is a conflict of interest between the incumbents that affects both the amount of resources transferred and the final expenditure for the actual provision of the public good. This strategic interaction between the two governments could be considered as a proxy for the real forms of a bargaining about the allocation of resources from the central state to regions and provinces, typical of many developed countries where partial reforms of decentralisation took place.

To the best of the author’s knowledge, this is the first attempt to take this feature into account. The usual assumption found in literature is that voters only observe the total amount of public good provided (see for example Joanis, 2014).

The analysis highlights how this conflict of interest damages the provision of a public good, allowing to underline what are the main drivers of the central governments’ decision to transfer. In particular, the paper shows how party alignment between local and central incumbent increases both transfer and public good provision.

The paper is organised as follows. A survey of the relevant literature is carried out in section 2; section 3 introduces the stylised theoretical framework and section 4 presents the main results. Section 5 explores an extension of the model taking into account the presence of two local authorities rather than a single one. Finally, section 6 concludes the paper by discussing limits and possible extensions.

2 Related Literature

Decentralisation has been an increasing topic in the last decades, in particular with the growth of the “second generation fiscal federalism” literature (Oates, 2005; Weingast, 2009). In order to explain the unsatisfactory results of decentralisation reforms, this strand of literature has abandoned the benevolent social planner assumption of the first generation of fiscal federalism models, and focuses more on political economy and corruption problems related to centralisation. \(^3\)

As a matter of fact, the definition provided by Devarajan et al. (2009) is very broad and leaves space for ambiguity, and indeed many refinements have been provided in the literature. The one mainly connected with this paper distinguishes whether the incompleteness of the decentralisation is related to the collection of revenues or the expenditure of such revenues. The first case falls under the definition of partial fiscal decentralisation, defined by Brueckner (2009) as a regime in which the spending authority is devolved to the subnational level while financing relies on transfers from the central government. The second case is defined as partial expenditure decentralisation, described by Joanis (2014) as a situation where after the implementation of decentralisation reforms, the central government remains involved in the provision of the local public good targeted by decentralisation.

\(^3\)For a comprehensive discussion of the literature on political decentralisation see Lockwood (2006), Mookherjee (2015).
The model developed in this paper analyses a situation of partial fiscal decentralisation (fulfilling the definition of Brueckner, 2009), but follows Joanis (2014) in assuming that voters are aware that the public good provision depends on the contribution of both authorities but they are not able to measure the exact contribution of each government.

Despite the relatively scarce theoretical literature, both directly and indirectly, partial decentralisation is the subject of a large and still growing empirical literature. The distinction between fiscal and expenditure forms of partial decentralisation applies also to the empirical studies. Moreover, the occurrence of one case or the other seems to be correlated with the degree of development of countries.

Partial expenditure decentralisation is more common in developing countries, where, even though large decentralisation reforms have been de jure implemented, they are often incomplete because de facto central government has still an active role. This phenomenon has generated a large number of case studies, triggering a growing empirical literature.

For example, Estache, Garsous, and Seroa da Motta (2016) studied sanitation policies in the state of São Paulo in Brazil, where the Constitution divides the mandate related to water pollution among mayors and governor, assigning to the former the control of the local sanitation services, specifically sewage treatment, and to the latter the responsibility for the quality of water bodies. In studying the case of Bolivian decentralisation reforms, Inchauste (2009) argues that the lack of a clear improvement for either the poorest segments of the population and the poorest municipalities can be linked to the misalignment of responsibilities between different levels of government. Caldeira, Foucault, and Rota-Graziosi (2012) describe a similar pattern in decentralisation reforms in Benin, where some of the most important public services, like education and water management, are provided in a regime of “shared competency”.

Partial fiscal decentralisation is more common in developed countries, and particularly in European countries where, following the emergence of the vertical subsidiarity paradigm, important expenditure decentralisation reforms have been implemented, however not always followed by clear or complete decentralisation on the financing side. As a consequence, despite the full autonomy in the expenditure and regulatory functions, central governments still play a key role through the fiscal channel (see for example Ambrosiano & Bordignon, 2006).4

This phenomenon is mainly captured by the large literature studying the dynamic of intergovernmental grants in response to political economy factors. In particular, numerous studies conducted on several countries have consistently highlighted a positive correlation between political alignment and intergovernmental grants. For example, see the works of Levitt and Snyder Jr (1995) for the U.S., Solé-Ollé and Sorribas-Navarro (2008) for Spanish municipalities, Case (2001) for Albania, Migueis (2013) for Portugal. More recently Bracco, Lockwood, Porcelli,  

4Another issue related to the decrease in accountability due to a partial fiscal decentralisation is the reduction in transparency over tax decisions by different governments. The intuition is that partial decentralisation may reduce voters’ ability to hold the right politician accountable for the amount of taxes they pay in exchange for the number of services they receive. In the end, they will be less able to select good politicians, undermining the assumption of Oates’ theorem (see for example Bordignon, Grembi, & Piazza, 2017, who provide an interesting analysis for the Italian municipal taxation).
and Redoano (2015) studied the case of Italian municipalities, arguing that the main reason for this positive relationship is that the national government will use grants to increase the reelection probability of its aligned Mayors. Closely related to these studies, Joanis (2016) studied the evolution of Canada’s intergovernmental fiscal arrangements in spite of changes in federal and provincial electoral variables, showing a strong influence of political consideration on the evolution of those arrangements.

3 The model

There are two levels of government, a “central government” (indexed by the subscript $c$) and a “local government” (indexed by the subscript $l$) each of them controlled by an incumbent, who is a rent-seeking politician also caring about being reelected. The local government is formally in charge of the provision of the local public good $\theta$, while the central government provides a relevant part of the funding through direct transfers.

Since the focus of this work is on the allocation of spending, the paper assumes that the aggregate public revenue is given by $B$ that, without loss of generality, can be normalised to 1; let $\theta$ be measured by the monetary amount of public resources devoted to its provision.\(^5\)

The local government can spend its resources in two ways: it can assign a share $g$ to the actual provision of the local public good, and a share $m$ to influence voters’ perception about its own contribution to the provision of $\theta$.

At the beginning of the period, the local government is endowed with a share $b \in (0,1)$ of the total revenue to which is then added the amount $t$ that it receives from the central government as a direct unconstrained transfer.

