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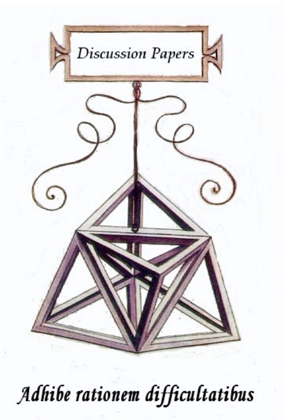

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From the TFR to the IFR approach for the multidimensional analysis of poverty and living conditions

Discussion Paper n. 252

2019

Discussion Paper n. 252, presentato: **Dicembre 2019**

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Si prega di citare così:

Cheli B., Lemmi A., Pannuzi N., Regoli A. (2019), "From the TFR to the IFR approach for the multidimensional analysis of poverty and living conditions", Discussion Papers del Dipartimento di Economia e Management – Università di Pisa, n. 252 (http://www.ec.unipi.it/ricerca/discussion-papers).

Discussion Paper
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Bruno Cheli, Achille Lemmi, Nicoletta Pannuzi, Andrea Regoli

From the TFR to the IFR approach for the multidimensional analysis of poverty and living conditions

Abstract

Most of the methods designed for the analysis of poverty share two main limitations: i) they are unidimensional, i.e. they refer to only one proxy of poverty, namely equivalent income or consumption expenditure; ii) they need to dichotomise the population into the *poor* and the *non poor* by means of the so called *poverty line*. This reductionism simplifies the analysis, but also wipes out all the complexity of this phenomenon which, on the contrary, should also be object of study. The *Totally Fuzzy and Relative* (TFR) approach proposed by Cheli and Lemmi (1995) allows us to overcome these limitations and to analyse poverty in a multidimensional perspective avoiding the use of arbitrary threshold values.

In this paper we aim to remark how the binary distinction between the "poor" and "non poor" states is too sharp, since deprivation is likely to occur by degrees. Starting from this consideration, we retrace and update the fuzzy sets approach for measuring multidimensional poverty, which leads to the Integrated Fuzzy and Relative method (Betti et al., 2006a, 2006b, 2008).

Keywords: poverty measurement; multidimensional poverty; fuzzy set; living conditions analysis

JEL: I32

From the TFR to the IFR approach for the multidimensional analysis of poverty and living conditions

1. Multidimensional poverty measurement: the-state-of-art

The spectrum of poverty measurement methods is wide and varies from purely monetary indicators to more or less sophisticated models based also on non-monetary measures (UNECE, 2017; Aaberge et al., 2017; Atkinson, 2017; Atkinson et al., 2017; Alkire and Foster, 2011). Measurement choices are often implicit despite they can have a profound impact on results and related policies (Berger et al., 2018; Atkinson, 2019; Jenkins, 2018; Ravallion, 2016; Brucker et al., 2015; Haughton and Khandker, 2009).

The most traditional approach focuses on measuring monetary poverty, which includes the conventional poverty analyses using information on household income or consumption expenditure, and more recently also on wealth (Kuypers and Marx, 2018; Brewer et al., 2017, Serafino and Tonkin, 2017; Jolliffe and Espen, 2016; OECD, 2013; Azpitarte, 2012; Brandolini et al., 2010; Meyer and Sullivan, 2011).

In this way poverty is defined as a condition of lack of strictly economic resources, indeed monetary, under the hypothesis that income or, more generally, monetary resources are able to interpret and capture all the aspects that contribute to determining the conditions of material hardship. Economic resources allow people to satisfy their needs and pursue many other goals that they deem important.

The advantages and disadvantages of this approach are well known (Neckermann et al., 2017; Nelson, 2012), but – across all - its simplicity and direct policy links (especially with monetary transfer policies) still make it the most common and widespread approach to measure poverty and monitor its dynamics.

Measures of poverty based on income or consumption expenditure are still the only available, for example, in the United States, in Japan or in Canada, while in Europe the income-based poverty indicator (the so called *at-risk-of poverty*) is available since 1995 and in 2004 was juxtaposed and integrated with the material deprivation and low-work-intensity indicators (Eurostat, 2012). For several decades, in fact, it has been widely acknowledged that reality is much more complex and that poverty, specifically, is a phenomenon concerning a plurality of dimensions (Stiglitz et al., 2018; World Bank, 2018; Ravallion, 2011). The problem of multidimensionality is mainly represented by the identification of the relevant dimensions, the availability of adequate and reliable information on them and the way of putting the information together (Ferreira and Lugo, 2013; Alkire and Foster, 2011; Lustig, 2011; Labonté, 2011; Maasoumi and Lugo, 2008; Duclos et al., 2006). Therefore, for many years, also this limit has led to the predominance of one-dimensional poverty measures, especially within the official statistics.

