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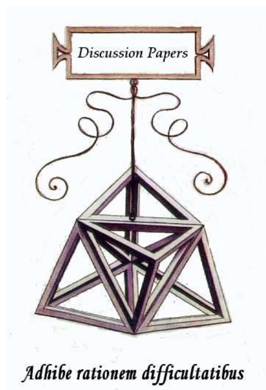

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Discussion Paper
n. 278

Domenico Buccella - Luciano Fanti - Luca Gori

The R&D investment decision game with product differentiation

Abstract

This article extends the classical d'Aspremont and Jacquemin's (1988, 1990) cost-reducing R&D model with spill-overs to allow quantity-setting firms (Cournot rivalry) to play the non-cooperative R&D investment decision game with horizontal product differentiation. Unlike Bacchiega et al. (2010), who identify a parametric region – defined by the extent of technological spill-overs and the efficiency of R&D activity – in which the game is a prisoner's dilemma (self-interest and mutual benefit of cost-reducing innovation conflict), this work shows that product differentiation changes the game into a deadlock (self-interest and mutual benefit do not conflict), irrespective of the parameter scale (thus, holding also in the absence of spill-over effects). The social welfare when the degree of product differentiation is high enough and a deadlock characterises investing in cost-reducing R&D is larger than when firms do not invest in R&D, irrespective of the technological spill-overs extent and the R&D activity's efficiency. These findings suggest that investing in R&D challenges the improvement of interventions aimed at favouring product differentiation. These results also hold for price-setting firms (Bertrand rivalry).

Keywords: Process innovation; Nash equilibrium; Social welfare

JEL: D43, L13, O31

1. Introduction

This article revisits the influential d'Aspremont and Jacquemin (1988, 1990), AJ henceforth, model of cost-reducing R&D with spill-overs by considering a non-cooperative three-stage game in which firms endogenously choose whether to invest in R&D – along the line of Bacchiega et al. (2010) – and consumers have preferences characterising horizontally differentiated products (Singh and Vives, 1984).

In their pioneering contributions, AJ build a two-stage game in which two identical firms invest in R&D at the first stage turning to be Cournot competitors in a market for homogeneous goods at the second stage. The authors consider three cases under which firms (i) non-cooperatively compete at both stages (two-stage non-cooperative game), (ii) cooperate at both stages (two-stage cooperative solution), and (iii) collude at the R&D stage and compete at the market stage (two-stage mixed game). The process innovation R&D investment flows and reduces the investing firms' marginal cost; however, it also exogenously spills over reducing, in turn, the rival's marginal cost. Specifically, AJ's article aims at comparing the magnitude of cost-reducing technical advance achieved when firms conduct R&D either competitively or cooperatively, finding that cooperative R&D leads to greater technological advance than competitive R&D for sufficiently large spill-over effects.

Later, Henriques (1990) and Suzumura (1992) extend AJ in two distinct directions. In the first route, Henriques (1990) augments AJ's findings by providing stability conditions showing that the main results obtained by AJ's quantity-setting duopoly are meaningful only when the solution of the non-cooperative game is stable in the sense of Seade (1980). This implies the existence of thresholds – in the parameter space defined by the extent of technological spill-overs and the efficiency of R&D activity (as also clarified by Bacchiega et al., 2010) – such that the reaction functions in the R&D space should adequately cross. In particular, for any given level of the extent of technological spill-overs, the efficiency of the R&D technology should be low enough to avoid over-investment in R&D that in turn would increase the degree of competition (by reducing average variable costs of production, increasing output and lowering market price) thereby eroding profits in both cases of strategic substitutability (little technological spill-overs) and complementarity (large technological spill-overs) of R&D investments.[1] In the second route, Suzumura (1992) applies AJ's idea to a general class of oligopoly models both in the cases of cooperative and non-cooperative R&D with spill-overs.

Subsequent extensions by, amongst others, Kamien et al. (1992), Ziss (1994), De Bondt (1996), Amir (2000), Amir et al. (2003) and Lambertini and Rossini (2009), have typically shown that cooperative R&D decisions amongst firms competing in the product market are socially beneficial. In this literature spill-overs are exogenous, i.e., a fixed fraction of a firm's R&D investment exogenously flows to competitors, so that each firm has no direct control over the extent of disclosure.

There exists, however, another branch of the literature assuming firms can endogenously control spill-overs aiming at investigating whether owners decide about information sharing. The works belonging to this literature can be divided into two groups. One group (Poyago-Theotoky, 1999; Atallah, 2004; Lambertini et al., 2004) studies the case in which firms decide on information sharing after they invested in R&D (i.e., spill-overs do not affect the extent of R&D investments). The main result of this branch of literature is to have firms choosing to keep their R&D knowledge secret and thus R&D spill-overs are absent (non-disclosure). The second group (Gersbach and Schmutzler, 2003; Gil-Moltó et al., 2005; Piga and Poyago-Theotoky, 2005; Milliou, 2009) considers the possibility of firms choosing whether sharing R&D outcomes before they invest in R&D (i.e., spill-

[1] In the case of strategic substitutability (resp. complementarity) of the R&D activity, the intensity of the R&D externality is small (resp. large) and the amount of R&D investment x_i of firm i and x_j and of firm j ($i = \{1,2\};\ i \neq j$) are negatively (resp. positively) related in the R&D space, i.e., the R&D reaction curves are downward-sloping (resp. upward-sloping). The stability conditions require that $\left|\frac{dx_i}{dx_j}\right| < 1$ in both cases. Poyago-Theotoky (1999) interpreted strategic substitutability (resp. complementarity) as resembling the case in which R&D-related information disclosure is small (resp. large), and firms follow similar (resp. distinct) research paths.

overs do affect the extent of R&D investments). Protecting or sharing knowledge depends on price or quantity competition (the first paper) on location (second and third papers), and if the extent of R&D spill-overs is not too strong, firms may let R&D knowledge flow to competitors (the last paper). The main result of this branch of literature is to have firms choosing to disclose R&D knowledge with the rivals.[2]

Though the AJ framework is widely used in the industrial organization literature as a basic textbook model for process innovation in oligopolistic contexts, the analysis of the properties and solutions of the (three-stage) R&D investment decision game played by non-cooperative and selfish firms is rather scant. Indeed, while AJ and all of the subsequent aforementioned works just analyse exogenously given contexts with R&D investments (irrespective of whether the focus is on either the analysis of cooperation versus competition in the R&D phase or the endogeneity or exogeneity of spill-overs), the underlying game played by firms choosing whether to invest in R&D *in the absence of spill-over effects* is a prisoner's dilemma (there exists conflict between self-interest and mutual benefit to undertake cost-reducing innovation). In other words, R&D investments result to be profit-reducing.

Despite showing this result in the simplified AJ's two-stage model seems to be rather straightforward, it has never been explicitly noted. As an exception, Bacchiega et al. (2010) add the investment-decision stage to the two-stage AJ's game. *By assuming R&D spill-over effects occur*, those authors pinpoint the existence of: (1) a parameter region – defined by the interplay between the extent of technological spill-overs and the efficiency of the R&D activity – in which the three-stage R&D game (played by the firm that non-cooperatively choose both R&D investment at the R&D stage and output at the market stage) is a prisoner's dilemma (low values of R&D spill-overs); (2) conversely and more importantly, a parameter region where the prisoner's dilemma is solved and the game turns to be an anti-prisoner's dilemma (larger values of R&D spill-overs), in which no conflict between self-interest and mutual benefit to undertake cost-reducing innovation exists. However, Bacchiega et al. (2010) choose to restrict the analysis to the case of homogeneous products and quantity-setting firms. Moreover, they also neglect to consider the R&D cost conditions related to the symmetric subgame I/I and the asymmetric subgame I/NI needed to define the parametric space in which the non-cooperative version of the R&D game is feasible.

The present work aims at generalising the variety of products allowing capturing the case of (horizontally) differentiated products and augmenting the analysis to consider all the relevant constraints of the R&D game, assuming both Cournot and Bertrand rivalries. Definitively, the article proposes to question whether the unpleasant prisoner's dilemma (on the firm side) is a robust feature of the R&D duopoly game framed in the AJ set-up. It explicitly shows that horizontal product differentiation solves the dilemma obtained by Bacchiega et al. (2010) and let the game become a deadlock, *irrespective of the extent of technological spill-overs and the efficiency of the R&D activity*. It also represents an attempt to provide a thoughtful analysis of the non-cooperative version of the R&D model developed by AJ – which is surprisingly missing in the IO literature – by clarifying (a) the role played by the relevant thresholds (the stability conditions and the R&D cost conditions) so that the feasibility conditions of the model are accurately disentangled to assure the non-violation of several bounds (e.g., the non-negativity of the R&D costs), and (b) the parameter configurations under which the game is a prisoner's or an anti-prisoner's dilemma. As is known, the prisoner's dilemma of the R&D game in the AJ setting reveals the sub-optimality of the non-cooperative solution (alternatively, cooperation is Pareto improving). Thus, the AJ's basic model always shows (regardless of the spill-overs effects) the unpleasant result that non-cooperation is harmful. This also explains why much of the subsequent literature has focused on the cooperative solution. Unlike the existing literature, this article shows that an appropriate generalisation with product differentiation allows the non-cooperative and unilateral R&D investment behaviour to turn to be better off for players.

Our results differ from those of Bacchiega et al (2010) in a crucial respect: in their work, the prisoner's dilemma vanishes if and only if the extent of technological spill-overs is sufficiently high,

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[2] There exists another relevant branch of the literature augmenting the AJ model to the analysis of R&D subsidies, i.e., Hinloopen (1997, 2000) and Amir et al. (2019).

which, in turn, would require that firms disclose (or – equivalently – they are unable to keep closed) the information on the results of their R&D investment. However, the non-disclosure (i.e., keeping secret) R&D-related result in the AJ setting is in the unilateral interest of each non-cooperative firm. *Under product differentiation, the prisoner's dilemma can vanish also in the absence of R&D spill-overs in both cases of quantity-setting and price-setting duopolies.* Moreover, the welfare analysis also reveals that, at the non-cooperative Pareto efficient Nash equilibrium with product differentiation in which both firms invest in cost-reducing R&D, firms and consumers are better off than without R&D.

Our findings suggest that investing in R&D challenges the improvement of interventions aimed at fostering product differentiation. Though this policy may seem to be detrimental for consumers as it strengthens firms' market power, it also and encourages R&D investments (at least starting with a relatively high degree of differentiation) in turn allowing firms to increase their market share at the expense of the rival's share in the market for the product of their variety (and thus consumers' surplus increases in aggregate terms as total supply increases due to the stimulus of the higher variety). This eventually contributes to increasing profits and thus social welfare compared to the scenario where products are perceived as homogeneous.

When products are perfect substitutes, an increase in the degree of R&D spill-overs always induces the investing firm to reduce its investment effort (to prevent the rival from taking advantage by free-riding on its investment activity). An increase in product heterogeneity resulting from consumer preferences tends to promote R&D strategic complementarity as the degree of competition in the product market reduces and profits increases. Therefore, there is room for the joint use of resources devoted to R&D (i.e., the R&D externality of one firm favours the R&D investment of the rival and vice versa) if the efficiency of R&D activity is high enough. Finally, product complementarity promotes cooperative behaviour in the product market by letting firms act as if they were maximising joint profits, and this in turn eventually leads each of them to benefit from R&D complementarity.

The remainder of the article is organised as follows. Section 2 outlines the model and discusses the main ingredients of the R&D investment decision game with product differentiation. Section 3 concentrates on quantity-setting firms.[3] Section 4 concludes the article. The Appendix provides analytical details and the proofs of the propositions.

2. The model

The starting point of our analysis can be spelt out by following the pioneering idea of cost-reducing innovation developed by AJ, augmented almost two decades later by Bacchiega et al. (2010), who consider the non-cooperative R&D investment decision game played by two quantity-setting firms.

Unlike Bacchiega et al. (2010), this section aims at developing the investment decision game in an AJ-like setting (including the stability conditions as in Henriques, 1990) by considering horizontal product differentiation à la Singh and Vives (1984).

Consider an industry where two quantity-setting firms, i and j ($i = \{1,2\}$; $i \neq j$), face the perspective of investing in R&D and then sell goods of variety i and j, respectively, in the product market. The linear (inverse) demand for product of variety i is given by $p_i = a - b(q_i + dq_j)$, where p_i denotes the price of product i (representing the marginal willingness to pay of consumers towards products of firm i), q_i, and q_j are the quantities of product of variety i and variety j produced by firm i and firm j, respectively, $a > 0$ is a positive parameter representing the market size, $b > 0$ measures the slope of the market demand being part of its elasticity, and $-1 \leq d \leq 1$ measures the degree of product differentiation as consumers perceive in the market (Singh and Vives, 1984). Positive (resp. negative) values of d refer to product substitutability (resp. complementarity). When $d = 0$ goods

[3] For space constraints, the analysis of Bertrand competition in the AJ setting is available upon request. In this regard, we pinpoint that the results of the quantity competition model qualitatively hold for price-setting firms (with quantitative differences, especially about the thresholds identified by the stability conditions and the R&D cost conditions).

are totally differentiated, i.e., each firm acts as a monopolist. The case $d = 1$ refers to homogeneous goods and resembles the model developed by Bacchiega et al. (2010). This demand structure comes from the usual specification of quadratic utility for consumers' preferences, that is $U(q_i, q_j) = a(q_i + q_j) - \frac{b}{2}(q_i^2 + q_j^2 + 2dq_iq_j)$, as proposed by Dixit (1979) and subsequently used, amongst many others, by Singh and Vives (1984). For reasons of analytical tractability (and without loss of generality), we normalise $a = b = 1$ henceforth. Therefore, the indirect demand for product of variety i under horizontal differentiation is:

$$p_i = 1 - q_i - dq_j,\ i,j = \{1,2\},\ i \neq j. \tag{1}$$

The total cost of production and the cost of R&D effort of firm i are respectively given by the functions $C_i(q_i, x_i, x_j)$ and $X_i(x_i)$, where x_i and x_j represent the R&D effort (investment) firm i and firm j exert, respectively. Following AJ, these functions can be specified using the expressions:

$$C_i(q_i, x_i, x_j) = (w - x_i - \beta x_j)q_i,\ \ i,j = \{1,2\}, i \neq j, \tag{2}$$

and

$$X_i(x_i) = \frac{g}{2}x_i^2,\ \ i,j = \{1,2\}, i \neq j, \tag{3}$$

where $g > 0$ is a parameter measuring R&D efficiency. It scales up/down R&D investment total costs and represents an exogenous index of technological progress measuring, for example, the appearance of a new, cost-effective technology, weighting the degree at which the available technology for process innovation affects investment decisions and firm's profits. A reduction in g can be interpreted as a technological advance so that investing in R&D becomes cheaper (i.e., the efficiency of R&D investment increases). In addition, $\beta \in [0,1]$ captures the extent of spill-overs (externality) of the R&D investment activity of firm j exogenously flowing as a cost-reducing device towards firm i (i.e., the amount of information that firm j exogenously discloses). We assume that both firms symmetrically share this characteristic of the extent of technological spill-overs. This scenario represents the standard case of exogenous spill-overs – with respect to which a fixed fraction of a firm's R&D process innovation exogenously flows to competitors, so that each firm has no direct control over the extent of disclosure for, e.g., technological reasons – and directly follows AJ and the subsequent contributions by Henriques (1990), Suzumura (1992), Kamien et al. (1992), De Bondt (1996) and Bacchiega et al. (2010). When $\beta = 0$ there are no R&D externalities, resembling the case of non-disclosure of R&D information. When $\beta = 1$, R&D information is fully shared, so that R&D disclosure is at its (exogenous) highest intensity.

The expression representing the firm's technology in Eq. (2) implies that the unitary cost of production should be positive so that $w - x_i - \beta x_j > 0$ should always hold, where $0 < w < 1$ measures the unitary technology of production cost irrespective of R&D investments. Moreover, the expression representing the cost of R&D effort in Eq. (3) reveals diminishing returns in the R&D technology exerted by firm i. Therefore, each firm sustains the cost of R&D effort with a technology displaying decreasing returns to scale to achieve the benefit of reducing the total unit costs of production with constant returns to scale.

Definitively, selfish firms are engaged in a three-stage non-cooperative *R&D investment decision game with horizontal product differentiation* and complete information in which they must choose whether to invest in R&D activities at stage one (*the investment-decision stage*). At stage two (*the R&D stage*), firms choose the extent of process innovation R&D investment (if they invest) or, alternatively, they do not invest in R&D. At stage three (*the market stage*), firms choose the quantity (Section 3) or the price of the goods in the Bertrand scenario, available upon request. As usual, the game is solved by adopting the backward induction logic.

3. The R&D investment decision game with product differentiation: Cournot competition

This section initiates the analysis of the non-cooperative R&D investment decision game with horizontal product differentiation in a quantity-setting (Cournot) rivalry.

3.1. The symmetric subgame in which firms do not invest in R&D (NI/NI)

Consider the possibility that firms symmetrically choose not to invest in R&D, that is $x_i = 0$ and $X_i(x_i) = 0$ $(i = \{1,2\}, i \neq j)$. By using Eqs. (1) and (2) the profit function of firm i is:

$$\Pi_i^{NI/NI} = p_i q_i - C_i(q_i, 0,0) = \left(1 - q_i - dq_j - w\right)q_i, \tag{4}$$

where the upper script NI/NI stands for universal no R&D investments. At the market stage of the game, each firm chooses the amount of output to maximise profits. Maximisation of (4) with respect to q_i leads to the following downward-sloping reaction function of firm i in the output space:

$$\frac{\partial \Pi_i^{NI/NI}}{\partial q_i} = 0 \Leftrightarrow \overline{q}_i^{NI/NI}\left(q_j\right) = \frac{1-w-dq_j}{2}. \tag{5}$$

Using Eq. (5) together with the symmetric counterpart for firm j allows to obtain the system of reaction functions in the (q_i, q_j) space, whole solution $\overline{q}_i^{NI/NI}\left(q_j\right)$ $(i = \{1,2\}, i \neq j)$ leads firm i to produce the following equilibrium quantity (denoted with an asterisk) of product of variety i (which is symmetric to the quantity of product of variety j produced by firm j):

$$q_i^{*NI/NI} = \frac{1-w}{2+d}. \tag{6}$$

From the expression in (6) one can easily get the equilibrium value of the market price of product of variety i and profits of firm i, which are respectively:

$$p_i^{*NI/NI} = \frac{1+w(1+d)}{2+d}. \tag{7}$$

and

$$\Pi_i^{*NI/NI} = \left(\frac{1-w}{2+d}\right)^2. \tag{8}$$

Clearly, a change in consumers' tastes generating an increase in the degree of product differentiation $(d \downarrow)$ causes, ceteris paribus, an increase in the demand directed to both firms for the product of their own variety (at the expense of the rival's share) also strengthening the market power by eventually increasing profits.