Then, the budget constraint of the local government can be written as:

$$g + m \leq b + t.$$  

The reason to distinguish between $g$ and $\theta$ is due to a particular feature of this model. In fact, this paper assumes that incumbents extract a rent from each monetary unit spent in the provision of public goods onto which they retain total control. This means that the local incumbent extracts a rent from any monetary unit spent in the provision of $\theta$. Thus the level of the local public good actually provided to citizens is lower than the amount of resources devoted to its production.

Let $\alpha$ be the rent that the incumbent extracts from any monetary unit spent to provide $\theta$. It follows that

$$\theta = (1 - \alpha)g.$$  

Therefore, on the one hand, the rent-extracting motive of the local incumbent is defined by the share $\alpha$ of the total expenditure in public good. On the other hand, the incumbent has a holding office motive, that can be considered as a mixture of economic benefits connected to the position and ego motives, and it is measured by $r \in (0,1)$ (see for example Ferejohn, 1986; Maskin & Tirole, 2004).

As it is standard in the literature, by assuming a linear utility function for the local incumbent, it follows:

$$U_l = \alpha g + p(\theta)r,$$  

\(^5\)Or similarly that the production function of $\theta$ is linear in the amount of funds.
where \( p(\theta) \) is the probability of being re-elected at the end of the game.

The central government is endowed with the remainder of the total revenues \((1 - b)\). The incumbent can spend this budget in two ways: it can transfer a share \( t \) to the local government to finance the provision of \( \theta \) and a share \( G \) to finance a generic public expenditure at the central level.\(^6\)

The budget constraint of the central government is then

\[
G + t \leq 1 - b. \tag{4}
\]

Since the amount of resources transferred to the local government is under the control of the local incumbent, it follows that the central incumbent can extract a rent only out of the expenditure in \( G \).

Therefore, let \( \beta \) be the equivalent of \( \alpha \) for the central incumbent and, for the sake of simplicity, assume that the pay off premium for being in office is equal for both levels.

Assuming that the central incumbent too has a linear utility function, it follows:

\[
U_c = \beta G + P(\theta)r, \tag{5}
\]

where \( P(\theta) \) is the central incumbent’s probability of being re-elected.

3.1 Reelection probability

At the end of the game, both a central and a local election take place, and voters decide whether or not to reelect the incumbents on the basis of the level of the local public good they have received.

Following the literature on partial decentralisation (Joanis, 2014), assume that voters can only observe the value of \( \theta \) without any knowledge about how much local and central governments contribute. Moreover, this paper extends this idea by assuming that they form a belief about the relative governments’ contributions, which are endogenous.

In order to keep the analysis simple, an exogenous functional form for the re-election probability is assumed, such that it is increasing in \( \theta \) and takes into account the belief about the relative contribution of different branches. Defining \( \sigma \) as the voters’ belief about the share of local government’s contribution to \( \theta \), let \( p(\theta) \) take the form:

\[
p(\theta) = \phi_l + \sigma \theta, \tag{6}
\]

and \( P(\theta) \) the form:

\[
P(\theta) = \phi_c + (1 - \sigma)\theta. \tag{7}
\]

The first terms \( \phi_k \) represent a constant baseline re-election probability independent from the provision of \( \theta \). This considers the fact that, in general, the provision of \( \theta \) would not be the only issue at stake. Assume it to be exogenous, specific for each level of government, and such that \( \phi_l \in [0, \Phi_l] \) and \( \phi_c \in [0, \Phi_c] \).\(^7\)

\(^6\)In this paper, \( G \) will be treated as unproductive public expenditure, meaning that, when they need to decide about the reelection of the central incumbent, voters do not take into account the level of public expenditure at the central level.

\(^7\)See Appendix B for a detailed discussion.
3.1.1 Voters belief about contribution

Voters can observe the level of the public good provided, $\theta$, but they cannot observe the exact contribution of each governmental authority. However, they may form an opinion about the contributions. The formation of this opinion is in reality closely connected to political actions between the parties involved; these actions may involve political debates, but also public expenditure to influence voters.

This paper takes into account the case in which only the local government can directly affect the formation of this opinion. Despite being a simplification, this assumption is not so unreasonable for two reasons. The first reason is that the local authority is the one that actually provides $\theta$, which combined with the difficulties in understanding public budgets, makes it is easier for voters to recognise the economic effort of the one that actually spent the money rather than the effort of the one who just transferred it. The second reason is that local politicians are much closer to voters than national ones, therefore it is much easier for them to convey their message to voters than it is for national politicians.

Summing up, even if it may be a simplification to assume that only the local politicians are able to influence voters’ perception about contribution, it seems quite reasonable to think that, ceteris paribus, they are more effective.

Local government can influence voters’ perception of its own contribution to the provision of $\theta$ by spending a share $m$ out of its budget to increase the perception of its contribution. The expenditure in $m$ is directed to the financing of a “side good”, defined as a good which does not increase voters utility, but since it is connected to the provision of $\theta$, it works as a signal for voters about the contribution of the local government. This is a broad definition and can include different ways to influence voters; the simplest case of side good is the political advertisement to voters, to inform them that a public good is provided, thanks to the local government. More sophisticated situations can also fit this definition, such as the inclusion of accessory features that, while not increasing the functionality of a public good, indicate the cost and economic commitment of local government; this second case is more common if the public good is, for example, an infrastructural project.

However, the effectiveness of $m$ is not constant and may depend on the institutional situation of the specific constituency. In particular, it seems reasonable to think that if there is party alignment between the local and the central incumbent, the expenditure in $m$ will be less effective with respect to the case in which there is no alignment.

To understand why this could be the case, one may think that since voters cannot observe the amount of money transferred from the central government to the local one, the local incumbent could have the incentive to pretend that the transfer has been lower than in reality, claiming that despite the cut from the central government, the local government guaranteed the provision of $\theta$. This message is more effective when the two incumbents are political opponents rather than allies.

In order to capture this alignment, suppose that the political spectrum can be represented by a unitary segment, where 0 corresponds to the most leftwing position and 1 corresponds to the most rightwing position. Then the political position of each incumbent corresponds to a point in this segment.
Let $\delta \in [0, 1]$ be the measure of the distance between the two points representing the incumbents’ political positions. This means that $\delta$ measures the degree of political misalignment between the two incumbents. Thus if $\delta = 0$, incumbents are perfectly aligned, while if $\delta = 1$ incumbents are at the extreme of the political space.

Hence $\sigma$ can be written as:

$$\sigma(m) = \delta m.$$  

Local government does not spend at all in the side good $m$ and its reelection probability is equal to $\phi_l$.