Measures of poverty that exceed the exclusive use of monetary indicators have been developed in different conceptual contexts, referring to different terms such as exclusion, inclusion or social cohesion, deprivation or "capability" poverty (Bak, 2018; Garner and Short, 2015). This last approach, which arises from the fundamental contribution of Amartya Sen (1993), is what defines the most complete and robust conceptual reference for the development of multidimensional poverty measures. Nevertheless, the theoretical literature on the multidimensional measurement is still far from consolidation.

Finally, it is now commonly accepted that income poor people do not necessarily have to be deprived in other dimensions. For example, in countries where some public services (such as education or health care services) are provided for free or at subsidized prices by the national social security system, even household with very low levels of income may have good health conditions and high levels of education, or, in other words, an acceptable standard of living. There is an emerging consensus that multidimensional measures of poverty should complement monetary ones. Despite the fascination represented by the possibility of calculating a composite indicator that facilitates spatial and temporal comparisons and between population subgroups, the excessive aggregation of dimensions and indicators results in an extremely significant informative and analytical loss, even threatening to frustrate the use of multidimensional measures. Keeping the dimensions distinct or considering them as appropriate aggregations can help in identifying the most critical or serious aspects of poverty and in putting in place more targeted and effective intervention policies.

The consensus on the fact that poverty must be seen and measured as a multidimensional phenomenon, is also enshrined in the 2030 UN Agenda for Sustainable Development which identifies the reduction of poverty in "all its forms and dimensions" among the objectives to be achieved. In particular, the 1.2 target of the SDGs (Sustainable Development Goals) indicators states that by 2030 at least the share of men, women and children of all ages living in poverty should be halved, considered in all its dimensions and calculated according to national definitions (United Nations, 2015; Loewe and Rippin, 2016).

Several multidimensional poverty indicators have been developed during time and they can be broadly classified into two groups: i) *aggregate indicators* and ii) *composite indicator*s. In the first case, the information for each item is considered at the individual level and an individual multidimensional measure of poverty is calculated and subsequently aggregated to provide a collective measure. In the second case, the information for each item is considered at the aggregate level and then it is combined. In other words, the combination across dimensions is made at the individual level in the first case and at the aggregate level in the second.

Two main examples are represented by the “people at risk-of-poverty or social exclusion” indicator, developed at the European level and published by Eurostat as a EU2020 indicator[1], and the Multidimensional Poverty Index (MPI), developed by the Oxford Poverty and Human Development Initiative (OPHI). Even if they are conceptually different, both approaches need to define threshold levels (for one or more indicators) in order to classify each unit as poor or non-poor.

In synthesis, whatever the approach, the choice of the indicators is intertwined with the definition of the respective deprivation thresholds and in multivariate analyses, these problems may be amplified by the consideration of intangible dimensions for which it may be even more contentious to identify minimum thresholds (Thorbecke, 2013).

1 An alternative indicator has been proposed by Guio, Gordon and Marlier, who suggested a list of 13 items (six of them already included in the previous indicator) and also a separate indicator for children aged less than 16 years based on 18 items (Guio et al. 2016). For a Critical Evaluation of the EU 2020 Poverty and Social Exclusion indicators see Maître et al. (2013).

In this paper we do not want to discuss all the relevant key points which are preliminary to any discussion of the methods for the multidimensional analysis of poverty: the selection of the relevant dimensions; the indicators used to measure people's achievements in these dimensions, and the related issue of the choice of deprivation thresholds; the weights assigned to each dimension. We simply want to remark how the binary distinction between a "bad state" and a "good state" is too sharp, since deprivation is likely to occur by degrees. Starting from this consideration, we retrace and update the fuzzy sets approach for measuring multidimensional poverty, which leads to the Integrated Fuzzy and Relative method.

2. Poverty as a fuzzy concept

The fuzzy sets approach is a valid instrument to measure multidimensional poverty and it offers the additional advantage of overcoming the use of unavoidably arbitrary poverty thresholds. In such a way, it avoids extreme simplifications and the loss of statistical information, which derive from a rigid poor/non poor dichotomy.

Moreover, a number of contributions to poverty research agree in stating that the fuzzy sets approach can be used to analyse poverty in a consistent way with the Sen's capability approach (Sen, 1985).

