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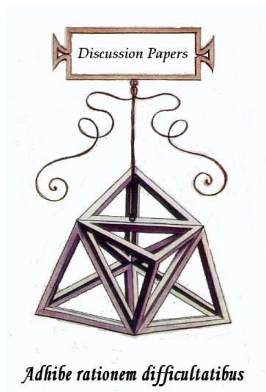

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Product quality and product compatibility in network industries

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Abstract

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Keywords Network externality; Product compatibility; Cournot duopoly; Quality differential

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1. Introduction

Markets for network products (goods and services) are of increasing relevance and size in contemporary economics. The key characteristic of those products is that the consumers' utility depends not just on the use of the individual but also on the number of other users. In this case, positive consumption externalities, which generate the so-called bandwagon effect, exist.[1] Telephones, mobile phones, and the hardware and software of computers are clear examples of network goods; telecommunication services, internet-related activities such as online video players and banking exemplifies network services. As Katz and Shapiro (1985) pinpoint, network effects can play a relevant role in strategic competitive markets and decisions concerning R&D innovation (Cabral, 1990; Buccella et al., 2022a; for a survey, see Shy, 2011). Additional significant characteristics of network industries are the presence of possible product compatibility[2] and the provision of goods and services of different quality.[3] Our paper precisely focuses on those aspects.

As anecdotal evidence, consider first the classical case of Microsoft and Apple products in the computer market. Microsoft computers are nowadays based on the Windows operating system (OS), while Apple uses the macOS. Compatibility issues exist, but historically Apple computers can work with Microsoft software, while Microsoft computers are incompatible with Apple software.

Consider now the market for smartphones. Moving from a product that uses the operating system Android to an Apple product that runs with the iOS system, compatibility issues appear 1) for the use of applications and 2) because of different file formats. Moreover, peripheral devices are, in most cases, incompatible. Nonetheless, once again, Apple devices are usually (at least in part) hardware and software compatible with Android devices, while the opposite does not hold. Moreover, in the same segment of the market, Apple devices are usually perceived as being of higher quality compared to those of other brands, and consumers' willingness to pay for the most recent version of the iPhone series (iPhone 14 Pro, price 999USD) is much higher than that for Motorola devices (Motorola Edge Fusion 30, price 599,99USD).

To the best of our knowledge, the economics literature has paid scarce attention to the endogenization of the choice of compatibility according to an appropriate game-theoretic approach. Many strong results – nowadays common wisdom, like the one that equilibrium prices and profits are higher under compatibility – have been obtained only

[1] Network goods can generate negative consumption externalities as well. A typical example is the so-called Snob Effect: the desire to not have a good because almost everyone else has it (see Buccella et al., 2022b).
[2] Another feature sometimes considered is the presence of switching costs burdened by consumers in the case in which they change over from one brand to another when the two brands are not compatible. This feature will be considered in future works. See Shy (2001) for an exhaustive treatment of the network goods issue.
[3] Some articles considering the issue of quality in network industries are, for example, Choi (2002) and Reme (2019), but they focus on issues at all different from those investigated in the present paper.

by comparing the polar situations of compatibility and incompatibility, and often considering only the specific model of spatial competition without network effects (see, e.g., Matutes and Regibeau, 1988; Economides, 1989; Choi and Kim, 2015). Recently, Stadler et al. (2022) develop a two-stage model in the spatial competition frame with network effects; however, their firm's decisional variable is "in which degree" and not "whether" to be compatible, as the present work does.

This article sharply departs from the previous literature by choosing an appropriate formulation of the endogenous game and a standard quantity competition framework for network industries. The latter has been largely investigated and applied to many issues such as the managerial delegation (e.g., Hoernig, 2012; Bhattacharjee and Pal, 2013; Chirco and Scrimitore, 2013; Nakamura, 2020) the corporate social responsibility (Fanti and Buccella, 2016a), the unionized oligopolies (Fanti and Buccella, 2016b) and the codetermination in the labour market (e.g., Fanti and Gori, 2019). However, the issue of compatibility and its strategic use have been so far neglected.[4] Moreover, the presence of firms of different quality has not been considered in such a framework.

This article aims at filling these gaps by providing an analysis based on a non-cooperative game-theoretic setting with complete information. Hence, we properly endogenize the choice of compatibility versus non-compatibility and specifically investigate the effects – regarding choice and properties – of the presence of low- and high-quality firms in the same network industry. For doing this, it considers a strategic two-stage non-cooperative game in which two firms simultaneously choose: (i) in the first stage, whether to produce compatible network goods;[5] and (ii) in the second stage, the output in the product market.[6] We denote such a game as the "compatibility decision game" (CDG henceforth).

By assuming homogenous and heterogeneous qualities, results show that the unique stable sub-game perfect Nash equilibrium (SPNE) is product compatibility. However, while with homogenous quality the SPNE is also Pareto efficient (CDG is a deadlock), with a sufficiently significant heterogeneous quality the SPNE is Pareto inefficient as a common incompatibility choice would benefit the high-quality firm.

The rationale for the latter result is as follows: in the case of compatible products, when the quality progressively worsens (resp. improves) the reduction (resp. increase) in the output of the low- (resp. high-) quality firm is less than whether it had chosen to

[4] For instance, Naskar and Pal (2020) and Shrivastav (2021) analyse the implications of network compatibility on process innovation in a differentiated network goods duopoly, but compatibility is exogenously assumed in their models.

[5] For simplicity we will discuss the duopolistic quantity competition under full compatibility (when the product or the components produced by all firms are compatible with each other) and under incompatibility (when the product or no components produced by different firms are compatible) as is standard in the literature on compatibility between competing products (e.g., Matutes and Regibeau, 1988; Economides, 1989). Recent exceptions are Stadler et al. (2022) and Buccella et al. (2023), who also consider the case of partial compatibility.

[6] Our approach considers a static game. Since the issue of compatibility has been also considered under a dynamic aspect (e.g., Katz and Shapiro, 1986a; Heinrich, 2017), we may consider an appropriate dynamic game as one interesting further direction of research.

produce non-compatible products; in particular, when its quality becomes sufficiently high, the high-quality firm produces even a lower output than if it had chosen to be non-compatible. Indeed, the article also shows that some established effects of the heterogeneous quality in standard markets significantly modify in presence of network goods, depending on whether these goods are compatible.

Finally, the article argues – regarding the high-quality firm – that a remedy to escape the dilemma in which the firm is entrapped ("bad" equilibrium) does exist: the profit gain obtained by the high-quality firm in the case of a switch from the SPNE, in which both firms produce compatible products, to the situation of common incompatibility is high enough that there is room for a side-payment towards the low-quality firm to let them play the incompatibility strategy, although incompatibility reduces the consumer surplus, and the side-payment can be under the scrutiny of the Anti-trust authorities.

The rest of the article proceeds as follows. Section 2 sets up the model by also presenting and discussing the main results about output and profits in the polar cases of full compatibility and incompatibility. Section 3 provides the analysis of the endogenous CGD with product quality. Section 4 concentrates on the social welfare outcomes corresponding to the SPNE. Section 5 outlines the conclusions. Appendix A presents some analytical details and Appendix B concentrates on the model with asymmetric installed bases (history matters). Finally, Appendix C and appendix D respectively show the results of the CDG with asymmetric marginal costs and output commitment of one of the two firms.

2. The model

The model presented in this section directly departs from Katz and Shapiro (1985), who were first in studying network externality and product compatibility in a quantity-setting strategic competitive market. That work was followed by two other relevant articles (Katz and Shapiro, 1986a, 1986b) on the topic of product compatibility in oligopolistic industries. In these contributions, the degree of compatibility was exogenously given. The industrial organisation literature has also concentrated on the case of endogenous compatibility by letting firms choose the extent of the degree of compatibility of their products with aim at maximising profits (e.g., Economides, 1989; Kim and Choi 2015; Stadler et al., 2022). The present article goes one step further and considers the degree of compatibility as a strategic variable. To this purpose, it adds to the literature the compatibility decision stage to a Cournot duopoly, in which each firm produces low-quality or high-quality products (as were perceived by customers) and strategically chooses whether they should be compatible with those of its rival. This will allow us to pinpoint a complete set of sub-game perfect Nash equilibria emerging in the compatibility decision game with product quality and then study whether product compatibility can emerge endogenously as a market outcome in a strategic context. Results depends on the extent of the network externality, which can be positive (bandwagon effect) or negative (snob effect). To simplify the narrative and have well-defined outcomes and economic intuition, we consider the polar cases of full compatibility versus no compatibility.

The timing of the non-cooperative compatibility decision game with product quality and complete information is the following. At the first (decision) stage each firm chooses to let products being fully compatible or incompatible. At the second (market) stage, each firm competes à la Cournot in the product market and then chooses the quantity to maximise profits.

The section now outlines the main features of the model. Consider a Cournot duopoly in which firms produce homogeneous network goods (Katz and Shapiro, 1985). The network effect (consumption externality) can be positive (e.g., mobile communications, software, internet-related activities, online social networks, fashion, etc) or negative (e.g., traffic congestion or network congestion over limited bandwidth). Under a positive (resp. negative) externality an increasing number of users increases (resp. reduces) the individual utility and thus the value of the goods for each consumer thus causing the so-called bandwagon (resp. snob) effect. To tackle the issue of strategic product compatibility with product quality we follow the main narrative of Katz and Shapiro (1985) and assume that firms are unable to commit themselves to a given output level and thus consumers form their expectations on total sales, which are fulfilled at equilibrium according to the standard rational expectations hypothesis. The microeconomic foundations of the (linear) market demands with network externality and product compatibility follow the recent article by Buccella et al. (2022)[7] and adds consumers' preferences for low- and high-quality product.

More formally, there exists a continuum of identical consumers with identical preferences described by a separable utility function $V = U + m$, where m is the numeraire good produced by a competitive industry and U is a twice continuously differentiable function that evaluates the individual welfare from the consumption of network goods q_i and q_j produced by a duopolistic industry. The utility function V, therefore, is quasi-linear in m so that all the related properties about the demand of m and the demand of the network goods q_i and q_j hold (Amir et al., 2017; Choné and Linnemer, 2020). Consumers evaluate differently the quality of the products produced by firm i and firm j $(i, j = \{1,2\},\ i \neq j)$, which are framed in a quantity-setting duopoly. The function U follows the usual specification of a quadratic utility. Therefore, one gets:

$$U = aq_i + bq_j - \frac{1}{2}(q_i^2 + q_j^2 + 2q_iq_j) + n[q_i(y_i + k_iy_j) + q_j(y_j + k_jy_i)] - \frac{n}{2}(y_i^2 + y_j^2 + 2k_iy_jk_jy_i), \quad (1)$$

where $a > 0$ (resp. $b > 0$) is the market size of the product of network i (resp. j) representing an index of product quality. If $a > b$ (resp. $a < b$) then the product of network i is perceived, ceteris paribus, of higher (resp. lower) quality than the product of network j by customers. In addition, $-1 \leq n \leq 1$ is the strength of the network effect ($n = 0$ represents the standard case of non-network goods) and the higher the absolute value of n, the stronger the effect of network goods. Positive (resp. negative) values of n reflect a positive (resp. negative) consumption externality capturing the bandwagon (resp. snob) effect. The variable q_i (resp. q_j) denotes the quantity of goods produced by firm i (resp. firm j). The parameter $k_i \in [0,1]$ measures the degree of

[7] See also Naskar and Pal (2020) and Shrivastav (2021).

compatibility of the network of product j towards the network of product i. Pairwise, the parameter $k_j \in [0,1]$ measures the degree of compatibility of the network of product i with the network of product j. In addition, y_i and y_j denotes the consumers' expectations about the equilibrium output produced by firm i and firm j, respectively. For analytical tractability, and without loss of generality, we will consider henceforth the case of symmetric compatibility $k_i = k_j = k$, which resembles the case of common standardisation (Stadler et al., 2022).

The representative consumer maximises $V = U + m$ subject to the budget constraint $p_i q_i + p_j q_j + m = R$, where q_i and q_j are the control variables of the problem, p_i and p_j represent the price of (i.e., the marginal willingness to pay of the representative consumer for) product of variety i and variety j, respectively, and R is the consumer's exogenous nominal income. This income is assumed to be high enough to avoid income effects on the demand of q_i and q_j (i.e., the goods entering non-linearly in V).

