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Transition into adulthood, marriage and timing of life in a stable population framework

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Abstract

The process of transition into adulthood, as an irreversible process which, during the lifetime of individuals, usually begins with the end of formal education, is reconsidered within a multistate generalisations of stable population theory. The chosen theoretical framework is inspired by a recent mathematical development by Inaba (1996). Two mathematical frameworks are developed, a simpler one based on constant rates, and a more general one embedding ages structure as well. The analysis of the simpler model gives a clear-cut threshold result. The general model permits to clarify several some general questions related with the impact of changes in patterns of transition to adulthood on reproductivity. Some preliminary results on the effects of the delayed role of transition into adulthood on reproductivity are finally given, using data from the recent italian national survey "Indagine Multiscopo sulle Famiglie".

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1. Introduction: transition into adulthood and marriageability: consequences for reproductivity

In some Western European countries of the Mediterranean Area, Italy for instance, the process of family formation, which takes course generally within a legal marriage (Castiglioni and Dalla Zuanna, 1994), has significantly slowed during the last decades. In the meanwhile, the same countries are experiencing below-replacement fertility at bottom levels. In the case of Italy, the connection between the slowering of the process of family formation on the one side and that of lowering fertility on the other one, has been linked by some scholars to the diffusion of psychological refuse for long-term choices which may affect both the family formation and the reproduction phases (Micheli, 1996; De Sandre, 1997). Other scholars underline changes in gender relationships (Blossfeld and De Rose, 1992). Another stylised fact that may be put forth is that, in the same geographic areas and historical contexts, the length of full-time education for the young adults has been steadily increasing, especially with regard to women. For both sexes, the entrance into the labour market has been postponed. In general, one may say that, over time, the whole process of transition into adulthood has been increasingly delayed.

Studies focusing on the process of transition into adulthood show that the typical sequence of involved events is the following: the end of (formal) education comes first, then followed by the first job, then the first cohabitation/marriage (in some cases preceded by leaving the parental home), and finally, at last, the birth of the first child (Kiernan, 1991; Corijn, 1996). Having a steady, married or unmarried, cohabitation seems to be an almost necessary condition in order to become a parent - or, at least, "pregnancy remains strongly linked with union formation" (Santow and Bracher, 1996). Entering the labour market seems to be an almost necessary condition to be able to leave the parental home in order to live with a partner, and, finally, having completed formal education seems to be an almost necessary condition to enter the labour market. The distance between the end of formal education of an individual and the formation of her/his own family may be shorter or longer, but the sequence seems to be well established.

The general hypothesis adopted in the present work will be that the fact of beginning the transition into adulthood, thanks to the end of the process of formal education, gives also rise to a transition from the state of being "not marriageable" to the state of being "marriageable". This last distinction was introduced in a fundamental paper by Coale and McNeil (1972), who show that the frequency of first marriage in a female cohort can be computed by assuming that people enters in a "marriageable" state at a certain age, according to a normal distribution, and then marries after some exponentially distributed delays. While Coale and McNeil explicitly argue that "in contemporary populations of Western European origin (...) we may conjecture that the age of becoming marriageable is the age at which serious dating, or going steady begins", in this paper it is assumed that the beginning of the transition into adulthood is the marker of becoming marriageable. [1]

It thus seems of interest to investigate the effects of changes in the timing of transition into adulthood on the patterns of family formation and reproduction of a population. Among the possible approaches, one which seems particularly fruitful is based on multistate stable population theory. This area has been the object of a renewed theoretical interest in recent time, especially by Inaba, who developed, and provided a detailed mathematical characterisation of, several models aimed to study the different facets of the complex interaction between marriage and reproduction. Inaba (1996) has considered a one sex irreversible three state (single $\rightarrow$ married $\rightarrow$ widowed or divorced) model for human reproduction via first marriage, whereas Inaba (1993) adds to the basic scheme the possibility of iterative marriage. Finally Inaba (1993) attacks, in a more theoretical vein, the more difficult case of a true multistate two-sex interaction. Inaba's models are very general in that not only chronological age but also duration-dependent transitions state are systematically considered. They therefore represent an appropriate background for the modelling of transition into adulthood and the investigation of its relations with reproduction.