The equilibrium values of consumers' surplus ($CS^{*NI/NI}$) and producers' surplus ($PS^{*NI/NI}$) that can be obtained in the NI/NI subgame are summarised as follows:

$$CS^{*NI/NI} = \frac{1}{2}\left[\left(q_i^{*NI/NI}\right)^2 + \left(q_j^{*NI/NI}\right)^2 + 2dq_i^{*NI/NI}q_j^{*NI/NI}\right] = \frac{(1-w)^2(1+d)}{(2+d)^2}. \tag{9}$$

and

$$PS^{*NI/NI} = \Pi_i^{*NI/NI} + \Pi_j^{*NI/NI} = 2\left(\frac{1-w}{2+d}\right)^2. \tag{10}$$

Therefore, social welfare under NI/NI, $W^{*NI/NI}$, is:

$$W^{*NI/NI} = CS^{*NI/NI} + PS^{*NI/NI} = \frac{(1-w)^2(3+d)}{(2+d)^2}. \tag{11}$$

3.2. The symmetric subgame in which both firms invest in R&D (I/I)

Consider now the possibility that firms symmetrically choose to invest in R&D, that is $x_i > 0$ and $X_i(x_i) > 0$ $(i = \{1,2\}, i \neq j)$. Therefore, by using Eqs. (1), (2) and (3) the profit function of firm i becomes:

$$\Pi_i^{I/I} = p_i q_i - C_i\left(q_i, x_i, x_j\right) - X_i(x_i) = \left(1 - q_i - dq_j\right)q_i - \left(w - x_i - \beta x_j\right)q_i - \frac{g}{2}x_i^2, \tag{11}$$

where the upper script I/I stands for positive R&D investments of both firms. At the market stage of the game, each firm chooses the amount of output to maximise profits. Maximisation of (11) with respect to q_i leads to the following downward-sloping reaction function of firm i in the (q_i, q_j) space as a function also of R&D efforts x_i and x_j, that is:

$$\frac{\partial \Pi_i^{I/I}}{\partial q_i} = 0 \Leftrightarrow \overline{q}_i^{I/I}\left(q_j, x_i, x_j\right) = \frac{1-w-dq_j+x_i+\beta x_j}{2}. \tag{12}$$

Eq. (12) reveals that an increase in the R&D investment of firm i shift outwards its own reaction function the thereby contributing to increase production, whereas an increase in the R&D investment of the rival (firm j) shift outwards firm i's reaction function with a lower intensity as x_j can be beneficial for q_i only whether there exists positive R&D externalities ($\beta > 0$), so that R&D-related information can flow to competitors partially if $0 < \beta < 1$ or totally if $\beta = 1$ generating benefits without payments to the rival. In the last case, the x_j-related externality contributes (at its highest intensity) exactly as x_i as a device fostering production of firm i. Using Eq. (12) together with the symmetric counterpart for firm j allows to obtain the system of reaction functions in the (q_i, q_j) space as a function of the R&D effort. The solution of the system of output reaction functions $\overline{q}_i^{I/I}(q_j, x_i, x_j)$ ($i = \{1,2\}$, $i \neq j$) allows us to get the following equilibrium output obtained at the third stage of the symmetric subgame I/I:

$$\overline{\overline{q}}_i^{I/I}(x_i, x_j) = \frac{(1-w)(2-d)+(2-d\beta)x_i+(2\beta-d)x_j}{(2-d)(2+d)}. \tag{13}$$

Eq. (13) shows that output production of firm i depends on its own R&D investments (due to a twofold reason) as well as on R&D investments carried out by firm j (due to R&D the externality). On one hand, the R&D investment undertaken by firm i allows for a direct increase in the amount of its own output production (whose intensity is weighted by the coefficient 2) due to the strategic interaction with the rival. Therefore, firm i increases production through this channel. On the other hand, if products are substitutes ($d > 0$), there exists a mitigating effect of x_i on q_i because of the amount of R&D-related information externalities flowing from firm i to firm j (whose intensity is equal to $d\beta$). Therefore, firm i reduces production through this channel. However, the strength of the latter effect can never counterbalance the strength of the former, including under the most extreme conditions, i.e., when the products are perfect substitutes ($d = 1$) and disclosure is at its maximum intensity ($\beta = 1$). Definitively, an increase in x_i always causes an increase in q_i if products are substitutes. Product complementarity ($d < 0$), instead, sharply modifies the incentives on firm i's flowing through R&D towards firm j. Specifically, the mitigating effect becomes a strengthening one, so that there exists a positive effect on firm i's output of its own R&D externality flowing towards the rival. It is like that firm i would have the advantage of agreeing with firm j to disclose R&D information. This effect reaches its maximal effectiveness when products are perfect complements ($d = -1$) and disclosure is at its maximum intensity ($\beta = 1$). Additionally, the R&D disclosure of both firms allows for a further increase in output production of firm i through the R&D investments of firm j if and only if the extent of technological spill-overs is sufficiently large ($\beta > \frac{d}{2}$, i.e., x_i and x_j are strategic complements), turning otherwise to a reduction if the extent of technological spill-overs is sufficiently small ($\beta < \frac{d}{2}$, i.e., x_i and x_j are strategic substitutes). This holds when products are substitutes and the degree of product substitutability tends to counterbalance the positive feedback effect of the spill-overs, reaching its maximal intensity under product substitutability ($d = 1$). This implies that the higher the degree of product substitutability, the higher the need for firms to disclose R&D information to increase their own output in the product market. Differently, product complementarity works exactly out in the opposite direction thereby letting output production always being positively correlated with the amount R&D investment of the rival.

Substituting out Eq. (13) together its counterpart for firm j in Eq. (11) allows to obtain firm i's profits as a function of R&D efforts x_i and x_j, i.e., $\Pi_i^{I/I}(x_i, x_j)$. Following the tradition initiated by AJ in the non-cooperative version of the process investment R&D duopoly, firms maximise profits non-cooperatively at the second (R&D) stage of the game by choosing the amount of cost-reducing investment. Formally, this implies that:

$$\frac{\partial \Pi_i^{I/I}(x_i,x_j)}{\partial x_i} = 0 \Leftrightarrow \overline{x}_i^{I/I} = \frac{2(2-d\beta)\left[(2-d)(1-w)+(2\beta-d)x_j\right]}{g[16-d^2(8-d^2)]+2d\beta(4-d\beta)-8}. \tag{14}$$

Using Eq. (14) together with the corresponding counterpart for firm j allows to get the system of reaction functions in the R&D space, that is (x_i, x_j). Solving the system of the R&D reaction functions allows us to obtain the amount of equilibrium investment (denoted as usual with an asterisk) following the process innovation effort of firm i at the second stage of the game (and consequently the symmetrical firm j's response), that is:

$$x_i^{*I/I} = \frac{2(1-w)(2-d\beta)}{g(2-d)(2+d)^2-2(1+\beta)(2-d\beta)}. \tag{15}$$

From Eq. (15), $x_i^{*I/I} > 0$ if and only if the denominator is positive, that is $g > \frac{2(1+\beta)(2-d\beta)}{(2-d)(2+d)^2} := g_{SC}^{\beta_{high}}(\beta, d)$, as will be clear later (see Eq. (19)), where the subscript SC denotes "Stability Condition".

The second-order condition for a maximum (concavity) requires that $\left.\frac{\partial^2 \Pi_i^{I/I}(x_i,x_j)}{\partial x_i^2}\right|_{x_i=x_i^{*I/I}} < 0$. This implies that the inequality

$$g > \frac{2(2-d\beta)^2}{(2-d)^2(2+d)^2} := g_{SOC}(\beta, d) \text{ (second-order condition)}, \tag{16}$$

must hold to guarantee that the solution to the profit maximisation problem is economically meaningful, where the subscript SOC denotes "Second Order Condition". This condition boils down to $g > g_{SOC}(\beta, 1) := \frac{2}{9}(2-\beta)^2$ if $d = 1$, which replicates exactly the AJ's result in our normalised set up. The R&D equilibrium characterised by the expression in (15) is stable (in the sense of Seade, 1980) if and only if the reaction functions defined in the R&D space should adequately cross (Henriques, 1990). Indeed, Henriques (1990) and Bacchiega et al. (2010) find that the R&D reaction curves can be downward-sloping or upward-sloping depending on the relative size of β (the R&D externality). If they are downward-sloping (resp. upward-sloping), x_i and x_j are strategic substitutes (resp. complements). This holds when the R&D externality is small (resp. large). The stability conditions require that $\left|\frac{dx_i}{dx_j}\right| < 1$ in both cases of strategic substitutability and complementarity and thus lead to a relationship only between g and β in the case of perfect substitutability, i.e., $d = 1$ (Bacchiega et al., 2010). Differently, the R&D reaction curves in the case of product differentiation can be downward-sloping or upward-sloping depending on the relative size of β and d and consequently the stability conditions include three parameters (g, β and d). By computing the derivative

$$\frac{dx_i}{dx_j} = \frac{2(2\beta-d)(2-d\beta)}{g(2-d)^2(2+d)^2-2(2-d\beta)^2}, \tag{17}$$

the denominator is positive if and only if $g > g_{SOC}(\beta, d)$, which should always be fulfilled for concavity. Therefore, $\frac{dx_i}{dx_j} < 0$ if and only if $\beta < \frac{d}{2}$ (x_i and x_j are strategic substitutes) and $\frac{dx_i}{dx_j} > 0$ if and only if $\beta > \frac{d}{2}$ (x_i and x_j are strategic complements). These conditions boil down to those found by Bacchiega et al. (2010) under the assumption of perfect substitutability ($d = 1$). The stability conditions $\left|\frac{dx_i}{dx_j}\right| < 1$ in the R&D model with product differentiation require that one should impose:

$$g > \frac{2(1-\beta)(2-d\beta)}{(2-d)^2(2+d)} := g_{SC}^{\beta_{low}}(\beta, d) \text{ if } 0 \le \beta < \frac{d}{2}, \text{ (x_i and x_j are strategic substitutes)}, \tag{18}$$

and

$$g > \frac{2(1+\beta)(2-d\beta)}{(2-d)(2+d)^2} := g_{SC}^{\beta_{high}}(\beta, d) \text{ if } \frac{d}{2} < \beta \le 1, \text{ (x_i and x_j are strategic complements)}, \tag{19}$$

where $g_{SC}^{\beta_{low}}(\beta, d) > g_{SC}^{\beta_{high}}(\beta, d)$ for any $0 \le \beta \le \frac{d}{2}$ and $g_{SC}^{\beta_{low}}(\beta, d) < g_{SC}^{\beta_{high}}(\beta, d)$ for any $\frac{d}{2} \le \beta \le 1$. Therefore, the condition that guarantees positive R&D investments ($g > g_{SC}^{\beta_{high}}(\beta, d)$) from Eq. (15) and the second-order condition ($g > g_{SOC}(\beta, d)$) are fulfilled for any $0 \le \beta \le 1$ if the stability conditions are satisfied. We basically recall that the second-order condition and the stability conditions suggest that the efficiency of R&D activity should not be too high, i.e., parameter g should not be too low to avoid excessive R&D investments that would contribute to greatly reduce marginal and average production costs and increase output pushing down the market price of products of both varieties. This would reduce the market power of firms, the extent of which depends on the price elasticity of demand. Indeed, if $g > g_{SOC}(\beta, d)$, the percentage reduction in market price is high enough to let profits become negative (as will be clear from Eq. (24)).

A graph can help understanding this result. In this regard, Figure 1, drawn in the (β, g) space, depicts the second-order condition in (16), black line, and the stability conditions in (18) and (19), orange and green lines respectively, showing for the case of *homogeneous products* $(d = 1)$ that $g > g_{SOC}(\beta, d)$ holds for every g satisfying the stability conditions irrespective of the value of β for which (given d) x_i and x_j are strategic substitutes and strategic complements. If q_i and q_j are complements (negative values of d) the relevant stability condition is always given by the inequality in (19), i.e., x_i and x_j are strategic complements for any couple (β, g) in that case. An increase in the degree of product differentiation $(d \downarrow)$ changes the shape of the stability conditions in the (β, g) space. In particular, the β-threshold separating the region of strategic substitutability from the region of strategic complementarity, i.e., $\frac{d}{2}$, shifts leftward in the (β, g) space thus favouring strategic complementarity in the R&D effort that works therefore out as a device enforcing the R&D externality. Figure 2 contrasts Figure 1 and clearly shows this result for the case *heterogeneous products*, where Panel A refers to product substitutability $(d = 0.5)$ and Panel B to product complementarity $(d = -0.5)$. This can also be ascertained analytically by studying how $x_i^{*I/I}$ reacts to a change in β by comparing the cases of homogeneous and heterogeneous products. In fact,

$$\frac{\partial x_i^{*I/I}}{\partial \beta} = \frac{-2(1-w)\left[gd(2-d)(2+d)^2 - 2(2-d\beta)^2\right]}{\left[g(2-d)(2+d)^2 - 2(1+\beta)(2-d\beta)\right]^2}. \tag{20}$$

From Eq. (20), one can see that $\frac{\partial x_i^{*I/I}}{\partial \beta} < 0$ for any β and g when products are perfect substitutes $(d = 1)$ as $g > g_{SOC}(\beta, 1)$ must hold. In fact, as the externality of R&D investment becomes larger, each firm has an incentive to reduce its own amount of cost-reducing R&D investment as it can benefit from the externality resulting from the rival's investment. Instead, when products are differentiated $(d < 1)$ the effect on $x_i^{*I/I}$ of a change in β depends on whether products are substitutes $(d > 0)$ or complements $(d < 0)$. In the former case, $\frac{\partial x_i^{*I/I}}{\partial \beta} > 0$ if $g < \frac{g_{SOC}(\beta,d)}{d}$ and $\frac{\partial x_i^{*I/I}}{\partial \beta} < 0$ if $g > \frac{g_{SOC}(\beta,d)}{d}$, where $\frac{g_{SOC}(\beta,d)}{d} > g_{SOC}(\beta, d)$ for any $0 < d < 1$, and the difference between the two thresholds increases as the degree of product differentiation increases. An increase in product heterogeneity resulting from consumer preferences tends to promote R&D strategic complementarity as the degree of competition in the product market reduces and profits increases. Therefore, there is room for the joint use of resources devoted to R&D (i.e., the R&D externality of one firm favours the R&D investment of the rival and vice versa) if the efficiency of R&D activity is high enough (low values of g). In the latter case, instead, $\frac{\partial x_i^{*I/I}}{\partial \beta} > 0$ for any β and g as product complementarity promotes cooperative behaviour in the product market by letting firms act as if they were maximising joint profits, and this in turn eventually leads each of them to benefit from R&D complementarity.

The analysis made so far should be augmented with additional constraints on the side of the costs of production. Indeed, as we know from Eq. (2), the unitary production cost $w - x_i - \beta x_j$ as part of the total costs of production $C_i(q_i, x_i, x_j)$ must always be positive. Therefore, by using Eq. (15) the inequality $w - x_i - \beta x_j > 0$ is fulfilled if and only if:

$$g > \frac{2(1+\beta)(2-d\beta)}{w(2-d)(2+d)^2} := g_T^{I/I}(\beta, d, w), \text{ (R\&D cost condition)}, \tag{21}$$

where the subscript T stands for "Threshold".

The inequality in (21) must hold as an additional threshold in determining meaningful Nash equilibrium outcomes of the game, as will be clear from the analysis presented in Section 3.4. Figure 3 – plotted in the (β, g) space for four different values of w depicted in Panels A-D – however clarifies the behaviour of the threshold $g_T^{I/I}(\beta,d,w)$ for the subgame I/I (the blue line in the figure) by overlapping it to the stability conditions in (18) and (19) depicted in Figures 1 for the case of homogeneous products (the second-order condition was not drawn as it is always fulfilled once the stability conditions are satisfied). As can be seen, the shape of the threshold $g_T^{I/I}(\beta,d,w)$ also depends on w. Therefore, it is important to study when the threshold $g_T^{I/I}(\beta,d,w)$ is binding compared to the stability conditions *for the subgame I/I*.

Comparison of (19) and (21) easily reveals that $g_T^{I/I}(\beta,d,w) > g_{SC}^{\beta_{high}}(\beta,d)$ for any $w < 1$ and $g_T^{I/I}(\beta,d,1) \to g_{SC}^{\beta_{high}}(\beta,d)$ from above for $w \to 1$. Differently, comparison of (18) and (21) reveals that $g_T^{I/I}(\beta,d,w)$ can be higher or lower than $g_{SC}^{\beta_{low}}(\beta,d)$ depending on the relative size of β, d and w. Proposition 1 deepens this result and complements Figure 3 by showing that $g_T^{I/I}(\beta,d,w)$ can be binding in the (β,g) space depending on some conditions on the main parameters of the problem. Let us first define $\beta_T^{I/I} \coloneqq \frac{-(2-d)+w(2+d)}{2-d+w(2+d)}$ as a threshold value of the intensity of the R&D externality such that $g_T^{I/I}(\beta,d,w) = g_{SC}^{\beta_{low}}(\beta,d)$ in the (β,g) space. Then, $\beta_T^{I/I} \to \frac{d}{2}$ if $w \to 1$ and $\beta_T^{I/I} < \frac{d}{2}$ for any d and $w < 1$. In addition, $\beta_T^{I/I} < 0$ if $w < \frac{2-d}{2+d} \coloneqq w_T^{I/I}$ and $\beta_T^{I/I} > 0$ if $w > w_T^{I/I}$, where $w_T^{I/I} = \frac{1}{3}$ if $d = 1$, $w_T^{I/I} = 1$ if $d = 0$ and $w_T^{I/I} > 1$ if $d < 0$. Then, the following proposition holds.

Proposition 1. 1) If products are complements ($d < 0$) then $g_T^{I/I}(\beta,d,w)$ is binding in the (β,g) space for any for any $0 \leq \beta \leq 1$ and $0 < w < 1$ for the subgame I/I. 2.1) If products are substitutes ($d > 0$) and $w < w_T^{I/I}$ then $g_T^{I/I}(\beta,d,w)$ is binding in the (β,g) space for any β for the subgame I/I. 2.2) If products are substitutes ($d > 0$) and $w > w_T^{I/I}$ then 3.1) $g_{SC}^{\beta_{low}}(\beta,d)$ is binding in the (β,g) space for any $\beta < \beta_T^{I/I}$ for the subgame I/I, and 3.2) $g_T^{I/I}(\beta,d,w)$ is binding in the (β,g) space for any $\beta > \beta_T^{I/I}$ for the subgame I/I. 3) If products are substitutes ($d > 0$) and $w \to 1$ then $g_{SC}^{\beta_{low}}(\beta,d)$ is binding for any $0 \leq \beta < \frac{d}{2}$ and $g_{SC}^{\beta_{high}}(\beta,d)$ is binding for any $\frac{d}{2} < \beta \leq 1$ for the subgame I/I.

Proof. Appendix.

Proposition 1 clarifies the role of the unitary (and marginal) costs of productions in an R&D environment within the subgame I/I. Basically, it tells us that $g_T^{I/I}(\beta,d,w)$ tends to be binding for the subgame I/I when 1) products are complements as this kind of consumers' tastes favours R&D strategic complementarity by increasing the R&D investment of each firm, 2) products are substitutes but the unitary cost resulting from the technology of production of final goods is sufficiently low, and 3) product are substitutes and the extent of technological spill-overs is sufficiently high. In each of these cases, in fact, there are reasons for each firm to greatly reduce production costs. A huge reduction in the unitary (and marginal) production costs through R&D effort in fact implies a dramatic increase in output (as shown by the output reaction function Eq. (12)) and a corresponding reduction in the market price that erodes the market power of firms leading them to produce eventually quantities corresponding to the downward-sloping branch of their own total revenue function, where the price elasticity of demand is smaller than one, so that firms should reduce instead of increase production to maximise profits in that case. An increase in the degree of product differentiation ($d \downarrow$) has an ambiguous effect on $g_T^{I/I}(\beta,d,w)$ depending on the extent of the R&D externality (β). This can be ascertained by computing $\frac{\partial g_T^{I/I}(\beta,d,w)}{\partial d}$, whose sign is the same (and depends on the same thresholds) as the sign of $\frac{\partial x_i^{*I/I}}{\partial d}$. Indeed, this derivative is (*i*) positive (resp. negative) if $\frac{2}{3} < d < 1$

(resp. $-1 < d < \frac{2}{3}$) if $\beta = 0$, (*ii*) positive (resp. negative) if $\frac{3+\beta-\sqrt{(1-\beta)(9+7\beta)}}{2\beta} < d < 1$ (resp. $-1 < d < \frac{3+\beta-\sqrt{(1-\beta)(9+7\beta)}}{2\beta}$) for any $0 < \beta < \frac{1}{2}$, where $\frac{3+\beta-\sqrt{(1-\beta)(9+7\beta)}}{2\beta} > \frac{2}{3}$ and $\frac{3+\beta-\sqrt{(1-\beta)(9+7\beta)}}{2\beta} = 1$ if $\beta = \frac{1}{2}$, and (*iii*) negative for any $-1 < d < 1$ if $\frac{1}{2} < \beta \leq 1$. Therefore, product heterogeneity broadens the feasible parameter space bounded by the R&D cost condition ($g_T^{I/I}(\beta, d, w) \downarrow$) if the degree of product substitutability is sufficiently small (ranging from the cases of no externality up to half the strength of the externality of R&D information). If products are perceived as poorly differentiated, a reduction in d allows each firm to reduce R&D investment at the optimum (as product differentiation per se increases output and then reduces the need to invest in R&D) thus widening the parameter region in the (g, β) space in which the R&D cost condition is fulfilled. However, the larger the externality of the cost-reducing R&D activity (β), the smaller this effect.

Differently, product heterogeneity narrows the feasible parameter space bounded by the R&D cost condition ($g_T^{I/I}(\beta, d, w) \uparrow$) if the degree of product substitutability is sufficiently large. Unlike the previous case, in fact, when products of variety i and j are perceived as highly differentiated, a further reduction in d marks the start for an increase in R&D investment at the optimum to capture unilaterally the benefits of higher product differentiation, and the larger the externality of the cost-reducing R&D activity (β), the stronger this effect. Finally, if the extent of R&D externality is high enough ($\beta > \frac{1}{2}$) product differentiation always favours strategic complementarity of R&D effort and thus contributes to let the R&D cost condition become more and more binding in the (g, β) space for the subgame I/I.

We now move forward by continuing the equilibrium analysis of the subgame I/I. By using the symmetrical equilibrium R&D expression in (15) and substituting out for $x_i^{*I/I}$ in the equilibrium output obtained at the third stage of the game, one gets the amount of output produced by firm i ($i = \{1,2\}$, $i \neq j$) at equilibrium under I/I, that is:

$$q_i^{*I/I} = \frac{g(1-w)(2-d)(2+d)}{g(2-d)(2+d)^2 - 2(1+\beta)(2-d\beta)}. \tag{22}$$

Eq. (22) reveals that $g > g_{SC}^{\beta_{high}}(\beta, d)$ is sufficient to guarantee a positive output production for both firms. In addition, from the expressions in (15) and (22) one can easily get the equilibrium values of the market price of product of variety i and profits of firm i, which are respectively given by the following equations:

$$p_i^{*I/I} = \frac{g(2-d)(2+d)[1+w(1+d)] - 2(1+\beta)(2-d\beta)}{g(2-d)(2+d)^2 - 2(1+\beta)(2-d\beta)}, \tag{23}$$

where the denominator is positive if $g > g_{SC}^{\beta_{high}}(\beta, d)$, and

$$\Pi_i^{*I/I} = \frac{g(1-w)^2[g(2-d)^2(2+d)^2 - 2(2-d\beta)^2]}{[g(2-d)(2+d)^2 - 2(1+\beta)(2-d\beta)]^2}. \tag{24}$$

The expressions of the equilibrium price and equilibrium profits in (23) and (24) reveal that $p_i^{*I/I} > 0$ if $g > \frac{2(1+\beta)(2-d\beta)}{(2-d)(2+d)[1+w(1+d)]} := g_p^{I/I}(\beta, d, w)$ and $\Pi_i^{*I/I} > 0$ if $g > g_{SOC}(\beta, d)$. We note that $g_p^{I/I}(\beta, d, w) < g_{SC}^{\beta_{high}}(\beta, d)$ for any $0 < w < 1$ and $g_p^{I/I}(\beta, d, 1) \to g_{SC}^{\beta_{high}}(\beta, d)$ if $w \to 1$. Therefore, both thresholds are satisfied if either $g > g_T^{I/I}(\beta, d, w)$ or $g > g_{SC}^{\beta_{low}}(\beta, d)$ holds.