By using (1), one gets the (normalised) inverse market demand of firm i and firm j are (see also, e.g., Hoernig 2012; Chirco and Scrimitore, 2013; Bhattacharjee and Pal, 2014; Fanti and Buccella, 2016; Buccella et al., 2022), which are given by the following expressions:

$$p_i = a - q_i - q_j + n(y_i + ky_j), \tag{2}$$

and

$$p_j = b - q_j - q_i + n(y_j + ky_i), \tag{3}$$

For the sake of simplicity and without loss of generality, in what follows we will assume $b = 1$. Therefore, if $a > 1$ (resp. $a < 1$) the relative quality of the product of network i is larger than that of network j.

The generic firm i's profit function is given by:

$$\Pi_i = (p_i - w)q_i, \tag{4}$$

where $0 \leq w < 1$ is the constant average and marginal cost (i.e., the technology has constant returns to scale), which is set to zero henceforth without loss of generality as our aim is to deal with the most parsimonious modelling structure possible. To simplify the analysis, we build on the CDG with product quality by considering full compatibility ($k = 1$) versus incompatibility ($k = 0$) in turn avoiding partial compatibility. Although this choice may seem simplistic, it indeed replicates the case of endogenous compatibility, in which each firm can choose the degree of compatibility of their products at an intermediate stage of the game. In fact, the profit function is monotonically increasing in k when the network externality is positive, so that each of them has the incentive to produce goods to the highest degree of compatibility. Indeed, the existence of an interior optimal degree of compatibility, that is partial compatibility, (e.g., Stadler et al., 2022), or a corner solution implying full compatibility does not mean that the production of compatible products emerges as an endogenous SPNE of a non-cooperative compatibility decision game with complete information. The emergence of this kind of equilibrium depends on the firms' incentives at the first (decision) stage of the game, which is the main aim of the present article and the main innovation in the related literature.

The model now proceeds by considering the behaviour of firm i and firm j in each possible sub-game, i.e., the sub-game in which 1) both firms produce compatible goods, K/K, 2) both firms produce incompatible goods, NK/NK, 3) one firm produces compatible goods and the rival produces incompatible goods, K/NK and NK/K.

At the second stage of a generic sub-game, firm i maximises its profits by taking the quantity produced by the rival and the consumers' expectations about the equilibrium output as given. The same is done by the counterpart, firm j. More formally, one gets:

$$\frac{\partial \Pi_i}{\partial q_i} = 0 \Rightarrow RF_i(q_j), \tag{5}$$

and

$$\frac{\partial \Pi_j}{\partial q_j} = 0 \Rightarrow RF_j(q_i), \tag{6}$$

where $RF_i(q_j)$ and $RF_j(q_i)$ represent the reaction function of firm i and the reaction function of firm j, respectively, i.e., the quantity that firm i (resp. firm j) produces to maximise its profits as a function the quantity it expects the rival will produce given the network size and the amount of products of network i and network j consumers will expect to be produced.

From Eqs. (5) and (6) it is possible to derive the system of output reaction functions that can be used, by imposing the usual "rational expectation condition" such that $y_i = q_i$ and $y_j = q_j$, to compute the (exogenous) Nash equilibrium quantity produced by each firm. To this purpose, Table 1 summarises the outcomes (quantities) obtained by every firm at the Nash equilibrium in each sub-game at the second stage of the game (the market stage) and Table 2, instead, represents the payoff matrix that includes the corresponding profit function. The entries of Table 2 are used by each firm to study the convenience to play K or NK at the first (decision) stage of the CDG.

Table 1. Quantities in each sub-game of the CDG. Second stage.

Firm j → Firm i ↓	K	NK
K	$q_i^{*K/K}, q_j^{*K/K}$	$q_i^{*K/NK}, q_j^{*K/NK}$
NK	$q_i^{*NK/K}, q_j^{*NK/K}$	$q_i^{*NK/NK}, q_j^{*NK/NK}$

Table 2. The CDG (payoff matrix: profits). First stage.

Firm j → Firm i ↓	K	NK
K	$\Pi_i^{*K/K}, \Pi_j^{*K/K}$	$\Pi_i^{*K/NK}, \Pi_j^{*K/NK}$
NK	$\Pi_i^{*NK/K}, \Pi_j^{*NK/K}$	$\Pi_i^{*NK/NK}, \Pi_j^{*NK/NK}$

The entries of Table 1 are:

$$q_i^{*K/K} = \frac{a(2-n)-(1-n)}{3-2n} > 0 \text{ iff } a > \frac{1-n}{2-n} := a^{\circ}(n), \tag{7}$$

$$q_j^{*K/K} = \frac{2-n-a(1-n)}{3-2n} > 0 \text{ iff } a < \frac{2-n}{1-n} := a^{\circ\circ}(n), \tag{8}$$

$$q_i^{*NK/NK} = \frac{a(2-n)-1}{(3-n)(1-n)} > 0 \text{ iff } a > \frac{1}{2-n} \coloneqq a^{\circ\circ\circ}(n), \tag{9}$$

$$q_j^{*NK/NK} = \frac{2-a-n}{(3-n)(1-n)} > 0 \text{ iff } a < 2-n \coloneqq a^{\circ\circ\circ\circ}(n), \tag{10}$$

$$q_i^{*K/NK} = \frac{a(2-n)-(1-n)}{3(1-n)+n^2} > 0 \text{ iff } a > a^{\circ}(n), \tag{11}$$

$$q_j^{*K/NK} = \frac{2-a-n}{3(1-n)+n^2} > 0 \text{ iff } a < a^{\circ\circ\circ\circ}(n), \tag{12}$$

and

$$q_i^{*NK/K} = \frac{a(2-n)-1}{3(1-n)+n^2} > 0 \text{ iff } a > a^{\circ\circ\circ}(n), \tag{13}$$

$$q_j^{*NK/K} = \frac{2-n-a(1-n)}{3(1-n)+n^2} > 0 \text{ iff } a < a^{\circ\circ}(n), \tag{14}$$

where the feasibility conditions $a > a^{\circ}(n)$ and $a < a^{\circ\circ}(n)$ respectively imply, in the sub-game K/K, that the quality, as perceived by consumers, of firm i's products should be high enough to ensure, given the network size, a positive demand towards it, but low enough to prevent the rival from exiting the market due to the perceived relative insufficient quality of its products. Similar conditions exist in the sub-game NK/NK, in which $a > a^{\circ\circ\circ}(n)$ mush hold for firm i and $a < a^{\circ\circ}(n)$ for firm j. These conditions must be fulfilled, albeit in a cross-over manner, in the asymmetric sub-game K/NK and NK/K whereby, 1) in the former case, the perceived quality of the products of the firm producing compatible goods must be sufficiently high whereas that of the products of the firm producing incompatible goods must be sufficiently low to ensure that both firms do not exit the market, and 2) in the latter case the perceived quality of the products of the firm producing incompatible goods must be sufficiently high whereas that of the products of the firm producing compatible goods must be sufficiently low to ensure that both firms do not exit the market. The shape of the feasibility constraints depicted in the space (a, n) is drawn in Figure 1. The figure also reports the corresponding feasible (white area) and unfeasible (sand-coloured area) regions. Specifically, the constraints $a > a^{\circ}(n)$ and $a > a^{\circ\circ}(n)$ are always fulfilled for any $n > 0$, but they are both binding for any $n < 0$. Differently, the constraints $a > a^{\circ\circ\circ}(n)$ and $a < a^{\circ\circ\circ\circ}(n)$ are always fulfilled for any $n < 0$, but they are both binding for any $n > 0$. The higher the absolute value of n the narrower the space of the relative quality differential for which the model is feasible. This means that, in network industries, there is less room for the coexistence of low- and high-quality firms.

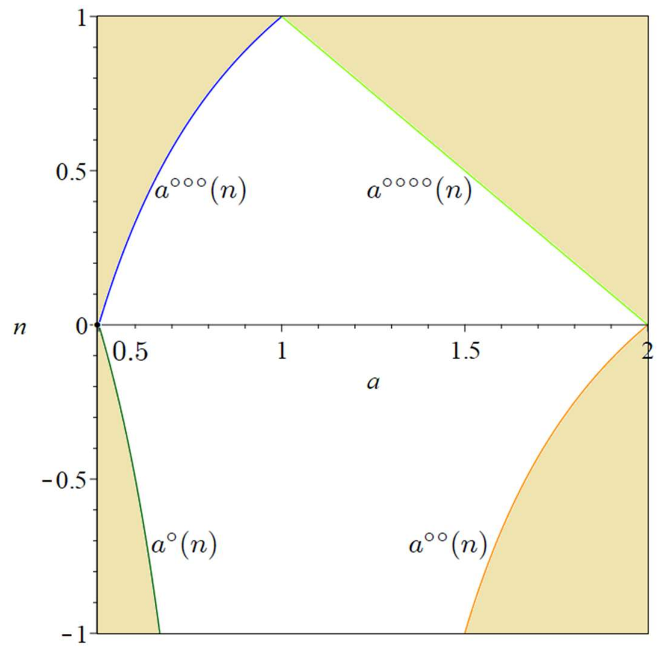

Figure 1. Feasible region (white area) and unfeasible region (sand-coloured) of the CDG with product quality in the space (a, n). The low-quality (high-quality) firm is i for any $a < 1$ (resp. $a > 1$). When $n = 0$, the range of feasible values of the relative product quality is $0.5 < a < 2$ in the horizontal axis. The feasible values of the relative product quality shrinks when the absolute value of n increases.

The entries of Table 2 are:

$$\Pi_i^{*K/K} = \frac{[a(2-n)-(1-n)]^2}{(3-2n)^2}, \tag{15}$$

$$\Pi_j^{*K/K} = \frac{[2-n-a(1-n)]^2}{(3-2n)^2}, \tag{16}$$

$$\Pi_i^{*NK/NK} = \frac{[a(2-n)-1]^2}{(3-n)^2(1-n)^2}, \tag{17}$$

$$\Pi_j^{*NK/NK} = \frac{(2-a-n)^2}{(3-n)^2(1-n)^2}, \tag{18}$$

$$\Pi_i^{*K/NK} = \frac{[a(2-n)-(1-n)]^2}{[3(1-n)+n^2]^2}, \tag{19}$$

$$\Pi_j^{*K/NK} = \frac{(2-a-n)^2}{[3(1-n)+n^2]^2}, \tag{20}$$

and

$$\Pi_i^{*NK/K} = \frac{[a(2-n)-1]^2}{[3(1-n)+n^2]^2}, \tag{21}$$

$$\Pi_j^{*NK/K} = \frac{[2-n-a(1-n)]^2}{[3(1-n)+n^2]^2}. \tag{22}$$

Lemma 1 and Lemma 2 clarify the role of the feasibility constraints in the CDG game with product quality given any value of the network externality.

Lemma 1. If the network externality is positive, the CDG with product quality is feasible for any $a^{\circ\circ\circ}(n) < a < a^{\circ\circ\circ\circ}(n)$.

Lemma 2. If the network externality is negative, the CDG with product quality is feasible for any $a^{\circ}(n) < a < a^{\circ\circ}(n)$.

The basic established effect of an increase in the quality index a in a Cournot duopoly is to increase (resp. reduce) output, market share and profit of firm i (resp. firm j). In a network industry this basic effect of the (relative) quality differential is affected by the degree of the network externality. The network effect affects the basic quality differential in a different way depending on whether products are compatible ($k = 1$) or incompatible $k = 0$. Specifically, the quality differential effect is magnified by the intensity of the network effect for both low- and high-quality firms when products are incompatible. Differently, the quality differential effect is mitigated for firm j (eventually, the network effect is maximal when the increase in the quality of the firm i has no effect on output of firm j. These effects are captured by the following expressions:

$$\frac{\partial q_i^{*NK/NK}}{\partial a} = \frac{2-n}{(3-n)(1-n)} > 0 \text{ and } \frac{\partial q_j^{*NK/NK}}{\partial a} = \frac{-1}{(3-n)(1-n)} < 0, \tag{23}$$

$$\frac{\partial^2 q_i^{*NK/NK}}{\partial a \partial n} = \frac{5-4n+n^2}{(3-n)^2(1-n)^2} > 0 \text{ and } \frac{\partial^2 q_j^{*NK/NK}}{\partial a \partial n} = \frac{-2(2-n)}{(3-n)^2(1-n)^2} < 0, \tag{24}$$

and

$$\frac{\partial q_i^{*K/K}}{\partial a} = \frac{2-n}{3-2n} > 0 \text{ and } \frac{\partial q_j^{*K/K}}{\partial a} = \frac{-(1-n)}{3-2n} < 0, \tag{25}$$

$$\frac{\partial^2 q_i^{*K/K}}{\partial a \partial n} = \frac{1}{(3-2n)^2} > 0 \text{ and } \frac{\partial^2 q_j^{*K/K}}{\partial a \partial n} = \frac{1}{(3-2n)^2} > 0, \tag{26}$$

Moreover, and more interestingly, the well-known positive network effect on output holds only when products are compatible. Unlike this, in the absence of compatibility, the higher the network effect, the lower the output of the low-quality firm. Indeed, a sufficiently high level of the network intensity may even induce that firm to exit the market. In other words, the firm producing incompatible products, if it is the low-quality producer, is, rather paradoxically, harmed by the positive consumption externality.[8]

Lemma 3. Under incompatibility, the established positive effect of the consumption externalities on output does no longer hold for the low-quality firm: the higher n, the lower the output when $a < \hat{a}(n)$ if firm i is the low-quality producer or $a > \breve{a}(n)$ if firm i is the low-quality producer.