This report represents a preliminary effort in this direction. We generalise Inaba's (1996) irreversible three state model, by explicitly recognising a fourth state to take into account the problem of transition into adulthood, and of marriageability. We also consider a simplified version of our four state model with age-independent transition rates which is reducible to ordinary differential equations (ODE's). The theoretical framework employed appears to be fruitful from several points of view such as: i)to provide clearcut analytical results capable to enlight the relationships between transition into adulthood and reproductivity; ii)to derive explicit formulas relating the reproductivity indices of the population with the indices characterising the process of transition into adulthood, as suggested by Inaba; iii) to perform model assisted macro-simulation aimed to evaluate the long term impact of changes in the patterns of transition into adulthood on reproduction, which is the main aim of our future research.

For instance, with reference to i), by resorting to the simpler constant rates model we can prove a typical threshold condition which clearly shows how a strongly delayed transition into adulthood can prevent the restoration of replacement level fertility even in case it could be possible to raise fertility of marriages well over the level of two children per couple. With reference to ii) the availability of explicit formulas for the reproductivity indices also in the more complex age structured case has the merit of clearly putting in evidence those demographic functions which are to be of crucial interest for the problem at point and seem to deserve to be carefully studied in field investigations. These are: i)the proportions ever married stratified with respect to the age of transition into adulthood, ii)the average age at marriage stratified with respect to the age of transition into adulthood, and finally iii)the conditional (on the age of transition into adulthood) birth rates. These formulas for the reproductivity indices are finally used to provide a preliminary application of our framework to recently collected italian data, in order to evaluate what we call here the "delayer" role of

[1] The delay between becoming marriageable and marrying may also vary according to specific conditions. In such a context, marriage and unmarried cohabitation may be treated under the same approach, if it is needed to refer to countries where legal marriage is not largely predominant.

transition into adulthood on reproduction. This work is intended to be preliminary to a more deep simulation work.

The present paper is organised as follows. In the second section some aspects of the role of the completion of the formal education as a marker of transition into adulthood, and its interrelationships with the process of family formation, are considered, even for what concerns empirical evidence. In the third section we introduce the aggregate framework with age-independent transition rate.s In the fourth section the general four-states age-structured stable population framework is developed. In the fifth section the implications due to the existence of the state of adult on reproductivity indices are developed. Finally the preliminary application of our framework to italian FFS data is presented in section six.

2. The end of formal education as a marker of transition into adulthood

In this paper, the end of formal education is selected as a primary marker of the beginning of the transition into adulthood, being also an irreversible transition from the state of being "not marriageable" to the state of being "marriageable". This appears to be a sensible choice, and not only for Mediterranean Europe, because:

a)sooner or later education will be completed (with the exception of death during the educational period). The end of formal education may thus be studied only for its tempo, because we may assume a unitary quantum. However, this is not a necessary condition for the models we present here;

b)not every woman enters the labour market, but it is empirically sensible to assume that every woman enters the marriage market only after having completed her formal education. As Blossfeld and De Rose (1992) state: "finishing education is expected to count as one of the important prerequisites for entering into adulthood status, and thereby entering into marriage and parenthood". While the nature of the influence of women's education on the process of family formation is still a matter of debate, the importance of women's education itself is not questioned. In Becker's (1991) work, the increasing educational level of women implies an increased human capital level, thus partially diminishing women's advantages from getting married (the main effect must be thus seen via an increased proportion never marrying). In Blossfeld and Huinink's (1991), the fact of prolonging the education process has the effect of postponing marriage, thus lowering the risk of getting married. A recent study on West Germany (Huinink and Mayer, 1995) found support for the hypothesis that "for women, it was educational participation and educational plus vocational attainment that increasingly influences their pattern of family formation". Santow and Bracher (1994) find the same results for Australian women, stating that "education depresses marriage rates by extending the period over which women are not viewed, and do not view themselves, as fully eligible for marriage". Broadly speaking, a large amount of empirical evidence shows the delaying role played by being involved in the educational system on the process of family formation (see also Klijzing, 1995), while the evidence on the effect of educational attainment after the end of formal education is not unidirectional (Santow and Bracher, 1996);

c)while other events might have been chosen as a marker (e.g. the entry into the labour force or the moment where leaving the parental home), the end of formal education has a specific characteristic: it marks the end of a period which is rarely compatible with adult roles[2].

