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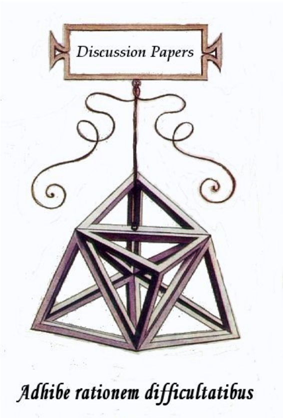

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Discussion Paper n. 248

2019

Discussion Paper n. 248, presentato: **Novembre 2019**

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Si prega di citare così:
Colucci D., Doni N. e Valori V. (2019), "Information revelation in procurement auctions: an equivalence result", Discussion Papers del Dipartimento di Economia e Management – Università di Pisa, n. 248 (http://www.ec.unipi.it/ricerca/discussion-paper).

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Information revelation in procurement auctions: an equivalence result

Abstract

Procurement auctions often involve quality considerations as a determinant of the final outcome. When the procurer has private information about qualities, various information policies may be used to affect the expected outcome. For auctions with two cost heterogeneous suppliers, this work defines a notion of duality between pairs of policies, and shows that dual policies are revenue equivalent.

Keywords: procurement, information revelation, discriminatory policy, asymmetric auctions

JEL: D44, D82, H57

Information revelation in procurement auctions: an equivalence result

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November 29, 2019

Abstract

Procurement auctions often involve quality considerations as a determinant of the final outcome. When the procurer has private information about qualities, various information policies may be used to affect the expected outcome. For auctions with two cost heterogeneous suppliers, this work defines a notion of duality between pairs of policies, and shows that dual policies are revenue equivalent.

1 Introduction

In recent years the range of auction formats that can be used in public procurement has significantly increased, so that the available procedures include on-line reverse auctions in which various information revelation policies are permitted (see e.g. General Service Administration [4], page 12; European Directives 2004/18/EC, art. 54). Even more dramatic has been the impact of such new procedures on business-to-business procurement. The information architecture underlying on-line auctions and specifically when and which information should be revealed to bidders is considered by many a compelling research question (see e.g. Teich *et al.* [10], Kostamis *et al.* [6], Rothkopf and Whinston [9]). The model studied in this paper assumes a buyer who takes into account non-price attributes when procuring a good or service from one of two possible suppliers (who differ in terms of cost). It is assumed herein that the buyer knows how much the goods produced by a given supplier fit her needs, and such fit is labeled simply as quality. Each supplier knows what the buyer decides to reveal him, which is her information policy. This kind of setup has been analyzed by Gal-Or et al. [3], who essentially focus on symmetric policies, i.e. in which the suppliers receive the same amount of information, and a few more authors.[1] However it is interesting to extend the analysis to asymmetric policies:[2] indeed there is a lively debate regarding the pros and cons of preferential treatment in public procurement auctions (see e.g. McAfee and McMillan [7], Rothkopf et al. [8], Hubbard and Paarsch [5]). Now suppose for example that the buyer discloses his own quality to one supplier only. This means she is granting him an informational advantage. What would be the consequences if instead of his own quality she revealed him the quality of his competitor?

[1] For example Colucci, Doni and Valori [2].

[2] In Colucci, Doni and Valori [1] we have studied the determination of the best information policy (including asymmetric cases), from the viewpoint of the buyer's expected revenue, in a related but simplified model.

This is an interesting question which we partially answer in this paper: in the case of symmetrically distributed qualities nothing would change, both in terms of the buyer's expected payoff and the suppliers'. So what is shown here is that, given our assumptions, for the suppliers the cardinality of their information sets is key, but the specific informational content thereof is not. In fact having a piece of information when the competitor has none (or having both, i.e. knowing both qualities when the competitor only knows one) boosts his odds of winning the auction, but whether it is his own or the opponent's quality that he actually knows is indifferent.

2 Model

Two suppliers j and k compete in an auction to provide a good or service which they supply at costs c_j, c_k (costs are common knowledge). Each tenders a bid p_i (for $i = j, k$) with the aim of maximizing his expected profit. The buyer's utility depends on the qualities q_j, q_k which she privately knows. She can decide to reveal such qualities to the bidders, in a way specified before the auction: this defines the information policy.