Figure 2 graphically depicts Lemma 3.

[8] We recall that in the case of incompatibility, one firm loses the positive externality represented by the rival's production, but the positive externality represented by its production is always carried out, and thus, at a first sight, a higher network effect should be beneficial.

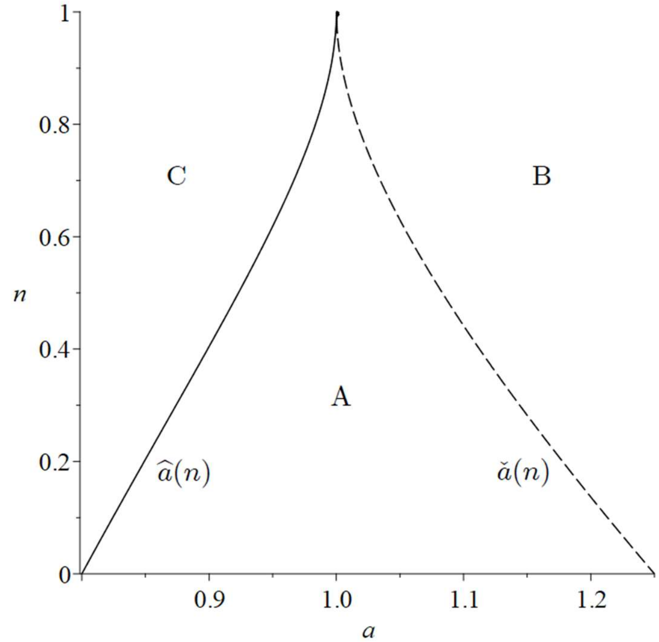

Figure 2. Plot of the critical curves $\hat{a}(n)$ and $\check{a}(n)$ in the plane (a, n). Area A: $\frac{\partial q_i^{*NK/NK}}{\partial n} > 0$ and $\frac{\partial q_j^{*NK/NK}}{\partial n} > 0$. Area B: $\frac{\partial q_i^{*NK/NK}}{\partial n} < 0$ and $\frac{\partial q_j^{*NK/NK}}{\partial n} < 0$. Area C: $\frac{\partial q_i^{*NK/NK}}{\partial n} < 0$ and $\frac{\partial q_j^{*NK/NK}}{\partial n} > 0$.

The ranking of the output levels under compatibility and incompatibility may change. This is univocally determined in the case of homogeneous quality ($a = 1$), bur depends on the relative size of the quality index and the network externality in the case of heterogeneous quality ($a \neq 1$). This is shown in Lemma 4.

Lemma 4. Under homogeneous quality ($a = 1$), the output under compatibility is higher than under incompatibility. Under heterogenous quality ($a \neq 1$), the output differential for the high-quality (resp. low-quality) firm is monotonically decreasing (resp. increasing) in the relative quality index a. For a sufficiently high-quality index, $a > a^*(n)$ regarding firm i or $a < a^{**}(n)$ regarding firm j, the output of the high-quality firm under incompatibility is higher than under compatibility.

Proof. See Appendix A.

Lemma 4 implies that output and profits under compatibility are higher than the corresponding values under incompatibility only if the quality index is not too high.

3. The CDG with product quality

This section studies the strategic choice of each firm at the first stage of the game, in which each of them must choose between K and NK strategically under product quality. The analysis is bounded by the feasible region shown in Figure 1 (white area).

Let us first define the following six profit differentials, which compare any deviation from any symmetric strategic situation for each player), where the first (resp. second) subscript denotes the pay-off differential for each type of deviation (resp. firm).

The profit differentials of firm i are:

$$\Delta\Pi_{A,i} \coloneqq \Pi_i^{*K/NK} - \Pi_i^{*NK/NK}, \tag{27}$$

$$\Delta\Pi_{B,i} \coloneqq \Pi_i^{*NK/K} - \Pi_i^{*K/K}, \tag{28}$$

and

$$\Delta\Pi_{C,i} \coloneqq \Pi_i^{*NK/NK} - \Pi_i^{*K/K}. \tag{29}$$

The profit differentials of firm j are:

$$\Delta\Pi_{A,j} \coloneqq \Pi_j^{*NK/K} - \Pi_j^{*NK/NK}, \tag{30}$$

$$\Delta\Pi_{B,j} \coloneqq \Pi_j^{*K/NK} - \Pi_j^{*K/K}, \tag{31}$$

and

$$\Delta\Pi_{C,j} \coloneqq \Pi_j^{*NK/NK} - \Pi_j^{*K/K}. \tag{32}$$

From Eqs. (27), (28) and (30), (31) the sign of the corresponding profit differentials are: 1) $\Delta\Pi_{A,i} > 0$, $\Delta\Pi_{B,i} < 0$ and $\Delta\Pi_{A,j} > 0$, $\Delta\Pi_{B,j} < 0$ for any $n > 0$ and $a^{\circ\circ\circ}(n) < a < a^{\circ\circ\circ\circ}(n)$; 2) $\Delta\Pi_{A,i} < 0$, $\Delta\Pi_{B,i} > 0$ and $\Delta\Pi_{A,j} < 0$, $\Delta\Pi_{B,j} > 0$ for any $n < 0$ and $a^{\circ}(n) < a < a^{\circ\circ}(n)$. Therefore, the sign these four differentials is uniquely determined throughout the feasible space. Unlike this, the sign of $\Delta\Pi_{C,i}$ and $\Delta\Pi_{C,j}$ change depending on the relative size of a and n. Let $a_{C,i}(n)$ and $a_{C,j}(n)$ be the values of the relative product quality such that $\Delta\Pi_{C,i} = 0$ and $\Delta\Pi_{C,j} = 0$, respectively, where

$$a_{C,i}(n) = a^{*}(n) \coloneqq \frac{5(1-n)+n^2}{4(1-n)+n^2} > 1, \tag{33}$$

and

$$a_{C,j}(n) = a^{**}(n) \coloneqq \frac{4(1-n)+n^2}{5(1-n)+n^2} < 1, \tag{34}$$

with $a^{\circ\circ\circ}(n) < a_{C,i}(n) < a^{\circ\circ\circ\circ}(n)$ and $a^{\circ\circ\circ}(n) < a_{C,j}(n) < a^{\circ\circ\circ\circ}(n)$ for any $n > 0$, and $a^{\circ}(n) < a_{C,i}(n) < a^{\circ\circ}(n)$ and $a^{\circ}(n) < a_{C,j}(n) < a^{\circ\circ}(n)$ for any $n < 0$. Then, 1) $\Delta\Pi_{C,i} < 0$ (resp. $\Delta\Pi_{C,i} > 0$) for any $n > 0$ and $a < a_{C,i}(n)$ (resp. $a > a_{C,i}(n)$), and $\Delta\Pi_{C,j} > 0$ (resp. $\Delta\Pi_{C,j} < 0$) for any $n > 0$ and $a < a_{C,j}(n)$ (resp. $a > a_{C,j}(n)$); 2) $\Delta\Pi_{C,i} > 0$ (resp. $\Delta\Pi_{C,i} < 0$) for any $n < 0$ and $a < a_{C,i}(n)$ (resp. $a > a_{C,i}(n)$), and $\Delta\Pi_{C,j} < 0$ (resp. $\Delta\Pi_{C,j} > 0$) for any $n < 0$ and $a < a_{C,j}(n)$ (resp. $a > a_{C,j}(n)$).

The analysis of the sign of these differentials offers any possible Nash equilibrium – with its properties – of the non-cooperative CDG game with product quality. Preliminarily, Proposition 1 shows the emergence of SPNE in the absence of quality differential ($a = 1$). This is also reported in Figure 3, which is a geometrical representation of the results detailed in Propositions 1 (homogeneous quality) and 2 (heterogeneous quality).

Proposition 1. Let $n > 0$. Under homogeneous product quality ($a = 1$), the unique Pareto efficient SPNE of the CDG is (K, K), which is an anti-prisoner's dilemma (deadlock) with symmetric firms. Let $n < 0$. Under homogeneous product quality ($a =$

1), the unique Pareto efficient SPNE of the CDG is (NK, NK), which is an anti-prisoner's dilemma (deadlock) with symmetric firms.

Proof. See Appendix A.

However, when the relative product quality (as perceived by customers) is different between the products of the two (networking) firms, the structure of the CDG changes. Specifically, when the relative quality index a is sufficiently small or large compared to the homogenous case, the SPNE of the CDG loses the property of (Pareto) efficiency for one of the two firms. This is because either the high-quality or low-quality firm worsens its profitability at the Nash equilibrium. The emergence of product compatibility and incompatibility at the SPNE depends on the sign of the network externality: the bandwagon effect (positive externality) favours the emergence of (K, K) as the unique SPNE of the CDG with product quality; the snob effect (negative externality) favours the emergence of (NK, NK) as the unique SPNE of the CDG with product quality. The Pareto efficiency property of the SPNE depends on the relative degree of the quality index. Proposition 2 and Figure 3 clarify these outcomes. We first recall that the quality index of the product of firm j is normalised to 1. Therefore, for any $a < 1$ (resp. $a > 1$) the low-quality (resp. high-quality) firm is i. The proposition now follows.

Proposition 2. Let $n > 0$. Under heterogeneous product quality ($a \neq 1$), the unique SPNE of the CDG is (K, K), which is 1) Pareto efficient for firm i (low-quality) and Pareto inefficient for firm j (high-quality) if $a^{\circ\circ\circ}(n) < a < a_{C,j}(n)$, 2) Pareto efficient for both firms if $a_{C,j}(n) < a < a_{C,i}(n)$, and 3) Pareto inefficient for firm i (high-quality) and Pareto efficient for firm j (low-quality) if $a_{C,i}(n) < a < a^{\circ\circ\circ\circ}(n)$. Let $n < 0$. Under heterogeneous product quality ($a \neq 1$), the unique SPNE of the CDG is (NK, NK), which is 4) Pareto efficient for firm i (low-quality) and Pareto inefficient for firm j (high-quality) if $a^{\circ}(n) < a < a_{C,j}(n)$, 5) Pareto efficient for both firms if $a_{C,j}(n) < a < a_{C,i}(n)$, and 6) Pareto inefficient for firm i (high-quality) and Pareto efficient for firm j (low-quality) if $a_{C,i}(n) < a < a^{\circ\circ}(n)$.

Proof. See Appendix A.

Knowing that $a_{C,i}(n) > a_{C,j}(n)$, from Proposition 1 and 2 one gets the following corollary.

Corollary 1. Irrespective of whether the network externality is positive or negative, the CDG changes from an anti-prisoner's dilemma (deadlock) to a prisoner's dilemma for firm i (resp. firm j) when the relative quality index a is sufficiently larger than 1, i.e., $a > a_{C,i}(n)$ (resp. smaller than one 1, i.e., $a < a_{C,j}(n)$).