### 3.2 Time of the game

The game is divided into two stages. In the first stage, the central incumbent decides how to allocate the share of public revenue assigned to the central government ($1 - b$) between discretionary spending ($G$) and intergovernmental transfer ($t$).

In the second stage, given the total amount of resources available ($b + t$), the local incumbent decides how much to spend for the provision of the local public good ($\theta$) and how much for the provision of the side good ($m$).

After the end of the game, both incumbents eventually face an election.

### 4 Solution of the game

#### 4.1 Equilibrium spending of the local government

In the last stage of the game, given the amount of transfers $t$ received from the national government, the local incumbent decides how much of its budget to devote to the production of $\theta$, and how much to invest on voters’ beliefs.\(^8\)

\[
\begin{aligned}
\max U_l &= \alpha g + r(\phi_l + \sigma \theta) \\
\text{s.t.} & \\
& b + t = g + m \\
& \theta = (1 - \alpha)g \\
& \sigma(m) = \delta m
\end{aligned}
\]

**Proposition 1.** Consider the maximisation problem of equation (9) and let $\hat{t} = \frac{\alpha}{2r(1 - \alpha)} - b$. Two cases are possible.

(i) If $t \geq \hat{t}$, the reaction functions of the local incumbent are

\[
\begin{aligned}
g^*(t) &= \frac{b + t}{2} + \frac{\alpha}{2r(1 - \alpha)} \\
m^*(t) &= \frac{b + t}{2} - \frac{\alpha}{2r(1 - \alpha)}
\end{aligned}
\]

(ii) If $t < \hat{t}$, the reaction functions of the local incumbent are:

\[
\begin{aligned}
g^*(t) &= b + t \\
m^*(t) &= 0
\end{aligned}
\]

---

\(^8\)All proofs are provided in Appendix A.
Therefore, if the local government receives enough transfers from the central government, the local incumbent will divert part of these resources to influence the opinion of voters. On the other hand, if resources transferred by the central government are extremely scarce, the local incumbent prefers to assign all the budget to the provision of the local public good, even at the cost of minimising the probability of being reelected in the following period.

The reason is that while both \( g \) and \( m \) positively affect the reelection probability of the incumbent, only \( g \) provides the rent. Thus, if the local budget is scarce, the increase in the reelection chances due to a positive expenditure in \( m \) is very small compared to the amount of rent that can be extracted from the public good.

Reaction functions are depicted in Figure 1.

Figure 1: Reaction functions
\((\alpha = 0.2, b = 0.4, r = 0.7, \delta = 0.6)\)

However, this suggests that, in order for this corner solution to appear, the rent extraction motive, \( \alpha \), has to be sufficiently high. Indeed, the following relations can be derived.

**Corollary 1.** Given the result of Proposition 1, two extreme cases are possible:

(i) if \( \frac{\alpha}{1 - \alpha} \geq \delta r \), the solution is never interior;

(ii) if \( \frac{\alpha}{1 - \alpha} \leq \delta rb \), the solution is always interior.

In order to avoid the possibility of corner solutions, the underlying assumption will be \( \alpha \leq \frac{\delta rb}{1 + \delta rb} \). This assumption simplifies the analysis and it appears reasonable because, since \( \alpha \) measures the share of public expenditure that is captured as rent, values of \( \alpha \) too high would be unrealistic.

The reaction functions in the well-behaved case are depicted in Figure 2.

Figure 2: Reaction functions, special case
\((\alpha = 0.1, b = 0.4, r = 0.7, \delta = 0.6)\)

Given equation (10) it directly follows that an increase in \( b \) and \( t \) has a positive effect on both \( g \) and \( m \). This is evident since, ceteris paribus, the higher the local
budget, either due to its own resources or due to an increase in transfers, the higher the expenditure in all sectors.

Conversely, the effect of the other parameters has an opposite sign on \( g \) and \( m \), as highlighted in the following Corollary.

**Corollary 2.** The expenditure in public good provision \( g(t) \)

(i) is higher when rent and political alignment are higher \( (\frac{\partial g^*(t)}{\partial \alpha} > 0, \frac{\partial g^*(t)}{\partial \delta} < 0) \),

(ii) is lower when the office holding motive is higher \( (\frac{\partial g^*(t)}{\partial \eta} > 0) \).

The converse holds for \( m(t) \).

The effect of rent extraction on \( g(t) \) is clear: since the local incumbent is rent-seeking, and the rent can be extracted only out of the provision of \( \theta \), the higher \( \alpha \) the higher the incentive for the local incumbent to increase the share of the public budget devoted to the local public good.

The positive effect of party alignment over the provision of the public good is due to the fact that the higher the alignment (i.e. the higher \( \delta \)), the less effective is the effort to persuade voters that the larger part of the credit for the provision of \( \theta \) should be recognised to the local government. Therefore the local incumbent will have the incentive to increase the spending \( g \). This can also be seen in the other way around. If the local incumbent belongs to a different party than the central one, voters will be more willing to believe that a lower level of \( \theta \) is due to the lack of transfer from the central government rather than to an opportunistic behaviour of the local one; this provides to the local incumbent the incentive to divert resources from \( g \) to \( m \).

Finally, an increase in the office holding motive of the local incumbent has a negative impact on the quantity of public good provided. This result may seem odd since, in principle, voters react positively to an increase of \( \theta \). The explanation is that without any rent motive, the “optimal” electoral motivated choice for the local incumbent would be to exactly split the budget between \( m \) and \( \theta \), since that maximises \( p(\theta) \). Because of the possibility to extract the rent, the local incumbent partly sacrifices the reelection chances devoting extra resources to the production of \( \theta \). Thus \( r \) represents the opportunity cost of extracting that rent. Therefore, the more the local incumbent cares about reelection, the closer will be the budget allocation to the “probability maximising” one.

**4.2 Central Government**

Consider now the first stage of the game, in which the central incumbent knows the reaction function of the local one and assigns its budget deciding the amount to transfer to the local authority.

The maximisation problem of the central incumbent is:

\[
\begin{align*}
\max_{U_c} & \quad U_c = \beta G + r[\phi_e + (1 - \sigma)\theta] \\
\text{s.t.} & \quad 1 - b = G + t
\end{align*}
\]  

(12)
Proposition 2. Consider the maximisation problem of equation (12). The optimal transfer \( t^* \) is equal to

\[
t^* = \frac{1}{\delta} - b - \frac{2\beta}{\delta r(1-\alpha)}.
\]

(13)

Two corner solutions are possible:

(i) if \( \beta < \bar{\beta} \), the optimal transfer is equal to \( 1 - b \);
(ii) if \( \beta > \bar{\beta} \), the optimal transfer is equal to 0.