The suppliers' prior information on qualities is that they are drawn independently from a known symmetric distribution on the unit interval. The winner is the supplier with the largest score $q_i - p_i$; the associated profit is $p_i - c_i$.

Each supplier acts so as to maximize expected net profit on the basis of his information set, which can consist in one of the following items: the qualities of both suppliers, one's own quality, the competitor's quality, or none of those. Formally, i's information set can be any of the elements of

$$I = \{\{q_j, q_k\}, \{q_j\}, \{q_k\}, \varnothing\}$$

and it is determined by the information policy. There are 16 possible revelation policies, i.e. as many as the number of possible ordered pairs one can extract from the set I. Such policies are organized into *dual* pairs as follows: the dual of a given policy is obtained substituting q_j for q_k and vice-versa in the information set of each supplier.

The following table summarizes the dual pairs (where for each policy the first piece of information is for player j and the second is for player k):

Dual pairs	
1) $\varnothing; \varnothing$	1) $\varnothing; \varnothing$
2) $\varnothing; \{q_j\}$	3) $\varnothing; \{q_k\}$
4) $\{q_j\}; \varnothing$	5) $\{q_k\}; \varnothing$
6) $\varnothing; \{q_j, q_k\}$	6) $\varnothing; \{q_k, q_j\}$
7) $\{q_j, q_k\}; \varnothing$	7) $\{q_k, q_j\}; \varnothing$
8) $\{q_j\}; \{q_k\}$	9) $\{q_k\}; \{q_j\}$
10) $\{q_j\}; \{q_j\}$	11) $\{q_k\}; \{q_k\}$
12) $\{q_j\}; \{q_j, q_k\}$	13) $\{q_k\}; \{q_k, q_j\}$
14) $\{q_j, q_k\}; \{q_j\}$	15) $\{q_k, q_j\}; \{q_k\}$
16) $\{q_j, q_k\}; \{q_j, q_k\}$	16) $\{q_k, q_j\}; \{q_k, q_j\}$

3 Results

We now prove that dual pairs of policies are revenue equivalent for both the buyer and the two suppliers. Observe that four of the sixteen policies are the same as their dual, namely those numbered 1, 6, 7 and 16 in the table above. The results below trivially apply to such policies, hence the proofs focus on the policies which differ from their dual.

Lemma 1 *Let $q_j, q_k, \hat{q}_j, \hat{q}_k$ be iid random variables with a symmetric density $f_{[0,1]}$, and a cumulative distribution function $F_{[0,1]}$. Then*

$$p_i\left(x,y\right)=\hat{p}_i\left(1-y,1-x\right)\ \textit{for all } x,y\in\left[0,1\right]\ \textit{ and } i=j,k$$

where $p_j\left(q_j,q_k\right)$ and $p_k\left(q_j,q_k\right)$ are the bidding functions associated to a given policy and $\hat{p}_j\left(\hat{q}_j,\hat{q}_k\right)$ and $\hat{p}_k\left(\hat{q}_j,\hat{q}_k\right)$ are the bidding functions associated to its dual.

Proof. We consider separately the case in which no one is completely informed and that in which one of the sellers has full information.

1) No one is completely informed

Define

$$\begin{aligned} G_j\left(q_j,q_k,p_j,p_k\right) &= \Pr\left(q_j-p_j>q_k-p_k|I_j\right) \\ G_k\left(q_j,q_k,p_j,p_k\right) &= \Pr\left(q_j-p_j<q_k-p_k|I_k\right) \end{aligned}$$

where I is the information set available to the bidder (i.e. $I=\varnothing,\{q_j\},\{q_k\}$). Let $g_i=\frac{\partial}{\partial p_i}G_i$ (for $i=j,k$). Similarly, define $\hat{G}_i\left(\hat{q}_j,\hat{q}_k,\hat{p}_j,\hat{p}_k\right)$ as the winning probability in the corresponding dual problem and $\hat{g}_i=\frac{\partial}{\partial p_i}\hat{G}_i$.