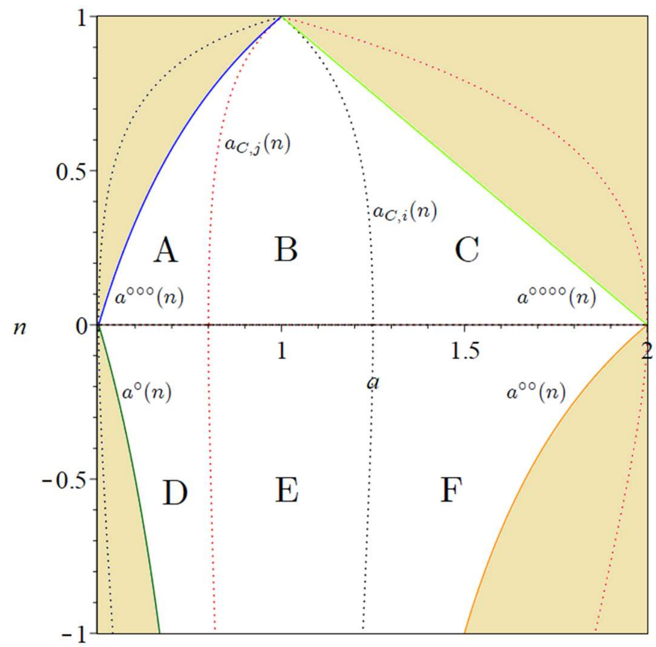

Figure 3. The CDG with product quality: SPNE in the plane (a, n). The sand-coloured region is the unfeasibility area. Area A: (K, K) is the unique SPNE, which is Pareto efficient for the low-quality firm i and Pareto inefficient for the high-quality firm j. Area B: (K, K) is the unique SPNE, which is Pareto efficient for both firms. Area C: (K, K) is the unique SPNE, which is Pareto efficient for the low-quality firm j and Pareto inefficient for the high-quality firm i. Area D: (NK, NK) is the unique SPNE, which is Pareto efficient for the low-quality firm i and Pareto inefficient for the high-quality firm j. Area E: (NK, NK) is the unique SPNE, which is Pareto efficient for both firms. Area F: (NK, NK) is the unique SPNE, which is Pareto efficient for the low-quality firm j and Pareto inefficient for the high-quality firm i.

The results detailed so far allow us to state the following comment: a relatively high product quality index (as perceived by customers) makes the high-quality firm being entrapped in a less profitable equilibrium when it produces compatible (resp. incompatible) goods under positive (resp. negative) network externalities. The intuition follows from Lemma 4 and Eqs. (23)-(26). When the network externality is positive, if the quality of the goods produced by the high-quality firm is high enough, and consequently the demand towards it is very high, a common incompatibility would be better off. This is because of (i) the support of the positive externality (through compatibility) to the demand of the rival is the highest, and (ii) the quality of the goods of the high-quality firm is no longer able – under compatibility – to reduce the output of the rival as the network externality sterilises the standard effect of the qualitative advantage under Cournot competition (i.e., when goods are strategic substitutes). The opposite holds under negative network externality.

Though this outcome does not favour compatibility with high quality of products, it is possible to escape the trap (dilemma or "bad" equilibrium) for the high-quality firm, as shown in Proposition 3 and Figure 4.

Proposition 3. Within the set of values of the quality differentials for which the high-quality firm is entrapped in a "bad" equilibrium under compatibility K (resp. incompatibility NK), there exists a subset of values of a such that deviating towards NK (resp. K) is profit improving to the extent that the high-quality firm can may make a side-payment (SP) to the (low-quality) rival to eliminate the incentive to produce compatible (resp. incompatible) goods when the network externality is positive (resp. negative). This holds for any $-1 < n < \frac{3}{2} - \frac{\sqrt{3}}{2} = 0.63397 \cong 0.634$.

Proof. See Appendix A.

Proposition 3 states that if n is positive there exists a side payment of the high-quality firm j to the low-quality firm i to produce incompatible goods for any $a < a_{min}^{SP}(n)$, and there exists a side payment of the high-quality firm i to the low-quality firm j to produce incompatible goods for any $a > a_{max}^{SP}(n)$. In addition, if n is negative there exists a side payment of the high-quality firm j to the low-quality firm i to produce compatible goods for any $a < a_{min}^{SP}(n)$, and there exists a side payment of the high-quality firm i to the low-quality firm j to produce compatible goods for any $a > a_{max}^{SP}(n)$.

Using the side payment requires that profits of the high-quality firm in the case of incompatibility are sufficiently high to overcompensate the rival firm to deviate towards the incompatibility if the network externality is positive and to deviate towards compatibility if the network externality is negative. In other words, to deviate from K to NK when the network externality is positive (resp. from NK to K when the network externality is negative), the profit gain of the high-quality firm must be larger than the profit loss of the low-quality firm, so that the former may transfer part of that gain to the latter to incentivise it to eliminate compatibility (resp. incompatibility). If the incompatibility scenario reduces the consumer surplus, the side-payment would be under the scrutiny of the Anti-Trust authority. In this case it is relevant the social welfare analysis stated in Section 4.

A numerical example can be useful to illustrate the main results. By assuming an intermediate level of the (positive and negative) network effect ($n = 0.5$ and $n = -0.5$), one can observe the emergence of a unique SPNE with different efficiency features when the relative quality index varies. These results are reported in Table 3 (positive externality) and Table 4 (negative externality).

Table 3. Main results of the CDG when $n = 0.5$.

Quality parameter: a	SPNE of the CDG and its properties
$a < a^{\circ\circ\circ}(n) \cong 0.667$	The CDG is unfeasible
$0.667 \cong a^{\circ\circ\circ}(n) < a < a_{min}^{SP}(n) = 0.704$	SPNE: (K, K) Pareto efficient for the low-quality firm i and Pareto inefficient for the high-quality firm j

	Firm j is worse off, but it can make a side payment to firm i to deviate towards (NK, NK)
$0.704 = a_{min}^{SP}(n) < a < a_{C,j}(n) = 0.818$	SPNE: (K, K) Pareto efficient for the low-quality firm i and Pareto inefficient for the high-quality firm j Firm j is worse off. No side payment
$0.818 = a_{C,j}(n) < a < a_{C,i}(n) = 1.222$	SPNE: (K, K) Pareto efficient for both firms
$1.222 = a_{C,i}(n) < a < a_{max}^{SP}(n) = 1.418$	SPNE: (K, K) Pareto inefficient for the high-quality firm i and Pareto efficient for the low-quality firm j Firm i is worse off. No side payment
$1.418 = a_{max}^{SP}(n) < a < a^{\circ\circ\circ\circ}(n) = 1.5$	SPNE: (K, K) Pareto inefficient for the high-quality firm i and Pareto efficient for the low-quality firm j Firm i is worse off, but it can make a side payment to firm j to deviate towards (NK, NK)
$a > a^{\circ\circ\circ\circ}(n) = 1.5$	The CDG is unfeasible

Table 4. Main results of the CDG when $n = -0.5$.

Quality parameter: a	SPNE of the CDG and its properties
$a < a^{\circ}(n) = 0.6$	The CDG is unfeasible
$0.6 = a^{\circ}(n) < a < a_{min}^{SP}(n) = 0.686$	SPNE: (NK, NK) Pareto efficient for the low-quality firm i and Pareto inefficient for the high-quality firm j Firm j is worse off, but it can make a side payment to firm i to deviate towards (K, K)
$0.686 = a_{min}^{SP}(n) < a < a_{C,j}(n) = 0.806$	SPNE: (NK, NK) Pareto efficient for the low-quality firm i and Pareto inefficient for the high-quality firm j Firm j is worse off. No side payment
$0.806 = a_{C,j}(n) < a < a_{C,i}(n) = 1.24$	SPNE: (NK, NK) Pareto efficient for both firms
$1.24 = a_{C,i}(n) < a < a_{max}^{SP}(n) = 1.45$	SPNE: (NK, NK) Pareto inefficient for the high-quality firm i and Pareto efficient for the low-quality firm j

	Firm i is worse off. No side payment
$1.45 = a_{max}^{SP}(n) < a < a^{\circ\circ}(n) \cong 1.667$	SPNE: (NK, NK) Pareto inefficient for the high-quality firm i and Pareto efficient for the low-quality firm j Firm i is worse off, but it can make a side payment to firm j to deviate towards (K, K)
$a > a^{\circ\circ}(n) \cong 1.667$	The CDG is unfeasible

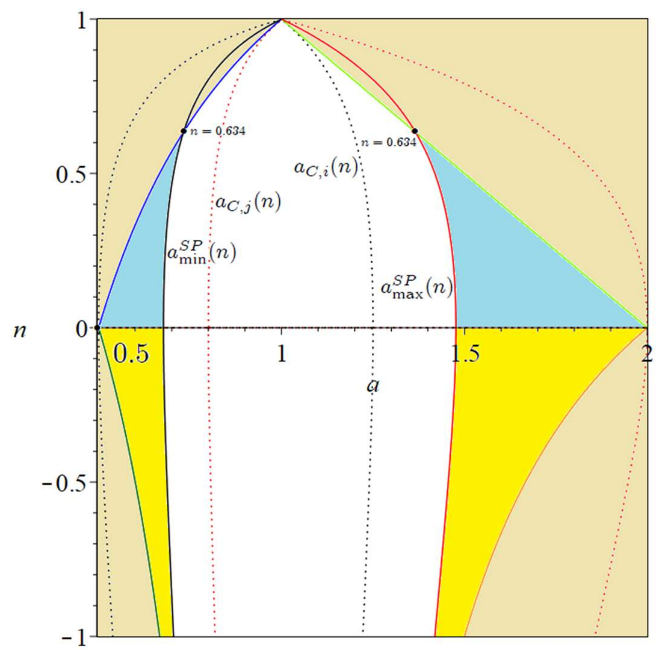

Figure 4. Side payment (light-blue and yellow regions) to escape the dilemma when the network externality is positive (the SPNE is (K, K)) or negative (the SPNE is (NK, NK)). The light-blue regions are the parametric areas in which there exists a side payment when $n > 0$. The yellow regions are the parametric areas in which there exists a side payment when $n < 0$. The side payment exists in both case $a > 1$ (the high-quality producer is firm i) and $a < 1$ (the low-quality producer is firm i).

4. Welfare analysis

This section goes one step further and studies the welfare properties of the CDG with product quality corresponding to the SPNE discussed so far. In doing so, it resembles to a geometrical representation like those pinpointed in the previous figures (deriving indeed by analytical inspections) by adding the curves representing the loci of points in which the consumer surplus and the social welfare are zero in the space (a, n). These loci allow us to separate the regions in plane in which the consumer surplus differential and the social welfare differential – between the cases (K, K) and (NK, NK) – are positive from those in which they are negative by also comparing them with the prevailing sub-game perfect Nash equilibrium and its efficiency properties.

The social welfare function at the Nash equilibrium (K, K) is defined as:

$$W^{*K/K} = \Pi_i^{*K/K} + \Pi_j^{*K/K} + CS^{*K/K}, \tag{35}$$

where $\Pi_i^{*K/K}$ and $\Pi_j^{*K/K}$ are given by the expressions in (15) and (16), respectively, and $CS^{*K/K} = \frac{1}{2}(a - p_i^{*K/K})q_i^{*K/K} + \frac{1}{2}(1 - p_j^{*K/K})q_j^{*K/K}$, which can be written as follows:

$$CS^{*K/K} = \frac{(1+a)(1-n)(q_i^{*K/K}+q_j^{*K/K})}{2(3-2n)}. \tag{36}$$

The social welfare function at the Nash equilibrium (NK, NK) is defined as:

$$W^{*NK/NK} = \Pi_i^{*NK/NK} + \Pi_j^{*NK/NK} + CS^{*NK/NK}, \tag{37}$$

where $\Pi_i^{*NK/NK}$ and $\Pi_j^{*NK/NK}$ are given by the expressions in (17) and (18), respectively, and $CS^{*NK/NK} = \frac{1}{2}(a - p_i^{*NK/NK})q_i^{*NK/NK} + \frac{1}{2}(1 - p_j^{*NK/NK})q_j^{*NK/NK}$, which can be written as follows:

$$CS^{*NK/NK} = \frac{q_i^{*NK/NK}\{1+a[(1-n)^2-n]\}+q_j^{*NK/NK}[a-n+(1-n)^2]}{2(3-n)(1-n)}. \tag{38}$$

Define $\Delta CS = CS^{*K/K} - CS^{*NK/NK}$ and $\Delta W = W^{*K/K} - W^{*NK/NK}$ being the consumer surplus differential and the social welfare differential, respectively. Then, 1) for any couple (a, n) such that $\Delta CS > 0$ (resp. < 0) consumers are better off under (K, K) than under (NK, NK) (resp. under (NK, NK) than under (K, K)), and 2) for any couple (a, n) such that $\Delta W > 0$ (resp. < 0) the society is better off under (K, K) than under (NK, NK) (resp. under (NK, NK) than under (K, K)).

Let $a_{max}^{CS}(n)$ and $a_{min}^{CS}(n)$ be the values of the relative quality index such that $\Delta CS = 0$ in the space (a, n) when $a > 1$ and $a < 1$, respectively, where

$$a_{max}^{CS}(n) := \frac{n^4-13n^3+50n^2-75n+39+(3-2n)(3-n)(1-n)\sqrt{6n-2n^2-3}}{(n-2)(3n^3-17n^2+33n-21)}. \tag{39}$$

and

$$a_{min}^{CS}(n) := -\frac{-n^4+13n^3-50n^2+75n-39+(3-2n)(3-n)(1-n)\sqrt{6n-2n^2-3}}{(n-2)(3n^3-17n^2+33n-21)}. \tag{40}$$

with $a_{max}^{CS}(n) > a_{min}^{CS}(n)$ for any n.