Analogously to what done with the local government, conditions are derived with respect to the rent-extraction parameter \( \beta \). If the rent extraction incentive is sufficiently low, the central incumbent prefers to maximise the reelection probability, thus transferring all the budget to the local government. On the other hand, if the rent that can be extracted is sufficiently high, the central incumbent prefers not to transfer anything to the local authority, maximising the rent extracted, at the cost of a low reelection probability.\(^9\)

Consistently with what done for \( \alpha \), in the proceedings of the paper it will be assumed that \( \beta \in [\bar{\beta}, \bar{\beta}] \).

4.2.1 Comparative statics

The first thing that is worth to notice when analysing equation (13) is that, when deciding the amount to transfer, the central incumbent takes perfectly into account the budget of the local one and reduces the transfer accordingly. Therefore, \( \text{ceteris paribus} \), the final budget of the local authority is constant with respect to \( b \).

Corollary 3. The amount of transfers to the local government, \( t^* \)

(i) is higher when career concern and alignment are higher \( \frac{\partial t^*}{\partial r} > 0, \frac{\partial t^*}{\partial \alpha} < 0 \),

(ii) is lower when rent extraction is higher \( \frac{\partial t^*}{\partial \alpha} < 0 \) and \( \frac{\partial t^*}{\partial \beta} < 0 \).

For the central incumbent rent extraction and career concern act in the opposite way with respect to the case of the local one. Since the rent is extracted from public expenditure under the direct control of the incumbent, there are two consequences. On the one hand, the higher the incumbent can extract, \( \beta \), the higher the incentive to sacrifice reelection probability in exchange for a higher direct public expenditure \( G \). Then resources transferred will be lower.

On the other hand, since the office holding motive is the opportunity cost of extracting that rent and since higher rent means lower chances of being reelected, the stronger the career concern of the incumbent, the larger the transfer.

The effect of political alignment is the one expected from the literature: the more the local incumbent is aligned to the central one (i.e. the lower \( \delta \)), the higher is the transfer received. What differs from the literature is the motivation for this result. \( \text{Ceteris paribus} \), the central incumbent is willing to provide a higher grant

\(^9\)It is worth noting that, even if the central incumbent does not transfer anything to the local authority, the reelection probability is always greater than the baseline \( \phi_c \) because, given equation (10), the local incumbent will spend the entire budget \( b \) in the provision of \( \theta \). It then follows that \( P(\theta) = \phi_c + (1-\alpha)b \), while \( p(\theta) = \phi_c \).
to an aligned local incumbent knowing that this money will be spent more for the production of $\theta$ rather than $m$, from which the central incumbent can benefit in terms of reelection probability.

Therefore, a constituency whose local incumbent is more aligned to the central government receives a larger transfer. The reason is that the money transferred to a more aligned region will be spent more for the production of $\theta$ rather than $m$, hence the central incumbent will be less damaged. This result is coherent with the empirical literature that finds a consistent alignment bias in the analysis of intergovernmental transfers. Finally, a remarkable result is that the central incumbent always prefers an honest local official rather than a dishonest one. Ex ante this could be unexpected because from equation (10) the central incumbent knows that a more corrupt local politician will spend more in $g$ and less in $m$. Therefore one might expect that, at least in some cases, the amount of transfer would increase the higher $\alpha$. However, this is never the case, meaning that the negative impact of a more corrupt local incumbent, namely that more resources are lost due to rent extraction, is always stronger than the positive one.

4.3 Optimal public good provision

Proposition 3. Given the regularity condition over $\alpha$ and $\beta$, the level of public good provided is equal to

$$\theta^* = \frac{r(1 - \alpha) + \alpha - 2\beta}{2\delta r}.$$ (14)

The provision of the public good is higher the higher the political alignment between incumbents ($\frac{\partial \theta^*}{\partial \alpha} < 0$) and the higher the rent extracted by the local incumbent ($\frac{\partial \theta^*}{\partial \alpha} > 0$).

The provision of the public good is lower the larger the rent of the central incumbent ($\frac{\partial \theta^*}{\partial \alpha} < 0$).

The effect of a variation in the office-holding motive is ambiguous and depends on the relative strength among the rent extraction of each incumbent ($\frac{\partial \theta^*}{\partial \alpha} \geq 0$ if $\beta \geq \frac{\alpha}{2}$).

The comparative statics for the level of public good highlights some insightful effects. Two of them, the effect of $\delta$ and $\beta$, were largely expected given the previous results.

Political alignment is confirmed to have a positive effect on the final public good provision. This result is in line with the literature, both theoretical and empirical, but the underlying motive is different from standard interpretations. The reason for which the aligned local government spends less in influencing the belief of voters is not that it internalises the damage caused to its aligned central incumbent, but just that the expenditure in $m$ is less credible and then less effective because voters are aware of their alignment.

The negative impact of $\beta$ over the public good provision comes directly from its negative effect over the transfer from the central to the local government.

The overall impact of $\alpha$, on the other hand, was ex ante less obvious. On the one hand $\alpha$ has a positive direct impact over the expenditure in the provision of the public good from the local incumbent but, on the other hand, it has an indirect negative impact on the amount of money that the local government receives from the central incumbent.
The intuition for this neat result is that while the local incumbent has a strong incentive in increasing the expenditure in $g$ due to the rent extraction, the central incumbent has a much weaker incentive to punish a more corrupt local incumbent. After all, despite the waste in public resources due to the higher rent extraction, a more corrupted local incumbent will, ceteris paribus, spend more for the public good and less for influencing voters, thus increasing the reelection chances of the central incumbent.

A particular attention is required in discussing the effect of the office holding motive, $r$. Similarly to the case of $\alpha$, this ambiguous net outcome is due to the opposite effects that $r$ has on incumbents, discussed earlier in the paper. For the local incumbent, an increase in $r$ has a negative effect on the expenditure for $\theta$. Spending less in $m$ to increase the expenditure in $g$ is not a probability maximising behaviour, therefore the higher $r$ the higher the opportunity cost of doing it. For the same reason it has a positive effect on the central incumbent: the higher the office holding motive, the higher will be the transfer to the local government.

However, this result firstly depends on the simplifying assumption that incumbents have the same payoff premium. If the payoffs differ for different offices, the two effects are separated, and each of them has an unambiguous impact.