The fuzzy sets approach takes into account the vagueness of the poverty concept, which does not mean to view it as semantically ambiguous (Chiappero Martinetti, 2008). The notion that poverty is an either/or condition was brought into question some fifty years ago by Watts (1968), who claimed that there is a continuous gradation as one crosses any particular poverty line. As Qizilbash (2006) argued, poverty meets all the following requirements of a vague or fuzzy predicate: i) there are "borderline" situations where individuals cannot be unequivocally classified as poor or non-poor; ii) there is not a sharp boundary to separate the poor from the non-poor; iii) the Sorites paradox holds: an increase by 1 Eurocent in the monetary resources of a poor may lift him above the poverty line while his real condition remains the same. Moreover, other sources of vagueness arise from the fact that, after all, a consensus is missing on a threshold below which an individual can be treated as definitely poor and that the variables used in poverty analyses (income, consumption and other deprivation indicators) are likely to be subject to measurement errors (Chakravarty, 2006; Zheng, 2015). These considerations lead to identify a grey area, where households and individuals are neither completely poor nor completely non-poor.

A phenomenon with such characteristics cannot be properly addressed through a crisp classification into poor and non–poor, based on the setting of a poverty threshold acting as a clear cut-off line, which constitutes the current practice of the poverty measurement. For example, based on the World Bank's $1.90-a-day (2011 PPP prices) International Poverty Line (World Bank, 2018), a person who commands over $1.91 is regarded as being undoubtedly non-poor just like another person whose resources are by far larger than the poverty line.

The fuzzy sets approach rejects the traditional dichotomy between poor and non-poor. The fuzziness is accounted for via a poverty membership function, measured on a scale from 0 to 1 - whereby 1 means full membership to the set of the poor and 0 full non-membership - which allows to deal with such a blurry (as opposed to sharp) vision of the poverty concept. The conventional classification into a rigid dichotomy can then be viewed as a special case of the fuzzy conceptualization of poverty, where the membership function equals 1 for those below the poverty line and 0 for those above it.

Moreover, the degree of membership can be considered as the relative risk of becoming poor in the future. Therefore, when the focus is on the people who are more vulnerable, the measures derived from a fuzzy logic lend themselves to be used fruitfully for policies aiming at preventing poverty. In the traditional approach, only letting the poverty line change allows to identify some "border areas" where people near the poverty line are located. For example, switching from a poverty line defined as 60% of the median equivalised income (which corresponds to the official Eurostat definition) to a threshold defined as 50% of the same income summary value allows to identify people who were classified as poor according to the former threshold and non-poor according to the latter threshold and, in general, to detect whether and to what extent the poverty measurement is sensitive to the choice of the threshold (Lemmi et al., 2019).

An early attempt to incorporate the fuzzy sets theory for poverty measurements and analyses at the methodological level (and in a multidimensional framework) was made by Cerioli and Zani (1990). Their original proposal was later developed by Cheli and Lemmi (1995) giving origin to the so called Totally Fuzzy and Relative (TFR) approach, who completely avoided the need of poverty thresholds, defining a membership function logically and economically founded. Several other proposals have been implemented according to different definitions of the membership function (Dombi, 1990; Chiappero Martinetti, 2000; Lelli, 2001; Clark and Quizilbash, 2002; Chakravarty, 2006; Belhadj, 2011; Abdullah, 2011; Zheng, 2015)

3. From the Totally Fuzzy and Relative approach to the Integrated Fuzzy and Relative approach

3.1 Fuzzy approach to the measurement of (unidimensional) monetary poverty

As already pointed out, the conventional unidimensional approach based on a poverty line can be viewed as a special case of the fuzzy approach, where the membership function equals 1 for those below the poverty line and 0 for those above the poverty line. If we consider a poverty measure based on income, in symbols the membership function (μ) may be seen as:

$$\mu(y_i) = 1 \;\; if \; y_i < z,$$
$$\mu(y_i) = 0 \;\; if \; y_i \geq z,$$

where y_i is the equivalised income of individual i, and z is the poverty line.

In order to move away from the poor/non-poor dichotomy, Cerioli and Zani (1990) proposed the introduction of a transition zone (z_1, z_2) between the two states, a zone over which the m.f. declines from 1 to 0 linearly:

$$\mu(y_i) = 1 \;\; if \; y_i < z_1,$$
$$\mu(y_i) = (z_2 - y_i)/(z_2 - z_1) \;\; if \, z_1 \le y_i < z_2,$$
$$\mu(y_i) = 0 \;\; if \; y_i \ge z_2.$$

Such an expedient allowed to partially overcome the rigid dichotomisation, but was still characterised by the use of two (instead of one) arbitrary poverty thresholds. Moreover, such membership function is simply defined *ad hoc* without any economic foundation.