The sellers have to solve the problem

$$\begin{cases} \max\limits_{p_j}\left(p_j-c_j\right)G_j\left(q_j,q_k,p_j,p_k\right) \\ \max\limits_{p_k}\left(p_k-c_k\right)G_k\left(q_j,q_k,p_j,p_k\right) \end{cases}$$

which gives the set of F.O.C.

$$\begin{cases} G_j\left(q_j,q_k,p_j,p_k\right)+\left(p_j-c_j\right)g_j\left(q_j,q_k,p_j,p_k\right)=0 \\ G_k\left(q_j,q_k,p_j,p_k\right)+\left(p_k-c_k\right)g_k\left(q_j,q_k,p_j,p_k\right)=0 \end{cases} \tag{1}$$

Likewise, in the corresponding dual case the sellers solve the problem

$$\begin{cases} \max\limits_{\hat{p}_j}\left(\hat{p}_j-c_j\right)\hat{G}_j\left(\hat{q}_j,\hat{q}_k,\hat{p}_j,\hat{p}_k\right) \\ \max\limits_{\hat{p}_k}\left(\hat{p}_k-c_k\right)\hat{G}_k\left(\hat{q}_j,\hat{q}_k,\hat{p}_j,\hat{p}_k\right) \end{cases}$$

which gives

$$\begin{cases} \hat{G}_j\left(\hat{q}_j,\hat{q}_k,\hat{p}_j,\hat{p}_k\right)+\left(\hat{p}_j-c_j\right)\hat{g}_j\left(\hat{q}_j,\hat{q}_k,\hat{p}_j,\hat{p}_k\right)=0 \\ \hat{G}_k\left(\hat{q}_j,\hat{q}_k,\hat{p}_j,\hat{p}_k\right)+\left(\hat{p}_k-c_k\right)\hat{g}_k\left(\hat{q}_j,\hat{q}_k,\hat{p}_j,\hat{p}_k\right)=0 \end{cases} \tag{2}$$

Making G (and $\hat{G}$) explicit for all the possible information sets we have

$$\begin{array}{ll} G_j|\left(I_j=\{q_j\}\right)=F\left(q_j+p_k-p_j\right) & G_k|\left(I_k=\{q_j\}\right)=1-F\left(q_j+p_k-p_j\right) \\ G_j|\left(I_j=\{q_k\}\right)=1-F\left(q_k+p_j-p_k\right) & G_k|\left(I_k=\{q_k\}\right)=F\left(q_k+p_j-p_k\right) \\ G_j|\left(I_j=\varnothing\right)=H\left(p_k-p_j\right) & G_k|\left(I_k=\varnothing\right)=1-H\left(p_k-p_j\right) \end{array} \tag{3}$$

where $H\left(p_k-p_j\right)=\int_{-\infty}^{+\infty}F\left(x+p_k-p_j\right)f\left(x\right)dx$. Observing that, by the symmetry of f,

$$F\left(x\right)=1-F\left(1-x\right)$$

from (3) we obtain that, if $\hat{q}_j = 1 - q_k$ and $\hat{q}_k = 1 - q_j$, then

$$\hat{G}_i(\hat{q}_j, \hat{q}_k, x, y) = \hat{G}_i(1 - q_k, 1 - q_j, x, y) = G_i(q_j, q_k, x, y) \qquad i = j, k \tag{4}$$

Regarding g_i (and $\hat{g}_i$) we have

$$\begin{array}{c} g_j|(I_j = \{q_j\}) = g_k|(I_k = \{q_j\}) = -f(q_j + p_k - p_j) \\ g_j|(I_j = \{q_k\}) = g_k|(I_k = \{q_k\}) = -f(q_k + p_j - p_k) \\ g_j|(I_j = \varnothing) = g_k|(I_k = \varnothing) = -h(p_k - p_j) \end{array} \tag{5}$$

where

$$h(p_k - p_j) = \int_{-\infty}^{+\infty} f(x + p_k - p_j) f(x) \, dx$$

and again, if $\hat{q}_j = 1 - q_k$ and $\hat{q}_k = 1 - q_j$, we have

$$\hat{g}_i(\hat{q}_j, \hat{q}_k, x, y) = \hat{g}_i(1 - q_k, 1 - q_j, x, y) = g_i(q_j, q_k, x, y) \qquad i = j, k \tag{6}$$