5 The case of two constituencies

The results derived in the previous section rely on the assumption that the two incumbents share exactly the same voters. This situation is clearly unrealistic since there would be no reason to have two different tiers of government covering exactly the same territorial area.

In reality, a higher tier of government is in charge of several lower ones, therefore, given the budget of that tier, a higher transfer to a given local authority often implies a lower transfer to another one.

This section provides an extension of the model, in which the central government is in charge of an election district divided into two constituencies, each of them controlled by a local government.

Since the reelection probability, the central incumbent now does not depend on the level of public good provided by just one local incumbent. Moreover, if regions are not symmetric, the opportunity for the central incumbent to behave opportunistically increases.

In particular, the expected result is that ceteris paribus, the incentive to transfer to a given constituency will be higher, the larger the share of the population in that region and the more the local incumbent is aligned to the central incumbent.

5.1 The environment

With respect to the basic model, assume now that there are two constituencies, \( \{1, 2\} \), with a different population. In particular let $\lambda \in (0, 1)$ be the share of the total population living in the constituency 1, and $(1 - \lambda)$ the share of population living in constituency 2.
In order to keep the analysis simple assume that there is no yardstick competition neither between voters nor between local incumbents. The first assumption has two implications. On the one hand, the reelection probability in each constituency depends exclusively on the level of the local public good provided in that constituency. On the other hand, voters’ belief about the local government’s contribution depends only on the level of expenditure \( m \) of their constituency. The second assumption implies that each local incumbent takes into account only the amount of transfer received.

Combining these two assumptions implies that each local incumbent faces the same maximisation problem of the simpler model. Given the amount of resources received from the central government, the local budget is allocated independently from what happens in the other constituency.

The time of the game is analogous as before. The sole difference is that now in the first stage the central incumbent has to decide not only how much to transfer to local governments, but also how much to transfer (if any transfer occurs) to each local government.

A representation of the temporal structure of the game is provided in figure 3.

It is now relevant how to consider these dual channels into the reelection probability. It is reasonable to assume that politicians care about their chances to be reelected in both regions, and even though the focus may be stronger on regions with larger population, the interest for the smaller ones will be rarely zero. The same is true for the reelection probability of the incumbent government in any region: no matter the decision of a central government, it will never be zero.

Given these observations, let \( P(\theta_1, \theta_2) \) be the central incumbent’s probability of winning the majority in the district. Let also \( P(\theta_1) \) and \( P(\theta_2) \) be the central incumbent’s probability of winning the majority in constituency 1 and 2 respectively. Due to the absence of yardstick competition between voters, \( P(\theta_1) \) and \( P(\theta_2) \) are independent from each other and can be then defined as in equation (7).\(^{10}\)

In order for the central incumbent to win the election in the district, it is not necessary to win the majority in each constituency, therefore winning the majority

\(^{10}\)For the sake of simplicity assume the baseline probabilities in both constituencies to be equal to 0.
in a more populated constituency will increase the chance of winning more than winning the majority in a less populated area. In this case, $\lambda \in (0,1)$ can be exploited to weight the importance of each constituency in increasing the chances of winning of the central incumbent.

Thus the central incumbent’s probability of winning the reelection, $P(\theta_1, \theta_2)$, can be written as

$$P(\theta_1, \theta_2) = \lambda P(\theta_1) + (1 - \lambda)P(\theta_2).$$

(15)

Therefore the one constituency case of the previous section corresponds to one of the two extreme cases in which either $\lambda = 1$ or $\lambda = 0$.

Given that reelection probability depends on the level of the public good provided and voters’ belief about central government’s contribution in each region, it follows by equation (7) that $P(\theta_1, \theta_2)$ takes the form:

$$P(\theta_1, \theta_2) = \lambda(1 - \sigma_1)\theta_1 + (1 - \lambda)(1 - \sigma_2)\theta_2.$$  

(16)

5.2 Local governments

As previously stated, given the no yardstick competition assumption, the game has not changed for each local incumbent.

Given the amount of transfer $t_i$ received from the central government, the share $b_i$ of total revenues and the rent extraction parameter $\alpha_i$, the reaction functions of the local incumbent of constituency $i$ are analogous to equation (10):

$$g_i^*(t) = \frac{b_i + t_i}{2} + \frac{\alpha_i}{2\delta_i r(1 - \alpha_i)},$$

(17)

and

$$m_i^*(t) = \frac{b_i + t_i}{2} - \frac{\alpha_i}{2\delta_i r(1 - \alpha_i)},$$

(18)

where all the observations made in previous sections are still valid.

5.3 Central Government

Consider now the first stage of the game. The central incumbent has to decide how much to transfer to local authority 1 and 2, $t_1$ and $t_2$ respectively, and how much to hold for the rent extraction motive.

Taking into account the reaction functions, the relative dimensions of the two constituencies and the share of total revenues of the central government, the optimisation problem of the central incumbent can be written as:

$$\begin{aligned} 
& \text{max} & & U_c = \beta G + r[\phi_c + \lambda(1 - \sigma_1)\theta_1 + (1 - \lambda)(1 - \sigma_2)\theta_2] \\
& \text{s.t.} & & 1 - (b_1 + b_2) = G + (t_1 + t_2) \\
& & & t_1 + t_2 \leq 1 - B \\
& & & t_1, t_2 \geq 0 
\end{aligned}$$

(19)

Given that constrains form a close and bounded region, a maximum always exists, however, the focus is now moved to the interior solution, conditioning the existence of such maximum on the relationship between the degree of party alignment between incumbents and the share of the population in the region.
Since the focus is on the relationship between $\delta_1$, $\delta_2$ and $\lambda$, assume, for the sake of simplicity, that both local incumbents extract the same rent and that local regions are equally endowed.

The following proposition characterises the existence of an interior equilibrium.

**Proposition 4.** Let $H$ be the problem defined by equation (19), and let $\alpha_1 = \alpha_2 = \alpha$ and $b_1 = b_2 = b$.

If

(i) $\lambda \in [\lambda^{\min}; \lambda^{\max}]$,

(ii) $\beta \in [\beta^{\min}; \min\{\beta_1^{\max}, \beta_2^{\max}\}]$,

then, the global maximum of $U_c$

\[
t_1^* = \frac{1}{\delta_1} - b - \frac{2\beta}{\lambda \sigma r(1-\alpha)}
\]

\[
t_2^* = \frac{1}{\delta_2} - b - \frac{2\beta}{(1-\lambda)\delta_2 r(1-\alpha)}
\]

(20)

is an interior solution of $H$.