In what has been called the 'Totally Fuzzy and Relative' approach, Cheli and Lemmi (1995) define the m.f. in the fuzzy set of the poor as:

$$\mu_i = 1 - F(y_i) \tag{1}$$

where *F*( ) is the cumulated distribution function of income, normalised so as to equal 1 for the poorest and 0 for the richest person in the population. In this way the degree of poverty of each household simply depends on the household's position in the income distribution (which justifies the name "totally relative") implying an ordinal approach to poverty measurement. Moreover, this approach can also be seen as "totally fuzzy" as it defines the degree of poverty in fuzzy terms for any level of income and not only for the income values in an arbitrary interval (z_1,z_2) as shown above.
Unlike the original *ad hoc* specification proposed by Cerioli and Zani, the degree of poverty of a generic individual *i* as specified in (1) has a precise economic meaning. In particular, it represents the proportion of individuals whose level of income is higher than y_i.

Equation (1) gives us the individual measure of deprivation of each statistical unit. By aggregating all these values, we obtain a collective index referred to the overall population which is given by:

$$FM = (1/n) \sum \mu_i \tag{2}$$

The expression above represents the fuzzy proportion[2] of poor according to monetary variable Y, where symbol *FM* stands for "Fuzzy Monetary"[3].

It must be noticed that the FM index specified above is always 0.5 approximately. In fact it represents the sample estimate of E[1 – F(y_i)] = 1 – E[F(y_i)], where *F*( ) is

[2] Although such an interpretation from the mathematical point of view is absolutely unexceptionable, from that of the statistical analysis, it risks being misleading, as it could lead to reasoning again in dichotomous terms of poor and not poor. For example, a FM value of 0.2 does not mean that 20% of the population is poor, but simply expresses the average level of poverty measured according to (2).

[3] The name "Fuzzy Monetary" was introduced later by Betti and Verma (1999) and Cheli and Betti (1999).

uniformly distributed in the interval [0,1] so that its mean is 0.5. At first glance this fact might appear to be a drawback as when we compute the FM index over different samples or populations we always get the same value. However, since we work in a purely relative context we cannot (and we don't aim to) obtain a cardinal poverty measure for the surveyed population. We just evaluate which subgroups are better off and which ones are worse off by comparing the values of the FM index (2) computed for each one of them.

In the case of international comparisons, we should consider the union of the various national samples and compute the individual memberships (1) on this pooled sample. Hence we would compare the values of the FM index computed for each subgroup represented by the corresponding national sample.

In order to facilitate the comparison with the conventional poverty rate, Cheli (1995) proposed to raise the m.f. as specified in (1) to some power $\alpha \geq 1$:

$$\mu_i = [1 - F(y_i)]^{\alpha} \tag{3}$$

Increasing the value of exponent α implies giving more weight to the poorer end of the income distribution. In fact, being in general $0<\mu_i<1$, as α increases μ decreases more for those who are less poor compared to those who are poorer. The value of α in equation (3) is arbitrary and implies a political choice. Since its introduction essentially aims at facilitating comparisons between the fuzzy measures and conventional ones, Cheli and Betti (1999) and Betti and Verma (1999) chose the parameter α so that the mean of the m.f. (that is FM) computed for the overall sample is equal to the official head count ratio (i.e. the proportion of the population below the poverty line[4]).

Betti and Verma (1999) define the Fuzzy Monetary indicator (FM), using a somewhat refined version of the above formulation (1), namely:

$$\mu_i = [1 - L(y_i)]^{\alpha} \tag{4}$$

where $L(y_i)$ represents the Lorenz curve of income for individual *i*. Hence, $1-L(y_i)$ represents the *share of the total equivalised income* received by all individuals less poor than the person concerned. It varies from 1 for the poorest, to 0 for the richest individual. $1-L(y_i)$ can be expected to be a more sensitive indicator of the actual disparities in income, compared to $1-F(y_i)$. This is illustrated in Figure 1 where the two m.f. specifications (3) and (4) are compared (for $\alpha=1$) by means of the Lorenz diagram.