As a consequence of (4) and (6) we have that, when $\hat{q}_j = 1 - q_k$ and $\hat{q}_k = 1 - q_j$, systems (1) and (2) are equivalent; hence their solution must satisfy

$$p_i(q_j, q_k) = \hat{p}_i(1 - q_k, 1 - q_j) = \hat{p}_i(\hat{q}_j, \hat{q}_k)$$

as claimed.

2) One bidder is completely informed

Let j be the uninformed bidder. Suppose, without loss of generality, that $I_j = \{q_j\}$ and $I_k = \{q_j, q_k\}$. Then, in the dual problem it must be $I_j = \{q_k\}$ and $I_k = \{q_j, q_k\}$. In this case, j wins if and only if his score is larger than k's valuation. So the problem for j is

$$\left\{ \max_{p_j} (p_j - c_j) \Pr(q_j - p_j > q_k - c_k | I_j) \right.$$

and the bidding strategies are the solution of

$$\left\{ \begin{array}{l} F(q_j + c_k - p_j) + (p_j - c_j) f(q_k + p_j - c_k) = 0 \\ p_k = \left\{ \begin{array}{cc} q_k - q_j + p_j & if \quad q_j - p_j < q_k - c_k \\ c_k & if \quad q_j - p_j > q_k - c_k \end{array} \right. \end{array} \right. \tag{7}$$

Similarly, in the dual problem the bidding strategies are the solution of

$$\left\{ \begin{array}{l} 1 - F(\hat{q}_k + \hat{p}_j - c_k) + (\hat{p}_j - c_j) f(\hat{q}_j + c_k - \hat{p}_j) = 0 \\ \hat{p}_k = \left\{ \begin{array}{cc} \hat{q}_k - \hat{q}_j + \hat{p}_j & if \quad \hat{q}_j - \hat{p}_j < \hat{q}_k - c_k \\ c_k & if \quad \hat{q}_j - \hat{p}_j > \hat{q}_k - c_k \end{array} \right. \end{array} \right. \tag{8}$$

As before, using the symmetry of f, we have that problems (7) and (8) are equivalent if $\hat{q}_j = 1 - q_k$ and $\hat{q}_k = 1 - q_j$. The claimed result follows immediately. ■

Lemma 2 *Under the assumptions of Lemma 1, if $\hat{q}_j = 1 - q_k$ and $\hat{q}_k = 1 - q_j$ then*

$$\begin{array}{ccc} q_j - p_j(q_j, q_k) & \gtrless & q_k - p_k(q_j, q_k) \\ & \Updownarrow & \\ \hat{q}_j - \hat{p}_j(\hat{q}_j, \hat{q}_k) & \gtrless & \hat{q}_k - \hat{p}_k(\hat{q}_j, \hat{q}_k) \end{array}$$

Proof. Without loss of generality consider the case

$$q_j - p_j(q_j, q_k) > q_k - p_k(q_j, q_k) \tag{9}$$

By Lemma 1 and using $\hat{q}_j = 1 - q_k$ and $\hat{q}_k = 1 - q_j$ we have

$$\hat{p}_i(\hat{q}_j, \hat{q}_k) = \hat{p}_i(1 - q_k, 1 - q_j) = p_i(q_j, q_k) \qquad i = j, k \tag{10}$$

hence, using (9) and (10) we obtain the desired result:

$$\hat{q}_j - \hat{p}_j(\hat{q}_j, \hat{q}_k) = 1 - q_k - p_j(q_j, q_k) > 1 - q_j - p_k(q_j, q_k) = \hat{q}_k - \hat{p}_k(\hat{q}_j, \hat{q}_k)$$

■

We now turn to the main result.