Moreover, a framework where the end of formal education is a necessary condition for entering the "marriageable" subpopulation may well apply also to developing countries, where the focus may be on estimation of the effect of rising female educational level on fertility.

It seems thus very interesting to study the effects of modifications of the timing of transition into adulthood on family formation and reproduction of a population, within a generalised one-sex stable population framework.

The approach followed in the present work is fundamentally aimed at the *macro* level, as it is the case for all studies based on stable population theory frameworks. In this sense, little attention is paid to the determinants of individual behaviour, which should rather have to be investigated with the use of event history analysis techniques. We are here mainly interested in one of the four directions for future research suggested by Marini (1984), who was interested to the analysis of the whole spectra of consequences of changes in the process of transition into adulthood for the societies a a whole. Our attention particularly focuses on the consequences of modifications in the timing of the process on population dynamics and reproduction. This is obviously a first step in a broader task.

3. Transition into adulthood, marriage, reproductivity: an elementary preliminary without age

As previously pointed out, the theoretical framework developed in the present work to investigate the relations between transition to adulthood and marriageability, first marriage and reproductivity is represented by a one-sex (the female one for simplicity) four states stable population formulation with "irreversible" transitions (fig. 1), which extends Inaba's 1996 model of a stable population with reproduction via first marriage. As represented in the flow diagram below, young individuals can not marry, since they do not become marriageable until they become adult. Moreover only married[3] individuals do reproduce.

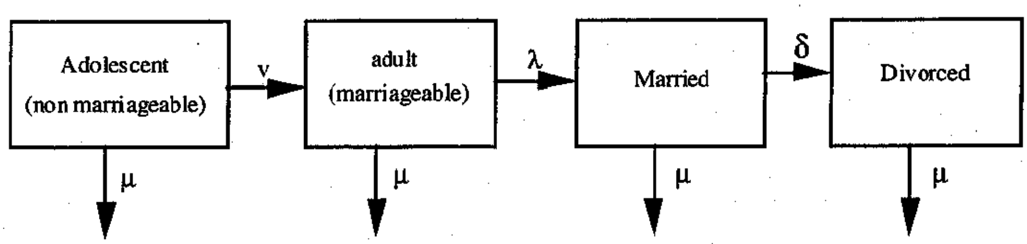


Fig. 1 Flow diagram of the basic four states model

[2] In fact, being involved in the military service normally has the same consequences (Marini, 1985).
[3] The word "married" can of course substituted by "in stable union".

Let us consider, as a pedagogic preliminary, an "aggregate" case, based on constant transition rates. Though oversimpified this case provides clearcut results, so enlightening the role of "sequential" stages within classical stable population frameworks.

Let us denote with $p_i(t)$ (i=0,...,3) the four state variables, i.e. the total number (or density) of female individuals in the four states considered: $p_0(t)$ is the number of "never adult" individuals at time t, $p_1(t)$ is the number of adult individuals, $p_2(t)$ the number of married individuals and finally $p_3(t)$ the number in the residual state (divorced or remarried). Furthermore let μ_i's be the state specific death rates per unit time, v the rate of transition to adulthood per unit time of young individuals, λ the rate at which adult individuals get marry per unit time, and δ the separation rate. Finally let m denote the fertility rate of the married population.

By assuming that all the involved rates are constant, a straightforward translation into continuous equations of the flow diagram of fig. 1, leads to the following system of linear ordinary differential equations:[4]

$$\begin{aligned} \dot{p}_0(t) &= mp_2(t) - (\mu_0 + v)p_0(t) \\ \dot{p}_1(t) &= vp_0(t) - (\mu_1 + \lambda)p_1(t) \\ \dot{p}_2(t) &= \lambda p_1(t) - (\mu_2 + \delta)p_2(t) \\ \dot{p}_3(t) &= \delta p_2(t) - \mu_3 p_3(t) \end{aligned} \qquad (3.1)$$