Let $a_{max}^{W}(n)$ and $a_{min}^{W}(n)$ be the values of the relative quality index such that $\Delta W = 0$ in the space (a, n) when $a > 1$ and $a < 1$, respectively, where

$$a_{max}^{W}(n) = -\frac{-4n^5+43n^4-179n^3+358n^2-345n+129+(3-2n)(3-n)(1-n)\sqrt{(3-2n)(15-12n+2n^2)}}{(n-2)(4n^4-33n^3+97n^2 123n+57)}. \tag{41}$$

and

$$a_{min}^{W}(n) = \frac{4n^5-43n^4+179n^3-358n^2+345n-129+(3-2n)(3-n)(1-n)\sqrt{(3-2n)(15-12n+2n^2)}}{(n-2)(4n^4-33n^3+97n^2 123n+\quad)}. \tag{42}$$

with $a_{max}^{W}(n) > a_{min}^{W}(n)$ for any n.

The analysis of the sign of ΔCS and ΔW gives a complete characterisation of both the consumer welfare and social welfare when a and n vary in the feasible region (sand-coloured). To this purpose, Figure 5 adds the loci given by Eqs. (39)-(42) to Figure 4 and then considers the policy implications that can emerge given the prevailing SPNE depending on whether the network externality is positive or negative. The figure is divided into several regions labelled A to H when $n > 0$ and B' to H' when $n < 0$. The meaning of each region is detailed below.

- Area A: SPNE (K, K), Pareto efficient for both firm i and firm j; $\Delta CS < 0$ and $\Delta W > 0$; no win-win solution, as consumers would be better under (NK, NK).
- Area B: SPNE (K, K), Pareto efficient for firm i (low-quality) and Pareto inefficient for firm j (high-quality); $\Delta CS > 0$ and $\Delta W < 0$; no win-win solution; there would be the possibility of side payment from firm j to firm i to produce incompatible goods, but consumers would be worse off; the side payment would be under the scrutiny of the anti-trust authority.
- Area C: SPNE (K, K), Pareto efficient for firm i (low-quality) and Pareto inefficient for firm j (high-quality); $\Delta CS > 0$ and $\Delta W > 0$; no win-win solution, as the high-quality firm j gets a lower payoff than it would get under NK; there would be the possibility of side payment from firm j to firm i to produce incompatible goods, but consumers would be worse off and social welfare would be reduced; the side payment would be under the scrutiny of the anti-trust authority.
- Area D: SPNE (K, K), Pareto efficient for firm i (low-quality) and Pareto inefficient for firm j (high-quality); $\Delta CS > 0$ and $\Delta W > 0$; no win-win solution, as the high-quality firm j gets a lower payoff than it would get under NK; no side payment.
- Area E: SPNE (K, K), Pareto efficient for both firm i and firm j; $\Delta CS > 0$ and $\Delta W > 0$; win-win solution; when n is positive, a relatively small product quality differential leads to a Pareto superior outcome for the society as long as n is not too high.
- Area F: SPNE (K, K), Pareto inefficient for firm i (high-quality) and Pareto efficient for firm j (low-quality); $\Delta CS > 0$ and $\Delta W > 0$; no win-win solution, as the high-quality firm i gets a lower payoff than it would get under NK; no side payment.
- Area G: SPNE (K, K), Pareto inefficient for firm i (high-quality) and Pareto efficient for firm j (low-quality); $\Delta CS > 0$ and $\Delta W > 0$; no win-win solution, as the high-quality firm i gets a lower payoff than it would get under NK; there would be the possibility of side payment from firm i to firm j to produce incompatible goods, but consumers would be worse off and social welfare would be reduced; the side payment would be under the scrutiny of the anti-trust authority.
- Area H: SPNE (K, K), Pareto inefficient for firm i (high-quality) and Pareto efficient for firm j (low-quality); $\Delta CS > 0$ and $\Delta W < 0$; no win-win solution; there would be the possibility of side payment from firm i to firm j to produce incompatible goods, but consumers would be worse off; the side payment would be under the scrutiny of the anti-trust authority.

- Area B': SPNE (NK, NK), Pareto efficient for firm i (low-quality) and Pareto inefficient for firm j (high-quality); $\Delta CS < 0$ and $\Delta W > 0$; no win-win solution; there would be the possibility of side payment from firm j to firm i to produce compatible goods, but consumers would be worse off; the side payment would be under the scrutiny of the anti-trust authority.
- Area C': SPNE (NK, NK), Pareto efficient for firm i (low-quality) and Pareto inefficient for firm j (high-quality); $\Delta CS < 0$ and $\Delta W < 0$; no win-win solution, as the high-quality firm j gets a lower payoff than it would get under K; there would be the possibility of side payment from firm j to firm i to produce compatible goods, but consumers would be worse off and social welfare would be reduced; the side payment would be under the scrutiny of the anti-trust authority.
- Area D': SPNE (NK, NK), Pareto efficient for firm i (low-quality) and Pareto inefficient for firm j (high-quality); $\Delta CS < 0$ and $\Delta W < 0$; no win-win solution, as the high-quality firm j gets a lower payoff than it would get under K; no side payment.
- Area E': SPNE (NK, NK), Pareto efficient for both firm i and firm j; $\Delta CS < 0$ and $\Delta W < 0$; win-win solution; when n is negative, a relatively small product quality differential leads to a Pareto superior outcome for the society.
- Area F': SPNE (NK, NK), Pareto inefficient for firm i (high-quality) and Pareto efficient for firm j (low-quality); $\Delta CS < 0$ and $\Delta W < 0$; no win-win solution, as the high-quality firm i gets a lower payoff than it would get under K; no side payment.
- Area G': SPNE (NK, NK), Pareto inefficient for firm i (high-quality) and Pareto efficient for firm j (low-quality); $\Delta CS < 0$ and $\Delta W < 0$; no win-win solution, as the high-quality firm i gets a lower payoff than it would get under K; there would be the possibility of side payment from firm i to firm j to produce compatible goods, but consumers would be worse off and social welfare would be reduced; the side payment would be under the scrutiny of the anti-trust authority.
- Area H': SPNE (NK, NK), Pareto inefficient for firm i (high-quality) and Pareto efficient for firm j (low-quality); $\Delta CS < 0$ and $\Delta W > 0$; no win-win solution; there would be the possibility of side payment from firm i to firm j to produce compatible goods, but consumers would be worse off; the side payment would be under the scrutiny of the anti-trust authority.

Therefore, quality heterogeneity tends to eliminate the win-win result from a societal perspective. This happens (when the relative quality differential is sufficiently low or high) mainly because of the entrapment in a dilemma ("bad" equilibrium) of the high-quality firm as the reduction of its profits at the Nash equilibrium with compatibility if n is positive (or incompatibility if n is negative) is strong enough to more than offsets the higher payoff of the firm and the consumer welfare. However, if there is the network effect is high enough, product quality homogeneity tends to reduce the welfare of the consumers, who would be better off if firms would produce incompatible goods. In the absence of quality heterogeneity, when the network effect is sufficiently strong consumers would always prefer incompatible products, which is a rather surprisingly outcome.

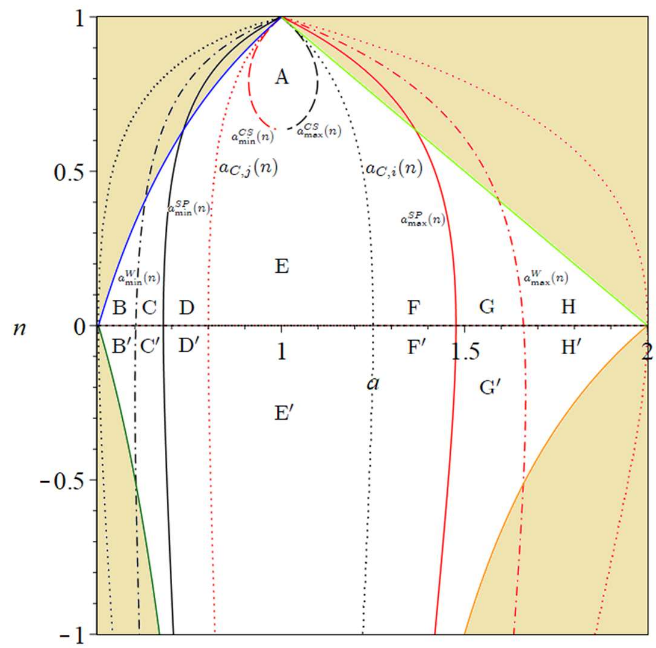

Figure 5. Consumer surplus and social welfare corresponding to the SPNE of the CGD with product quality.

5. Conclusions

The present article has been motivated by the importance of the issue of compatibility and by the widespread presence of firms producing goods perceived as being of heterogeneous quality in network markets. To this purpose, the article has developed a Cournot game-theoretic setting considering a two-stage non-cooperative duopoly with heterogeneous product quality, in which the degree of product compatibility is the strategic variable chosen by each firm at the first (decision) stage. Results first have shown that the unique SPNE of the game is full compatibility (resp. no compatibility) if the network externality is positive (resp. negative). However, in contrast with the standard case of homogeneous quality, the SPNE can be Pareto inefficient for one of the two firms (the high-quality firm), which is therefore entrapped in a dilemma. This leads to the remark that compatibility (resp. incompatibility) can be a "trap" for the firm with relatively high-quality products as perceived by customers under positive (resp. negative) network externalities. The article has eventually pinpointed the welfare outcome prevailing at the SPNE.

These findings may guide future empirical research on the network industry with some testable hypotheses: first, in network markets, there is less room for the coexistence of low- and high-quality firms, that is it should be more often observed a higher degree of qualitative homogeneity than in standard markets; second, in the absence of compatibility, it should be more often evidenced that in situations in which the network effect is more intense, the production of the low-quality firm is lower and that of the high-quality firm higher than in markets in which the network effect is weaker or absent; third, it should be more often found that firms with high-quality products (or cost advantages) and compatibility show output and profits lower than the corresponding firms producing incompatible products. It might also be worthwhile to point out that the game-theoretic setting adopted in this work shows similar results by considering exogenous partial compatibility.

Finally, we have also investigated the presence of another widespread feature of network industries, which may affect the consumers' demands and the Nash equilibrium outcomes: the presence of installed bases of different sizes. We found that the SPNE is again the compatibility choice, but it may be Pareto inefficient as the large/old firm with an installed base advantage is worse off and reduces its profits at the SPNE. Then, history matters about the effects of the choice of producing compatible or incompatible products. The corresponding analysis is presented in the Appendix. This result is, in some sense, like that shown under heterogeneous quality confirming the finding that the firm with "advantages" (regarding either product quality or historical dimension) worsens its profitability with uniform compatibility, but, unfortunately for it, it is entrapped in the "bad" equilibrium.

A possible future research agenda can include the existence of (fixed and/or variable) costs to implement the production of compatible products as well as the analysis of marginal cost differential between the two firms under homogeneous product quality. Another possible interesting direction to extend the present work is to include managerial delegation, corporate social responsibility and outsourcing, which are widespread in network industries and may greatly affect the results pinpointed here.

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Compliance with ethical standards

Disclosure of potential conflict of interest The authors declare that they have no conflict of interest.

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Declarations of interest None.