Proposition 3 *Under the assumptions of Lemma 1, for any possible choice of the information policy, the buyer's and the sellers' expected revenue are the same as those in the corresponding dual policy.*

Proof. Let J (resp. K) be the subset of $[0,1] \times [0,1]$ such that player j (resp. k) wins the auction whenever $(q_j, q_k) \in J$ (resp. K) in a given information scenario. Also, let $\hat{J}$ (resp. $\hat{K}$) be the subset of $[0,1] \times [0,1]$ such that player j (resp. k) wins the auction whenever $(q_j, q_k) \in \hat{J}$ (resp. $\hat{K}$) in the corresponding dual. By Lemma 2 we have

$$\begin{aligned} (q_j, q_k) &\in J \Longleftrightarrow (1 - q_k, 1 - q_j) \in \hat{J} \\ (q_j, q_k) &\in K \Longleftrightarrow (1 - q_k, 1 - q_j) \in \hat{K} \end{aligned} \tag{11}$$

The buyer's expected revenue is

$$EU_B = \int_J (q_j - p_j(q_j, q_k)) f(q_j) f(q_k) dq_j dq_k + \int_K (q_k - p_k(q_j, q_k)) f(q_j) f(q_k) dq_j dq_k$$

while in the dual it is

$$\widehat{EU}_B = \int_{\hat{J}} (\hat{q}_j - \hat{p}_j(\hat{q}_j, \hat{q}_k)) f(\hat{q}_j) f(\hat{q}_k) d\hat{q}_j d\hat{q}_k + \int_{\hat{K}} (\hat{q}_k - \hat{p}_k(\hat{q}_j, \hat{q}_k)) f(\hat{q}_j) f(\hat{q}_k) d\hat{q}_j d\hat{q}_k$$

Consider now $\widehat{EU}_B$. We can separate the part regarding qualities from that regarding bids

$$\begin{aligned} \widehat{EU}_B &= \int_{\hat{J}} \hat{q}_j f(\hat{q}_j) f(\hat{q}_k) d\hat{q}_j d\hat{q}_k + \int_{\hat{K}} \hat{q}_k f(\hat{q}_j) f(\hat{q}_k) d\hat{q}_j d\hat{q}_k \\ &\quad - \int_{\hat{J}} \hat{p}_j(\hat{q}_j, \hat{q}_k) f(\hat{q}_j) f(\hat{q}_k) d\hat{q}_j d\hat{q}_k - \int_{\hat{K}} \hat{p}_k(\hat{q}_j, \hat{q}_k) f(\hat{q}_j) f(\hat{q}_k) d\hat{q}_j d\hat{q}_k \\ &\equiv S_1 - S_2 \end{aligned} \tag{12}$$

where S_1 denotes the first two integrals and S_2 the last two. Observing that

$$\hat{J} = \left(\hat{J} \cap J\right) \cup \left(\hat{J} \cap K\right), \; \hat{K} = \left(\hat{K} \cap J\right) \cup \left(\hat{K} \cap K\right)$$

we can write

$$\begin{aligned} S_1 &= \int_{(\hat{J} \cap J)} \hat{q}_j f(\hat{q}_j) f(\hat{q}_k) d\hat{q}_j d\hat{q}_k + \int_{(\hat{J} \cap K)} \hat{q}_j f(\hat{q}_j) f(\hat{q}_k) d\hat{q}_j d\hat{q}_k \\ &\quad + \int_{(\hat{K} \cap J)} \hat{q}_k f(\hat{q}_j) f(\hat{q}_k) d\hat{q}_j d\hat{q}_k + \int_{(\hat{K} \cap K)} \hat{q}_k f(\hat{q}_j) f(\hat{q}_k) d\hat{q}_j d\hat{q}_k \end{aligned} \tag{13}$$