The total population $n(t) = p_0(t) + p_1(t) + p_2(t) + p_3(t)$ satisfies the ODE:

$$\dot{n}(t) = mp_2(t) - \mu n(t) \qquad (3.2)$$

ODE systems as (3.1), though not frequently used in demography, are quite common in mathematical biology and population dynamics of infectious diseases (for instance Anderson and May 1991). In compact form the system (3.1) may be represented as:

$$\dot{P}(t) = M_a P(t) \qquad (3.3)$$

where M_a is the matrix:

$$M_a = \begin{pmatrix} -(\mu_0 + v) & 0 & m & 0 \\ v & -(\mu_1 + \lambda) & 0 & 0 \\ 0 & \lambda & -(\mu_2 + \delta) & 0 \\ 0 & 0 & \delta & -\mu_3 \end{pmatrix} \qquad (3.4)$$

The matrix M_a has $(-\mu_3)$ as an eigenvalue. The remaining eigenvalues are solutions of the cubic equation:

[4] The system (3.1) may also be derived from the general system of the next section.

$$P(K)=\left((\mu_0+v)+K\right)\left((\mu_1+\lambda)+K\right)\left((\mu_2+\delta)+K\right)-mv\lambda=0$$

ie:

$$P(K)=K^3+aK^2+bK+c=0 \qquad (3.5)$$

where:

$$a=(\mu_0+v)+(\mu_1+\lambda)+(\mu_2+\delta)=A+B+C>0$$
$$b=(\mu_0+v)(\mu_1+\lambda)+(\mu_0+v)(\mu_2+\delta)+(\mu_1+\lambda)(\mu_2+\delta)=AB+AC+BC>0$$
$$c=(\mu_0+v)(\mu_1+\lambda)(\mu_2+\delta)-mv\lambda=ABC-D$$

The demographically relevant features of the system (3.3) are studied by noting that M_a is a Metzler matrix. Metzler matrices are positivity preserving operators in continuous time: they play the same role played by positive matrices for discrete dynamical systems and possess their specific version of the Pérron-Frobenius theorems apparatus (Luenberger 1979, ch. 6). Hence, in particular, the matrix (3.4) has a unique dominant eigenvalue K_0, to which it belongs a demographically meaningful (i.e.: non negative) eigenvector, and furthermore all remaining eigenvalues of M_a have real part which is less than K_0.

It is easy to show that the sign of the dominant eigenvalue K_0, which plays the role of the Lotka's intrinsic rate of growth of our population and hence determines its long-term behaviour, only depends on the sign of the coefficient of the constant term c in (3.5). Hence, depending on whether $c \gtreqless 0$, i.e. depending on whether:

$$mv\lambda-(\mu_0+v)(\mu_1+\lambda)(\mu_2+\delta)\gtreqless 0 \quad (3.6)$$

we will have in turn i)stable exponential growth, ii)perfect stationarity, iii)stable exponential decay. The threshold condition (3.6), which separates the case of Lotka's stable growth from stable decay can be represented in the following form:

$$\frac{m}{\mu_2+\delta}\cdot\frac{v}{\mu_0+v}\cdot\frac{\lambda}{\mu_1+\lambda}>1 \qquad (3.7)$$

The left side of (3.7) defines the net reproductive rate of the population (the basic reproduction rate in the epidemiological jargon):

$$R_0=\frac{m}{\mu_2+\delta}\cdot\frac{v}{\mu_0+v}\cdot\frac{\lambda}{\mu_1+\lambda} \qquad (3.8)$$

In fact the first term of the factorization (3.8) is given by the product between the average number of children produced by a married women per unit time (m) times the expected duration of her sejour-time in the married state ($1/(\mu_2+\delta)$). Hence the quantity $m/(\mu_2+\delta)$ represents the average number of children produced by a married woman during her sejour in the married state. This latter quantity has in turn to be multiplied by the conditional probability to reach the adult state ($v/(\mu_0+v)$), and by the conditional probability to marry being adult ($\lambda/\mu_1+\lambda$), in order to take into account the existence of mortality and other states.