Appendix A. Analytical details

Proof of Lemma 4. The proof follows by using the following line of reasoning: $q_i^{*K/K} - q_i^{*NK/NK} \overset{\geq}{\underset{<}{=}} 0 \Leftrightarrow a \overset{\leq}{\underset{>}{=}} a^*(n) := \frac{5(1-n)+n^2}{4(1-n)+n^2} \geq 1$ and $q_j^{*K/K} - q_j^{*NK/NK} \overset{\geq}{\underset{<}{=}} 0 \Leftrightarrow a \overset{\geq}{\underset{<}{=}} a^{**}(n) := \frac{4(1-n)+n^2}{5(1-n)+n^2} \leq 1$. **Q.E.D.**

Proof of Proposition 1. The proof follows from the sign of the profit differentials. If $n > 0$ then $\Delta\Pi_{A,i} > 0$, $\Delta\Pi_{B,i} < 0$, $\Delta\Pi_{C,i} < 0$, $\Delta\Pi_{A,j} > 0$, $\Delta\Pi_{B,j} < 0$ and $\Delta\Pi_{C,j} < 0$. If $n < 0$ then $\Delta\Pi_{A,i} < 0$, $\Delta\Pi_{B,i} > 0$, $\Delta\Pi_{C,i} > 0$, $\Delta\Pi_{A,j} < 0$, $\Delta\Pi_{B,j} > 0$ and $\Delta\Pi_{C,j} > 0$. **Q.E.D.**

Proof of Proposition 2. The proof follows by considering the sign of the profit differentials. If $n > 0$ and $a^{\circ\circ\circ}(n) < a < a_{C,j}(n)$ then $\Delta\Pi_{A,i} > 0$, $\Delta\Pi_{B,i} < 0$, $\Delta\Pi_{C,i} < 0$ and $\Delta\Pi_{A,j} > 0$, $\Delta\Pi_{B,j} < 0$, $\Delta\Pi_{C,j} > 0$; if $n > 0$ and $a_{C,j}(n) < a < a_{C,i}(n)$ then $\Delta\Pi_{A,i} > 0$, $\Delta\Pi_{B,i} < 0$, $\Delta\Pi_{C,i} < 0$ and $\Delta\Pi_{A,j} > 0$, $\Delta\Pi_{B,j} < 0$, $\Delta\Pi_{C,j} < 0$; if $n > 0$ and $a_{C,i}(n) < a < a^{\circ\circ\circ\circ}(n)$ then $\Delta\Pi_{A,i} > 0$, $\Delta\Pi_{B,i} < 0$, $\Delta\Pi_{C,i} > 0$ and $\Delta\Pi_{A,j} > 0$, $\Delta\Pi_{B,j} < 0$, $\Delta\Pi_{C,j} < 0$. If $n < 0$ and $a^{\circ}(n) < a < a_{C,j}(n)$ then $\Delta\Pi_{A,i} < 0$, $\Delta\Pi_{B,i} > 0$, $\Delta\Pi_{C,i} > 0$ and $\Delta\Pi_{A,j} < 0$, $\Delta\Pi_{B,j} > 0$, $\Delta\Pi_{C,j} < 0$; if $n < 0$ and $a_{C,j}(n) < a < a_{C,i}(n)$ then $\Delta\Pi_{A,i} < 0$, $\Delta\Pi_{B,i} > 0$, $\Delta\Pi_{C,i} > 0$ and $\Delta\Pi_{A,j} < 0$, $\Delta\Pi_{B,j} > 0$, $\Delta\Pi_{C,j} > 0$; if $n < 0$ and $a_{C,i}(n) < a < a^{\circ\circ}(n)$ then $\Delta\Pi_{A,i} < 0$, $\Delta\Pi_{B,i} > 0$, $\Delta\Pi_{C,i} < 0$ and $\Delta\Pi_{A,j} < 0$, $\Delta\Pi_{B,j} > 0$, $\Delta\Pi_{C,j} > 0$. **Q.E.D.**

Proof of Proposition 3. Let $\Delta SP_i := \Pi_i^{*NK/NK} - \Pi_i^{*K/K}$ be the side payment differential of firm i and $\Delta SP_j := \Pi_j^{*NK/NK} - \Pi_j^{*K/K}$ be the side payment differential of firm j. If $a > 1$ then firm i is the high-quality producer and firm j is the low-quality producer. If $a < 1$ then firm i is the low-quality producer and firm j is the high-quality producer. Let $n > 0$ (resp. $n < 0$). Then

1) for any $a > 1$ within the feasible region, the high-quality firm i can make a side payment to the low-quality firm j if and only if $\Delta SP_i + \Delta SP_j > 0$ (resp. < 0) if and only if $a > a_{max}^{SP}(n) > a_{C,i}(n)$, where

$$a_{max}^{SP}(n) := -\frac{-2n^4+(18+2\sqrt{3})n^3-(60+\quad\sqrt{3})n^2+(84+\quad\sqrt{3})n-42-\ \sqrt{3}}{2n^4-18n^3+57n^2-78n+39}, \tag{A.1}$$

is the value of the relative quality index (as a function of the degree of the network effect) such that $\Delta SP_i + \Delta SP_j = 0$ when $a > 1$, and

2) for any $a < 1$ within the feasible region, the high-quality firm j can make a side payment to the low-quality firm i if and only if $\Delta SP_i + \Delta SP_j > 0$ (resp. < 0) if and only if $a < a_{min}^{SP}(n) < a_{C,j}(n)$, where

$$a_{min}^{SP}(n) := \frac{2n^4+(-18+2\sqrt{3})n^3+(60\qquad\sqrt{3})n^2-(84-18\sqrt{3})n+42-\ \sqrt{3}}{2n^4-18\ \ ^3+57n^2-78n+39}, \tag{A.2}$$

is the value of the relative quality index (as a function of the degree of the network effect) such that $\Delta SP_i + \Delta SP_j = 0$ when $a < 1$, and $a_{max}^{SP}(n) > a_{min}^{SP}(n)$ for any $-1 < n < 0.634$. If $n > 0.634$ then $a_{min}^{SP}(n) > a^{\circ\circ\circ}(n)$ and $a_{max}^{SP}(n) > a^{\circ\circ\circ\circ}(n)$. **Q.E.D.**

Appendix B. Compatibility and the dimension of the installed base: when history matters

The cumulated sold products of larger and older firms are clearly higher than those of younger and smaller ones. This is indeed relevant especially in network industries as consumers enjoy – given the positive network effect – not only the consumption of products currently produced but also the stock accumulated in the past. In the words of Besen and Farrell (1994, p. 118): "A final characteristic of network markets is that history matters". The main difference between standard and network markets is that while in the former only the existing consumer preferences and producer technologies matter to explain the market outcomes, in the latter "market equilibria often cannot be understood without knowing the pattern of technology adoption in earlier periods" (Besen and Farrell, 1994, pp. 118-119), as consumers prefer product compatibility with the installed base.[9]

Therefore, in addition to the direct network effects of the model analysed in the main text, there are also indirect effects that can increase the consumers' demand. Indeed, the larger the number of consumers that are using a certain type of hardware, the more likely the provision of compatible hardware. This implies that the size of the installed base may increase the positive consumption externality. This also means that the larger and older the firm, the larger is its installed base. In other words, if the dimension of products sold in the past matters, history also matters for the choice of compatible or incompatible goods.

In this case, the network of firm i depends not only on the expected sales in the considered period, i.e., y_i and y_j but also on its installed base, which is defined as $I_i > 0$, and the installed base of rival, which is defined as $I_j > 0$ (if products are fully compatible between them). The installed base is the number of products sold to users in the past.[10] Then, the only change with respect to the model discussed in the main text involves the inverse demand functions (2) and (3), which are respectively re-written in the following way:

$$p_i = a - q_i - q_j + n(I_i + k_i I_j + y_i + k_i y_j), \tag{B.1}$$

and

[9] The fact that buyers prefer compatibility with the installed base, may imply dynamical unexpected effects, such as "better products that arrive later may be unable to displace poorer, but earlier standards" (Besen and Farrell, 1994,118-119), as reported by David (1985) for the case of the QWERTY typewriter keyboard and by Besen (1992) for the case of the difficulties to convince AM radio users to switch to the superior FM band immediately after World War II.

[10] This way of considering the past production sold in the network market is standard in the literature, e.g., Stadler et al. (2022), who consider however, a uniform degree of compatibility, while we allow for a different degree.

$$p_j = 1 - q_j - q_i + n(I_j + k_j I_i + y_j + k_j y_i), \tag{B.2}$$

where we will assume henceforth that product quality is homogeneous, that is $a = 1$. This is indeed useful to stress the similarities of the SPNE outcomes with the CDG with product quality studied so far, which in this model are driven by the size of the relative value of the installed base of the two firms. Standard calculations lead to equilibrium output and profits in the four different sub-games, similarly to the main text, which are summarised in Table B.1 and Table B.2. To follow up on what was done previously, we consider full compatibility (K) versus incompatibility (NK).

Table B.1. Quantities in each sub-game of the CDG with installed base. Second stage.

Firm j → Firm i ↓	K	NK
K	$q_i^{*K/K}(I_i, I_j)$, $q_j^{*K/K}(I_i, I_j)$	$q_i^{*K/NK}(I_i, I_j)$, $q_j^{*K/NK}(I_i, I_j)$
NK	$q_i^{*NK/K}(I_i, I_j)$, $q_j^{*NK/K}(I_i, I_j)$	$q_i^{*NK/NK}(I_i, I_j)$, $q_j^{*NK/NK}(I_i, I_j)$

Table B.2. The CDG with installed base (payoff matrix: profits). First stage.

Firm j → Firm i ↓	K	NK
K	$\Pi_i^{*K/K}(I_i, I_j)$, $\Pi_j^{*K/K}(I_i, I_j)$	$\Pi_i^{*K/NK}(I_i, I_j)$, $\Pi_j^{*K/NK}(I_i, I_j)$
NK	$\Pi_i^{*NK/K}(I_i, I_j)$, $\Pi_j^{*NK/K}(I_i, I_j)$	$\Pi_i^{*NK/NK}(I_i, I_j)$, $\Pi_j^{*NK/NK}(I_i, I_j)$

The entries of Table A.1 are:

$$q_i^{*K/K}(I_i, I_j) = \frac{1+n(2-n)I_i - n(1-n)I_j}{3-2n} > 0, \tag{B.3}$$

iff $I_i > \frac{n(1-n)I_j - 1}{n(2-n)} := I_i{}^{\circ}(n)$ for any $n > 0$ and $I_i < I_i{}^{\circ}(n)$ for any $n < 0$,

$$q_j^{*K/K}(I_i, I_j) = \frac{1-n(1-n)I_i + n(2-n)I_j}{3-2n} > 0, \tag{B.4}$$

iff $I_i < \frac{1+n(2-n)I_j}{n(1-n)} := I_i{}^{\circ\circ}(n)$ for any $n > 0$ and $I_i > I_i{}^{\circ\circ}(n)$ for any $n < 0$,

$$q_i^{*NK/NK}(I_i, I_j) = \frac{1-n+n(2-n)I_i - nI_j}{(3-n)(1-n)} > 0, \tag{B.5}$$

iff $I_i > \frac{nI_j + n - 1}{n(2-n)} := I_i{}^{\circ\circ\circ}(n)$ for any $n > 0$ and $I_i < I_i{}^{\circ\circ\circ}(n)$ for any $n < 0$,

$$q_j^{*NK/NK}(I_i, I_j) = \frac{1-n+n(2-n)I_j - nI_i}{(3-n)(1-n)} > 0, \tag{B.6}$$

iff $I_i < \frac{1-n+n(2-n)I_j}{n} := I_i{}^{\circ\circ\circ\circ}(n)$ for any $n > 0$ and $I_i > I_i{}^{\circ\circ\circ\circ}(n)$ for any $n < 0$,

$$q_i^{*K/NK}(I_i, I_j) = \frac{1+n(2-n)I_i - n(1-n)I_j}{3(1-n)+n^2} > 0, \tag{B.7}$$

iff $I_i > I_i{}^{\circ}(n)$ for any $n > 0$ and $I_i < I_i{}^{\circ}(n)$ for any $n < 0$,

$$q_j^{*K/NK}(I_i, I_j) = \frac{1-n+n(2-n)I_j - nI_i}{3(1-n)+n^2} > 0, \tag{B.8}$$

iff $I_i < I_i{}^{\circ\circ\circ\circ}(n)$ for any for any $n > 0$ and $I_i > I_i{}^{\circ\circ\circ\circ}(n)$ for any $n < 0$, and

$$q_i^{*NK/K}(I_i, I_j) = \frac{1-n+n(2-n)I_i-nI_j}{3(1-n)+n^2} > 0, \quad \text{(B.9)}$$

iff $I_i > I_i^{\circ\circ\circ}(n)$ for any $n > 0$ and $I_i < I_i^{\circ\circ\circ}(n)$ for any $n < 0$,

$$q_j^{*NK/K}(I_i, I_j) = \frac{1-n(1-n)I_i+n(2-n)I_j}{3(1-n)+n^2} > 0, \quad \text{(B.10)}$$

iff $I_i < I_i^{\circ\circ}(n)$ for any $n > 0$ and $I_i > I_i^{\circ\circ}(n)$ for any $n < 0$.