Moreover, it may be noted that while the mean of $1-F(y_i)$ values is always ½, as we already pointed out, the mean of $1-L(y_i)$ values equals $(1+G)/2$, where G is the Gini

[4] In Cheli and Betti (1999) and Betti and Verma (1999) the official head count ratio is calculated on ECHP data, by applying the modified OECD equivalence scale and setting the poverty line at 60% of the median equivalent income.

index of the distribution. Moreover, the more concentrated is income the greater is 1-$L(y_i)$ compared to 1-$F(y_i)$.

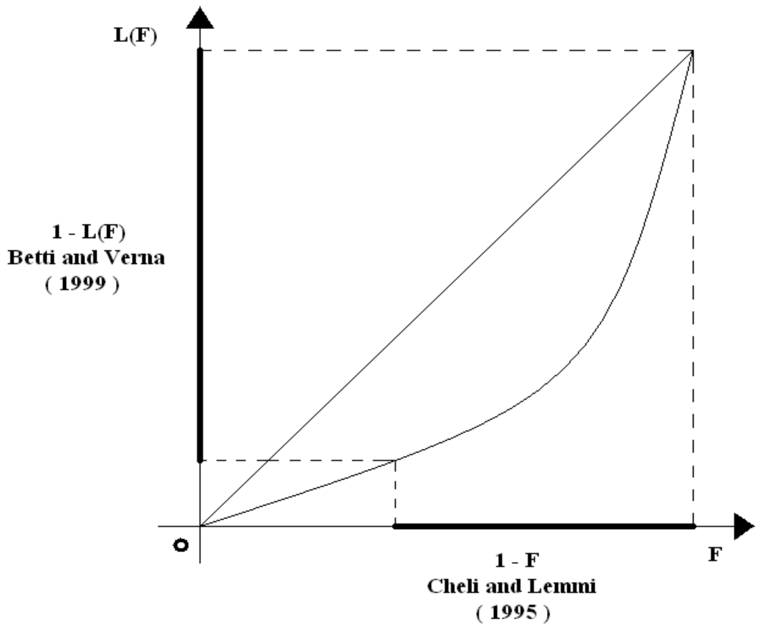

Figure 1. *Membership functions used by Cheli and Lemmi (1995), and Betti and Verma (1999)*

However, recognising that both specifications of the m.f. are meaningful, effective and theoretically founded, Betti *et al.* (2006a) proposed to combine them into an "Integrated Fuzzy and Relative" (IFR) approach. The resulting measure, which can be seen as a generalisation of both components, is defined as:

$$\mu_i = [1 - F(y_i)]^{\alpha-1}[1 - L(y_i)] \tag{5}$$

Again, parameter α can be chosen so that the mean of the m.f. (equation 6) is equal to the official head count ratio:

$$FM = (1/n)\sum \mu_i = \frac{\alpha + G_\alpha}{\alpha \cdot (\alpha+1)} \tag{6}$$

It is elegant that the Fuzzy Monetary (FM) measure as defined above is expressible in terms of the generalised Gini index G_α, which corresponds to the standard Gini coefficient when α =1 and is defined (in the continuous case) as:

$$G_\alpha = \alpha \cdot (\alpha + 1) \cdot \int_0^1 [1 - F(y)]^{\alpha-1}[F(y) - L(y)]dF, \tag{7}$$

In practice, the generalised Gini index weights the distance $[F(y) - L(y)]$ between the line of perfect equality and the Lorenz curve by a function of the individual's position in the income distribution, giving more weight to its poorer end.

3.2 Fuzzy approach to the measurement of (multidimensional) non monetary poverty ("Fuzzy Supplementary")

The standard of living of households and persons can also be measured by means of a variety of non monetary indicators referring to different domains (or dimensions), such as housing conditions, possession of durable goods, the general financial situation, perception of hardship, expectations, etc. Quantification and combination of a large set of indicators of this kind involves a number of steps, models and assumptions.

Firstly, from all the available indicators a selection has to be made of those ones which are substantively meaningful and useful for the analysis of deprivation. This is a substantive as well as a statistical question.

Secondly, it is useful to identify the underlying dimensions and to group the indicators accordingly. Taking into account the manner in which different indicators may be grouped together adds to the richness of the analysis, whereas ignoring such dimensionality can result in misleading conclusions (Whelan et al. 2001).