Figure 4, provides a representation of Proposition 4.

![Figure 4: The case of two regions: optimal and total transfers](image)

(a) optimal transfers $t_1^*$, $t_2^*$  
(b) total transfers $t_1^* + t_2^*$

Figure 4: The case of two regions: optimal and total transfers

($\alpha = 0.2$, $\beta = 0.1$, $b = 0.1$, $r = 0.75$, $\delta_1 = 0.7$, $\delta_2 = 0.8$)

Figure 4 represents the amount of money transferred by the central government to each local government. If the share of voters in a region is too small, the central incumbent would rather eliminate the transfer to that region. On the contrary, it might be the case that the optimal transfer region is too large; in that case a corner solution arises (in this case it happens with $t_2^*$). Figure 4b represents the corresponding total amount of budget that the central government transfers to local one. It is worth to notice that at the extremes of $\lambda$ the figures corresponds, given that for extreme values of $\lambda$ only one region receives a positive transfer. Moreover it can be noticed that the central government budget can be better off if transfers are granted to both regions, rather than to just one region.

It is possible to recognise that all the comparative statics done for the one region case are still valid. Focus now the analysis on the impact of changes of $\lambda$ and $\delta_i$ on the optimal transfers.
The first effect is that the higher the share of population present in a region, the higher the transfer that it receives from the central government.

\[
\frac{\partial t_1^*}{\partial \lambda} = \frac{2\beta}{\lambda^2(1-\alpha)\delta_1 r} > 0
\]

\[
\frac{\partial t_2^*}{\partial \lambda} = \frac{-2\beta}{(1-\lambda)^2(1-\alpha)\delta_2 r} < 0
\]

Focusing on the relation between \( \delta_i \) and \( \lambda \), it is easy to see that, \textit{ceteris paribus}, a higher party alignment can compensate a reduction in size. In fact, on the one hand, given the same size, more aligned regions receive a larger transfer, and, on the other hand, more aligned incumbents receive a positive transfer for a smaller relative dimension.

The simplest way to show this result is to analyse the corner solution \( t_1^* = 0 \).

\( t_1^* = 0 \) implies:

\[
-b + \frac{1}{\delta_1} - \frac{2\beta}{\delta_1 \lambda r(1-\alpha)} = 0
\]

which gives the condition:

\[
\delta_1 = \frac{1}{b} - \frac{2\beta}{\lambda \delta_1 r(1-\alpha)} \equiv \delta_1^{max}
\]

where \( \delta_1^{max} \) is the maximum value of \( \delta_1 \) (so the minimum level of political alignment) that, given the population of region 1, guarantees a non negative transfer. It is immediate to see that higher \( \lambda \) the higher \( \delta_1^{max} \).

The implication is that if a region is relatively small, it is likely that it will receive a grant from the government only if it is very aligned, while if a region is big, it will receive a grant even if the local incumbent is not very aligned with the central one. This may provide an explanation for the fact that big municipalities are always able to intercept grants from regions and central government, despite the political alignment of the Mayor.

6 Conclusions

Political decentralisation is a key topic which is basically focused on the hierarchical structure of governments. The key question is always whether this multi-level structure and shared responsibility are improving the efficiency of local public good provision and there are several ways to focus on this efficiency.

A relative unexplored efficiency issue is the information problem that emerges among voters on the spending side due to the shared responsibility. Conceptually this paper builds on the idea of the informational problem caused by a partial expenditure decentralisation of Joanis (2014), though treating this voters’ uncertainty as a strategic variable of the model.

In each constituency, a local and a central incumbent seek reelection by the same voters, even though in different elections, through the provision of a local public good. Local government provides the public good out of its budget spending, while

\[\text{The analysis is symmetric for region 2.}\]
the central government finances that budget through a transfer out of its resources. Voters observe only the final level of the public good provided and form a belief about the contribution of each government. The local authority can influence this belief by diverting a share of the public budget for advertisement.

This partial decentralisation creates a diverging incentive for the incumbents. By characterising the optimal level of both transfer and spending in the local public good, the paper highlights how party alignment is a key factor in both. In particular, it shows that a high party alignment between incumbents is beneficial in terms of resources transferred and spending in the local public good. Even though this result is consistent with the existing literature, the driver of this result is not an altruistic behaviour of politicians in the same party, rather the lower capacity of the local government to influence voters’ belief in its favour. In fact, the greater the alignment, the less voters are willing to believe in a conflict between the two tiers, thus the local government’s effort to increase its consensus at the expense of the central government is less effective. Therefore, ceteris paribus, the local incumbent prefers to increase the expenditure in the provision of the public good.

The model is then extended to include two asymmetric constituencies to take into account the incentive for an opportunistic redistribution of transfers by the central incumbent.

Finally, due to the simplicity of the model many issues have been left aside. In particular, the inefficiency at the heart of this process might be compensated by the standard advantages of decentralisation well identified by the literature. Moreover the framework set up in this paper leaves room for theoretical refinements: in particular, the model could be extended to provide a micro foundation of voters’ behaviour and to allow a comparison of this model with a baseline non-decentralised model.

Appendices

A Proofs

Proof of Proposition 1.

Equation (10) is derived from the maximisation of $U_l$ under the binding budget constraint. The reaction functions have also to fulfil the non-negativity constraint for both $g$ and $m$.

(i) $g \geq 0$ implies:

$$\frac{b + t}{2} + \frac{\alpha}{2\delta r(1 - \alpha)} \geq 0,$$

which is true since it is the sum of two non negative quantities.

(ii) $m \geq 0$ implies:

$$\frac{b + t}{2} - \frac{\alpha}{2\delta r(1 - \alpha)} \geq 0,$$

which is true only if

$$t \geq \frac{\alpha}{\delta r(1 - \alpha)} - b \equiv \hat{t}$$
Thus for \( t \geq \hat{t} \) the maximisation of \( U_l \) under the budget constraint fulfils both non-negativity constraints, hence the reaction functions are well defined. If \( t < \hat{t} \) the result is a corner solution where \( g = b + t \) and \( m = 0 \).

**Proof of Corollary 1.**

On the one hand, condition (i) can be immediately derived by imposing \( \hat{t} > 1 - b \). If this is the case, then for every possible \( t \), it is always the case that \( t < \hat{t} \). Hence there are only corner solutions.

On the other hand, condition (ii) directly derives by imposing \( \hat{t} < 0 \). If this is the case, then for every \( t \), it is always the case that \( t > \hat{t} \). Therefore there are only interior solutions.