The equilibrium values of consumers' surplus ($CS^{*I/I}$) and producers' surplus ($PS^{*I/I}$) that can be obtained in the I/I subgame are summarised as follows:

$$CS^{*I/I} = \frac{1}{2}\left[\left(q_i^{*I/I}\right)^2 + \left(q_j^{*I/I}\right)^2 + 2dq_i^{*I/I}q_j^{*I/I}\right] = \frac{g^2(1-w)^2(1+d)(2-d)^2(2+d)^2}{[g(2-d)(2+d)^2 - 2(1+\beta)(2-d\beta)]^2}. \tag{25}$$

and

$$PS^{*I/I} = \Pi_i^{*I/I} + \Pi_j^{*I/I} = \frac{2g(1-w)^2[g(2-d)^2(2+d)^2 - 2(2-d\beta)^2]}{[g(2-d)(2+d)^2 - 2(1+\beta)(2-d\beta)]^2}. \tag{26}$$

Therefore, social welfare under I/I, $W^{*I/I}$, is given by:

$$W^{*I/I} = CS^{*I/I} + PS^{*I/I} = \frac{g(1-w)^2[g(3+d)(2-d)^2(2+d)^2 - 4(2-d\beta)^2]}{[g(2-d)(2+d)^2 - 2(1+\beta)(2-d\beta)]^2}. \quad (27)$$

Comparison between Eqs. (11) and (27) allows the get the following proposition.

Proposition 2. Social welfare under I/I is larger than social welfare under NI/NI.

Proof. Appendix.

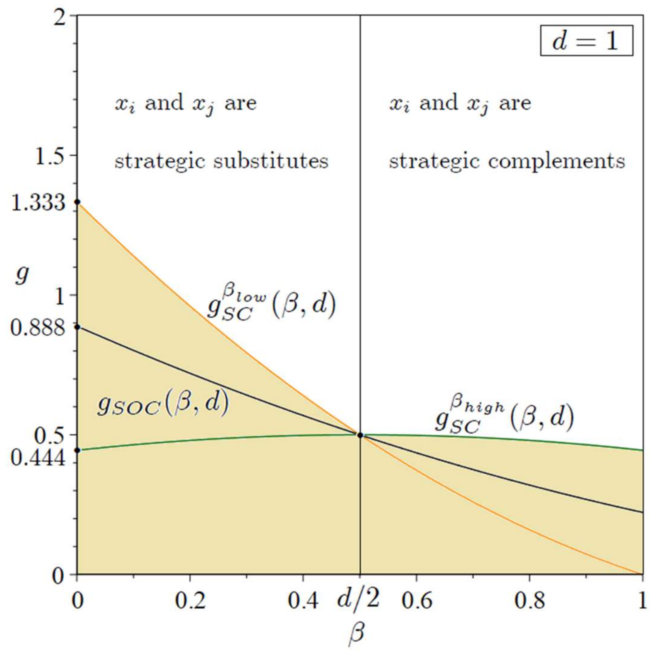
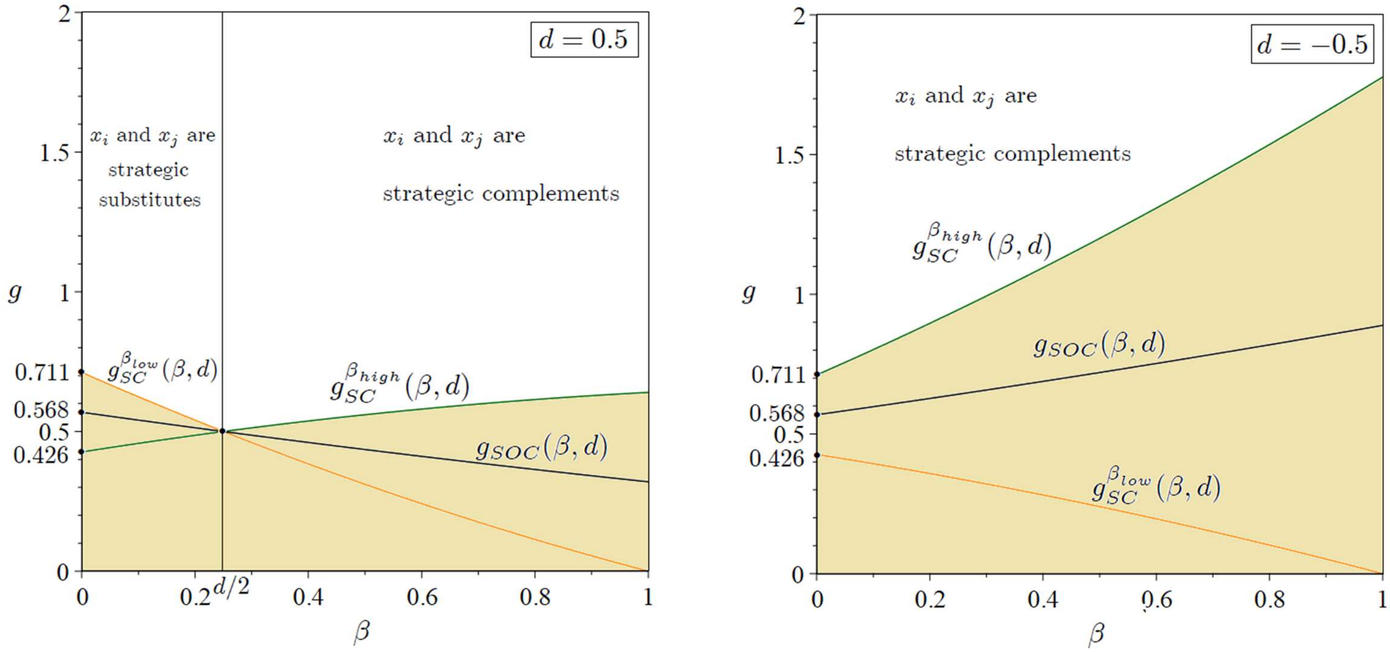
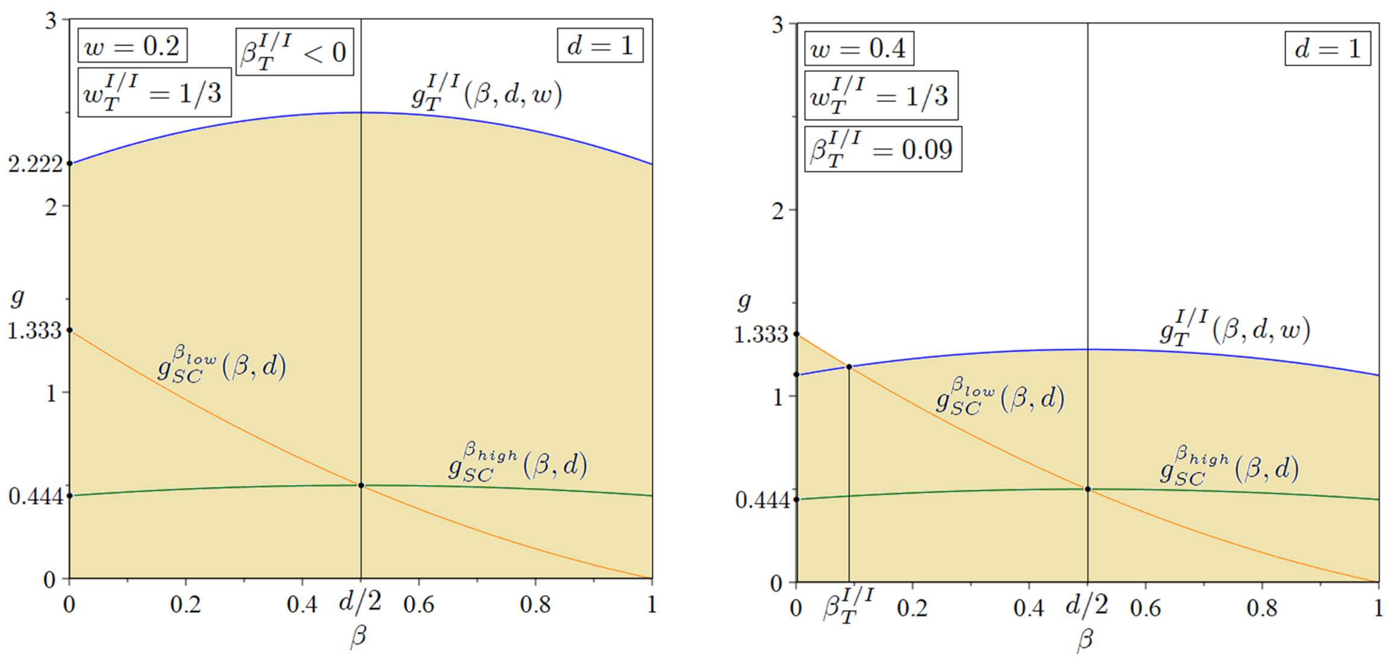


Figure 1. Second order condition (black line) and stability conditions (orange and green lines) in the (β, g) space when products are homogeneous $(d = 1)$. The sand-coloured area represents the parametric region of unfeasibility.

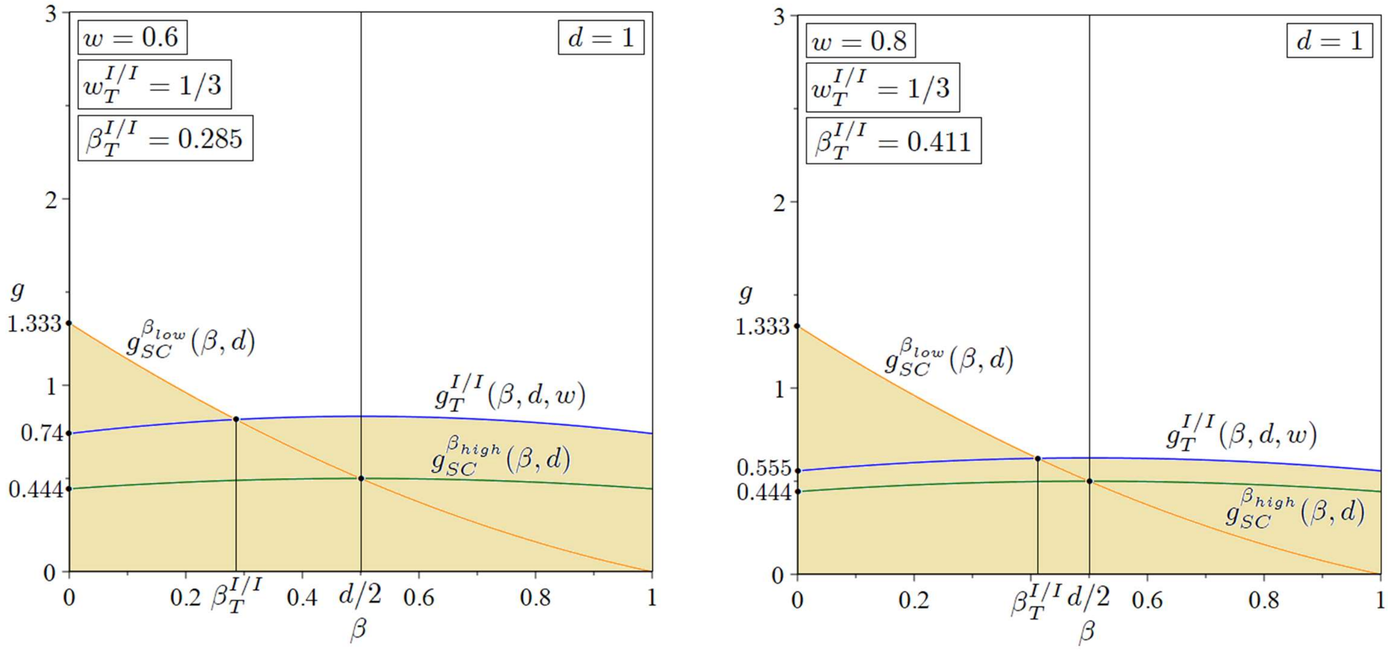


(A) (B)

Figure 2. Second order condition (black line) and stability conditions (orange and green lines) in the (β, g) space when products are substitutes ($d = 0.5$), Panel A, and complements ($d = -0.5$), Panel B. The sand-coloured area represents the parametric region of unfeasibility.



(A) (B)



(C) (D)

Figure 3. The R&D cost condition (blue line) and the stability conditions (orange and green lines) for different values of the unitary cost resulting from the technology of production ($0 < w < 1$) and product homogeneity ($d = 1$): Panel A: $w = 0.2$. Panel B: $w = 0.4$. Panel C: $w = 0.6$. Panel D: $w = 0.8$. The sand-coloured area represents the parametric region of unfeasibility.

3.3. The asymmetric subgame in which only one firm invest in R&D (I/NI)

This section continues the analysis made so far by considering the asymmetric subgame in which one firm invests in R&D (say, firm i) and the rival does not (say, firm j). As the R&D investment of firm i is positive and the R&D investment of firm j is zero, we have that $x_i > 0$ and $X_i(x_i) > 0$, and $x_j = 0$ and $X_j(x_j) = 0$. Therefore, on one hand, firm i invests in process innovation allowing to reduce its own average and marginal production costs but does not benefit from any stream of knowledge (externality) related to firm j's R&D activity, which is absent in this case. On the other hand, firm j does not invest in process innovation but it can benefit from a stream of knowledge (externality) related to firm i's R&D activity, whose extent is measured by the parameter β. Therefore, by using Eqs. (1), (2) and (3) the profit functions of firm i and firm j read now respectively as follows:

$$\Pi_i^{I/NI} = p_i q_i - C_i(q_i, x_i, 0) - X_i(x_i) = (1 - q_i - dq_j)q_i - (w - x_i)q_i - \frac{g}{2}x_i^2, \tag{28}$$

and

$$\Pi_j^{I/NI} = p_j q_j - C_j(q_j, 0, x_i) = (1 - q_j - dq_i)q_j - (w - \beta x_i)q_j, \tag{29}$$

where the upper script I/NI stands for positive R&D investments of firm i and no R&D investments of firm j. At the market stage of the game, each firm chooses the optimal amount of output production by maximising profits. Therefore, the maximisation of (28) and (29) with respect to q_i and q_j, respectively, leads to the following downward-sloping output reaction function of firm i and firm j in the (q_i, q_j) space as a function also of the R&D effort exerted by firm i (x_i), that is:

$$\frac{\partial \Pi_i^{I/NI}}{\partial q_i} = 0 \Leftrightarrow \overline{q}_i^{I/NI}(q_j, x_i) = \frac{1 - w - dq_j + x_i}{2}, \tag{30}$$

and

$$\frac{\partial \Pi_j^{I/NI}}{\partial q_j} = 0 \Leftrightarrow \overline{q}_j^{I/NI}(q_i, x_i) = \frac{1-w-dq_i+\beta x_i}{2}. \quad (31)$$

Eq. (30) and (31) reveal that the output reaction functions of both firms i and j in the asymmetric subgame are similar, differing however in one crucial respect: an increase in the R&D investment of firm i, in fact, shift outwards both reaction functions, thereby contributing to increase production of firm i and firm j, but the extent of the upward shift of the output reaction function of firm i – that sustains process innovation R&D – is larger than the upward shift of the output reaction function of firm j – that does not incur in R&D costs and can benefit only partially ($\beta < 1$) by the positive R&D externality generated by the rival. The extent of the upward shift of the reaction function of firm j is the same as that of firm i only under full disclosure of R&D-related information by firm i ($\beta = 1$). The solution of the system of output reaction functions $\overline{q}_i^{I/NI}(q_j, x_i)$ and $\overline{q}_j^{I/NI}(q_i, x_i)$ ($i = \{1,2\}$, $i \neq j$) allows us to get the following equilibrium output obtained by firms i and j at the third stage of the asymmetric subgame I/NI, that is:

$$\overline{\overline{q}}_i^{I/NI}(x_i) = \frac{(1-w)(2-d)+(2-d\beta)x_i}{(2-d)(2+d)}. \quad (32)$$

and

$$\overline{\overline{q}}_j^{I/NI}(x_i) = \frac{(1-w)(2-d)+(2\beta-d)x_i}{(2-d)(2+d)}. \quad (33)$$

A direct comparison of Eqs. (32) and (33) with Eq. (13) allows to conclude that if products are substitutes ($d > 0$), depending on the extent of technological spill-overs, the amount of output production of the investing firm i at the third stage of the game in the asymmetric subgame I/NI is smaller (resp. larger) that the amount of output production of firm i at the third stage of the game in the symmetric subgame I/I. In particular, $\overline{\overline{q}}_i^{I/I}(x_i, x_j) < \overline{\overline{q}}_i^{I/NI}(x_i)$ if $\beta < \frac{d}{2}$ (i.e., x_i and x_j are strategic substitutes) and $\overline{\overline{q}}_i^{I/I}(x_i, x_j) > \overline{\overline{q}}_i^{I/NI}(x_i)$ if $\beta > \frac{d}{2}$ (i.e., x_i and x_j are strategic complements). In addition, the amount of output production of the non-investing firm j at the third stage of the game in the asymmetric subgame I/NI is always smaller than the amount of output production obtained by the corresponding firm at the third stage of the game in the symmetric subgame I/I. Differently, if products are complements ($d < 0$) the output production of both firms in the symmetric subgame I/I is always larger than the output production of the investing and non-investing firms in the asymmetric subgame I/NI.

Substituting out Eqs. (32) and (33) in the profit equations (28) and (29) allows to obtain profits of the investing and non-investing firms as a function of the R&D effort x_i, i.e., $\Pi_i^{I/NI}(x_i)$ and $\Pi_j^{I/NI}(x_i)$. At the second (R&D) stage of the game, the investing firm i maximises its own profits $\Pi_i^{I/NI}(x_i)$ by choosing the amount of cost-reducing investment. Formally, this implies that:

$$\frac{d\Pi_i^{I/NI}(x_i)}{dx_i} = 0 \Leftrightarrow x_i^{*I/NI} = \frac{2(1-w)(2-d)(2-d\beta)}{g(2-d)^2(2+d)^2-2(2-d\beta)^2}. \quad (34)$$

As only firm i does invest in R&D in this subgame, there are no R&D reaction functions so that $x_i^{*I/NI}$ represents the amount of equilibrium investment following the process innovation effort of firm i. The denominator of the expression of $x_i^{*I/NI}$ in (34) should be positive to guarantee positive R&D effort. By imposing $x_i^{*I/NI} > 0$ one gets $g > \frac{2(2-d\beta)^2}{(2-d)^2(2+d)^2} \coloneqq g_{SOC}(\beta, d)$, which coincides with the second-order condition in (16). Therefore, the condition $x_i^{*I/NI} > 0$ is always fulfilled once the stability conditions in (18) and (19) and the R&D cost condition in (21) resulting from the symmetric subgame in which both firms do invest in process innovation are satisfied. Differentiating the expression in (34) with respect to β gives $\frac{\partial x_i^{*I/NI}}{\partial \beta} < 0$. In fact, as the externality of the R&D investment generated by the investing firm becomes larger, the rival can free ride at a higher degree so firm i is incentivised to reduce its own amount of cost-reducing R&D investment, otherwise it would favour to much the rival by reducing its own production cost.

Like the symmetric subgame I/I, we should augment the analysis by considering the additional constraints on the side of the costs of production of both the investing firm i and non-investing firm j by explicitly accounting for their own R&D cost conditions resulting from the inequalities $w - x_i > 0$ for the investing firm i and $w - \beta x_i > 0$ for the non-investing free-riding firm j, as part of their own total costs of production $C_i(q_i, x_i, 0)$ and $C_j(q_j, 0, x_i)$, respectively. This can be specialised by substituting out $x_i^{*I/NI}$ from (34) into the last inequalities showing that they are fulfilled if and only if:

$$g > \frac{2(2-d\beta)[2-d+dw(1-\beta)]}{w(2-d)^2(2+d)^2} := g_T^{I/NI}(\beta, d, w), \text{ (R\&D cost condition of the investing firm).} \quad (35)$$

and

$$g > \frac{2(2-d\beta)[2w(1-\beta)+\beta(2-d)]}{w(2-d)^2(2+d)^2}, \text{ (R\&D cost condition of the non-investing firm).} \quad (36)$$

The condition in (36) is never binding as $g_T^{I/NI}(\beta, d, w) > \frac{2(2-d\beta)[2w(1-\beta)+\beta(2-d)]}{w(2-d)^2(2+d)^2}$ is always fulfilled in the asymmetric subgame I/NI. Differently, the expression in (35) adds another (the last!) threshold to the stability condition (18), i.e., $g_{SC}^{\beta_{low}}(\beta, d)$, and the R&D cost condition of the symmetric subgame I/I (21), i.e., $g_T^{I/I}(\beta, d, w)$, in determining the feasible region for the emergence of meaningful Nash equilibrium outcomes of the investment decision game with product differentiation (which is drawn as the red line in Figures 4-7). The comparison amongst the shapes of the relevant constraints of the game $g_{SC}^{\beta_{low}}(\beta, d)$, $g_T^{I/I}(\beta, d, w)$ and $g_T^{I/NI}(\beta, d, w)$ as well as their position in the (β, g) space is a complex exercise depending, amongst others, on the relative size of w. Therefore, to avoid lengthening the exposition further with additional graphs, we leave the complete geometrical analysis to Section 3.4, dealing with the study of the Nash equilibria of the game and related discussion. However, the Proposition 3 below resembles Proposition 1 and examines further this issue analytically by definitively clarifying the position of $g_{SC}^{\beta_{low}}(\beta, d)$, $g_T^{I/I}(\beta, d, w)$ and $g_T^{I/NI}(\beta, d, w)$ in the (β, g) space depending on β, d and w, showing therefore which constraint is definitively binding *for the R&D investment decision game*.