The entries of Table B.2 are:

$$\Pi_i^{*K/K}(I_i, I_j) = \frac{[1+n(2-n)I_i-n(1-n)I_j]^2}{(3-2n)^2}, \quad \text{(B.11)}$$

$$\Pi_j^{*K/K}(I_i, I_j) = \frac{[1-n(1-n)I_i+n(2-n)I_j]^2}{(3-2n)^2}, \quad \text{(B.12)}$$

$$\Pi_i^{*NK/NK}(I_i, I_j) = \frac{[1-n+n(2-n)I_i-nI_j]^2}{(3-n)^2(1-n)^2}, \quad \text{(B.13)}$$

$$\Pi_j^{*NK/NK}(I_i, I_j) = \frac{[1-n+n(2-n)I_j-nI_i]^2}{(3-n)^2(1-n)^2}, \quad \text{(B.14)}$$

$$\Pi_i^{*K/NK}(I_i, I_j) = \frac{[1+n(2-n)I_i-n(1-n)I_j]^2}{[3(1-n)+n^2]^2}, \quad \text{(B.15)}$$

$$\Pi_j^{*K/NK}(I_i, I_j) = \frac{[1-n+n(2-n)I_j-nI_i]^2}{[3(1-n)+n^2]^2}, \quad \text{(B.16)}$$

and

$$\Pi_i^{*NK/K}(I_i, I_j) = \frac{[1-n+n(2-n)I_i-nI_j]^2}{[3(1-n)+n^2]^2}, \quad \text{(B.17)}$$

$$\Pi_j^{*NK/K}(I_i, I_j) = \frac{[1-n(1-n)I_i+n(2-n)I_j]^2}{[3(1-n)+n^2]^2}. \quad \text{(B.18)}$$

Proposition B.1 summarises the main SPNE outcomes of the CDG with installed base. The results of the proposition are also reported in Figure B.1, which represents a rigorous geometrical portrait of these outcomes. We recall that if $I_i > I_j$ (resp. $I_i < I_j$) then firm i (resp. firm j) is the larger/older firm in the market.[11] For simplicity, and without loss of generality, we assume $I_j = 1$. Then, we get the following result.

Proposition B.1. Let $n > 0$. Under asymmetric installed base ($I_i \neq 1$), the unique SPNE of the CDG is (K, K), which is 1) Pareto efficient for firm i (small) and Pareto inefficient for firm j (large) if $I_i^{\circ\circ\circ}(n) < I_i < I_{C,j}(n)$, 2) Pareto efficient for both firms if $I_{C,j}(n) < I_i < I_{C,i}(n)$, and 3) Pareto inefficient for firm i (large) and Pareto efficient for firm j (small) if $I_{C,i}(n) < I < I_i^{\circ\circ\circ\circ}(n)$. Let $n < 0$. Under asymmetric installed base ($I_i \neq 1$), the unique SPNE of the CDG is (NK, NK), which is 4) Pareto inefficient for firm i (small) and Pareto efficient for firm j (large) if $I_i^{\circ\circ}(n) < I < I_{C,i}(n)$, 5) Pareto efficient for both firms if $I_{C,i}(n) < I < I_{C,j}(n)$, and 6) Pareto efficient for firm i (large) and Pareto inefficient for firm j (small) if $I_{C,j}(n) < I < I_i^{\circ}(n)$.

[11] The analysis of the game straightforwardly follows the line of reasoning used in the main text and then details are omitted.

Proof. The proof follows by considering the sign of the profit differentials. If $n > 0$ and $I_i^{\circ\circ\circ}(n) < I < I_{C,j}(n)$ then $\Delta\Pi_{A,i}(I_i, I_j) > 0$, $\Delta\Pi_{B,i}(I_i, I_j) < 0$, $\Delta\Pi_{C,i}(I_i, I_j) < 0$ and $\Delta\Pi_{A,j}(I_i, I_j) > 0$, $\Delta\Pi_{B,j}(I_i, I_j) < 0$, $\Delta\Pi_{C,j}(I_i, I_j) > 0$; if $n > 0$ and $I_{C,j}(n) < I < I_{C,i}(n)$ then $\Delta\Pi_{A,i}(I_i, I_j) > 0$, $\Delta\Pi_{B,i}(I_i, I_j) < 0$, $\Delta\Pi_{C,i}(I_i, I_j) < 0$ and $\Delta\Pi_{A,j}(I_i, I_j) > 0$, $\Delta\Pi_{B,j}(I_i, I_j) < 0$, $\Delta\Pi_{C,j}(I_i, I_j) < 0$; if $n > 0$ and $I_{C,i}(n) < I < I_i^{\circ\circ\circ\circ}(n)$ then $\Delta\Pi_{A,i}(I_i, I_j) > 0$, $\Delta\Pi_{B,i}(I_i, I_j) < 0$, $\Delta\Pi_{C,i}(I_i, I_j) > 0$ and $\Delta\Pi_{A,j}(I_i, I_j) > 0$, $\Delta\Pi_{B,j}(I_i, I_j) < 0$, $\Delta\Pi_{C,j}(I_i, I_j) < 0$. If $n < 0$ and $I_i^{\circ\circ}(n) < I < I_{C,i}(n)$ then $\Delta\Pi_{A,i}(I_i, I_j) < 0$, $\Delta\Pi_{B,i}(I_i, I_j) > 0$, $\Delta\Pi_{C,i}(I_i, I_j) < 0$ and $\Delta\Pi_{A,j}(I_i, I_j) < 0$, $\Delta\Pi_{B,j}(I_i, I_j) > 0$, $\Delta\Pi_{C,j}(I_i, I_j) > 0$; if $n < 0$ and $I_{C,i}(n) < I < I_{C,j}(n)$ then $\Delta\Pi_{A,i}(I_i, I_j) < 0$, $\Delta\Pi_{B,i}(I_i, I_j) > 0$, $\Delta\Pi_{C,i}(I_i, I_j) > 0$ and $\Delta\Pi_{A,j}(I_i, I_j) < 0$, $\Delta\Pi_{B,j}(I_i, I_j) > 0$, $\Delta\Pi_{C,j}(I_i, I_j) > 0$; if $n < 0$ and $I_{C,j}(n) < I < I_i^{\circ}(n)$ then $\Delta\Pi_{A,i}(I_i, I_j) < 0$, $\Delta\Pi_{B,i}(I_i, I_j) > 0$, $\Delta\Pi_{C,i}(I_i, I_j) > 0$ and $\Delta\Pi_{A,j}(I_i, I_j) < 0$, $\Delta\Pi_{B,j}(I_i, I_j) > 0$, $\Delta\Pi_{C,j}(I_i, I_j) < 0$. **Q.E.D.**

The proposition reveals that when the network externality is positive, the unique SPNE of the CDG with asymmetric installed base is (K, K), which results to be Pareto inefficient for the large firm. This firm is indeed entrapped in a dilemma ("bad" equilibrium) when the relative size of the installed base is sufficiently small ($I_i < I_{C,j}(n)$) or when it is sufficiently high ($I_i > I_{C,i}(n)$). In the former case, firm j is entrapped in a dilemma ("bad" equilibrium). In the latter case, firm i is entrapped in a dilemma ("bad" equilibrium), where $I_{C,i}(n)$ and $I_{C,j}(n)$ are the values of the relative installed base such that $\Delta\Pi_{C,i}(I_i, I_j) = 0$ and $\Delta\Pi_{C,j}(I_i, I_j) = 0$, respectively, which are defined as follows:

$$I_{C,i}(n) := \frac{1-n+nI_j[5(1-n)+n^2]}{n[4(1-n)+n^2]}, \tag{B.19}$$

and

$$I_{C,j}(n) = \frac{n-1+nI_j[4(1-n)+n^2]}{n[5(1-n)+n^2]}. \tag{B.20}$$

Corollary B.1. From the expressions in (B.19) and (B.20), the larger the positive network effect, the more likely the entrapment in the "bad" equilibrium for the large firm. If the network effect is sufficiently high, it is enough to have a small differential in the installed bases of the two firms for the compatibility to be profit-reducing for the large firm.

Then, the large/old firm can be entrapped in a "bad" equilibrium (dilemma) if it is sufficiently big/old. Therefore, the production of compatible products when the installed base is high benefits the smaller rival. Unfortunately, the SPNE is (K, K), which is the rational outcome obtained in the market.

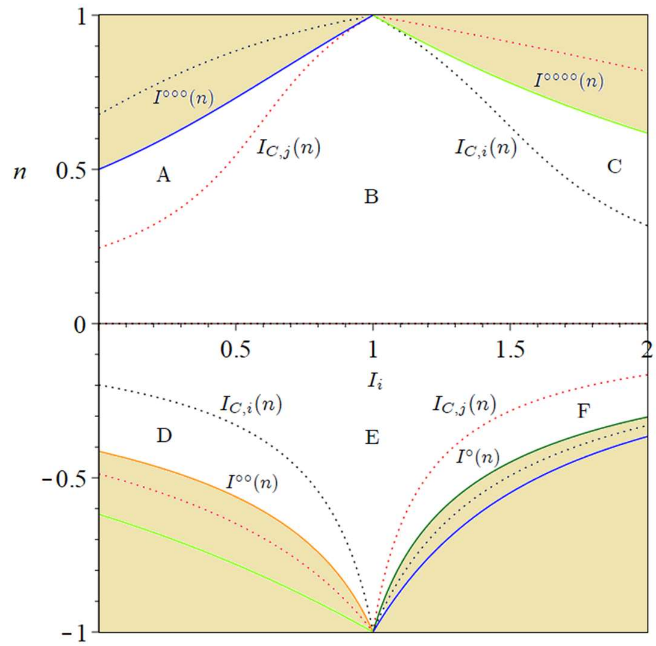

Figure B.1. The CDG with asymmetric installed base when $I_j = 1$: SPNE in the space (I_i, n). The sand-coloured region is the unfeasible parameter space.

- Area A: (K, K) is the unique SPNE, which is Pareto efficient for firm i (small) and Pareto inefficient for firm j (large).
- Area B: (K, K) is the unique SPNE, which is Pareto efficient for both firms.
- Area C: (K, K) is the unique SPNE, which is Pareto inefficient for firm i (large) and Pareto efficient for firm j (small).
- Area D: (NK, NK) is the unique SPNE, which is Pareto inefficient for firm i (small) and Pareto efficient for firm j (large).
- Area E: (NK, NK) is the unique SPNE, which is Pareto efficient for both firms.
- Area F: (NK, NK) is the unique SPNE, which is Pareto efficient for firm i (large) and Pareto inefficient for firm j (small).

Appendix C. The CDG with asymmetric marginal costs

This appendix continues the study of the extensions of the model presented in the main text and considers the CDG with heterogeneous (or asymmetric) marginal costs between firm i and firm j with homogeneous (instead of heterogeneous) product quality ($a = 1$). Thus, the only difference from the main model, in which the marginal cost was homogeneous and normalised to zero, concerns the technological efficiency. Specifically, we assume that firm i is the most efficient incurring a marginal cost of production of $c_i = 0$, and firm j is the least efficient incurring a positive marginal cost of production of $c_j > 0$. The analysis proceeds in the same way as that considered presented in the main text, so we do not present the analytical details here, but rather directly write down the payoff matrix (Table C.1), from which profit differentials can easily be calculated, and the feasibility conditions, which are given by $c_j < c^{\circ}(n) \coloneqq \frac{1}{2-n}$ and $c_j < c^{\circ\circ}(n) \coloneqq \frac{1-n}{2-n}$. The former (resp. latter) guarantees that the production of the least efficient firm in the sub-game K/K (resp. NK/NK) is positive. When these inequalities are fulfilled, a positive production in each sub-game is guaranteed.

Table C.1. The CDG with asymmetric marginal costs (payoff matrix: profits). First stage.