Individual items indicating non-monetary deprivation often take the form of simple 'yes/no' dichotomies (such as the presence or absence of enforced lack of certain goods or facilities). However, some items may involve three or more ordered categories, reflecting different degrees of deprivation. Consider the general case of c=1 to C ordered categories of some deprivation indicator, with c=1 representing the most deprived and c=C the least deprived situation. Let c_i be the category to which individual *i* belongs. Cerioli and Zani (1990), assuming that the rank of the categories represents an equally-spaced metric variable, assigned to any individual *i* a deprivation score as follows:

$$d_i = (C - c_i)/(C - 1) \;\; ; \;\; 1 \le c_i \le C \qquad (8)$$

In terms of fuzzy sets theory, the expression above represents the degree of membership in the fuzzy subset of the poor in relation to the sole information provided by the single indicator considered.

Cheli and Lemmi (1995) proposed an improvement of (8) by replacing the simple ranking of the categories with their *distribution function* in the population:

$$d_i = [1 - F(c_i)]/[1 - F(1)] \qquad (9)$$

In such a way, deprivation in terms of any specific item is measured in the same way than income poverty (according to (1)) so that the deprivation score of each household or individual depends only on how many have less and how many have more than she does[5].

Note that the above two formulations for d_i are identical in the case of a dichotomous indicator (C=2), which is by far the most common situation. In this case d_i is dichotomous too and takes values 1 (in case of deprivation) or 0 (otherwise).

The procedure for aggregating over a group of items is also the same for the two formulations: a weighted sum is taken over items (k): $\mu_i = \sum w_k d_{k,i} / \sum w_k$, where the w_k are item-specific weights, taken in the above references as[6] $w_k = \ln\left(\frac{1}{\bar{d}_k}\right)$. For dichotomous indicators, $\bar{d}_k$ the mean of individual values (d_i) for item k, simply equals the proportion of individuals who are deprived on that item.

Betti et al. (2006a, 2006b) proposed instead to treat the non-monetary scores in a way entirely analogous to that for fuzzy monetary (FM) measures, described in the previous section[7]. On the basis of this approach, the membership function corresponding to equation (5) would be:

$$\mu_i = [1 - F(s_i)]^{\alpha-1}[1 - L(s_i)] \quad ; \ \alpha \geq 1 \tag{10}$$

where s_i represents the endowment of the overall supplementary items evaluated for individual i, $F(s_i)$ and $L(s_i)$ are the corresponding values of the distribution function and the Lorenz curve (of s) respectively, whereas α is a parameter to be determined so as to make the mean of the non-monetary deprivation measure (10) numerically identical to the official head-count ratio.

In order to construct the fuzzy supplementary index defined in equation (10) one has to begin by grouping the items into 'dimensions' as described above. Within each dimension δ, deprivation scores ($d_{k,i}$) are assigned as in (8) [8].

The score for any individual i - that reflects her global endowment (or non-deprivation) - for what concerns dimension δ ($s_{\delta i}$) is computed as:

$$s_{\delta i} = \sum_{k \in \delta} w_k (1 - d_{k,i}) / \sum_{k \in \delta} w_k \tag{11}$$

where the weights, instead of being calculated as $w_k = \ln\left(\frac{1}{\bar{d}_k}\right)$, are defined as a product of two factors:

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[5] Dividing by 1-F(1) is necessary to assure that d_i always takes value 0 in correspondence to the lowest category (i.e. lowest risk of poverty) and 1 in correspondence to the highest one.

[6] According to the relative concept of poverty, the underlying principle is that of giving more importance to the items that are more diffused (and for which, symmetrically, deprivation is lower) and therefore more representative of the prevailing lifestyle in society. This particular specification of the weights that was adopted in all the early applications the TFR method is just one among many possible alternatives. In this respect a comparative study of different weight specifications was done by Filippone et al. (2001).

[7] This idea had been originally proposed by Betti and Verma (1999) who applied it to specification (4).

[8] When item k is of the binary type (i.e. present or absent), $d_{k,i}$ is correspondingly equal to 0 or 1, as explained above in this section.

$$w_k = w_k^{(a)} \cdot w_k^{(b)} \quad (12)$$

where $w_k^{(a)}$ depends only on the distribution of item k in the population and is defined as a decreasing function of the proportion of deprived in this item; on the other side, $w_k^{(b)}$ considers the correlation between k and the other items (in the dimension concerned) in order to reduce the redundancy produced by highly correlated indicators[9].

The deprivation score can be also obtained for the overall situation covering all the indicators:

$$s_i = \sum_k w_k (1 - d_{k,i}) / \sum_k w_k \quad (13)$$

4. Income poverty and non-monetary deprivation in combination: Manifest and Latent deprivation

In the IFR approach the two measures – FM_i concerning income poverty (equation 5), and FS_i concerning the overall non-monetary deprivation (equation 10) - may be combined to construct composite measures which indicate the extent to which income poverty and non-monetary deprivation overlap.