Let us first consider the relationship between $g_T^{I/I}(\beta, d, w)$ and $g_T^{I/NI}(\beta, d, w)$ and define

$$\beta_{T_1}^{I/NI}(d, w) := \frac{dw}{2-d(1-w)} < 1, \quad (37)$$

as a threshold value of the intensity of the R&D externality such that $g_T^{I/NI}(\beta, d, w) = g_T^{I/I}(\beta, d, w)$ in the (β, g) space. If products are complements ($d < 0$) then $\beta_{T_1}^{I/NI}(d, w) < 0$ and $g_T^{I/I}(\beta, d, w) > g_T^{I/NI}(\beta, d, w)$ for any $0 \le \beta \le 1$. If products are substitutes ($d > 0$) then $\beta_{T_1}^{I/NI}(d, w) > 0$ and 1) $g_T^{I/NI}(\beta, d, w) > g_T^{I/I}(\beta, d, w)$ for any $0 \le \beta < \beta_{T_1}^{I/NI}(d, w)$, 2) $g_T^{I/I}(\beta, d, w) > g_T^{I/NI}(\beta, d, w)$ for any $\beta_{T_1}^{I/NI}(d, w) < \beta \le 1$. If $w \to 1$ then $\beta_{T_1}^{I/NI}(d, 1) \to \frac{d}{2}$, $g_T^{I/I}(\beta, d, 1) \to g_{SC}^{\beta_{high}}(\beta, d)$, and $g_T^{I/NI}(\beta, d, 1) > g_{SC}^{\beta_{high}}(\beta, d) = g_T^{I/I}(\beta, d, 1)$ for any $0 \le \beta < \frac{d}{2}$ and $g_T^{I/NI}(\beta, d, 1) < g_{SC}^{\beta_{high}}(\beta, d) = g_T^{I/I}(\beta, d, 1)$ for any $\frac{d}{2} < \beta \le 1$.

Consider now the relationship between $g_{SC}^{\beta_{low}}(\beta, d)$ and $g_T^{I/NI}(\beta, d, w)$ and define

$$\beta_{T_2}^{I/NI}(d, w) := \frac{d-2(1-w)}{2w} < 1, \quad (38)$$

as a threshold value of the intensity of the R&D externality such that $g_T^{I/NI}(\beta, d, w) = g_{SC}^{\beta_{low}}(\beta, d)$ in the (β, g) space, where $\beta_{T_2}^{I/NI}(d, w) < \beta_{T_1}^{I/NI}(d, w)$ and

$$d_T^{I/NI}(w) := 2(1-w), \quad (39)$$

as a threshold value of the degree of product differentiation such that $\beta_{T_2}^{I/NI}(d,w)=0$, where $d_T^{I/NI}(0)=2$ if $w\to 0$, $d_T^{I/NI}\left(\frac{1}{2}\right)=1$ if $w=\frac{1}{2}$ and $d_T^{I/NI}(1)=0$ if $w\to 1$, so that $d_T^{I/NI}(w)>1$ for any $0<w<\frac{1}{2}$ and $0<d_T^{I/NI}(w)\leq 1$ for any $\frac{1}{2}\leq w<1$.

By looking at the expression of $\beta_{T_2}^{I/NI}(d,w)$ in (38) it is clear that $\beta_{T_2}^{I/NI}(d,w)<0$ for any $0<w<1$ and $-1\leq d\leq 0$, $\beta_{T_2}^{I/NI}(d,w)<0$ for any $0<w<\frac{1}{2}$ and $0\leq d\leq 1$, $\beta_{T_2}^{I/NI}(d,w)<0$ for any $\frac{1}{2}\leq w<1$ and $0\leq d<d_T^{I/NI}(w)$, and $\beta_{T_2}^{I/NI}(d,w)>0$ for any $\frac{1}{2}\leq w<1$ and $d_T^{I/NI}(w)\leq d\leq 1$. Therefore,

1) if products are complements ($d<0$) then $\beta_{T_2}^{I/NI}(d,w)<0$ and $g_T^{I/NI}(\beta,d,w)>g_{SC}^{\beta_{low}}(\beta,d)$ for any $0\leq\beta\leq 1$ and $0<w<1$ in the (β,g) space;
2) if products are substitutes ($d>0$), $0<w<\frac{1}{2}$ and $0<d\leq 1$ then $\beta_{T_2}^{I/NI}(d,w)<0$ and $g_T^{I/NI}(\beta,d,w)>g_{SC}^{\beta_{low}}(\beta,d)$ for any $0\leq\beta\leq 1$;
3) if products are substitutes ($d>0$), $\frac{1}{2}\leq w<1$ and $0<d<d_T^{I/NI}(w)$ then $\beta_{T_2}^{I/NI}(d,w)<0$ and $g_T^{I/NI}(\beta,d,w)>g_{SC}^{\beta_{low}}(\beta,d)$ for any $0\leq\beta\leq 1$;
4) if products are substitutes ($d>0$), $\frac{1}{2}\leq w<1$ and $d_T^{I/NI}(w)\leq d\leq 1$ then $\beta_{T_2}^{I/NI}(d,w)\geq 0$ and 1) $g_T^{I/NI}(\beta,d,w)<g_{SC}^{\beta_{low}}(\beta,d)$ for any $0\leq\beta<\beta_{T_2}^{I/NI}(d,w)$ and 2) $g_T^{I/NI}(\beta,d,w)>g_{SC}^{\beta_{low}}(\beta,d)$ for any $\beta_{T_2}^{I/NI}(d,w)<\beta\leq 1$;
5) If products are substitutes ($d>0$) and $w\to 1$ then $d_T^{I/NI}(w)\to 0$, $\beta_{T_2}^{I/NI}(d,1)\to\frac{d}{2}$, $g_T^{I/I}(\beta,d,1)\to g_{SC}^{\beta_{high}}(\beta,d)$, and $g_{SC}^{\beta_{low}}(\beta,d)>g_T^{I/NI}(\beta,d,1)>g_{SC}^{\beta_{high}}(\beta,d)=g_T^{I/I}(\beta,d,1)$ for any $0\leq\beta<\frac{d}{2}$ and $g_{SC}^{\beta_{high}}(\beta,d)=g_T^{I/I}(\beta,d,1)>g_T^{I/NI}(\beta,d,1)>g_{SC}^{\beta_{low}}(\beta,d)$ for any $\frac{d}{2}<\beta\leq 1$.

These arguments together with those used to state Proposition 1 allow us to write down the following proposition to definitively clarify which constraint is binding in the (β,g) space for the investment decision game.

Proposition 3. The relevant constraints of the R&D investment decision game with quantity competition and product differentiation are the following.

[1] If products are complements ($d<0$) then $g_T^{I/I}(\beta,d,w)$ is binding in the (β,g) space for any $0\leq\beta\leq 1$ and $0<w<1$ for the investment decision game.

[2] If products are substitutes ($d>0$) and $0<w<\frac{1}{2}$ then $g_T^{I/NI}(\beta,d,w)$ is binding for any $0\leq\beta<\beta_{T_1}^{I/NI}(d,w)$ and $g_T^{I/I}(\beta,d,w)$ is binding for any $\beta_{T_1}^{I/NI}(d,w)<\beta\leq 1$ for the investment decision game.

[3] If products are substitutes ($d>0$), $\frac{1}{2}\leq w<1$ and $d_T^{I/NI}(w)\leq d\leq 1$ then $g_{SC}^{\beta_{low}}(\beta,d)$ is binding for any $0\leq\beta<\beta_{T_2}^{I/NI}(d,w)$, $g_T^{I/NI}(\beta,d,w)$ is binding for any $\beta_{T_2}^{I/NI}(d,w)<\beta<\beta_{T_1}^{I/NI}(d,w)$ and $g_T^{I/I}(\beta,d,w)$ is binding $\beta_{T_1}^{I/NI}(d,w)<\beta\leq 1$ for the investment decision game.

[4] If products are substitutes ($d > 0$), $\frac{1}{2} \leq w < 1$ and $0 < d < d_T^{I/NI}(w)$ then $g_T^{I/NI}(\beta, d, w)$ is binding for any $0 \leq \beta < \beta_{T_1}^{I/NI}(d, w)$ and $g_T^{I/I}(\beta, d, w)$ is binding for any $\beta_{T_1}^{I/NI}(d, w) < \beta \leq 1$ for the investment decision game.

[5] If products are substitutes ($d > 0$) and $w \to 1$ then $g_{SC}^{\beta_{low}}(\beta, d)$ is binding for any $0 \leq \beta < \frac{d}{2}$ and $g_{SC}^{\beta_{high}}(\beta, d)$ is binding for any $\frac{d}{2} < \beta \leq 1$ for the investment decision game.

Proof. Appendix.

Proposition 3 allows us to make it clear the constraints emerging in the non-cooperative version of the d'Aspremont and Jacquemin's (1988, 1990) cost-reducing R&D model with spill-overs with homogeneous or heterogeneous products and let the R&D investment decision game be meaningful. These constraints are the stability condition emerging when x_i and x_j are strategic substitutes or strategic complements, the R&D cost condition emerging in the symmetric subgame I/I and the R&D cost condition emerging in the asymmetric subgame I/NI. Basically, the proposition tells us that $g_T^{I/I}(\beta, d, w)$ is always binding for the R&D investment decision game when products are complements (irrespective of the other parameters of the model). This is because this kind of consumers' tastes favours R&D strategic complementarity by incentivising both firms to invest in R&D to increase their own profits. Differently, product substitutability increases the complexity and let spill-overs and the unitary cost become relevant in determining the feasibility conditions.

Specifically, when the unitary production cost is sufficiently small ($0 < w < \frac{1}{2}$), the relevant constraint is $g_T^{I/NI}(\beta, d, w)$ if the R&D externality is low enough ($0 \leq \beta < \beta_{T_1}^{I/NI}(d, w)$). In this case, in fact, there are conditions for one (and only one) firm to invest in R&D as the non-investing firm (in the asymmetric subgame) cannot free ride at a high degree, so the investing firm is incentivised to keep its own cost-reducing investment high (whose size however reduces as far as β increases). When the unitary production cost is sufficiently high and products are perceived ad highly differentiated by customers ($\frac{1}{2} \leq w < 1$ and $0 < d < d_T^{I/NI}(w)$) but the extent of technological spill-overs is sufficiently high ($\beta_{T_1}^{I/NI}(d, w) < \beta \leq 1$) the incentive for only one firm to invest in R&D is reduced as it would be making its rival benefit too much through the degree of R&D externality by the cost-reducing investment activity. Therefore, the rival starts investing to avoid losing the opportunity to increase profits through the cost-reducing R&D effort, and the relevant constraint becomes $g_T^{I/I}(\beta, d, w)$.

When the unitary production cost is sufficiently high and products are perceived as poorly differentiated by customers ($\frac{1}{2} \leq w < 1$ and $d_T^{I/NI}(w) \leq d \leq 1$), i.e., they tend to be highly substitutable or perfect substitutes, the stability condition $g_{SC}^{\beta_{low}}(\beta, d)$ becomes the relevant constraint when the extent of technological spill-overs is sufficiently small ($0 \leq \beta < \beta_{T_2}^{I/NI}(d, w)$) as the R&D cost conditions tend to be always fulfilled in this case (i.e., thy become less important in defining the boundaries of the feasible region of the R&D investment decision game). Increases in the degree of the R&D externality however let the R&D cost conditions become the relevant thresholds to define meaningfulness of the R&D investment decision game for the same reasons discussed so far.

We now continue the equilibrium analysis of the asymmetric subgame I/NI. By using the expression in (34) and substituting out for $x_i^{*I/NI}$ in the equilibrium output obtained at the third stage of the game, one gets the amount of output produced by the investing firm i and the non-investing firm i ($i = \{1,2\}$, $i \neq j$) at equilibrium under I/NI, that is:

$$q_i^{*I/NI} = \frac{g(1-w)(2-d)^2(2+d)}{g(2-d)^2(2+d)^2-2(2-d\beta)^2}, \quad (40)$$

and

$$q_j^{*I/NI} = \frac{(1-w)\left[g(2-d)^2(2+d)-2(1-\beta)(2-d\beta)\right]}{g(2-d)^2(2+d)^2-2(2-d\beta)^2}. \quad (41)$$

Eqs. (40) and (41) reveal that the denominator is positive for any $g > g_{SOC}(\beta, d)$, which is always fulfilled if the other constraints discussed in Proposition 3 hold. Therefore, $q_i^{*I/NI} > 0$ always holds and $q_j^{*I/NI} > 0$ if and only if $g > g_{SC}^{\beta_{low}}(\beta, d)$, which is one of the relevant constraints discussed so far.

From the equations determining the equilibrium quantities in the subgame I/NI as expressed in (40) and (41), one can get the equilibrium values of the market price of product of variety i and variety j and the corresponding profit equations, that is:

$$p_i^{*I/NI} = \frac{g(2-d)^2(2+d)[1+w(1+d)]-2(2-d\beta)[2-d+d\ \ (1-\beta)]}{g(2-d)^2(2+d)^2-2(2-d\beta)^2}, \quad (42)$$

$$p_j^{*I/NI} = \frac{g(2-d)^2(2+d)[1+w(1+d)]-2(2-d\beta)[1+w(1-\beta)+\beta(1-d)]}{g(2-d)^2(2+d)^2-2(2-d\beta)^2}, \quad (43)$$

$$\Pi_i^{*I/NI} = \frac{g(1-w)^2(2-d)^2}{g(2-d)^2(2+d)^2-2(2-d\beta)^2}, \quad (44)$$

and

$$\Pi_j^{*I/NI} = \frac{(1-w)^2\left[g(2-d)^2(2+d)-2(1-\beta)(2-d\beta)\right]^2}{[g(2-d)^2(2+d)^2-2(2-d\beta)^2]^2}. \quad (45)$$

Eqs. (42) and (43) easily reveal that $p_i^{*I/NI} > 0$ if $g > \frac{2(2-d\beta)[2-d+dw(1-\beta)]}{(2-d)^2(2+d)[1+w(1+d)]} := g_{p_i}^{I/NI}(\beta, d, w)$ and $p_j^{*I/NI} > 0$ if $g > \frac{2(2-d\beta)[1+w(1-\beta)+\beta(1-d)]}{(2-d)^2(2+d)[1+w(1+d)]} := g_{p_j}^{I/NI}(\beta, d, w)$.

In addition, $g_T^{I/NI}(\beta, d, w) > g_{p_i}^{I/NI}(\beta, d, w)$, $g_T^{I/I}(\beta, d, w) > g_{p_i}^{I/NI}(\beta, d, w)$, $g_T^{I/NI}(\beta, d, w) > g_{p_j}^{I/NI}(\beta, d, w)$, $g_T^{I/I}(\beta, d, w) > g_{p_j}^{I/NI}(\beta, d, w)$ and both $g_{p_i}^{I/NI}(\beta, d, w)$ and $g_{p_j}^{I/NI}(\beta, d, w)$ are never binding in the (β, g) space and thus they are always fulfilled if the three constraints discussed in Proposition 3 are binding for any $-1 \le d \le 1$, $0 \le \beta \le 1$ and $0 < w < 1$.

If $w \to 1$ then $g_{p_i}^{I/NI}(\beta, d, 1) \to g_{SOC}(\beta, d)$ and $g_{p_j}^{I/NI}(\beta, d, 1) \to g_{SOC}(\beta, d)$, which are always fulfilled. Finally, from the expressions of the equilibrium profits in (44) and (45) one gets $\Pi_i^{*I/NI} > 0$ if $g > g_{SOC}(\beta, d)$ and $\Pi_j^{*I/NI} > 0$ and it is economically meaningful for any $g > g_{SOC}(\beta, d)$ and $g > g_{SC}^{\beta_{low}}(\beta, d)$, which are fulfilled for any β if the stability conditions are satisfied.

We avoid introducing the equilibrium values of consumers' surplus ($CS^{*I/NI}$) and producers' surplus ($PS^{*I/NI}$) for the I/NI subgame (along with the corresponding social welfare $W^{*I/NI}$) because, as we will see later, there are no asymmetric Nash equilibria in the R&D investment decision game and we do not want to lengthen the analysis further.

The next section studies the owner's choice whether to invest in R&D activities at stage one of the R&D game, and then provides a Nash equilibrium analysis along with the corresponding discussion.

3.4. The investment-decision stage under quantity competition: Nash equilibria and discussion

This section examines the first stage of the game, in which firms choose whether to invest in R&D in a non-cooperative quantity-setting environment à la d'Aspremont and Jacquemin's (1988, 1990), by also clarifying the role of the constraints (the stability conditions and the R&D cost conditions) of the R&D investment decisions game and stressing the differences between homogeneous and heterogeneous products.

Making use of the firms' profits in (8) for the symmetric subgame NI/NI, (24) for the symmetric subgame I/I, (44) and (45) for the asymmetric subgame I/NI, it is possible to build on the payoff matrix summarised in Table 1 regarding the Cournot R&D game with product differentiation (following the analysis of Bacchiega et al., 2010).

Table 1. The investment-decision game (payoff matrix). Cournot competition.

Firm 2 $\rightarrow$ Firm 1 $\downarrow$	I	NI
I	$\Pi_1^{*I/I}, \Pi_2^{*I/I}$	$\Pi_1^{*I/NI}, \Pi_2^{*I/NI}$
NI	$\Pi_1^{*NI/I}, \Pi_2^{*NI/I}$	$\Pi_1^{*NI/NI}, \Pi_2^{*NI/NI}$

To satisfy the technical restrictions and have well-defined equilibria in pure strategies for every strategic profile (one for each player), the analysis is restricted to the feasibility constraints discussed in Proposition 3 (which is assumed to be always satisfied henceforth). Then, to derive all the possible equilibria of the game, one must study the sign of the following profit differentials for $i = \{1,2\}, i \neq j$, that is:

$$\Delta\Pi_A = \Pi_i^{*I/NI} - \Pi_i^{*NI/NI} = \frac{2(1-w)^2(2-d\beta)^2}{(2+d)^2[g(2-d)^2(2+d)^2-2(2-d\beta)^2]} > 0, \tag{46}$$

$$\Delta\Pi_B = \Pi_i^{*NI/I} - \Pi_i^{*I/I} =$$

$$\frac{-2(1-w)^2(2-d\beta)^2}{[g(2-d)^2(2+d)^2-2(2-d\beta)^2]^2[g(2-d)(2+d)^2-2(1+\beta)(2-d\beta)]^2}$$
$$\begin{aligned}&\times\{d^8g^3 - 6d^6g^2\beta^2 - 16d^6g^3 + 12d^4g\beta^2 + 4d^6g^2 + 8d^5g^2\beta + 64d^4g^2\beta^2 \\ &-8d^2\beta^6 + 96d^4g^3 - 16d^4g\beta^2 - 32d^3g\beta^3 - 64d^2g\beta^4 - 56d^4g^2 - 64d^3g^2\beta \\ &-224d^2g^2\beta^2 + 16d^4\beta^4 + 32d\beta^5 + 64d^3g\beta - 256d^2g^3 + 32d^2g\beta^2 \\ &+256dg\beta^2 + 256d^2g^2 - 8d^2\beta^2 + 128dg^2\beta - 64d\beta^3 + 256g^2\beta^2 - 32\beta^4 - 64d^2g \\ &-128dg\beta + 256g^3 - 256g\beta^2 + 32d\beta - 384g^2 + 64\beta^2 + 192g - 32\} < 0\end{aligned}, \tag{47}$$

and

$$\Delta\Pi_C = \Pi_i^{*NI/NI} - \Pi_i^{*I/I} = \frac{2(1-w)^2(2-d\beta)\{g(2+d)^2[2(1-d)+\beta(4-d)]-2(1+\beta)^2(2-d\beta)\}}{(2+d)^2[g(2-d)(2+d)^2-2(1+\beta)(2-d\beta)]^2}. \tag{48}$$

Eqs. (46) and (47) reveal that $\Delta\Pi_A > 0$ and $\Delta\Pi_B < 0$ irrespective of the parameter scale, whereas from Eq. (48) we have that $\Delta\Pi_C$ can be positive or negative depending on the relative values of β, g and d. In this regard, let

$$g_C(\beta, d) := \frac{2(1+\beta)^2(2-d\beta)}{(2+d)^2[2(1-d)+\beta(4-d)]}, \tag{49}$$

be the threshold value of g as a function of the intensity of the spill-overs effect and the degree of product differentiation such that $\Delta\Pi_C = 0$. If $g < g_C(\beta, d)$ then $\Delta\Pi_C > 0$ and profits both firms under the strategic profile NI/NI are higher than profits under the strategic profile I/I. If $g > g_C(\beta, d)$ then $\Delta\Pi_C < 0$ and profits both firms under the strategic profile I/I are higher than profits under the strategic profile NI/NI.