A simple variables change ($\hat{q}_j = q_j$ and $\hat{q}_k = q_k$) in the two integrals over $\left(\hat{J} \cap J\right)$ and $\left(\hat{K} \cap K\right)$ gives

$$\begin{aligned} & \int_{\left(\hat{J}\cap J\right)} \hat{q}_j f\left(\hat{q}_j\right) f\left(\hat{q}_k\right) d\hat{q}_j d\hat{q}_k + \int_{\left(\hat{K}\cap K\right)} \hat{q}_k f\left(\hat{q}_j\right) f\left(\hat{q}_k\right) d\hat{q}_j d\hat{q}_k \\ = & \int_{\left(J\cap \hat{J}\right)} q_j f\left(q_j\right) f\left(q_k\right) dq_j dq_k + \int_{\left(K\cap \hat{K}\right)} q_k f\left(q_j\right) f\left(q_k\right) dq_j dq_k \end{aligned} \tag{14}$$

Instead, for the two integrals over $\left(\hat{J} \cap K\right)$ and $\left(\hat{K} \cap J\right)$ we can write

$$\begin{aligned} & \int_{\left(\hat{J}\cap K\right)} \hat{q}_j f\left(\hat{q}_j\right) f\left(\hat{q}_k\right) d\hat{q}_j d\hat{q}_k + \int_{\left(\hat{K}\cap J\right)} \hat{q}_k f\left(\hat{q}_j\right) f\left(\hat{q}_k\right) d\hat{q}_j d\hat{q}_k \\ = & \int_{\left(\hat{J}\cap K\right)} \hat{q}_j f\left(\hat{q}_j\right) f\left(\hat{q}_k\right) d\hat{q}_j d\hat{q}_k + \int_{\left(\hat{J}\cap K\right)} \left(1-\hat{q}_j\right) f\left(1-\hat{q}_k\right) f\left(1-\hat{q}_j\right) d\hat{q}_j d\hat{q}_k \end{aligned}$$

where in the second integral we have set $\hat{q}_j = 1 - \hat{q}_k$ and $\hat{q}_k = 1 - \hat{q}_j$. Finally, using the symmetry of f and summing up

$$\begin{aligned} & \int_{\left(\hat{J}\cap K\right)} \hat{q}_j f\left(\hat{q}_j\right) f\left(\hat{q}_k\right) d\hat{q}_j d\hat{q}_k + \int_{\left(\hat{J}\cap K\right)} \left(1-\hat{q}_j\right) f\left(1-\hat{q}_k\right) f\left(1-\hat{q}_j\right) d\hat{q}_j d\hat{q}_k \\ = & \int_{\left(\hat{J}\cap K\right)} f\left(\hat{q}_k\right) f\left(\hat{q}_j\right) d\hat{q}_j d\hat{q}_k \end{aligned}$$

Now, changing again variable ($\hat{q}_j = q_j$ and $\hat{q}_k = q_k$) and proceeding backward using the same set of arguments we have

$$\begin{aligned} & \int_{\left(\hat{J}\cap K\right)} f\left(\hat{q}_k\right) f\left(\hat{q}_j\right) d\hat{q}_j d\hat{q}_k = \int_{\left(K\cap \hat{J}\right)} f\left(q_k\right) f\left(q_j\right) dq_j dq_k \\ = & \int_{\left(K\cap \hat{J}\right)} q_k f\left(q_j\right) f\left(q_k\right) dq_j dq_k + \int_{\left(K\cap \hat{J}\right)} \left(1-q_k\right) f\left(1-q_k\right) f\left(1-q_j\right) dq_j dq_k \\ = & \int_{\left(K\cap \hat{J}\right)} q_k f\left(q_j\right) f\left(q_k\right) dq_j dq_k + \int_{\left(J\cap \hat{K}\right)} q_j f\left(q_j\right) f\left(q_j\right) dq_j dq_k \end{aligned} \tag{15}$$