We define *Manifest deprivation* as the propensity to both income poverty and non-monetary deprivation simultaneously. One may think of this as the more intense degree of deprivation. For any individual *i*, the measure of manifest deprivation (M_i) may be conceptualised as the degree of membership in the *intersection* of the two fuzzy states of deprivation and is specified as the *minimum* of the two memberships FM_i and FS_i as follows[10]:

$$M_i = \min (FM_i, FS_i) \quad (14)$$

On the other side we define *Latent deprivation* as a situation in which the individual is subject to at least one of the two, income poverty and/or non-monetary deprivation. One may think of this as the 'less intense' degree of deprivation. This measure may be conceptualised as the degree of membership in the *union* of the two fuzzy states of deprivation and is specified as the *maximum* of the two memberships FM_i and FS_i as follows:

$$L_i = \max (FM_i, FS_i) \quad (15)$$

[9] For a complete description of this weighting system see Betti and Verma (1999) or Betti et al. (2008).

[10] There are more than one way in which the fuzzy set intersection and union can be formulated, each representing an equally valid generalization of the corresponding crisp set operations. The reasons that justify the choice to use the *min* and *max* operators in this context are discussed in Betti et al. (2006a, 2006b).

Once the propensities to income poverty (FM_i) and non-monetary deprivation (FS_i) have been defined at the individual level (i), the corresponding collective measures for the whole population and for specific sub-populations can be obtained by averaging (14) and (15).

5. Longitudinal analysis of poverty conceptualized as a fuzzy state

The methodological development of longitudinal measures of poverty conceptualized as a fuzzy state was initiated by Cheli (1995) in the restricted context of two consecutive panel waves.

Later, Cheli and Betti (1999) and Betti et al. (2004) extended the analysis to *T* panel waves ($T \geq 2$), utilising a particular form of transition matrices between two fuzzy states. In parallel, Betti and Verma (1999, 2004) dropped the tool of transition matrices and focussed more on the choice of the rules for the manipulation of fuzzy sets - such as defining their complements, intersections, unions and aggregations – that seem to perform best in this particular context.

In the IFR approach[11] the procedure developed in Section 3 to represent multi-dimensional aspects of deprivation extends directly to the representation of its longitudinal aspects. In place of the two dimensions of deprivation (monetary and non-monetary), we have here fuzzy sets representing the state of poverty at two times.

5.1 Fuzzy longitudinal measures over two time periods

When analysing the situation over two periods, we are interested in combining the membership functions of four fuzzy states: poverty at both times 1 and 2; poverty at time 1 and non-poverty at time 2; non-poverty at time 1 and poverty at time 2; and finally, non-poverty at both times 1 and 2.

Longitudinal indices that measure persistent and transient poverty as well as movements into and out of the state of poverty can be identified in terms of these membership functions, as described in Table 1[12]. Here P_t represents the fuzzy set of the poor at time *t* and $\bar{P}_t$ its complement, that is the non-poor set.

For instance $\mu(P_1 \cap \bar{P}_2)$ is the joint membership in the fuzzy set of the poor at time 1 and in the non-poor at time 2. It represents the fuzzy proportion of people who are poor at time 1 and non poor at time 2. Therefore, we can interpret it as the fuzzy exit rate from poverty or the propensity to exit from poverty between times 1 and 2.

[11] See Betti et al. (2006b, 2008).

[12] The reasons for defining the joint m.f. as reported in Table 1 are extensively discussed and explained in Betti et al. (2006b). These authors use a different notation which is more compact but maybe less rigorous than the one we use here.

Table 1. *Longitudinal measures of interest over two time periods for a generic individual i*

	Measure	*Joint membership function*	*Description*
1	Never in poverty	$\mu(\bar{P}_1 \cap \bar{P}_2) \quad 1 \quad \max(\mu_1, \mu_2)$	Poverty at *neither* of the two years
2	Persistent in poverty	$\mu(P_1 \cap P_2) \quad \min(\mu_1, \mu_2)$	Poverty at *both* of the years
3	Exiting from poverty	$\mu(P_1 \cap \bar{P}_2) \quad \max(0, \mu_1 \quad \mu_2)$	Poverty at time 1, but non-poverty at time 2
4	Entering into poverty	$\mu(\bar{P}_1 \cap P_2) \quad \max(0, \mu_2 \quad \mu_1)$	Non-poverty at time 1, but poverty at time 2
5	Ever in poverty	$\mu(P_1 \cup P_2) \quad \max(\mu_1, \mu_2)$	Poverty at *at least one* of the two years