Eq. (49) highlights the most important difference between the R&D investment decision game with homogeneous products (Bacchiega et al., 2010) and the R&D investment decision game with heterogeneous products, as was already pinpointed in the introduction. Indeed, our results differ from those of Bacchiega et al (2010) in a crucial respect: in their work, the prisoner's dilemma vanishes if and only if the extent of technological spill-overs is positive and sufficiently high, which, in turn, would require that firms disclose (or – equivalently – they are unable to keep closed) the information on the results of their own R&D investment at a certain degree. However, the non-disclosure (i.e., keeping secret) R&D-related result in the AJ setting is in the unilateral interest of each non-cooperative firm. *The prisoner's dilemma can vanish also in the absence of R&D spill-overs if the* *degree of product differentiation is sufficiently high*. In fact, when $d = 1$ Eq. (49) boils down to $g_C(\beta, 1) := \frac{2(1+\beta)^2(2-\beta)}{27}$, so that $g_C(0,1) \to +\infty$ if $\beta \to 0$.

Therefore, g can be higher or lower than $g_C(\beta, 1)$ if and only if $0 < \beta \leq 1$ if products are homogeneous ($d = 1$). This implies that a in the case of no disclosure ($\beta = 0$) it is not possible to find a finite value of g to solve the prisoner's dilemma of the R&D game if products are homogeneous or perfect substitutes.

Differently, if products are heterogeneous (imperfect substitutes or complements) then $g_C(0, d) := \frac{2}{(1-d)(2+d)^2} > 0$ if $\beta = 0$ (no disclosure). Therefore, g can be higher or lower than $g_C(\beta, d)$ for any $0 \leq \beta \leq 1$ if products are heterogeneous ($-1 \leq d < 1$). This means that there exists a finite value of g that solves the prisoner's dilemma of the R&D game irrespective of the extent of the R&D spill-overs including the case of no disclosure, which is in the unilateral interest of each firm in the AJ setting. This result is clarified in the next Proposition 4 that concentrates on the case of imperfect substitutability ($d < 1$) avoiding treating the case of perfect substitutability dealt with by Bacchiega et al. (2010), according to which the prisoner's dilemma can vanish only whether firms are disclosing. This case is reported in Corollary 1 following Proposition 4. The shape of $g_C(\beta, d)$ (dotted line) and the shapes of the other relevant constraints of the model are depicted in Figures 4-7 in the parameter space (β, g) for different values of w and d, where Figures 4A and 4B refer to the case $0 < w < \frac{1}{2}$ (Points [2] and [3] of Proposition 4), Figures 5A and 5B refer to the case $\frac{1}{2} \leq w < \frac{5}{8}$ (Points [4], [5] and [6] of Proposition 4), Figures 6A and 6B refer to the case $\frac{2}{3} \leq w < 1$ (Points [11] and [12] of Proposition 4), and Figures 7A and 7B refer to the case $w \to 1$ (Points [13] and [14] of Proposition 4). The figures consider only the case of product substitutability. This is because the Nash equilibrium of the game in the case of product complementarity is univocally given by the Pareto efficient outcome (I,I) and skip Points [7], [8], [9] and [10] of Proposition 4 as the outcomes resemble those of Figure 5. These figures aim at helping the reader through the narrative of the analytical results. The sand-coloured region represents the unfeasible parameter space. This region is bounded by the constraints discussed above (Proposition 3) and tells us that – for any given value of the spill-overs – the efficiency of R&D activity should not be too high, i.e., parameter g should not be too low to avoid excessive R&D investments that would contribute to greatly reduce marginal and average production costs and increase output pushing down the market price of products of both varieties at too low a level. However, product differentiation favours technological progress by pushing downwards the stability conditions and the R&D cost conditions in the (β, g) space.

Define now some thresholds useful to disentangle the results of the points in which Proposition 4 is divided. Let

$$\bar{\beta}_C(d, w) := \frac{(8+d^2)(1-w)+d(10w-6)-\sqrt{(1-w)(2-d)^2[16+(1-w)d^2+(16w- \quad)d]}}{8w}, \tag{50}$$

be a threshold value of the intensity of the R&D externality such that $g_T^{I/NI}(\beta, d, w) = g_C(\beta, d)$ in the (β, g) space prevailing when $0 < w \leq \frac{5}{8}$, where $\bar{\beta}_C(d, w) \geq 0$ for any $\bar{d}_C(w) \leq d \leq 1$ and $\bar{\beta}_C(d, w) < 0$ for any $-1 \leq d < \bar{d}_C(w)$, and

$$\bar{d}_C(w) := \frac{3(1-w)-\sqrt{(1-w)(1+3w)}}{2-3w} > 0, \tag{51}$$

for any $0 < w \leq \frac{5}{8}$ represents a threshold value of the degree of product differentiation such that $\bar{\beta}_C(d, w) = 0$. Let

$$\bar{\bar{\beta}}_C(d) := \frac{3d-2}{6-d}, \tag{52}$$

be a threshold value of the intensity of the R&D externality such that $g_{SC}^{\beta_{low}}(\beta, d) = g_C(\beta, d)$ in the (β, g) space prevailing when $\frac{5}{8} < w < \frac{2}{3}$, where $\bar{\bar{\beta}}_C(d) \geq 0$ for any $\bar{\bar{d}}_C \leq d \leq 1$ and $\bar{\bar{\beta}}_C(d) < 0$ for any $-1 \leq d < \bar{\bar{d}}_C$, and

$$\bar{\bar{d}}_C = \frac{2}{3}. \tag{53}$$

represents a threshold value of the degree of product differentiation such that $\bar{\bar{\beta}}_C(d) = 0$. Let

$$d_{CC}(w) := 6 - 8w, \tag{54}$$

be a threshold value of the degree of product differentiation such that $\bar{\beta}_C(d, w) = \bar{\bar{\beta}}_C(d)$, where $d_{CC}(w) > 0$ for any $0 < w < \frac{3}{4}$ (so that it is certainly positive for any $0 < w < \frac{2}{3}$), $d_{CC}(w) \leq 1$ for any $\frac{5}{8} \leq w < 1$ and $d_{CC}(w) > 1$ for any $0 < w < \frac{5}{8}$. Therefore, if $0 < w < \frac{5}{8}$ then $\bar{\beta}_C(d, w) < \bar{\bar{\beta}}_C(d)$ for any $-1 \leq d \leq 1$; if $\frac{5}{8} \leq w < \frac{2}{3}$ then $\bar{\beta}_C(d, w) \geq \bar{\bar{\beta}}_C(d)$ for any $d_{CC}(w) \leq d \leq 1$ and $\bar{\beta}_C(d, w) < \bar{\bar{\beta}}_C(d)$ for any $-1 \leq d < d_{CC}(w)$. In addition, $d_{CC}(w) > d_T^{I/NI}(w) > \bar{d}_C(w) > \bar{\bar{d}}_C$ for any $0 < w < \frac{2}{3}$, $d_{CC}(w) < d_T^{I/NI}(w) < \bar{d}_C(w) < \bar{\bar{d}}_C$ for any $\frac{2}{3} < w < 1$, $\beta_{T_2}^{I/NI}(d, w) < \bar{\beta}_C(d, w)$ for any $d_T^{I/NI}(w) \leq d \leq 1$ and $\beta_{T_2}^{I/NI}(d, w) < 0$ for any $d < d_T^{I/NI}(w)$ if $\frac{1}{2} \leq w < \frac{5}{8}$, and $\bar{\bar{\beta}}_C(d) \leq \beta_{T_2}^{I/NI}(d, w)$ for any $d_{CC}(w) \leq d \leq 1$ and $\bar{\bar{\beta}}_C(d) > \beta_{T_2}^{I/NI}(d, w)$ for any $d < d_{CC}(w)$ if $\frac{5}{8} \leq w < \frac{2}{3}$.

The following proposition summarises the Nash equilibrium outcomes of the R&D investment decision game with product differentiation under quantity competition.

Proposition 4. The outcomes of the R&D investment decision game with quantity competition and product differentiation are the following.

[1] Products are complements ($d < 0$). Let $0 < w < 1$ hold. If $-1 \leq d < 0$ then (I,I) is the unique Pareto efficient Nash equilibrium and the R&D investment decision game with product differentiation is a deadlock for any $0 \leq \beta \leq 1$ and $g > g_T^{I/I}(\beta, d, w)$.

[2] Products are substitutes ($d > 0$). Let $0 < w < \frac{1}{2}$ hold. If $\bar{d}_C(w) \leq d < 1$ then [2.1] (I,I) is the unique Pareto inefficient Nash equilibrium and the R&D investment decision game with product differentiation is a prisoner's dilemma for any $0 \leq \beta \leq \bar{\beta}_C(d, w)$ and $g_T^{I/NI}(\beta, d, w) < g < g_C(\beta, d)$, and [2.2] (I,I) is the unique Pareto efficient Nash equilibrium and the R&D investment decision game with product differentiation is a deadlock for any $0 \leq \beta \leq \bar{\beta}_C(d, w)$ and $g > g_C(\beta, d)$, for any $\bar{\beta}_C(d, w) < \beta \leq \beta_{T_1}^{I/NI}(d, w)$ and $g > g_T^{I/NI}(\beta, d, w)$, and for any $\beta_{T_1}^{I/NI}(d, w) < \beta \leq 1$ and $g > g_T^{I/I}(\beta, d, w)$.

[3] Products are substitutes ($d > 0$). Let $0 < w < \frac{1}{2}$ hold. If $0 < d < \bar{d}_C(w)$ then (I,I) is the unique Pareto efficient Nash equilibrium and the R&D investment decision game with product differentiation is a deadlock for any $0 \leq \beta \leq \beta_{T_1}^{I/NI}(d, w)$ and $g > g_T^{I/NI}(\beta, d, w)$, and for any $\beta_{T_1}^{I/NI}(d, w) < \beta \leq 1$ and $g > g_T^{I/I}(\beta, d, w)$.

[4] Products are substitutes ($d > 0$). Let $\frac{1}{2} \leq w < \frac{5}{8}$ hold. If $d_T^{I/NI}(w) \leq d < 1$ then [4.1] (I,I) is the unique Pareto inefficient Nash equilibrium and the R&D investment decision game with product differentiation is a prisoner's dilemma for any $0 \leq \beta \leq \beta_{T_2}^{I/NI}(d, w)$ and $g_{SC}^{\beta_{low}}(\beta, d) < g < g_C(\beta, d)$, and for any $\beta_{T_2}^{I/NI}(d, w) < \beta \leq \bar{\beta}_C(d, w)$ and $g_T^{I/NI}(\beta, d, w) < g < g_C(\beta, d)$, and [4.2] (I,I) is the unique Pareto efficient Nash equilibrium and the R&D investment decision game with product differentiation is a deadlock for any $0 \leq \beta \leq \beta_{T_2}^{I/NI}(d, w)$ and $g > g_C(\beta, d)$, for any $\beta_{T_2}^{I/NI}(d, w) < \beta \leq \bar{\beta}_C(d, w)$ and $g > g_C(\beta, d)$, for any $\bar{\beta}_C(d, w) < \beta \leq \beta_{T_1}^{I/NI}(d, w)$ and $g > g_T^{I/NI}(\beta, d, w)$, and for any $\beta_{T_1}^{I/NI}(d, w) < \beta \leq 1$ and $g > g_T^{I/I}(\beta, d, w)$.

[5] Products are substitutes ($d > 0$). Let $\frac{1}{2} \leq w < \frac{5}{8}$ hold. If $\bar{d}_C(w) \leq d < d_T^{I/NI}(w)$ then [5.1] (I,I) is the unique Pareto inefficient Nash equilibrium and the R&D investment decision game with product differentiation is a prisoner's dilemma for any $0 \leq \beta \leq \bar{\beta}_C(d,w)$ and $g_T^{I/NI}(\beta,d,w) < g < g_C(\beta,d)$, and [5.2] (I,I) is the unique Pareto efficient Nash equilibrium and the R&D investment decision game with product differentiation is a deadlock for any $0 \leq \beta \leq \bar{\beta}_C(d,w)$ and $g > g_C(\beta,d)$, for any $\bar{\beta}_C(d,w) < \beta \leq \beta_{T_1}^{I/NI}(d,w)$ and $g > g_T^{I/NI}(\beta,d,w)$, and for any $\beta_{T_1}^{I/NI}(d,w) < \beta \leq 1$ and $g > g_T^{I/I}(\beta,d,w)$.

[6] Products are substitutes ($d > 0$). Let $\frac{1}{2} \leq w < \frac{5}{8}$ hold. If $0 < d < \bar{d}_C(w)$ then (I,I) is the unique Pareto efficient Nash equilibrium and the R&D investment decision game with product differentiation is a deadlock for any $0 \leq \beta \leq \beta_{T_1}^{I/NI}(d,w)$ and $g > g_T^{I/NI}(\beta,d,w)$, and for any $\beta_{T_1}^{I/NI}(d,w) < \beta \leq 1$ and $g > g_T^{I/I}(\beta,d,w)$.

[7] Products are substitutes ($d > 0$). Let $\frac{5}{8} \leq w < \frac{2}{3}$ hold. If $d_{CC}(w) \leq d < 1$ then [7.1] (I,I) is the unique Pareto inefficient Nash equilibrium and the R&D investment decision game with product differentiation is a prisoner's dilemma for any $0 \leq \beta \leq \bar{\bar{\beta}}_C(d)$ and $g_{SC}^{\beta_{low}}(\beta,d) < g < g_C(\beta,d)$, and [7.2] (I,I) is the unique Pareto efficient Nash equilibrium and the R&D investment decision game with product differentiation is a deadlock for any $0 \leq \beta \leq \bar{\bar{\beta}}_C(d)$ and $g > g_C(\beta,d)$, for any $\bar{\bar{\beta}}_C(d) < \beta \leq \beta_{T_2}^{I/NI}(d,w)$ and $g > g_{SC}^{\beta_{low}}(\beta,d)$, for any $\beta_{T_2}^{I/NI}(d,w) < \beta \leq \beta_{T_1}^{I/NI}(d,w)$ and $g > g_T^{I/NI}(\beta,d,w)$, and for any $\beta_{T_1}^{I/NI}(d,w) < \beta \leq 1$ and $g > g_T^{I/I}(\beta,d,w)$.

[8] Products are substitutes ($d > 0$). Let $\frac{5}{8} \leq w < \frac{2}{3}$ hold. If $d_T^{I/NI}(w) \leq d < d_{CC}(w)$ then [8.1] (I,I) is the unique Pareto inefficient Nash equilibrium and the R&D investment decision game with product differentiation is a prisoner's dilemma for any $0 \leq \beta \leq \beta_{T_2}^{I/NI}(d,w)$ and $g_{SC}^{\beta_{low}}(\beta,d) < g < g_C(\beta,d)$, and for any $\beta_{T_2}^{I/NI}(d,w) < \beta \leq \bar{\beta}_C(d,w)$ and $g_T^{I/NI}(\beta,d,w) < g < g_C(\beta,d)$, and [8.2] (I,I) is the unique Pareto efficient Nash equilibrium and the R&D investment decision game with product differentiation is a deadlock for any $0 \leq \beta \leq \beta_{T_2}^{I/NI}(d,w)$ and $g > g_C(\beta,d)$, for any $\beta_{T_2}^{I/NI}(d,w) < \beta \leq \bar{\beta}_C(d,w)$ and $g > g_C(\beta,d)$, for any $\bar{\beta}_C(d,w) < \beta \leq \beta_{T_1}^{I/NI}(d,w)$ and $g > g_T^{I/NI}(\beta,d,w)$, and for any $\beta_{T_1}^{I/NI}(d,w) < \beta \leq 1$ and $g > g_T^{I/I}(\beta,d,w)$.

[9] Products are substitutes ($d > 0$). Let $\frac{5}{8} \leq w < \frac{2}{3}$ hold. If $\bar{d}_C(w) \leq d < d_T^{I/NI}(w)$ then [9.1] (I,I) is the unique Pareto inefficient Nash equilibrium and the R&D investment decision game with product differentiation is a prisoner's dilemma for any $0 \leq \beta \leq \bar{\beta}_C(d,w)$ and $g_T^{I/NI}(\beta,d,w) < g < g_C(\beta,d)$, and [9.2] (I,I) is the unique Pareto efficient Nash equilibrium and the R&D investment decision game with product differentiation is a deadlock for any $0 \leq \beta \leq \bar{\beta}_C(d,w)$ and $g > g_C(\beta,d)$, for any $\bar{\beta}_C(d,w) < \beta \leq \beta_{T_1}^{I/NI}(d,w)$ and $g > g_T^{I/NI}(\beta,d,w)$, and for any $\beta_{T_1}^{I/NI}(d,w) < \beta \leq 1$ and $g > g_T^{I/I}(\beta,d,w)$.

[10] Products are substitutes ($d > 0$). Let $\frac{5}{8} \leq w < \frac{2}{3}$ hold. If $0 < d < \bar{d}_C(w)$ then (I,I) is the unique Pareto efficient Nash equilibrium and the R&D investment decision game with product differentiation

is a deadlock for any $0 \leq \beta \leq \beta_{T_1}^{I/NI}(d,w)$ and $g > g_T^{I/NI}(\beta,d,w)$, and for any $\beta_{T_1}^{I/NI}(d,w) < \beta \leq 1$ and $g > g_T^{I/I}(\beta,d,w)$.

[11] Products are substitutes ($d > 0$). Let $\frac{2}{3} \leq w < 1$ hold. If $\bar{\bar{d}}_C = \frac{2}{3} \leq d < 1$ then [11.1] (I,I) is the unique Pareto inefficient Nash equilibrium and the R&D investment decision game with product differentiation is a prisoner's dilemma for any $0 \leq \beta \leq \bar{\bar{\beta}}_C(d)$ and $g_{SC}^{\beta_{low}}(\beta,d) < g < g_C(\beta,d)$, and [11.2] (I,I) is the unique Pareto efficient Nash equilibrium and the R&D investment decision game with product differentiation is a deadlock for any $0 \leq \beta \leq \bar{\bar{\beta}}_C(d)$ and $g > g_C(\beta,d)$, for any $\bar{\bar{\beta}}_C(d) < \beta \leq \beta_{T_2}^{I/NI}(d,w)$ and $g > g_{SC}^{\beta_{low}}(\beta,d)$, for any $\beta_{T_2}^{I/NI}(d,w) < \beta \leq \beta_{T_1}^{I/NI}(d,w)$ and $g > g_T^{I/NI}(\beta,d,w)$, and for any $\beta_{T_1}^{I/NI}(d,w) < \beta \leq 1$ and $g > g_T^{I/I}(\beta,d,w)$.

[12] Products are substitutes ($d > 0$). Let $\frac{2}{3} \leq w < 1$ hold. If $0 < d < \frac{2}{3} = \bar{\bar{d}}_C$ then (I,I) is the unique Pareto efficient Nash equilibrium and the R&D investment decision game with product differentiation is a deadlock for any $0 \leq \beta \leq \beta_{T_2}^{I/NI}(d,w)$ and $g > g_{SC}^{\beta_{low}}(\beta,d)$, for any $\beta_{T_2}^{I/NI}(d,w) < \beta \leq \beta_{T_1}^{I/NI}(d,w)$ and $g > g_T^{I/NI}(\beta,d,w)$, and for any $\beta_{T_1}^{I/NI}(d,w) < \beta \leq 1$ and $g > g_T^{I/I}(\beta,d,w)$.

[13] Products are substitutes ($d > 0$). Let $w \to 1$ hold. If $\bar{\bar{d}}_C = \frac{2}{3} \leq d < 1$ then [13.1] (I,I) is the unique Pareto inefficient Nash equilibrium and the R&D investment decision game with product differentiation is a prisoner's dilemma for any $0 \leq \beta \leq \bar{\bar{\beta}}_C(d)$ and $g_{SC}^{\beta_{low}}(\beta,d) < g < g_C(\beta,d)$, and [13.2] (I,I) is the unique Pareto efficient Nash equilibrium and the R&D investment decision game with product differentiation is a deadlock for any $0 \leq \beta \leq \bar{\bar{\beta}}_C(d)$ and $g > g_C(\beta,d)$, for any $\bar{\bar{\beta}}_C(d) < \beta \leq \frac{d}{2}$ and $g > g_{SC}^{\beta_{low}}(\beta,d)$, and for any $\frac{d}{2} < \beta \leq 1$ and $g > g_{SC}^{\beta_{high}}(\beta,d)$.

[14] Products are substitutes ($d > 0$). Let $w \to 1$ hold. If $0 < d < \frac{2}{3} = \bar{\bar{d}}_C$ then (I,I) is the unique Pareto efficient Nash equilibrium and the R&D investment decision game with product differentiation is a deadlock for any $0 \leq \beta \leq \frac{d}{2}$ and $g > g_{SC}^{\beta_{low}}(\beta,d)$, and for any $\frac{d}{2} < \beta \leq 1$ and $g > g_{SC}^{\beta_{high}}(\beta,d)$.