Firm j $\rightarrow$ Firm i $\downarrow$	K	NK
K	$\Pi_i^{*K/K}(c_i,c_j)$, $\Pi_j^{*K/K}(c_i,c_j)$	$\Pi_i^{*K/NK}(c_i,c_j)$, $\Pi_j^{*K/NK}(c_i,c_j)$
NK	$\Pi_i^{*NK/K}(c_i,c_j)$, $\Pi_j^{*NK/K}(c_i,c_j)$	$\Pi_i^{*NK/NK}(c_i,c_j)$, $\Pi_j^{*NK/NK}(c_i,c_j)$

The entries of Table C.1 are:

$$\Pi_i^{*K/K}(c_i,c_j) = \frac{[1+c(1-n)]^2}{(3-2n)^2}, \tag{C.1}$$

$$\Pi_j^{*K/K}(c_i,c_j) = \frac{[1-c(2-n)]^2}{(3-2n)^2}, \tag{C.2}$$

$$\Pi_i^{*NK/NK}(c_i,c_j) = \frac{(1-n+c)^2}{(3-n)^2(1-n)^2}, \tag{C.3}$$

$$\Pi_j^{*NK/NK}(c_i,c_j) = \frac{[1-n-c(2-n)]^2}{(3-n)^2(1-n)^2}, \tag{C.4}$$

$$\Pi_i^{*K/NK}(c_i,c_j) = \frac{[1+c(1-n)]^2}{[3(1-n)+n^2]^2}, \tag{C.5}$$

$$\Pi_j^{*K/NK}(c_i,c_j) = \frac{[1-n-c(2-n)]^2}{[3(1-n)+n^2]^2}, \tag{C.6}$$

and

$$\Pi_i^{*NK/K}(c_i,c_j) = \frac{(1+c-n)^2}{[3(1-n)+n^2]^2}, \tag{C.7}$$

$$\Pi_j^{*NK/K}(c_i,c_j) = \frac{[1-c(2-n)]^2}{[3(1-n)+n^2]^2}. \tag{C.8}$$

Proposition C.1 summarises the main SPNE outcomes of the CDG with asymmetric marginal costs. The results of the proposition are also reported in Figure C.1, which represents a rigorous geometrical portrait of these results.

Proposition C.1. Let $n > 0$. Under asymmetric marginal costs ($c_i = 0$ and $c_j > 0$), the unique SPNE of the CDG is (K, K), which is 1) Pareto efficient for firm i (the most efficient) and firm j (the least efficient) if $0 < c_i < c_{C,i}(n)$, 2) Pareto inefficient for firm i (the most efficient) and Pareto efficient for firm j (the least efficient) if $c_{C,i}(n) < c_i < c^{\circ\circ}(n)$. Let $n < 0$. Under asymmetric marginal costs ($c_i = 0$ and $c_j > 0$), the unique SPNE of the CDG is (NK, NK), which is 3) Pareto efficient for firm i (the most efficient) and firm j (the least efficient) if $0 < c_i < c_{C,i}(n)$, 2) Pareto inefficient for firm i (the most efficient) and Pareto efficient for firm j (the least efficient) if $c_{C,i}(n) < c_i < c^{\circ}(n)$, where

$$c_{C,i}(n) := \frac{1-n}{5(1-n)+n^2}, \tag{C.9}$$

is the threshold value of the marginal cost such that $\Delta\Pi_{C,i}(c_i, c_j) = 0$.

Proof. The proof follows by considering the sign of the profit differentials. If $n > 0$ and $0 < c_i < c_{C,i}(n)$ then $\Delta\Pi_{A,i}(c_i, c_j) > 0$, $\Delta\Pi_{B,i}(c_i, c_j) < 0$, $\Delta\Pi_{C,i}(c_i, c_j) < 0$ and $\Delta\Pi_{A,j}(c_i, c_j) > 0$, $\Delta\Pi_{B,j}(c_i, c_j) < 0$, $\Delta\Pi_{C,j}(c_i, c_j) < 0$; if $n > 0$ and $c_{C,i}(n) < c_i < c^{\circ\circ}(n)$ then $\Delta\Pi_{A,i}(c_i, c_j) > 0$, $\Delta\Pi_{B,i}(c_i, c_j) < 0$, $\Delta\Pi_{C,i}(c_i, c_j) > 0$ and $\Delta\Pi_{A,j}(c_i, c_j) > 0$, $\Delta\Pi_{B,j}(c_i, c_j) < 0$, $\Delta\Pi_{C,j}(c_i, c_j) < 0$. If $n < 0$ and $0 < c_i < c_{C,i}(n)$ then $\Delta\Pi_{A,i}(c_i, c_j) < 0$, $\Delta\Pi_{B,i}(c_i, c_j) > 0$, $\Delta\Pi_{C,i}(c_i, c_j) > 0$ and $\Delta\Pi_{A,j}(c_i, c_j) < 0$, $\Delta\Pi_{B,j}(c_i, c_j) > 0$, $\Delta\Pi_{C,j}(c_i, c_j) > 0$; if $n < 0$ and $c_{C,i}(n) < c_i < c^{\circ}(n)$ then $\Delta\Pi_{A,i}(c_i, c_j) < 0$, $\Delta\Pi_{B,i}(c_i, c_j) > 0$, $\Delta\Pi_{C,i}(c_i, c_j) < 0$ and $\Delta\Pi_{A,j}(c_i, c_j) < 0$, $\Delta\Pi_{B,j}(c_i, c_j) > 0$, $\Delta\Pi_{C,j}(c_i, c_j) > 0$. **Q.E.D.**

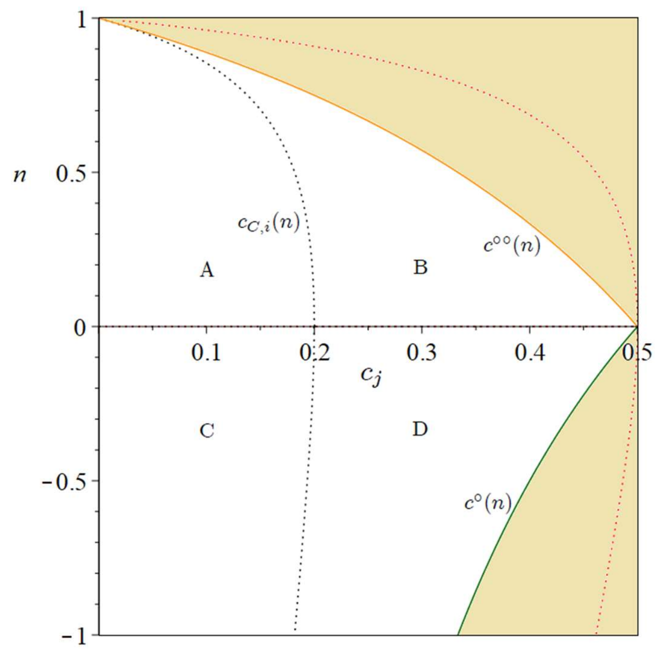

Figure C.1. The CDG with asymmetric marginal costs when $c_i = 0$ and $c_j > 0$: SPNE in the space (c_j, n). The sand-coloured region is the unfeasible parameter space.

- Area A: (K, K) is the unique SPNE, which is Pareto efficient for firm i (the most efficient) and firm j (the least efficient).
- Area B: (K, K) is the unique SPNE, which is Pareto inefficient for firm i (the most efficient) and Pareto efficient for firm j (the least efficient).
- Area C: (NK, NK) is the unique SPNE, which is Pareto efficient for firm i (the most efficient) and firm j (the least efficient).
- Area D: (NK, NK) is the unique SPNE, which is Pareto inefficient for firm i (the most efficient) and Pareto efficient for firm j (the least efficient).

Then, the most efficient firm can be entrapped in a "bad" equilibrium (dilemma) if the marginal cost of the least efficient firm is sufficiently high. Therefore, the production of compatible (resp. incompatible) products when the marginal cost is high benefits the least efficient rival when the network externality is positive (resp. negative). Unfortunately, the SPNE is (K, K) (resp. (NK, NK)) which is the rational outcome obtained in the market. By considering together both quality differential and cost differential, the outcomes of CDG are qualitatively the same, unless the appearance of a region in which the Nash equilibrium in pure strategy does not exists. For economy of space, we do not report this analysis here, but it is available on request.

Appendix D. The CDG with output commitment

This appendix follows the main analysis of Katz and Shapiro (1985), in which firm i commits itself to an announced output level and firm j does not commit itself to an announced output (asymmetric output commitment) and then evaluates the firm's incentives to produce compatible and incompatible goods at the first stage. In doing so, we consider homogeneous quality ($a = 1$) and no marginal cost ($c = 0$). We briefly outline here the main difference to the model presented in the main text, i.e., the demands for firm i and firm j. The (normalised) inverse demand of firm i (commitment) when the rival does not commit is:

$$p_i = 1 - q_i - q_j + n(q_i + k_i y_j), \tag{D.1}$$

and the (normalised) inverse demand of firm j (no commitment) when the rival commits is:

$$p_j = 1 - q_j - q_i + n(y_j + k_j q_i), \tag{D.2}$$

where y_i $(i, j = \{1,2\},\ i \neq j)$ denotes the consumers' expectations about the equilibrium output produced by firm i. We recall that 1) when both firms produce compatible goods $k_i = k_j = 1$ (K/K), 2) when both firms produce incompatible goods $k_i = k_j = 0$ (NK/NK), and 3) when only one of the two firms produces compatible goods $k_i = 1$ and $k_j = 0$ (K/NK) or vice versa (NK/K). As usual, the section does not report the analytical details, which are available on request, and directly presents the output matrix at the second stage (Table D.1) and the payoff matrix at the first stage (Table D.1) of the CDG when firm i commits itself and firm j does not commit itself to an announced output level as well as the feasibility condition, which is simply given by $n < 1/2$. When this inequality is fulfilled, the output produced by firms in all sub-games is positive.

Table D.1. The CDG with output commitment (output matrix). Second stage.

Firm j $\rightarrow$ Firm i $\downarrow$	K	NK
K	$\frac{1}{(3-n)(1-n)}, \frac{1}{3-n}$	$\frac{1}{(3-2n)(1-n)}, \frac{1-2n}{(3-2n)(1-n)}$
NK	$\frac{1}{3-2n}, \frac{1}{3-2n}$	$\frac{1-n}{3(1-n)^2-n^2}, \frac{1-2n}{3(1-n)^2-n^2}$

Table D.2. The CDG with output commitment (payoff matrix: profits). Second stage.

Firm j $\rightarrow$ Firm i $\downarrow$	K	NK
K	$\frac{1}{(3-n)^2(1-n)}, \frac{1}{(3-n)^2}$	$\frac{1}{(3-2n)^2(1-n)}, \frac{(1-2n)^2}{(3-2n)^2(1-n)^2}$
NK	$\frac{1-n}{(3-2n)^2}, \frac{1}{(3-2n)^2}$	$\frac{(1-n)^3}{[3(1-n)^2-n^2]^2}, \frac{(1-2n)^2}{[3(1-n)^2-n^2]^2}$

Proposition D.1 summarises the main SPNE outcomes of the CDG with asymmetric output commitment.

Proposition D.1. Let $n > 0$. Under asymmetric output commitment, the unique SPNE of the CDG is (K, K), which is 1) Pareto efficient for firm i (that commits itself to an announced output level) and firm j (that does not commit itself to an announced output level) if $0 < n < \frac{3}{2} - \frac{1}{2}\sqrt{5} \cong 0.381$, 2) Pareto inefficient for firm i (that commits itself to an announced output level) and Pareto efficient for firm j (that does not commit itself to an announced output level) if $0.381 < n < 0.5$. Let $n < 0$. Under asymmetric output commitment, the unique SPNE of the CDG is (NK, NK), which is 3) Pareto efficient for firm i (that commits itself to an announced output level) and firm j (that does not commit itself to an announced output level) if $-1 < n < 0$.

Proof. The proof follows by considering the sign of the profit differentials. If $n > 0$ and $0 < c_i < 0.381$ then $\Delta\Pi_{A,i}(n) > 0$, $\Delta\Pi_{B,i}(n) < 0$, $\Delta\Pi_{C,i}(n) < 0$ and $\Delta\Pi_{A,j}(n) > 0$, $\Delta\Pi_{B,j}(n) < 0$, $\Delta\Pi_{C,j}(n) < 0$; if $n > 0$ and $0.381 < c_i < 0.5$ then $\Delta\Pi_{A,i}(n) > 0$, $\Delta\Pi_{B,i}(n) < 0$, $\Delta\Pi_{C,i}(n) > 0$ and $\Delta\Pi_{A,j}(n) > 0$, $\Delta\Pi_{B,j}(n) < 0$, $\Delta\Pi_{C,j}(n) < 0$. If $n < 0$ then $\Delta\Pi_{A,i}(n) < 0$, $\Delta\Pi_{B,i}(n) > 0$, $\Delta\Pi_{C,i}(n) > 0$ and $\Delta\Pi_{A,j}(n) < 0$, $\Delta\Pi_{B,j}(n) > 0$, $\Delta\Pi_{C,j}(n) > 0$. **Q.E.D.**

The proposition reveals that, for a high positive network size, the firm that announces production and commits itself to it would find it more profitable to produce incompatible goods. This is because the commitment binds the firm to increase production, and this eventually reduces profits. Unfortunately for this firm, the SPNE that emerges in the market is to produce compatible goods and then the committing firm is worse off and entrapped in a "bad" equilibrium.

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