5.2 Longitudinal measures over more than two time periods

The analysis of the persistence of poverty over time requires the specification of joint membership functions of the type:

$$I_T = \mu(P_1 \cap P_2 \cap \ldots \cap P_T) \text{ and } U_T = \mu(P_1 \cup P_2 \cup \ldots \cup P_T)$$

where the first expression is the membership in the intersection of T cross-sectional fuzzy sets, whereas the second expression refers to their union. We calculate them as follows:

$$I_T = \min(\mu_1, \mu_2, \ldots \mu_t, \ldots, \mu_T)$$

$$U_T = \max(\mu_1, \mu_2, \ldots \mu_t, \ldots, \mu_T)$$

I_T represents the individual's propensity to be poor at all T periods, whereas U_T is the propensity to be poor at *at least one* of the T periods. On the other hand, the propensity to be *never poor* over all T periods is the complement of U_T, that is $\bar{U}_T = 1 - U_T$. The same result is obtained by considering the intersection of non-poor sets:

$$\bar{I}_T = \min(\bar{\mu}_1, \bar{\mu}_2, \ldots \bar{\mu}_t, \ldots \bar{\mu}_T) = 1 - \max(\mu_1, \mu_2, \ldots \mu_t, \ldots, \mu_T) = 1 - U_T.$$

Summing up we define the following longitudinal measures:

- *Never in poverty*: degree of membership in the set 'non poor for all the T years' = $\bar{I}_T = \bar{U}_T = 1 - U_T$
- *Any time poverty*: degree of membership in the set 'poor for at least one year' = U_T
- *Continuous poverty*: degree of membership in the set 'poor for all the T years' = I_T

• *Persistent poverty* as the propensity to be poor over at least a majority of the T years, i.e. over at least *t* years, with *t*=int(T/2)+1, the smallest integer strictly larger than (T/2).

For instance, for T = 4 or 5 years period, 'persistent' would refer to poverty for at least 3 years; for T = 6 or 7, it would refer to poverty for at least 4 years, etc. The required propensity to persistent poverty is the [int(T/2)+1]th largest value in the sequence[13] $(\mu_1, \mu_2, \ldots \mu_t, \ldots, \mu_T)$.

5.3 Rates of exit and re-entry

Given the state of poverty at time 1, and also at a later time (t−1), what is the proportion exiting from poverty (*exit rate*) at time t = 2, 3, …? Given the state of poverty at time 1, but of non-poverty at a later time (t−1), what is the proportion which has re-entered poverty (*re-entry rate*) at time t = 3, 4, ….?

In conventional analysis, the above rates are computed simply from the count of persons in various states. For instance, for the exit rate, the numerator is the count of persons who are poor at both times 1 and (t−1), but are non-poor at time t; the denominator is the count of all persons who are poor at times 1 and t−1 (and are present in the sample at time t).

For re-entry rate, the numerator is the count of persons poor at time 1, non-poor at time (t−1), but poor again at time t. The denominator is the count of persons who are poor at time 1 and non-poor at time t−1 (and are present in the sample at time t). The construction of these measures using fuzzy m.f.'s is also straightforward. With μ_t as a person's propensity to poverty at time t, the person's contribution of these rates is as follows.

Exit rate:

Numerator $\mu(P_1 \cap P_{t-1} \cap \bar{P}_t) = max[0, \min(\mu_1, \mu_{t-1}) - \mu_t]$

Denominator $\mu(P_1 \cap P_{t-1}) = \min(\mu_1, \mu_{t-1})$.

Re-entry rate:

Numerator $\mu(P_1 \cap \bar{P}_{t-1} \cap P_t) = \mu[(P_1 \cap P_t) \cap \bar{P}_{t-1}] = max[0, \min(\mu_1, \mu_t) - \mu_{t-1}]$

Denominator $\mu(P_1 \cap \bar{P}_{t-1}) = \max(0, \mu_1 - \mu_{t-1})$

The corresponding rates for the population are computed by simply averaging the above individual contributions.

[13] With the conventional poor/non-poor dichotomy, any individual spends some specified number of years between 0 and T in the state of poverty during the interval T. With poverty treated as a matter of degree, any particular individual is seen as contributing to the *whole distribution*, from 0 to T, of the number of years spent in poverty.

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