Proof. Appendix.

Corollary 1. Products are perfect substitutes ($d = 1$). [1] Let $0 < w < \frac{1}{2}$ hold. Then [1.1] (I,I) is the unique Pareto inefficient Nash equilibrium and the R&D investment decision game with product differentiation is a prisoner's dilemma for any $0 \leq \beta \leq \bar{\beta}_C(1,w)$ and $g_T^{I/NI}(\beta,1,w) < g < g_C(\beta,1)$, and [1.2] (I,I) is the unique Pareto efficient Nash equilibrium and the R&D investment decision game with product differentiation is a deadlock for any $0 < \beta \leq \bar{\beta}_C(1,w)$ and $g > g_C(\beta,1)$, for any $\bar{\beta}_C(1,w) < \beta \leq \beta_{T_1}^{I/NI}(1,w)$ and $g > g_T^{I/NI}(\beta,1,w)$, and for any $\beta_{T_1}^{I/NI}(1,w) < \beta \leq 1$ and $g > g_T^{I/I}(\beta,1,w)$. [2] Let $\frac{1}{2} \leq w < \frac{5}{8}$ hold. Then [2.1] (I,I) is the unique Pareto inefficient Nash equilibrium and the R&D investment decision game with product differentiation is a prisoner's dilemma for any $0 \leq \beta \leq \beta_{T_2}^{I/NI}(1,w)$ and $g_{SC}^{\beta_{low}}(\beta,1) < g < g_C(\beta,1)$, and for any $\beta_{T_2}^{I/NI}(1,w) < \beta \leq \bar{\beta}_C(1,w)$ and $g_T^{I/NI}(\beta,1,w) < g < g_C(\beta,1)$, and [2.2] (I,I) is the unique Pareto efficient Nash equilibrium and the R&D investment decision game with product differentiation is a deadlock for any $0 < \beta \leq \beta_{T_2}^{I/NI}(1,w)$ and $g > g_C(\beta,1)$, for any $\beta_{T_2}^{I/NI}(1,w) < \beta \leq \bar{\beta}_C(1,w)$ and $g > g_C(\beta,1)$, for any $\bar{\beta}_C(1,w) < \beta \leq \beta_{T_1}^{I/NI}(1,w)$ and $g > g_T^{I/NI}(\beta,1,w)$, and for any $\beta_{T_1}^{I/NI}(1,w) < \beta \leq 1$ and

$g > g_T^{I/I}(\beta, 1, w)$. [3] Let $\frac{5}{8} \leq w < \frac{2}{3}$ hold. Then [3.1] (I,I) is the unique Pareto inefficient Nash equilibrium and the R&D investment decision game with product differentiation is a prisoner's dilemma for any $0 \leq \beta \leq \bar{\bar{\beta}}_C(1)$ and $g_{SC}^{\beta_{low}}(\beta, 1) < g < g_C(\beta, 1)$, and [3.2] (I,I) is the unique Pareto efficient Nash equilibrium and the R&D investment decision game with product differentiation is a deadlock for any $0 < \beta \leq \bar{\bar{\beta}}_C(1)$ and $g > g_C(\beta, 1)$, for any $\bar{\bar{\beta}}_C(1) < \beta \leq \beta_{T_2}^{I/NI}(1, w)$ and $g > g_{SC}^{\beta_{low}}(\beta, 1)$, for any $\beta_{T_2}^{I/NI}(1, w) < \beta \leq \beta_{T_1}^{I/NI}(1, w)$ and $g > g_T^{I/NI}(\beta, 1, w)$, and for any $\beta_{T_1}^{I/NI}(1, w) < \beta \leq 1$ and $g > g_T^{I/I}(\beta, 1, w)$. [4] Let $\frac{2}{3} \leq w < 1$ hold. Then [4.1] (I,I) is the unique Pareto inefficient Nash equilibrium and the R&D investment decision game with product differentiation is a prisoner's dilemma for any $0 \leq \beta \leq \bar{\bar{\beta}}_C(1)$ and $g_{SC}^{\beta_{low}}(\beta, 1) < g < g_C(\beta, 1)$, and [4.2] (I,I) is the unique Pareto efficient Nash equilibrium and the R&D investment decision game with product differentiation is a deadlock for any $0 < \beta \leq \bar{\bar{\beta}}_C(1)$ and $g > g_C(\beta, 1)$, for any $\bar{\bar{\beta}}_C(1) < \beta \leq \beta_{T_2}^{I/NI}(1, w)$ and $g > g_{SC}^{\beta_{low}}(\beta, 1)$, for any $\beta_{T_2}^{I/NI}(1, w) < \beta \leq \beta_{T_1}^{I/NI}(1, w)$ and $g > g_T^{I/NI}(\beta, 1, w)$, and for any $\beta_{T_1}^{I/NI}(1, w) < \beta \leq 1$ and $g > g_T^{I/I}(\beta, 1, w)$. [5] Let $w \to 1$ hold. Then [5.1] (I,I) is the unique Pareto inefficient Nash equilibrium and the R&D investment decision game with product differentiation is a prisoner's dilemma for any $0 \leq \beta \leq \bar{\bar{\beta}}_C(1)$ and $g_{SC}^{\beta_{low}}(\beta, 1) < g < g_C(\beta, 1)$, and [5.2] (I,I) is the unique Pareto efficient Nash equilibrium and the R&D investment decision game with product differentiation is a deadlock for any $0 \leq \beta \leq \bar{\bar{\beta}}_C(1)$ and $g > g_C(\beta, 1)$, for any $\bar{\bar{\beta}}_C(1) < \beta \leq \frac{d}{2}$ and $g > g_{SC}^{\beta_{low}}(\beta, 1)$, and for any $\frac{d}{2} < \beta \leq 1$ and $g > g_{SC}^{\beta_{high}}(\beta, 1)$.

Proof. Appendix.

Propositions 3 and 4 and Corollary 1 definitively identify the parametric conditions under which the stability conditions and R&D cost conditions are binding for the R&D investment decision game, showing that a sufficiently high degree of product differentiation is capable to solve the prisoner's dilemma allowing (i) to let the R&D investment decisions game become a deadlock irrespective of the intensity of the R&D spill-overs, *including the case of no disclosure* (Proposition 4), and (ii) to pinpoint under perfect substitutability that the prisoner's dilemma can be solved if and only if *firms are disclosing on their R&D activity* (Corollary 1).

Though there are several parametric conditions in which either a stability condition or an R&D cost condition is binding, the outcomes of the R&D investment decision game belong to two standard paradigms: the prisoner's dilemma (when the degree of product differentiation is sufficiently low) and the anti-prisoner's dilemma (when the degree of product differentiation is sufficiently high). In the former case, investing in R&D represents a dominant strategy, the Nash equilibrium (I,I) is Pareto inefficient and thus there exists a conflict between self-interest and mutual benefit to undertake cost-reducing innovation. In the latter case, investing in R&D represents a dominant strategy, the Nash equilibrium (I,I) is Pareto efficient and thus there exists no conflict between self-interest and mutual benefit to undertake cost-reducing R&D. Other things being equal, product differentiation – by increasing firm's profits – helps to relax the tightness of the relevant constraints in the (β, g) space, contributes pushing down the stability conditions and the R&D cost conditions, in turn, allowing firms to increase the efficiency of the R&D activity (lower values of g) and reduce the need to sharing R&D related information (lower values of β), eventually opening the route for a solution to the prisoner's dilemma identified by Bacchiega et a. (2010). Figures 4-7 provide qualitative support to the analysis carried out so far, by helping to clarify (for different values of the unitary cost w) the role of product differentiation in the AJ setting, showing specifically the role of the stability conditions and the R&D cost conditions in determining the Nash equilibrium outcomes of the game in the case of homogeneous products ($d = 1$) and heterogeneous products ($d < 1$). Amongst other things, the figures clearly show that product differentiation pushes downwards all the constraints thus increasing the potential of technological improvements ($g \downarrow$) for any given value of the extent of the spill-overs.

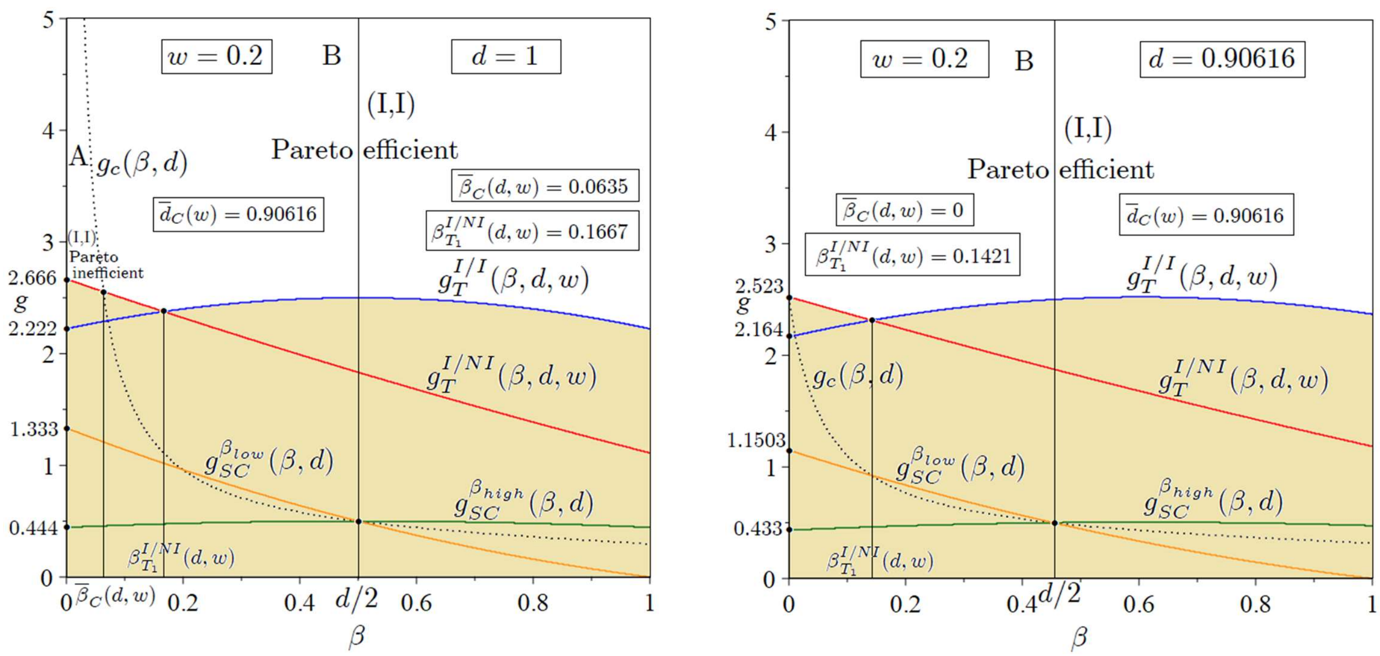
When the degree of product differentiation is low ($d \uparrow$), the results of Bacchiega et al. (2010) generally hold, so that – for a given value of the efficiency of R&D activity, g – the Nash equilibrium outcome is Pareto inefficient when β is sufficiently low and Pareto efficient when β becomes larger. However, unlike the case of homogeneous products studied by Bacchiega et al. (2010), there exists a finite value of g such that the game turns to be a deadlock *also in the absence of spill-over effects* ($\beta = 0$). Let us begin with a parametric configuration such that the R&D game is a prisoner's dilemma with a low spill-overs effect. In that case, I is a dominant strategy of each rational and selfish player (allowing to obtain the best outcome regardless of the rival's choice), but the Nash equilibrium (I,I) is sub-optimal as firms have a joint incentive to coordinate towards NI though each of them has a unilateral incentive to play I and invest in R&D. Given the payoff matrix no one is interested in playing NI if the rival plays NI. This is because everyone prefers to unilaterally invest in process innovation, increase output and obtain a higher profit. No one is also interested in playing NI even when the rival plays I. This is because each player prefers to forgo being NI rather than be the only one to play NI, which leads to the worst possible outcome as a portion of its profits is eroded by the rival that is investing in R&D. Thus, regardless of the rival's activity no one will play NI, and everyone will forgo obtaining a higher profit by becoming an investing firm. However, if both players had decided to cooperate to become a non-investing firm, they would be better off. Thus, by making decisions that guarantee each player the best outcome unilaterally, both players are worse off than they would have been if they had both chosen to play NI: the pursuit of individual success can thus lead to a collective failure. Can we then expect that both players, aware that they may get being disappointed by this result, will reach an agreement to jointly play NI? No. Players' choices are consistent if only if no one should regret after knowing the rivals' strategy. In an R&D game with this parameter configuration, players make consistent decisions when they choose to play I. After both firms have chosen to play I, no one will regret it as anyone who had decided to play NI unilaterally would have been worse off. In contrast, players would have made conflicting decisions if they had both chosen to cooperate and play NI. In this case, each would have regretted their choice as playing I unilaterally would have been better off resulting in a higher payoff. On one hand, we must expect players to be able to achieve an agreement prescribing consistent choices (R&D investments) because everyone is aware that no one after the agreement will be interested in the violation if the rival complies with it. On the other hand, we should not expect players to be able to achieve an agreement prescribing choices that are not mutually consistent (no R&D investment). This is because everyone is aware that no one will be interested in complying with that agreement if the rival complies with it.

In this setting, an increase in the degree of R&D spill-overs allows (i) an increase in profits of the non-investing firm in the asymmetric sub-game, and (ii) a reduction in profits of the investing firm in the asymmetric sub-game. Indeed, the former player gains from the free-riding activity at the expense of the investing rival (the latter). However, both increase their own profits under the strategic profile I. Therefore, the game becomes a deadlock and there is no conflict between self-interest and mutual benefit to undertake cost-reducing R&D.

When the degree of product differentiation increases ($d \downarrow$), the market share (in the markets for the relevant products) of each firm tends to increase and firms' profits become larger than when the degree of product differentiation was lower. Under this parameter configuration, there is no need to disclose, both firms invest in R&D and the R&D game becomes a deadlock irrespective of the parameter scale.

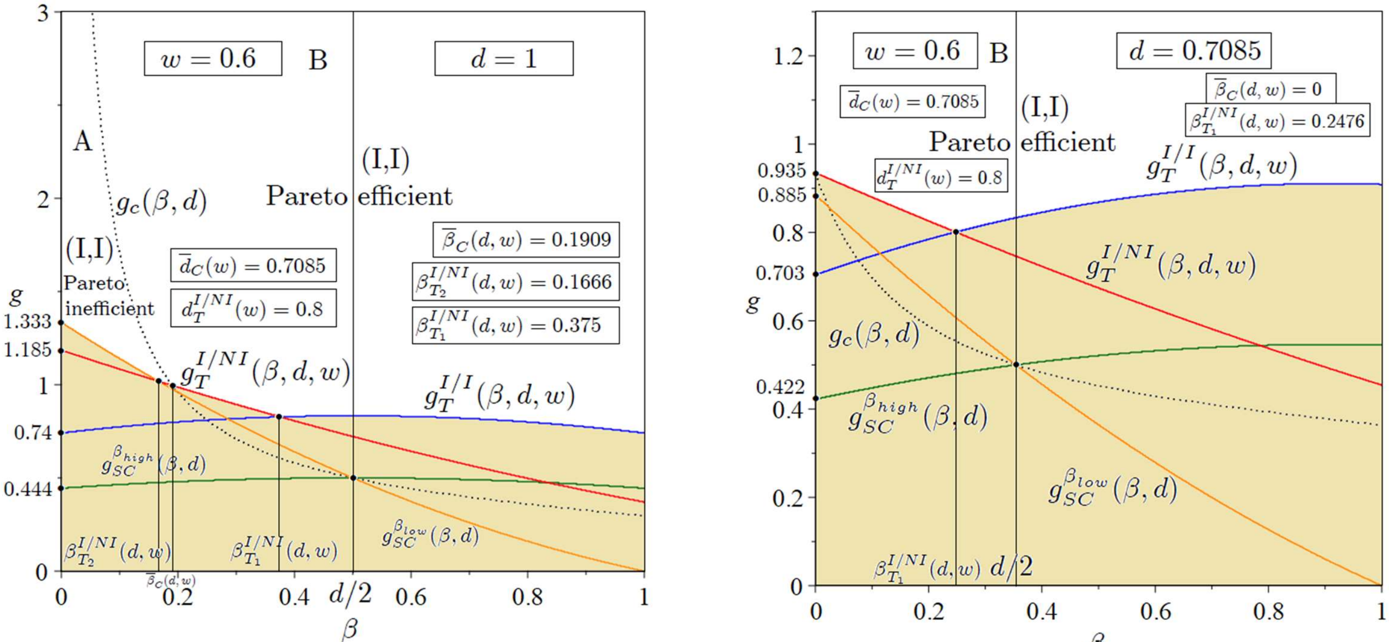
It is now important to turn to the study of social welfare following the results of Proposition 2, i.e., $W^{*I/I} > W^{*NI/NI}$ for any $0 \leq \beta \leq 1$. Indeed, though in the absence of spill-overs and homogeneous products R&D investments improve social welfare, firms are worse off (the game is a prisoner's dilemma). Differently, product differentiation represents a win-win result as social welfare under I/I is larger than under NI/NI, but firms are better off (the game is an anti-prisoner's dilemma).

In addition, the government should intervene to favour product differentiation. This is because $W^{*I/I}$ is a monotonic decreasing function of d, so that increasing the degree of product differentiation ($d \downarrow$) is welfare improving. This is because both profits of the investing firms and consumers' surplus increase when d decreases, though the latter is the result of two opposing effects (in the case of linear demand): 1) it reduces due to the reduction in the number of available goods; 2) it increases due to the increase in the variety of products available in the markets.



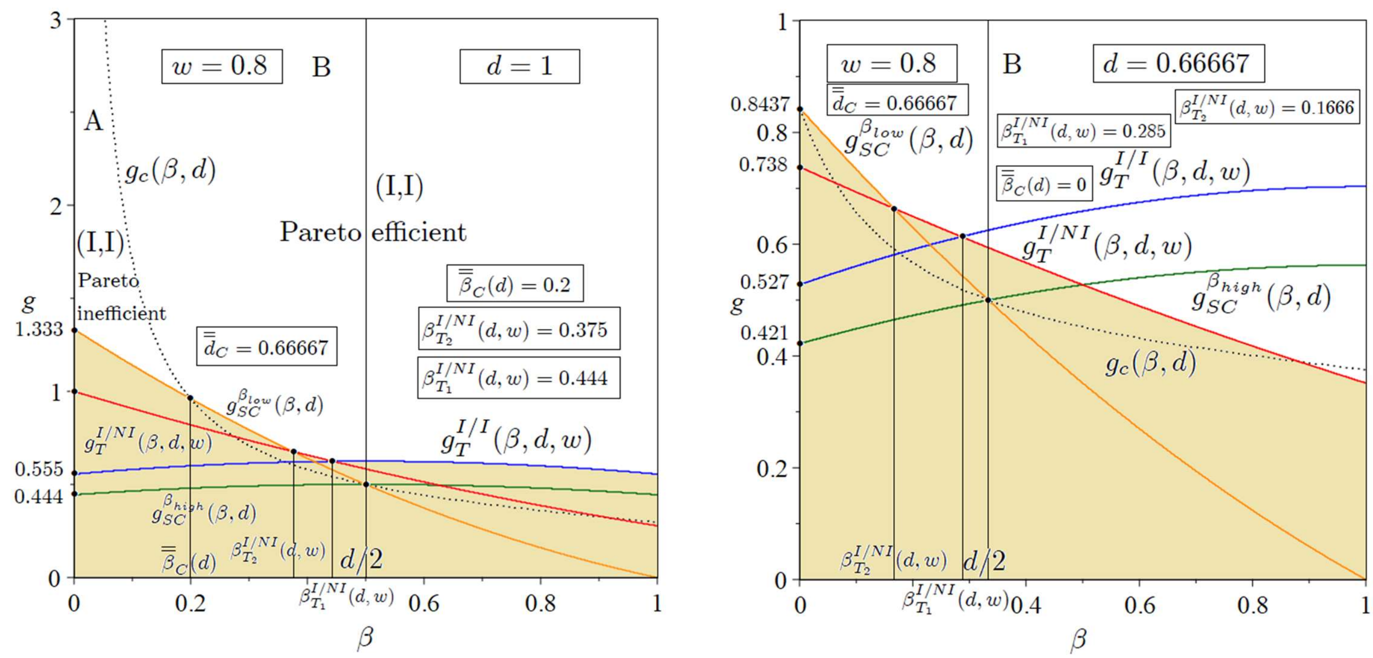
(A) (B)

Figure 4. The R&D investment decision game: Nash equilibrium outcomes when $0 < w < \frac{1}{2}$ ($w = 0.2$) and $d = 1 > \bar{d}_C(w)$ (Panel A) and $d = \bar{d}_C(w) = 0.90616$ (Panel B). The sand-coloured region represents the parametric area of unfeasibility in the (β, g) space. In Panel A, the R&D game is a prisoner's dilemma (area A) and an anti-prisoner's dilemma (area B). In Panel B, the prisoner's dilemma is solved, and the R&D game is anti-prisoner's dilemma irrespective of the parameter scale (area B).