**Proof of Corollary 2.**

Given equation (10), the first derivatives of \( g(t) \) w.r.t \( \alpha, r \) and \( \delta \) are respectively:

\[
\frac{\partial g^*(t)}{\partial \alpha} = \frac{1}{2\delta(1 - \alpha)^2} > 0 \\
\frac{\partial g^*(t)}{\partial r} = -\frac{\alpha}{2\delta(1 - \alpha)r^2} < 0, \\
\frac{\partial g^*(t)}{\partial \delta} = -\frac{\alpha}{2\delta^2(1 - \alpha)r} < 0
\]  
(27)

while partial derivatives of \( m(t) \) have exactly the opposite sign:

\[
\frac{\partial m^*(t)}{\partial \alpha} = -\frac{1}{2\delta(1 - \alpha)^2} < 0 \\
\frac{\partial m^*(t)}{\partial r} = \frac{\alpha}{2\delta(1 - \alpha)r^2} > 0. \\
\frac{\partial m^*(t)}{\partial \delta} = \frac{\alpha}{2\delta^2(1 - \alpha)r} > 0
\]  
(28)

**Proof of Proposition 2.**

Given equation (13), that immediately derives from the constrained maximisation of \( U_c \), the two conditions on \( \beta \) follow by imposing that \( t^* \) belongs to the allowed space \([0, \hat{t}]\).

\( t^* \geq 0 \) requires \( \frac{1}{\delta} - b - \frac{2\beta}{\delta r(1 - \alpha)} \geq 0 \), that rearranged with respect to \( \beta \) gives

\[
\beta \leq \frac{1}{2} r(1 - \alpha)(1 - \delta b) \equiv \bar{\beta}.
\]  
(29)

Therefore for \( \beta > \bar{\beta} \), \( t^* \) assumes negative values.

On the other hand, \( t^* \leq 1 - b \) requires \( \frac{1}{\delta} - b - \frac{2\beta}{\delta r(1 - \alpha)} \leq 1 - b \), and rearranging with respect to \( \beta \),

\[
\beta \geq \frac{1}{2} r(1 - \alpha)(1 - \delta) \equiv \underline{\beta}.
\]  
(30)

Therefore for \( \beta < \underline{\beta} \), \( t^* \) assumes values greater than \( 1 - b \).

---

12It is worth noting that, as long as \( b \leq 1, \bar{\beta} \leq \underline{\beta} \). Thus for every combination of the parameters, it always exists a value of \( \beta \) such that \( t^* \) is an interior solution.
We can notice that, since $\beta \leq \bar{\beta}$ as long as $b \leq 1$, an internal solution for the problem always exists.

To better underline the impact of the assumption made over $\alpha$, reconsider now the condition for an internal solution derived in the Corollary 1 by imposing the additional condition $t^* \geq t$.

$$\frac{1}{\delta} - b - \frac{2\beta}{\delta(1-\alpha)r} \geq -b + \frac{\alpha}{\delta(1-\alpha)r}$$

The result is an additional condition over $\beta$:

$$\beta \leq \frac{1}{2} r(1-\alpha) - \frac{\alpha}{2} \equiv \hat{\beta}. \quad (31)$$

It can be easily seen that if $\alpha \leq \frac{\delta b}{1+\delta b}$ then $\beta < \bar{\beta} < \hat{\beta}$, meaning that if $\beta$ is such that it guarantees an interior solution for $t^*$, it also guarantees an interior solution for $g(t)$.

For values of $\alpha \in \left(\frac{\delta b}{1+\delta b}, \frac{\delta r}{1+\delta r}\right)$, $\beta < \bar{\beta} < \hat{\beta}$, meaning that the absence of a corner solution for $t^*$ does not guarantee that there is not a corner solution for $g(t)$.

Finally, if $\alpha \geq \frac{\delta r}{1+\delta r}$ it follows that $\bar{\beta} < \hat{\beta}$, and consequently either there is a corner solution for $t^*$, or there is a corner solution for $g(t)$.

**Proof of Corollary 3.**

Given equation (13), the first derivatives of $t^*$ w.r.t $\alpha, r$ and $\delta$ are respectively:

$$\frac{\partial t^*}{\partial \alpha} = \frac{-2\beta}{\delta(1-\alpha)^2r} < 0$$

$$\frac{\partial t^*}{\partial r} = \frac{2\beta}{\delta(1-\alpha)r^2} > 0$$

$$\frac{\partial t^*}{\partial \beta} = \frac{2}{\delta(1-\alpha)r} < 0$$

$$\frac{\partial t^*}{\partial \delta} = \frac{2\beta - r(1-\alpha)}{\delta^2(1-\alpha)r} < 0 \quad (32)$$

For the sake of precision, if $\beta > \frac{1}{2} r(1-\alpha)$ the derivative of $t$ with respect to $\delta$ is positive. However, for these values of $\beta$ the problem has not an internal maximum, so this case can be ruled out.

**Proof of Proposition 3.**

The first step of the proof is to derive equation (14).

Assuming regularity condition on $\alpha$ and $\beta$, by putting together equations (10) and (13) it follows that

$$g^* = \frac{1}{2\delta} - \frac{2\beta - \alpha}{2\delta r(1-\alpha)} \quad (33)$$

and

$$m^* = \frac{1}{2\delta} - \frac{2\beta + \alpha}{2\delta r(1-\alpha)} \quad (34)$$

Equation (14) follows directly by equations (33) and (2).
The second step is to analyse the different comparative statics on $\theta^*$. The partial derivative of $\theta^*$ with respect to $\delta$ is equal to
\[
\frac{\partial \theta^*}{\partial \delta} = - \frac{r(1 - \alpha) - 2\beta + \alpha}{2\delta^2 r}.
\] (35)
\[
\frac{\partial \theta^*}{\partial \delta < 0} \text{ implies } \beta \leq \frac{1}{2} r(1 - \alpha) + \frac{\alpha}{2}, \text{ which under the regularity conditions of } \alpha \text{ and } \beta \text{ is always true.}^{13}
\]

The partial derivative of $\theta^*$ with respect to $\alpha$ is equal to
\[
\frac{\partial \theta^*}{\partial \alpha} = \frac{1 - r}{2\delta r},
\] (36)
which is always positive.

The partial derivative of $\theta^*$ with respect to $\beta$ is equal to
\[
\frac{\partial \theta^*}{\partial \beta} = - \frac{1}{\delta r},
\] (37)
which is always negative.