Finally, by substitution of (14) and (15) in (13) we end up with

$$\begin{aligned} S1 = & \int_{\left(J\cap \hat{J}\right)} q_j f\left(q_j\right) f\left(q_k\right) dq_j dq_k + \int_{\left(J\cap \hat{K}\right)} q_j f\left(q_j\right) f\left(q_j\right) dq_j dq_k \\ + & \int_{\left(K\cap \hat{J}\right)} q_k f\left(q_j\right) f\left(q_k\right) dq_j dq_k + \int_{\left(K\cap \hat{K}\right)} q_k f\left(q_j\right) f\left(q_k\right) dq_j dq_k \\ = & \int_J q_j f\left(q_j\right) f\left(q_k\right) dq_j dq_k + \int_K q_k f\left(q_j\right) f\left(q_k\right) dq_j dq_k \end{aligned} \tag{16}$$

As regards S_2 we have

$$S_2 = \int_{\hat{J}} \hat{p}_j\left(\hat{q}_j, \hat{q}_k\right) f\left(\hat{q}_j\right) f\left(\hat{q}_k\right) d\hat{q}_j d\hat{q}_k + \int_{\hat{K}} \hat{p}_k\left(\hat{q}_j, \hat{q}_k\right) f\left(\hat{q}_j\right) f\left(\hat{q}_k\right) d\hat{q}_j d\hat{q}_k$$

now, changing variables ($\hat{q}_j = 1 - q_k$ and $\hat{q}_k = 1 - q_j$) and using (11)

$$\begin{aligned}
S_2 &= \int_J \hat{p}_j (1 - q_k, 1 - q_j) f (1 - q_k) f (1 - q_j) dq_j dq_k & (17) \\
&+ \int_K \hat{p}_k (1 - q_k, 1 - q_j) f (1 - q_k) f (1 - q_j) dq_j dq_k \\
&= \int_J p_j (q_j, q_k) f (q_k) f (q_j) dq_j dq_k + \int_K p_k (q_j, q_k) f (q_k) f (q_j) dq_j dq_k & (18)
\end{aligned}$$

where the last equality is a consequence of Lemma 1 and of the symmetry of f. Finally, by substitution of (16) and (18) in (12) we obtain the desired result

$$\widehat{EU}_B = EU_B$$

Turning now to the sellers' expected revenue, we have

$$EU_j = \int_J (p_j (q_j, q_k) - c_j) f (q_j) f (q_k) dq_j dq_k \quad \text{and} \quad \widehat{EU}_j = \int_{\hat{J}} (\hat{p}_j (\hat{q}_j, \hat{q}_k) - c_j) f (\hat{q}_j) f (\hat{q}_k) d\hat{q}_j d\hat{q}_k$$

Using again the change of variables ($\hat{q}_j = 1 - q_k$ and $\hat{q}_k = 1 - q_j$) and the same arguments used to get to (18) we have

$$\begin{aligned}
\widehat{EU}_j &= \int_{\hat{J}} (\hat{p}_j (\hat{q}_j, \hat{q}_k) - c_j) f (\hat{q}_j) f (\hat{q}_k) d\hat{q}_j d\hat{q}_k \\
&= \int_J (\hat{p}_j (1 - q_k, 1 - q_j) - c_j) f (1 - q_k) f (1 - q_j) dq_j dq_k \\
&= \int_J (p_j (q_j, q_k) - c_j) f (q_k) f (q_j) dq_j dq_k = EU_j
\end{aligned}$$

■

4 Conclusions

This paper addresses information policies in a procurement auction setting where the buyer is the sole judge of the suppliers' quality in providing a good or service. The suppliers are different both in terms of how much their product fits the procurer's needs and cost-wise. What the suppliers know ex-ante about their perceived quality is that it is going to be the realization of a random variable. Given this context the buyer may decide to reveal one or both qualities to one or both the suppliers. Such information is clearly precious for the suppliers who have to bid in an auction to be awarded the procurement contract. A notion of duality within pairs of information policies (i.e. choices of what exactly to reveal the bidders) is defined in the paper and it is shown that, assuming a symmetric probability distribution of the qualities, two policies that are the dual of one another are equivalent in terms of the expected payoff for both the buyer and the suppliers. From the policy maker's point of view (or the modeler's) an implication of this result is that, in a context such as the present one, i.e. when assuming symmetrically distributed qualities is reasonable, one only needs to consider (and choose from) half of the whole set of available policies. Whether this result is robust to more general settings remains an open question.

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