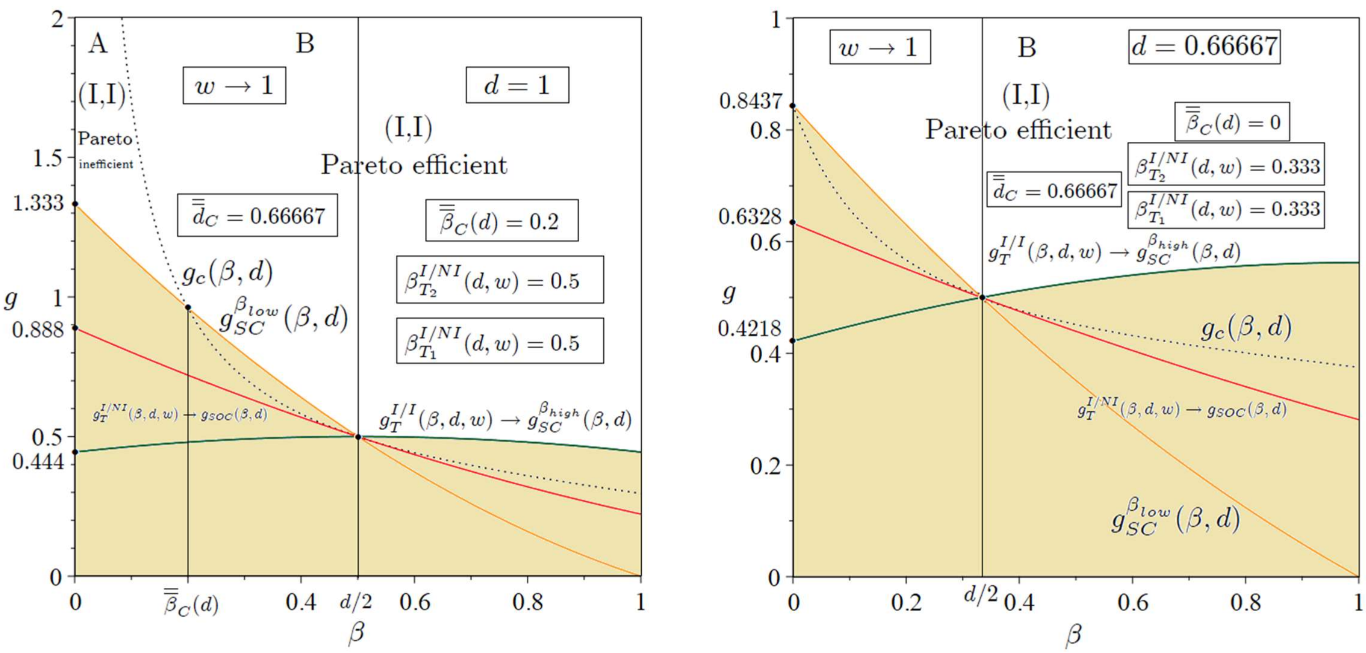
(A)

Figure 5. The R&D investment decision game: Nash equilibrium outcomes when $\frac{1}{2} \leq w < \frac{5}{8}$ ($w = 0.6$) and $d = 1 > \bar{d}_C(w)$ (Panel A) and $d = \bar{d}_C(w) = 0.7085$ (Panel B). The sand-coloured region represents the parametric area of unfeasibility in the (β, g) space. In Panel A, the R&D game is a prisoner's dilemma (area A) and an anti-prisoner's dilemma (area B). In Panel B, the prisoner's dilemma is solved, and the R&D game is an anti-prisoner's dilemma irrespective of the parameter scale (area B).



(A) (B)

Figure 6. The R&D investment decision game: Nash equilibrium outcomes when $\frac{2}{3} \leq w < 1$ ($w = 0.8$) and $d = 1 > \bar{\bar{d}}_C = \frac{2}{3}$ (Panel A) and $d = \bar{\bar{d}}_C = \frac{2}{3}$ (Panel B). The sand-coloured region represents the parametric area of unfeasibility in the (β, g) space. In Panel A, the R&D game is a prisoner's dilemma (area A) and an anti-prisoner's dilemma (area B). In Panel B, the prisoner's dilemma is solved, and the R&D game is an anti-prisoner's dilemma irrespective of the parameter scale (area B).



(A) (B)

Figure 7. The R&D investment decision game: Nash equilibrium outcomes when $w \to 1$ and $d = 1 > \bar{\bar{d}}_C = \frac{2}{3}$ (Panel A) and $d = \bar{\bar{d}}_C = \frac{2}{3}$ (Panel B). The sand-coloured region represents the parametric area of unfeasibility in the (β, g) space. In Panel A, the R&D game is a prisoner's dilemma (area A) and an anti-prisoner's dilemma (area B). In Panel B, the prisoner's dilemma is solved, and the R&D game is an anti-prisoner's dilemma irrespective of the parameter scale (area B).

4. Conclusions

This article takes d'Aspremont and Jacquemin (1988, 1990) seriously and deepens the analysis of the non-cooperative version of the cost-reducing R&D model with spill-overs using a game-theoretic approach. The work aims at complementing Bacchiega et al. (2010), who concentrate on homogeneous products and identify an interplay between the extent of technological spill-overs and the efficiency of the R&D activity in determining whether the game is a prisoner's dilemma or an anti-prisoner's dilemma. However, in their work, the prisoner's dilemma vanishes if and only if the extent of technological spill-overs is sufficiently high, which, in turn, would require that firms disclose (or – equivalently – they are unable to keep closed) the information on the results of their R&D investment. Generalising on the assumption of product differentiation à la Singh and Vives (1984) allows solving the dilemma also in the *absence of R&D spill-overs* in both cases of quantity and price competition.

Unlike the previous literature, this article (i) provides a thoughtful and detailed analysis of the role of the constraints needed to define the feasibility of the R&D investment decision game with homogeneous and heterogeneous products, (ii) identifies the parametric regions in which the game is a prisoner's dilemma or an anti-prisoner's dilemma, and (iii) shows the conditions for making the prisoner's dilemma disappear.

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Compliance with ethical standards

Disclosure of potential conflict of interest The authors declare that they have no conflict of interest.

Funding The authors declare that this study was not funded by any institution.

Informed consent Informed consent was obtained from all individual participants included in the study.

Declarations of interest None.

Appendix

Proof of Proposition 1. 1) If $d < 0$, then $w_T^{I/I} > 1$ and $\beta_T^{I/I} < 0$ for any $0 < w < 1$. Therefore, $g_T^{I/I}(\beta, d, w) > g_{SC}^{\beta_{low}}(\beta, d)$ for any $0 \leq \beta \leq 1$ and $0 < w < 1$. 2) If $d > 0$, then $w_T^{I/I} < 1$, and $\beta_T^{I/I} < 0$ for any $w < w_T^{I/I}$ and $\beta_T^{I/I} > 0$ for any $w > w_T^{I/I}$. Therefore, 2.1) if $w < w_T^{I/I}$ then $g_T^{I/I}(\beta, d, w) > g_{SC}^{\beta_{low}}(\beta, d)$ for any β, and 2.2) if $w > w_T^{I/I}$ then $g_T^{I/I}(\beta, d, w) < g_{SC}^{\beta_{low}}(\beta, d)$ for any $\beta < \beta_T^{I/I}$ and $g_T^{I/I}(\beta, d, w) > g_{SC}^{\beta_{low}}(\beta, d)$ for any $\beta > \beta_T^{I/I}$. 3) If $d > 0$ and $w \to 1$ then $\beta_T^{I/I} \to \frac{d}{2}$ and $g_T^{I/I}(\beta, d, 1) \to g_{SC}^{\beta_{high}}(\beta, d)$ from above for any $0 \leq \beta \leq 1$. Therefore, $g_{SC}^{\beta_{low}}(\beta, d) > g_{SC}^{\beta_{high}}(\beta, d)$ for any $0 \leq \beta \leq \frac{d}{2}$ and $g_{SC}^{\beta_{low}}(\beta, d) < g_{SC}^{\beta_{high}}(\beta, d)$ for any $\frac{d}{2} \leq \beta \leq 1$. **Q.E.D.**

Proof of Proposition 2. By computing the difference $W^{*I/I} - W^{*NI/NI}$ and solving for g allows to get the threshold $g_W(\beta, d) := \frac{(3+d)(2-d\beta)(1+\beta)^2}{(2+d)^2[4-d-d^2+6\beta-d^2\beta]} > 0$, which is smaller than $g_{SC}^{\beta_{high}}(\beta, d)$ for any $0 \leq \beta \leq 1$, $-1 < d < 1$. Therefore, $W^{*I/I} > W^{*NI/NI}$ either for any $g > g_{SC}^{\beta_{low}}(\beta, d)$ or for any $g > g_T^{I/I}(\beta, d, w)$. **Q.E.D.**

Proof of Propositions 3.

Point [1]. If products are complements ($d < 0$), then $g_T^{I/I}(\beta, d, w) > g_T^{I/NI}(\beta, d, w) > g_{SC}^{\beta_{low}}(\beta, d)$ for any $0 \leq \beta \leq 1$ and $0 < w < 1$.

Points [2] and [4]. If products are substitutes ($d > 0$) and $0 < w < \frac{1}{2}$ or if products are substitutes ($d > 0$), $\frac{1}{2} \leq w < 1$ and $0 < d < d_T^{I/NI}(w)$ then $\beta_{T_2}^{I/NI}(d, w) < 0$, $\beta_{T_1}^{I/NI}(d, w) > 0$, $g_T^{I/NI}(\beta, d, w) > g_{SC}^{\beta_{low}}(\beta, d)$ for any $0 \leq \beta \leq 1$, $g_T^{I/NI}(\beta, d, w) > g_T^{I/I}(\beta, d, w)$ for any $0 \leq \beta < \beta_{T_1}^{I/NI}(d, w)$, and $g_T^{I/I}(\beta, d, w) > g_T^{I/NI}(\beta, d, w)$ for any $\beta_{T_1}^{I/NI}(d, w) < \beta \leq 1$. Therefore,

$g_T^{I/NI}(\beta, d, w)$ is binding for any $0 \leq \beta < \beta_{T_1}^{I/NI}(d, w)$ and $g_T^{I/I}(\beta, d, w)$ is binding for any $\beta_{T_1}^{I/NI}(d, w) < \beta \leq 1$.

Point [3]. If products are substitutes ($d > 0$), $\frac{1}{2} \leq w < 1$ and $d_T^{I/NI}(w) \leq d \leq 1$ then $\beta_{T_1}^{I/NI}(d, w) > \beta_{T_2}^{I/NI}(d, w) > 0$, $g_{SC}^{\beta_{low}}(\beta, d) > g_T^{I/NI}(\beta, d, w) > g_T^{I/I}(\beta, d, w)$ for any $0 \leq \beta < \beta_{T_2}^{I/NI}(d, w)$, $g_T^{I/NI}(\beta, d, w) > g_{SC}^{\beta_{low}}(\beta, d)$ and $g_T^{I/NI}(\beta, d, w) > g_T^{I/I}(\beta, d, w)$ for any $\beta_{T_2}^{I/NI}(d, w) < \beta < \beta_{T_1}^{I/NI}(d, w)$, and $g_T^{I/I}(\beta, d, w) > g_T^{I/NI}(\beta, d, w)$ for any $\beta_{T_1}^{I/NI}(d, w) < \beta \leq 1$. Therefore, $g_{SC}^{\beta_{low}}(\beta, d)$ is binding for any $0 \leq \beta < \beta_{T_2}^{I/NI}(d, w)$, $g_T^{I/NI}(\beta, d, w)$ is binding for any $\beta_{T_2}^{I/NI}(d, w) < \beta < \beta_{T_1}^{I/NI}(d, w)$ and $g_T^{I/I}(\beta, d, w)$ is binding for any $\beta_{T_1}^{I/NI}(d, w) < \beta \leq 1$.

Point [5]. If products are substitutes and $w \to 1$, then $\beta_{T_1}^{I/NI}(d, 1) > 0$ and $\beta_{T_1}^{I/NI}(d, 1) \to \frac{d}{2}$, $\beta_{T_2}^{I/NI}(d, 1) > 0$ and $\beta_{T_2}^{I/NI}(d, 1) \to \frac{d}{2}$, $g_T^{I/I}(\beta, d, 1) \to g_{SC}^{\beta_{high}}(\beta, d)$ from above and $g_T^{I/NI}(\beta, d, 1) \to g_{SOC}(\beta, d)$ for any $0 < d \leq 1$, and $g_{SC}^{\beta_{low}}(\beta, d) > g_T^{I/NI}(\beta, d, 1) > g_{SC}^{\beta_{high}}(\beta, d)$ for any $0 \leq \beta < \frac{d}{2}$ and $g_{SC}^{\beta_{high}}(\beta, d) > g_T^{I/NI}(\beta, d, 1) > g_{SC}^{\beta_{low}}(\beta, d)$ for any $\frac{d}{2} < \beta \leq 1$. **Q.E.D.**

Proof of Propositions 4.

Point [1]. If products are complements ($d < 0$) then $g_T^{I/I}(\beta, d, w)$ is binding for any $0 \leq \beta \leq 1$ and $0 < w < 1$ in the (β, g) space (from Proposition 3). Moreover, $g_C(\beta, d) < g_T^{I/I}(\beta, d, w)$ for any $-1 \leq d < 0$, $0 \leq \beta \leq 1$ and $0 < w < 1$. If $\beta = 0$ then $g_C(0, d) < g_T^{I/I}(0, d, w)$ as $2(1-w) - d(2-w) > 0$ for any $-1 \leq d < 0$ and $0 < w < 1$. In addition, $\frac{\partial g_C(\beta,d)}{\partial \beta} < 0$ and $\frac{\partial g_T^{I/I}(\beta,d,w)}{\partial \beta} > 0$ for any $-1 \leq d < 0$, $0 \leq \beta \leq 1$ and $0 < w < 1$. Therefore, $\Delta\Pi_A > 0$, $\Delta\Pi_B < 0$ and $\Delta\Pi_C < 0$ for any $0 \leq \beta \leq 1$ and $g > g_T^{I/I}(\beta, d, w)$.

Points [2] and [3]. If products are substitutes ($d > 0$) and $0 < w < \frac{1}{2}$ then $g_T^{I/NI}(\beta, d, w)$ is binding for any $0 \leq \beta < \beta_{T_1}^{I/NI}(d, w)$ and $g_T^{I/I}(\beta, d, w)$ is binding for any $\beta_{T_1}^{I/NI}(d, w) < \beta \leq 1$ in the (β, g) space (from Proposition 3). In addition, $\frac{\partial g_C(\beta,d)}{\partial \beta} < 0$ for any $0 \leq \beta \leq 1$ and $0 < w < \frac{1}{2}$. If $\bar{d}_C(w) \leq d < 1$ then $g_C(\beta, d) \geq g_T^{I/NI}(\beta, d, w)$ for any $0 \leq \beta \leq \bar{\beta}_C(d, w)$ and $g_C(\beta, d) < g_T^{I/NI}(\beta, d, w)$ for any $\bar{\beta}_C(d, w) < \beta \leq 1$, where $\bar{\beta}_C(d, w) > 0$ and $\bar{\beta}_C(d, w) < \beta_{T_1}^{I/NI}(d, w)$ for any $0 < w < \frac{1}{2}$. Therefore, $\Delta\Pi_A > 0$, $\Delta\Pi_B < 0$ and $\Delta\Pi_C > 0$ for any $0 \leq \beta \leq \bar{\beta}_C(d, w)$ and $g_T^{I/NI}(\beta, d, w) < g < g_C(\beta, d)$, and $\Delta\Pi_A > 0$, $\Delta\Pi_B < 0$ and $\Delta\Pi_C < 0$ for any $0 \leq \beta \leq \bar{\beta}_C(d, w)$ and $g > g_C(\beta, d)$, for any $\bar{\beta}_C(d, w) < \beta \leq \beta_{T_1}^{I/NI}(d, w)$ and $g > g_T^{I/NI}(\beta, d, w)$, and for any $\beta_{T_1}^{I/NI}(d, w) < \beta \leq 1$ and $g > g_T^{I/I}(\beta, d, w)$. If $0 < d < \bar{d}_C(w)$ then $\bar{\beta}_C(d, w) < 0$ and $g_C(\beta, d) < g_T^{I/NI}(\beta, d, w)$ for any $0 \leq \beta \leq \beta_{T_1}^{I/NI}(d, w)$ and $g > g_T^{I/NI}(\beta, d, w)$, and $g_C(\beta, d) < g_T^{I/I}(\beta, d, w)$ for any $\beta_{T_1}^{I/NI}(d, w) < \beta \leq 1$ and $g > g_T^{I/I}(\beta, d, w)$. Therefore, $\Delta\Pi_A > 0$, $\Delta\Pi_B < 0$ and $\Delta\Pi_C < 0$ for any $0 \leq \beta \leq \beta_{T_1}^{I/NI}(d, w)$ and $g > g_T^{I/NI}(\beta, d, w)$, and for any $\beta_{T_1}^{I/NI}(d, w) < \beta \leq 1$ and $g > g_T^{I/I}(\beta, d, w)$.

Points [4], [5] and [6]. If products are substitutes ($d > 0$), $\frac{1}{2} \leq w < \frac{5}{8}$ and $d_T^{I/NI}(w) \leq d < 1$ then $g_{SC}^{\beta_{low}}(\beta, d)$ is binding for any $0 \leq \beta \leq \beta_{T_2}^{I/NI}(d, w)$, $g_T^{I/NI}(\beta, d, w)$ is binding for any $\beta_{T_2}^{I/NI}(d, w) <$

$\beta < \beta_{T_1}^{I/NI}(d,w)$ and $g_T^{I/I}(\beta,d,w)$ is binding $\beta_{T_1}^{I/NI}(d,w) < \beta \leq 1$ in the (β,g) space (from Proposition 3). In addition, $\frac{\partial g_C(\beta,d)}{\partial \beta} < 0$ for any $0 \leq \beta \leq 1$ and $\frac{1}{2} \leq w < \frac{5}{8}$. Then, $g_C(\beta,d) > g_{SC}^{\beta_{low}}(\beta,d) > g_T^{I/NI}(\beta,d,w)$ for any $0 \leq \beta \leq \beta_{T_2}^{I/NI}(d,w)$, $g_C(\beta,d) \geq g_T^{I/NI}(\beta,d,w)$ for any $\beta_{T_2}^{I/NI}(d,w) \leq \beta \leq \bar{\beta}_C(d,w)$ and $g_C(\beta,d) < g_T^{I/NI}(\beta,d,w)$ for any $\bar{\beta}_C(d,w) < \beta \leq 1$, where $\bar{\beta}_C(d,w) > 0$ and $\beta_{T_2}^{I/NI}(d,w) < \bar{\beta}_C(d,w) < \beta_{T_1}^{I/NI}(d,w)$ for any $\frac{1}{2} \leq w < \frac{5}{8}$. Therefore, $\Delta\Pi_A > 0$, $\Delta\Pi_B < 0$ and $\Delta\Pi_C > 0$ for any $0 \leq \beta \leq \beta_{T_2}^{I/NI}(d,w)$ and $g_{SC}^{\beta_{low}}(\beta,d) < g < g_C(\beta,d)$, and for any $\beta_{T_2}^{I/NI}(d,w) < \beta \leq \bar{\beta}_C(d,w)$ and $g_T^{I/NI}(\beta,d,w) < g < g_C(\beta,d)$, and $\Delta\Pi_A > 0$, $\Delta\Pi_B < 0$ and $\Delta\Pi_C < 0$ for any $0 \leq \beta \leq \beta_{T_2}^{I/NI}(d,w)$ and $g > g_C(\beta,d)$, for any $\beta_{T_2}^{I/NI}(d,w) < \beta \leq \bar{\beta}_C(d,w)$ and $g > g_C(\beta,d)$, for any $\bar{\beta}_C(d,w) < \beta \leq \beta_{T_1}^{I/NI}(d,w)$ and $g > g_T^{I/NI}(\beta,d,w)$, and for any $\beta_{T_1}^{I/NI}(d,w) < \beta \leq 1$ and $g > g_T^{I/I}(\beta,d,w)$. If products are substitutes $(d > 0)$, $\frac{1}{2} \leq w < \frac{5}{8}$ and $\bar{d}_C(w) \leq d < d_T^{I/NI}(w)$ then $g_T^{I/NI}(\beta,d,w)$ is binding for any $0 \leq \beta \leq \beta_{T_1}^{I/NI}(d,w)$ and $g_T^{I/I}(\beta,d,w)$ is binding $\beta_{T_1}^{I/NI}(d,w) < \beta \leq 1$ in the (β,g) space (from Proposition 3). In addition, $\frac{\partial g_C(\beta,d)}{\partial \beta} < 0$ for any $0 \leq \beta \leq 1$ and $\frac{1}{2} \leq w < \frac{5}{8}$. Then, $g_C(\beta,d) > g_T^{I/NI}(\beta,d,w)$ for any $0 \leq \beta < \bar{\beta}_C(d,w)$, $g_C(\beta,d) \leq g_T^{I/NI}(\beta,d,w)$ for any $\bar{\beta}_C(d,w) \leq \beta < \beta_{T_1}^{I/NI}(d,w)$ and $g_C(\beta,d) < g_T^{I/NI}(\beta,d,w)$ for any $\bar{\beta}_C(d,w) < \beta \leq 1$, where $\bar{\beta}_C(d,w) > 0$ and $\bar{\beta}_C(d,w) < \beta_{T_1}^{I/NI}(d,w)$ for any $\frac{1}{2} \leq w < \frac{5}{8}$. Therefore, $\Delta\Pi_A > 0$, $\Delta\Pi_B < 0$ and $\Delta\Pi_C > 0$ for any $0 \leq \beta \leq \bar{\beta}_C(d,w)$ and $g_T^{I/NI}(\beta,d,w) < g < g_C(\beta,d)$, and $\Delta\Pi_A > 0$, $\Delta\Pi_B < 0$ and $\Delta\Pi_C < 0$ for any $0 \leq \beta \leq \bar{\beta}_C(d,w)$ and $g > g_C(\beta,d)$, for any $\bar{\beta}_C(d,w) < \beta \leq \beta_{T_1}^{I/NI}(d,w)$ and $g > g_T^{I/NI}(\beta,d,w)$, and for any $\beta_{T_1}^{I/NI}(d,w) < \beta \leq 1$ and $g > g_T^{I/I}(\beta,d,w)$. If products are substitutes $(d > 0)$, $\frac{1}{2} \leq w < \frac{5}{8}$ and $0 < d < \bar{d}_C(w)$ then $g_T^{I/NI}(\beta,d,w)$ is binding for any $0 \leq \beta \leq \beta_{T_1}^{I/NI}(d,w)$ and $g_T^{I/I}(\beta,d,w)$ is binding $\beta_{T_1}^{I/NI}(d,w) < \beta \leq 1$ in the (β,g) space (from Proposition 3). In addition, $\frac{\partial g_C(\beta,d)}{\partial \beta} < 0$ for any $0 \leq \beta \leq 1$ and $\frac{1}{2} \leq w < \frac{5}{8}$. Then, $g_C(\beta,d) < g_T^{I/NI}(\beta,d,w)$ for any $0 \leq \beta \leq \beta_{T_1}^{I/NI}(d,w)$ and $g > g_T^{I/NI}(\beta,d,w)$, and $g_C(\beta,d) < g_T^{I/I}(\beta,d,w)$ for any $\beta_{T_1}^{I/NI}(d,w) < \beta \leq 1$ and $g > g_T^{I/I}(\beta,d,w)$. Therefore, $\Delta\Pi_A > 0$, $\Delta\Pi_B < 0$ and $\Delta\Pi_C < 0$ for any $0 \leq \beta \leq \beta_{T_1}^{I/NI}(d,w)$ and $g > g_T^{I/NI}(\beta,d,w)$, and for any $\beta_{T_1}^{I/NI}(d,w) < \beta \leq 1$ and $g > g_T^{I/I}(\beta,d,w)$.