The last result shows that in a population in which fertility is below replacement, no policy aimed to take fertility back to the "zero growth level" which is based on a reduction of the age at marriage and/or of the age of transition into adulthood can be successful. Vice-versa by suitably acting on such parameters can reveal to be an effective policy for taking down to stationarity a population experiencing stable exponential growth. An important example could be for instance a policy of systematically raising alphabetisation and education in developing countries. The last result permits to give simple answers to question such as: given the present value of the average age of entering the adult state, and given the state of mortality, what are the combinations of values of the marriage rate λ and the birth rate m which ensure stationarity?

Although the chronological age is only implicit in the model (3.1) with constant rates, the long stable term age-stage-structure of model (3.1) can be determined analytically. This can be done by considering an "enlarged" model recognising age as well (see the details in the appendix). In the standard case in which mortality is state-independent (i.e.: $\mu_i=\mu$ for all i), the population weights $w_i(a)=p_i(a,t)/n(a,t)$ which emerge in the long term stable regime satisfy the system (see the appendix):

$$\begin{aligned} &\frac{d}{da}w_0(a) = -vw_0(a) \\ &\frac{d}{da}w_1(a) = vw_0(a) - \lambda w_1(a) \\ &\frac{d}{da}w_2(a) = \lambda w_1(a) - \delta w_2(a) \\ &w_3(a) = 1 - \left(w_0(a) + w_1(a) + w_2(a)\right) \end{aligned} \tag{3.9}$$

Hence the long-term weights are given by

$$\begin{aligned} &w_0(a) = e^{-va} \\ &w_1(a) = \frac{v}{v-\lambda}\left(e^{-\lambda a} - e^{-va}\right) \\ &w_2(a) = \frac{v\lambda}{v-\lambda}e^{-\delta a}\left[\frac{1-e^{-(\lambda-\delta)a}}{\lambda-\delta} - \frac{1-e^{-(v-\delta)a}}{v-\delta}\right] \\ &w_3(a) = 1 - \Sigma_0^2 w_i(a) \end{aligned} \tag{3.10}$$

It finally is to be noticed that the above model (this is true also for the fully age structured problem considered in the next section), can easily be enlarged to explicitly introduce the parity structure existing in the population, as done still in Inaba (1996).

4. The fully age-structured model: stable distribution with respect to age and stages of life

Here we introduce our age structured model for the transition into adulthood. The formulation, which adds a fourth state (that of adult) to Inaba's (1996) "irreversible"formulation, rests on four basic population densities, ie: i)$p_0(a,t)$, the density of "never adult" individuals aged a at time t, ii)$p_1(c,t;\eta)$, the density at time t of individuals who entered into adulthood since c years, at the age of η, iii)$p_2(\tau,t;\eta,\xi)$, denoting the density of individuals married since τ years, who became adult at the age of η and married at the age of ξ, $p_3(a,t)$, the residual class, constituted by individuals who are not in the first three states, i.e. who are widowed or divorced (in this last case they could also be remarried, but it is assumed they do not contribute anymore to fertility).

The backbone of the model is given by the following system of Ross-McKendrick-Von Foerster (RMKF) balance equations:

$$\begin{cases} \left(\dfrac{\partial}{\partial a}+\dfrac{\partial}{\partial t}\right)p_0(a,t) = -[\mu(a)+v(a)]p_0(a,t) \\ \left(\dfrac{\partial}{\partial c}+\dfrac{\partial}{\partial t}\right)p_1(c,t;\eta) = -[\mu(c+\eta)+\lambda(c+\eta,\eta)]p_1(c,t,\eta) \\ \left(\dfrac{\partial}{\partial \tau}+\dfrac{\partial}{\partial t}\right)p_2(\tau,t;\eta,\xi) = -[\mu(\xi+\tau)+\delta(\tau,\xi)]p_2(\tau,t;\eta,\xi) \\ \left(\dfrac{\partial}{\partial a}+\dfrac{\partial}{\partial t}\right)p_3(a,t) = \iint_{\tau+\xi=a} \delta(\tau,\xi)p_2(\tau,t;\eta,\xi)d\tau d\eta - \mu(a)p_3(a,t) \end{cases} \tag{4.1}$$

where:

$$\frac{\partial}{\partial a}+\frac{\partial}{\partial t};\quad \frac{\partial}{\partial c}+\frac{\partial}{\partial t};\quad \frac{\partial}{\partial \tau}+\frac{\partial}{\partial t};$$

are the involved RMKF aging operators. In particular: i)$\mu(a)$ is the age dependent (but time independent) mortality rate (or force of mortality)[5], ii)$v(a)$ is the age-dependent rate of transition to adulthood, iii)$\lambda(a,\eta)=\lambda(c+\eta,\eta)$ is the age specific force of first marriage, which is assumed to be influenced by the age of entrance into adulthood, and, iv)$\delta(\tau,\xi)$ is the force of dissolution of couples married τ years before when the female was aged ξ.

The system (4.1) has to be completed with the following boundary conditions:

[5] We disregarded the further complication due to explicit consideration of age-state dependent forces of mortality, which imply shifts in the frailty regime of the individuals every time they change state.

$$\begin{cases} p_0(0,t) = B(t) = \int_0^{\beta}\int_{\eta}^{\beta}\int_0^{\beta-\xi} p_2(\tau,t;\xi,\eta)m(\tau;\xi,\eta)d\tau d\xi d\eta \\ p_1(0,t;\eta) = v(\eta)p_0(\eta,t) \\ p_2(0,t;\eta,\xi) = \lambda(\xi,\eta)p_1(\xi-\eta,t;\eta) \\ p_3(0,t) = 0 \end{cases} \qquad (4.2)$$

The interpretation of the conditions (4.2) is the following. The first one says that the number of individuals aged zero in the first state is simply the total number of births, which by assumption are all due to women in the married state (state 2). Moreover, β is the upper bound of the fertile age span and $m(\tau;\xi,\eta)$ defines the marital fertility rate at marriage duration τ for a married women entered into the adult state at the age of η and subsequently married at the age of ξ ($\xi>\eta$).
The boundary condition in the second of equations (4.2) tells us that the number of adult women with duration zero of permanence in the adult state and age at adulthood η at time t, is given by the number of transitions to adulthood of individuals aged η at time t. Moreover, the number, always at time t, of individuals with marriage duration zero, who entered the adult state at age η and married at ξ is given by the corresponding number of weddings of $p_1(c,t;\eta)$ individuals at the age $\xi=\eta+c$. The boundary condition in the fourth density is identically zero.
In addition, to close the model, a set of initial conditions, under the form of a set of prescribed initial distribution, has to be assigned:

$$p_0(a,0) = H_0(a) \quad p_1(c,0;\eta) = H_1(c,\eta) \quad p_1(\tau,0;\eta,\xi) = H_2(\tau;\eta,\xi) \quad p_3(a,0) = H_3(a) \quad (4.3)$$

The following relations, which relate the population densities $p_i(.,t)$ with the chronological age distributions in the four states hold:

$$\begin{aligned} p_1(a,t) &= \int_0^a p_1(a-\eta,t;\eta)d\eta \\ p_2(a,t) &= \int_0^a\int_0^{\xi} p_2(a-\xi,t;\eta,\xi)d\xi d\eta \\ p_3(a,t) &= n(a,t) - \sum_{i=0}^{2} p_i(a,t) \end{aligned} \qquad (4.4)$$

We notice again that, for purpose of generality, it has been assumed that the fertility rates depend not only on the duration of marriage, but also on the age at marriage (as in Inaba 1996) and on the age of transition into the adult state. This last assumption seems to be completely reasonable in that individuals who are adult, and possibly economically independent since a much shorter time, would have probably experienced very different job experiences, incomes and so on compared to individuals who became adult since a longer time.

The mathematical properties of the general model defined by the equations (4.1) plus the conditions (4.2), (4.3) can be studied by applying Inaba's approach based on the reduction of (4.1) - (4.2) - (4.3) to a traditional renewal equation (see Inaba 1996 for the detailed treatment of the three states model), i.e to a traditional Lotka's birth equation, whose behaviour is well known. Inaba's approach seems to be completely general and potentially applicable, as long as we are concerned with "irreversible" multistate processes, to problems characterised by an arbitrary number of states. Since we are not interested in mathematical details the approach followed here is heuristic. The most relevant steps of the reduction to a renewal equation are the following.