Finally a particular attention is required to analyse the effect of $r$. The partial derivative is equal to:
\[
\frac{\partial \theta^*}{\partial r} = \frac{2\beta - \alpha}{2\delta r^2},
\] (38)
where it is easy to see that if $\beta > \frac{\alpha}{2}$ then $\frac{\partial \theta^*}{\partial r} > 0$ while if $\beta < \frac{\alpha}{2}$ then $\frac{\partial \theta^*}{\partial r} < 0$.

**Proof of Proposition 4.**

The first step is to prove that equation (20) is the global maximum of $U_c$, i.e. that $U_c$ is concave. The gradient of $U_c$ is
\[
\nabla U_c(t_1, t_2) = \begin{pmatrix}
\frac{1}{2} \lambda r(1 - \alpha_1)(1 - (b_1 + t_1)) - \beta \\
\frac{1}{2} (1 - \lambda) r(1 - \alpha_2)(1 - (b_2 + t_2)) - \beta
\end{pmatrix},
\] (39)
from which can be derived the corresponding Hessian matrix:
\[
H_{U_c}(t_1, t_2) = \begin{pmatrix}
-\frac{1}{2} \lambda \delta_1 r(1 - \alpha_1) & 0 \\
0 & -\frac{1}{2} (1 - \lambda) \delta_2 r(1 - \alpha_2)
\end{pmatrix}.
\] (40)
It is possible to see that the Hessian is negative-definite, then $U_c$ is concave. It follows that the necessary condition for the maximum
\[
\nabla(U_c) = 0,
\] (41)
is also sufficient.

Writing the corresponding system
\[
\begin{cases}
\lambda r(1 - \alpha_1)(1 - (b_1 + t_1)) = \beta \\
(1 - \lambda) r(1 - \alpha_2)(1 - (b_2 + t_2)) = \beta
\end{cases},
\] (42)
\textit{\textsuperscript{13}One way to immediately verify this claim is to confront this condition with the one given by equation (31).}
and imposing the simplification $\alpha_1 = \alpha_2 = \alpha$ and $b_1 = b_2 = b$, it follows:

$$
\begin{align*}
    t_1^* &= \frac{1}{\delta_1} - b - \frac{2\beta}{\lambda \delta_1 r(1 - \alpha)} \\
    t_2^* &= \frac{1}{\delta_2} - b - \frac{2\beta}{(1 - \lambda) \delta_2 r(1 - \alpha)}.
\end{align*}
$$

(43)

The second step is to derive the regularity condition under which the global maximum is an interior solution of $H$.

In order to be an interior solution, the couple $(t_1^*, t_2^*)$ has to fulfil both the non negativity constraints and the budget constraint, i.e:

$$
\begin{align*}
    &t_1 \geq 0 \\
    &t_2 \geq 0 \\
    &t_1 + t_2 \leq 1 - 2b.
\end{align*}
$$

(44)

The non negativity constraints imply:

$$
\begin{align*}
    \beta &\leq \frac{\lambda r(1 - \alpha)(1 - \delta_1 b)}{2} \equiv \beta_1^{\text{max}} \\
    \beta &\leq \frac{(1 - \lambda)r(1 - \alpha)(1 - \delta_2 b)}{2} \equiv \beta_2^{\text{max}}.
\end{align*}
$$

(45)

The budget constrains can be rewritten as:

$$
\begin{align*}
    \frac{1}{\delta_1} - \frac{2\beta}{\lambda \delta_1 r(1 - \alpha)} + \frac{1}{\delta_2} - \frac{2\beta}{(1 - \lambda) \delta_2 r(1 - \alpha)} \leq 1,
\end{align*}
$$

from which the third condition is obtained:

$$
\begin{align*}
    \beta \geq \frac{\lambda(1 - \lambda)r(1 - \alpha)(\delta_1 + \delta_2 - \delta_1 \delta_2 b)}{2(\lambda \delta_1 + (1 - \lambda) \delta_2)} \equiv \beta^{\text{min}}.
\end{align*}
$$

(46)

In order for the global maximum to be an interior solution, the following condition must be satisfied:

$$
\beta^{\text{min}} \leq \min\{\beta_1^{\text{max}}, \beta_2^{\text{max}}\}.
$$

(47)

Condition $\beta^{\text{min}} \leq \beta_1^{\text{max}}$ implies

$$
\lambda \geq \frac{1 - (1 - b)\delta_2}{2 - b\delta_1 - (1 - b)\delta_2} \equiv \lambda^{\text{min}},
$$

(48)

while condition $\beta^{\text{min}} \leq \beta_2^{\text{max}}$ implies

$$
\lambda \leq \frac{1 - b\delta_2}{2 - (1 - b)\delta_1 - b\delta_2} \equiv \lambda^{\text{max}}.
$$

(49)

These conditions imply that there exist values of $\lambda$ for which the global maximum is interior if and only if

$$
\lambda^{\text{min}} \leq \lambda^{\text{max}}.
$$

(50)

This condition is true for $b \leq \frac{1}{7}$ which is always true.

Then for $\lambda \in [\lambda^{\text{min}}, \lambda^{\text{max}}]$ the regularity condition for $\beta$ guarantees that the global maximum is an interior point.
B Regularity condition

Given \( g^* \) and \( m^* \), it follows that the reelection probabilities of incumbents are equal to:

\[
p = \phi_l + \frac{(1 - \alpha - 2\beta)^2 - \alpha^2}{4\delta r^2(1 - \alpha)}, \tag{B.1}
\]

and

\[
P = \phi_c + \frac{(1 - \alpha + \alpha)^2 - 4\beta^2}{4\delta r^2(1 - \alpha)}. \tag{B.2}
\]

in order to avoid corner solutions, conditions \( p \leq 1 \) and \( P \leq 1 \) are required, then

\[
\phi_l \leq 1 - \frac{(r(1 - \alpha) - 2\beta)^2 - \alpha^2}{4\delta r^2(1 - \alpha)} \equiv \Phi_l. \tag{B.3}
\]

\[
\phi_c \leq 1 - \frac{(r(1 - \alpha) + \alpha)^2 - 4\beta^2}{4\delta r^2(1 - \alpha)} \equiv \Phi_c. \tag{B.4}
\]

correspond to regularity conditions over \( \phi_c \) and \( \phi_l \). It may be that for certain values of the parameters, \( \Phi_l \) and \( \Phi_c \) are higher than one. In that case, there is no need for a lower bound, since there are no \( \phi_i \) values that may cause a corner solution.

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