Points [7], [8], [9] and [10]. If products are substitutes $(d > 0)$, $\frac{5}{8} \leq w < \frac{2}{3}$ and $d_{CC}(w) \leq d < 1$ then $g_{SC}^{\beta_{low}}(\beta,d)$ is binding for any $0 \leq \beta \leq \beta_{T_2}^{I/NI}(d,w)$, $g_T^{I/NI}(\beta,d,w)$ is binding for any $\beta_{T_2}^{I/NI}(d,w) < \beta < \beta_{T_1}^{I/NI}(d,w)$ and $g_T^{I/I}(\beta,d,w)$ is binding $\beta_{T_1}^{I/NI}(d,w) < \beta \leq 1$ in the (β,g) space (from Proposition 3). In addition, $\frac{\partial g_C(\beta,d)}{\partial \beta} < 0$ for any $0 \leq \beta \leq 1$ and $\frac{5}{8} \leq w < \frac{2}{3}$. Then, $g_C(\beta,d) > g_{SC}^{\beta_{low}}(\beta,d) > g_T^{I/NI}(\beta,d,w)$ for any $0 \leq \beta < \bar{\bar{\beta}}_C(d)$, $g_{SC}^{\beta_{low}}(\beta,d) > g_C(\beta,d) > g_T^{I/NI}(\beta,d,w)$ for any $\bar{\bar{\beta}}_C(d) \leq \beta < \beta_{T_2}^{I/NI}(d,w)$, $g_C(\beta,d) < g_T^{I/NI}(\beta,d,w)$ for any $\beta_{T_2}^{I/NI}(d,w) \leq \beta < \beta_{T_1}^{I/NI}(d,w)$, $g_C(\beta,d) < g_T^{I/I}(\beta,d,w)$ for any $\beta_{T_1}^{I/NI}(d,w) \leq \beta \leq 1$, where $\bar{\bar{\beta}}_C(d) > 0$ and $\bar{\bar{\beta}}_C(d) < \beta_{T_2}^{I/NI}(d,w) < \beta_{T_1}^{I/NI}(d,w)$ for any $\frac{1}{2} \leq w < \frac{5}{8}$. Therefore, $\Delta\Pi_A > 0$, $\Delta\Pi_B < 0$ and $\Delta\Pi_C > 0$ for any $0 \leq \beta \leq \bar{\bar{\beta}}_C(d)$ and $g_{SC}^{\beta_{low}}(\beta,d) < g < g_C(\beta,d)$, and $\Delta\Pi_A > 0$, $\Delta\Pi_B < 0$ and $\Delta\Pi_C < 0$ for any $0 \leq \beta \leq \bar{\bar{\beta}}_C(d)$ and $g > g_C(\beta,d)$, for any $\bar{\bar{\beta}}_C(d) \leq \beta < \beta_{T_2}^{I/NI}(d,w)$ and $g > g_{SC}^{\beta_{low}}(\beta,d)$, for any

$\beta_{T_2}^{I/NI}(d,w) \le \beta < \beta_{T_1}^{I/NI}(d,w)$, $g > g_T^{I/NI}(\beta,d,w)$, and for any $\beta_{T_1}^{I/NI}(d,w) \le \beta \le 1$ and $g > g_T^{I/I}(\beta,d,w)$. If products are substitutes ($d > 0$), $\frac{5}{8} \le w < \frac{2}{3}$ and $d_T^{I/NI}(w) \le d < d_{CC}(w)$ then $g_{SC}^{\beta_{low}}(\beta,d)$ is binding for any $0 \le \beta \le \beta_{T_2}^{I/NI}(d,w)$, $g_T^{I/NI}(\beta,d,w)$ is binding for any $\beta_{T_2}^{I/NI}(d,w) < \beta < \beta_{T_1}^{I/NI}(d,w)$ and $g_T^{I/I}(\beta,d,w)$ is binding $\beta_{T_1}^{I/NI}(d,w) < \beta \le 1$ in the (β,g) space (from Proposition 3). In addition, $\frac{\partial g_C(\beta,d)}{\partial\beta} < 0$ for any $0 \le \beta \le 1$ and $\frac{5}{8} \le w < \frac{2}{3}$. Then, $g_C(\beta,d) > g_{SC}^{\beta_{low}}(\beta,d) > g_T^{I/NI}(\beta,d,w)$ for any $0 \le \beta \le \beta_{T_2}^{I/NI}(d,w)$, $g_C(\beta,d) \ge g_T^{I/NI}(\beta,d,w)$ for any $\beta_{T_2}^{I/NI}(d,w) \le \beta \le \bar{\beta}_C(d,w)$ and $g_C(\beta,d) < g_T^{I/NI}(\beta,d,w)$ for any $\bar{\beta}_C(d,w) < \beta \le 1$, where $\bar{\beta}_C(d,w) > 0$ and $\beta_{T_2}^{I/NI}(d,w) < \bar{\beta}_C(d,w) < \beta_{T_1}^{I/NI}(d,w)$ for any $\frac{5}{8} \le w < \frac{2}{3}$. Therefore, $\Delta\Pi_A > 0$, $\Delta\Pi_B < 0$ and $\Delta\Pi_C > 0$ for any $0 \le \beta \le \beta_{T_2}^{I/NI}(d,w)$ and $g_{SC}^{\beta_{low}}(\beta,d) < g < g_C(\beta,d)$, and for any $\beta_{T_2}^{I/NI}(d,w) < \beta \le \bar{\beta}_C(d,w)$ and $g_T^{I/NI}(\beta,d,w) < g < g_C(\beta,d)$, and $\Delta\Pi_A > 0$, $\Delta\Pi_B < 0$ and $\Delta\Pi_C < 0$ for any $0 \le \beta \le \beta_{T_2}^{I/NI}(d,w)$ and $g > g_C(\beta,d)$, for any $\beta_{T_2}^{I/NI}(d,w) < \beta \le \bar{\beta}_C(d,w)$ and $g > g_C(\beta,d)$, for any $\bar{\beta}_C(d,w) < \beta \le \beta_{T_1}^{I/NI}(d,w)$ and $g > g_T^{I/NI}(\beta,d,w)$, and for any $\beta_{T_1}^{I/NI}(d,w) < \beta \le 1$ and $g > g_T^{I/I}(\beta,d,w)$. If products are substitutes ($d > 0$), $\frac{5}{8} \le w < \frac{2}{3}$ and $\bar{d}_C(w) \le d < d_T^{I/NI}(w)$ then $g_T^{I/NI}(\beta,d,w)$ is binding for any $0 \le \beta \le \beta_{T_1}^{I/NI}(d,w)$ and $g_T^{I/I}(\beta,d,w)$ is binding $\beta_{T_1}^{I/NI}(d,w) < \beta \le 1$ in the (β,g) space (from Proposition 3). In addition, $\frac{\partial g_C(\beta,d)}{\partial\beta} < 0$ for any $0 \le \beta \le 1$ and $\frac{5}{8} \le w < \frac{2}{3}$. Then, $g_C(\beta,d) > g_T^{I/NI}(\beta,d,w)$ for any $0 \le \beta < \bar{\beta}_C(d,w)$, $g_C(\beta,d) \le g_T^{I/NI}(\beta,d,w)$ for any $\bar{\beta}_C(d,w) \le \beta < \beta_{T_1}^{I/NI}(d,w)$ and $g_C(\beta,d) < g_T^{I/NI}(\beta,d,w)$ for any $\bar{\beta}_C(d,w) < \beta \le 1$, where $\bar{\beta}_C(d,w) > 0$ and $\bar{\beta}_C(d,w) < \beta_{T_1}^{I/NI}(d,w)$ for any $\frac{5}{8} \le w < \frac{2}{3}$. Therefore, $\Delta\Pi_A > 0$, $\Delta\Pi_B < 0$ and $\Delta\Pi_C > 0$ for any $0 \le \beta \le \bar{\beta}_C(d,w)$ and $g_T^{I/NI}(\beta,d,w) < g < g_C(\beta,d)$, and $\Delta\Pi_A > 0$, $\Delta\Pi_B < 0$ and $\Delta\Pi_C < 0$ for any $0 \le \beta \le \bar{\beta}_C(d,w)$ and $g > g_C(\beta,d)$, for any $\bar{\beta}_C(d,w) < \beta \le \beta_{T_1}^{I/NI}(d,w)$ and $g > g_T^{I/NI}(\beta,d,w)$, and for any $\beta_{T_1}^{I/NI}(d,w) < \beta \le 1$ and $g > g_T^{I/I}(\beta,d,w)$. If products are substitutes ($d > 0$), $\frac{5}{8} \le w < \frac{2}{3}$ and $0 < d < \bar{d}_C(w)$ then $g_T^{I/NI}(\beta,d,w)$ is binding for any $0 \le \beta \le \beta_{T_1}^{I/NI}(d,w)$ and $g_T^{I/I}(\beta,d,w)$ is binding $\beta_{T_1}^{I/NI}(d,w) < \beta \le 1$ in the (β,g) space (from Proposition 3). In addition, $\frac{\partial g_C(\beta,d)}{\partial\beta} < 0$ for any $0 \le \beta \le 1$ and $\frac{5}{8} \le w < \frac{2}{3}$. Then, $g_C(\beta,d) < g_T^{I/NI}(\beta,d,w)$ for any $0 \le \beta \le \beta_{T_1}^{I/NI}(d,w)$ and $g > g_T^{I/NI}(\beta,d,w)$, and $g_C(\beta,d) < g_T^{I/I}(\beta,d,w)$ for any $\beta_{T_1}^{I/NI}(d,w) < \beta \le 1$ and $g > g_T^{I/I}(\beta,d,w)$. Therefore, $\Delta\Pi_A > 0$, $\Delta\Pi_B < 0$ and $\Delta\Pi_C < 0$ for any $0 \le \beta \le \beta_{T_1}^{I/NI}(d,w)$ and $g > g_T^{I/NI}(\beta,d,w)$, and for any $\beta_{T_1}^{I/NI}(d,w) < \beta \le 1$ and $g > g_T^{I/I}(\beta,d,w)$.

Points [11] and [12]. If products are substitutes ($d > 0$), $\frac{2}{3} \le w < 1$ and $\frac{2}{3} \le d < 1$ then $g_{SC}^{\beta_{low}}(\beta,d)$ is binding for any $0 \le \beta \le \beta_{T_2}^{I/NI}(d,w)$, $g_T^{I/NI}(\beta,d,w)$ is binding for any $\beta_{T_2}^{I/NI}(d,w) < \beta < \beta_{T_1}^{I/NI}(d,w)$ and $g_T^{I/I}(\beta,d,w)$ is binding $\beta_{T_1}^{I/NI}(d,w) < \beta \le 1$ in the (β,g) space (from Proposition 3). In addition, $\frac{\partial g_C(\beta,d)}{\partial\beta} < 0$ for any $0 \le \beta \le 1$ and $\frac{2}{3} \le w < 1$. Then, $g_C(\beta,d) > g_{SC}^{\beta_{low}}(\beta,d)$ for any $0 \le \beta < \bar{\bar{\beta}}_C(d)$, $g_C(\beta,d) < g_{SC}^{\beta_{low}}(\beta,d)$ for any $\bar{\bar{\beta}}_C(d) \le \beta < \beta_{T_2}^{I/NI}(d,w)$, $g_C(\beta,d) < g_T^{I/NI}(\beta,d,w)$ for any $\beta_{T_2}^{I/NI}(d,w) \le \beta < \beta_{T_1}^{I/NI}(d,w)$, $g_C(\beta,d) < g_T^{I/I}(\beta,d,w)$ for any $\beta_{T_1}^{I/NI}(d,w) \le \beta \le 1$, where $\bar{\bar{\beta}}_C(d) > 0$ and $\bar{\bar{\beta}}_C(d) < \beta_{T_2}^{I/NI}(d,w) < \beta_{T_1}^{I/NI}(d,w)$ for any $\frac{2}{3} \le w <$

1. Therefore, $\Delta\Pi_A > 0$, $\Delta\Pi_B < 0$ and $\Delta\Pi_C > 0$ for any $0 \le \beta \le \bar{\bar{\beta}}_C(d)$ and $g_{SC}^{\beta_{low}}(\beta, d) < g < g_C(\beta, d)$, and $\Delta\Pi_A > 0$, $\Delta\Pi_B < 0$ and $\Delta\Pi_C < 0$ for any $0 \le \beta \le \bar{\bar{\beta}}_C(d)$ and $g > g_C(\beta, d)$, for any $\bar{\bar{\beta}}_C(d) \le \beta < \beta_{T_2}^{I/NI}(d, w)$ and $g > g_{SC}^{\beta_{low}}(\beta, d)$, for any $\beta_{T_2}^{I/NI}(d, w) \le \beta < \beta_{T_1}^{I/NI}(d, w)$, $g > g_T^{I/NI}(\beta, d, w)$, and for any $\beta_{T_1}^{I/NI}(d, w) \le \beta \le 1$ and $g > g_T^{I/I}(\beta, d, w)$. If products are substitutes ($d > 0$), $\frac{2}{3} \le w < 1$ and $0 < d < \frac{2}{3}$ then $g_{SC}^{\beta_{low}}(\beta, d)$ is binding for any $0 \le \beta \le \beta_{T_2}^{I/NI}(d, w)$, $g_T^{I/NI}(\beta, d, w)$ is binding for any $\beta_{T_2}^{I/NI}(d, w) < \beta < \beta_{T_1}^{I/NI}(d, w)$ and $g_T^{I/I}(\beta, d, w)$ is binding $\beta_{T_1}^{I/NI}(d, w) < \beta \le 1$ in the (β, g) space (from Proposition 3). In addition, $\frac{\partial g_C(\beta,d)}{\partial \beta} < 0$ for any $0 \le \beta \le 1$ and $\frac{2}{3} \le w < 1$. Then, $\bar{\bar{\beta}}_C(d) < 0$ and $g_C(\beta, d) < g_{SC}^{\beta_{low}}(\beta, d)$ for any $0 \le \beta \le \beta_{T_2}^{I/NI}(d, w)$, $g_C(\beta, d) < g_T^{I/NI}(\beta, d, w)$ and $g_C(\beta, d) < g_T^{I/I}(\beta, d, w)$ for any $\beta_{T_1}^{I/NI}(d, w) < \beta \le 1$. Therefore, $\Delta\Pi_A > 0$, $\Delta\Pi_B < 0$ and $\Delta\Pi_C < 0$ for any $0 \le \beta \le \beta_{T_2}^{I/NI}(d, w)$ and $g > g_{SC}^{\beta_{low}}(\beta, d)$, for any $\beta_{T_2}^{I/NI}(d, w) < \beta \le \beta_{T_1}^{I/NI}(d, w)$ and $g > g_T^{I/NI}(\beta, d, w)$, and for any $\beta_{T_1}^{I/NI}(d, w) < \beta \le 1$ and $g > g_T^{I/I}(\beta, d, w)$.

Points [13] and [14]. If products are substitutes ($d > 0$), $w \to 1$ and $\frac{2}{3} \le d < 1$ then $g_{SC}^{\beta_{low}}(\beta, d)$ is binding for any $0 \le \beta \le \frac{d}{2}$ and $g_{SC}^{\beta_{high}}(\beta, d)$ is binding for any $\frac{d}{2} < \beta \le 1$ in the (β, g) space (from Proposition 3). In addition, $\frac{\partial g_C(\beta,d)}{\partial \beta} < 0$ for any $0 \le \beta \le 1$ and $w \to 1$. Then, $g_C(\beta, d) > g_{SC}^{\beta_{low}}(\beta, d)$ for any $0 \le \beta < \bar{\bar{\beta}}_C(d)$, $g_C(\beta, d) \le g_{SC}^{\beta_{low}}(\beta, d)$ for any $\bar{\bar{\beta}}_C(d) \le \beta \le \frac{d}{2}$, $g_C(\beta, d) < g_{SC}^{\beta_{high}}(\beta, d)$ for any $\frac{d}{2} \le \beta \le 1$, where $\bar{\bar{\beta}}_C(d) > 0$ and $\bar{\bar{\beta}}_C(d) < \frac{d}{2}$ for any $w \to 1$. Therefore, $\Delta\Pi_A > 0$, $\Delta\Pi_B < 0$ and $\Delta\Pi_C > 0$ for any $0 \le \beta \le \bar{\bar{\beta}}_C(d)$ and $g_{SC}^{\beta_{low}}(\beta, d) < g < g_C(\beta, d)$, and $\Delta\Pi_A > 0$, $\Delta\Pi_B < 0$ and $\Delta\Pi_C < 0$ for any $0 \le \beta \le \bar{\bar{\beta}}_C(d)$ and $g > g_C(\beta, d)$, for any $\bar{\bar{\beta}}_C(d) \le \beta < \frac{d}{2}$ and $g > g_{SC}^{\beta_{low}}(\beta, d)$, and for any $\frac{d}{2} \le \beta \le 1$ and $g > g_{SC}^{\beta_{high}}(\beta, d)$. If products are substitutes ($d > 0$), $w \to 1$ and $0 < d \le \frac{2}{3}$ then $g_{SC}^{\beta_{low}}(\beta, d)$ is binding for any $0 \le \beta \le \frac{d}{2}$ and $g_{SC}^{\beta_{high}}(\beta, d)$ is binding for any $\frac{d}{2} < \beta \le 1$ in the (β, g) space (from Proposition 3). In addition, $\frac{\partial g_C(\beta,d)}{\partial \beta} < 0$ for any $0 \le \beta \le 1$ and $w \to 1$. Then, $g_C(\beta, d) < g_{SC}^{\beta_{low}}(\beta, d)$ for any $0 \le \beta < \frac{d}{2}$ and $g_C(\beta, d) \le g_{SC}^{\beta_{high}}(\beta, d)$ for any $\frac{d}{2} \le \beta \le 1$, where $\bar{\bar{\beta}}_C(d) < 0$ for any $w \to 1$. Therefore, $\Delta\Pi_A > 0$, $\Delta\Pi_B < 0$ and $\Delta\Pi_C < 0$ for any $0 \le \beta \le \frac{d}{2}$ and $g > g_{SC}^{\beta_{low}}(\beta, d)$, and for any $\frac{d}{2} \le \beta \le 1$ and $g > g_{SC}^{\beta_{high}}(\beta, d)$. **Q.E.D.**

Proof of Corollary 1. The proof of Corollary 1 follows the same line of reasoning and uses the same arguments as the proof Proposition 4 by assuming $d = 1$ and knowing that $g_C(\beta, 1) \coloneqq \frac{2(1+\beta)^2(2-\beta)}{27\beta}$, so that $g_C(0,1) \to +\infty$ if $\beta \to 0$. This implies that it is not possible to solve the prisoner's dilemma in the absence of R&D spill-overs if products are perfect substitutes. **Q.E.D.**

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