It is possible to show first that the overall age distribution:

$$n(a,t) = \sum_{i=1}^{3} p_i(a,t) \qquad (4.5)$$

satisfies a traditional Ross-McKendrick-VonFoerster PDE:

$$\left(\frac{\partial}{\partial a} + \frac{\partial}{\partial t}\right) n(a,t) = -\mu(a) p_0(a,t) \qquad (4.5')$$

The last result crucially depends on the assumption of state-independent mortality: otherwise (4.5') would depend on the average mortality rate over states.

As a second step we derive the formal solutions of the basic PDE's (4.1). We recall that the standard form of the Lotka's birth equation is the following (Keyfitz 1968, 1985):

$$B(t) = G(t) + \int_0^t B(t-a)\phi(a)da \qquad (4.5'')$$

where B(t) is the birth function at t, $\phi(a)$ the net maternity function (given by the product between the age specific fertility rate m(a) and the survival function up to age a, p(a)), and G(t) a function which depends on the (arbitrarily) chosen initial distribution of age. The previous form splits the births at time t into two components, i.e. a "core" one depending on births since time zero, plus a component depending only on the "initial" conditions of the problem. Here we only report, for brevity, the components of the formal solutions of the (4.1) depending on the births since time zero. We get (by integration along characteristics, amounting to a suitable application of the involved survival function):

$$p_0(a,t) = B(t-a) l(a) V(a) \qquad t > a$$

$$p_1(c,t;\eta) = B(t-c-\eta) l(\eta+c) V(\eta) v(\eta) \Lambda_\eta(c) \qquad t > \eta + c$$

$$(4.6)$$

$$p_2(\tau,t;\eta,\xi) = B(t-\tau-\xi) l(\eta) V(\eta) v(\eta) \frac{l(\xi)}{l(\eta)} \Lambda_\eta(\xi) \lambda(\xi,\eta) \frac{l(\xi+\tau)}{l(\xi)} \Delta_\xi(\tau) =$$
$$= B(t-\tau-\xi) l(\xi+\tau) V(\eta) v(\eta) \Lambda_\eta(\xi) \lambda(\xi,\eta) \Delta_\xi(\tau) \qquad t > \xi + \tau$$

The relationships (4.6) express the populations densities in the first three states in terms of the relevant density of past births (since time zero) and of the following generalised survival functions:

$$
\begin{aligned}
&l(a) = \exp\left(-\int_0^a \mu(u)du\right)\\
&V(a) = \exp\left(-\int_0^a v(u)du\right)\\
&\Lambda_\eta(c) = \exp\left(-\int_\eta^{\eta+c} \lambda(u;\eta)du\right) = \exp\left(-\int_0^c \lambda(\eta+s;\eta)ds\right)\\
&\Delta_\xi(\tau) = \exp\left(-\int_0^\tau \delta(u;\xi)du\right)
\end{aligned}
\qquad (4.7)
$$

The quantities (4.7) denote respectively: i)the "survival to natural mortality" function, expressing the probability for a woman to survive from birth from birth up to age a; ii)the "survival to adulthood" function, expressing the probability for a living woman to survive from birth up to age a without having entered the adult state, iii) the conditional survival function to marriage until age $a=\eta+c$ for a woman entered into the adult state at the age η; the conditional survival function to divorce until age $a=\tau+\xi$ for a woman married at the age ξ.
The latter quantity of (4.6):

$$p_2(\tau,t;\eta,\xi) = B(t-\tau-\xi)l(\xi+\tau)V(\eta)v(\eta)\Lambda_\eta(\xi)\lambda(\xi,\eta)\Delta_\xi(\tau) \qquad t>\xi+\tau$$

is the one necessary to compute the "core" part of the births renewal equation in model (4.1)-(4.3). The structure of the involved renewal equation will be:

$$B(t) = G_1(t) + \int_0^t B(t-a)\phi_1(a)da \qquad (8)$$

where G_1 is a function of time only, which plays the same role of the function G(t) in the basic renewal equation (4.5''), i.e. it embeds the initial age distributions, whereas the function ϕ_1 is the new net maternity function. In particular: $\phi_1(a)=l(a)m^*(a)$ where $m^*(a)$ is the age-related fertility rate, which for the present problem is defined as follows

$$
\begin{aligned}
m^*(a) &= \int_0^a\int_0^\xi V(\eta)v(\eta)\Lambda_\eta(\xi)\lambda(\xi,\eta)\Delta_\xi(a-\xi)m(a-\xi;\xi,\eta)d\eta d\xi =\\
&= \int_0^a V(\eta)v(\eta)\left[\int_\eta^a \Lambda_\eta(\xi)\lambda(\xi,\eta)\Delta_\xi(a-\xi)m(a-\xi;\xi,\eta)d\xi\right]d\eta
\end{aligned}
\qquad (4.7)
$$

The fact the overall births satisfy a Lotka-type equation implies that in the long term the total population n(a,t) will achieve a Lotka's stable state characterised by exponential evolution at a constant rate r, plus an unchanging age distribution (which in turn are independent on the initial age distribution). In particular we will have stable growth (r>0) or decay (r<0) depending on whether the net reproduction rate, defined as

$$R_0 = \int_0^\beta l(a)m^*(a)da =$$
$$= \int_0^a l(a)da \int_0^a V(\eta)v(\eta)\left[\int_\eta^a \Lambda_\eta(\xi)\lambda(\xi,\eta)\Delta_\xi(a-\xi)m(a-\xi;\xi,\eta)d\xi\right]d\eta \tag{4.8}$$

is greater or smaller than one.

As a final step, in order to characterise the long term behaviour of the population in the states 0,1,2,3, let us observe that, by performing the integrations (4.4) on the quantities (4.6) we get:

$$\begin{aligned}
&p_0(a,t) = n(a,t)V(a) \\
&p_1(a,t) = n(a,t)\Psi(a); \quad \Psi(a) = \int_0^a V(\eta)v(\eta)\Lambda_\eta(a-\eta)d\eta \\
&p_2(a,t) = n(a,t)\Gamma(a); \quad \Gamma(a) = \int_0^a\int_0^\xi \left[V(\eta)v(\eta)\Lambda_\eta(\xi)\lambda_\eta(\xi,\eta)\Delta_\xi(a-\xi)\right]d\eta d\xi
\end{aligned} \tag{4.9}$$

The latter relations, which hold only for sufficiently large t, recall that (this essentially is a consequence of positions of the problem) when the total population n(a,t) will achieve its stable age distribution, this will automatically force the densities $p_i(a,t)$ (i=0,1,2,3) in achieving a stable pattern as well. This leads to a stable distribution by age and state.

5. Consequences for reproductivity indices

Now we consider how the classical stable population reproduction indices, constructed in the "only age-dependent" case, such as the classical Net Reproduction Rate (NRR) and the Total Fertility Rate (TFR), modify when we explicitly consider the present, more complex, multistate case. In what follows we provide explicit formulas for the relevant reproduction indices by relying on Inaba's work (1996) and extending it to the present four state analysis.

For what concerns the classical net reproduction rate R_0, defined in (4.8) the following equivalent definitions holds (they differ only for the different definitions chosen for the same region of integration):

$$\begin{aligned}
R_0 &= \int_0^\beta\int_\eta^\beta\int_0^{\beta-\xi} m(\tau;\xi,\eta)l(\xi+\tau)V(\eta)v(\eta)\Lambda_\eta(\xi)\lambda(\xi,\eta)\Delta_\xi(\tau)d\tau d\xi d\eta \\
&= \int_0^\beta\int_0^{\beta-\tau}\int_0^\xi m(\tau;\xi,\eta)l(\xi+\tau)V(\eta)v(\eta)\Lambda_\eta(\xi)\lambda(\xi,\eta)\Delta_\xi(\tau)d\eta d\xi d\tau \\
&= \int_0^\beta\int_0^\xi\int_0^{\beta-\xi} m(\tau;\xi,\eta)l(\xi+\tau)V(\eta)v(\eta)\Lambda_\eta(\xi)\lambda(\xi,\eta)\Delta_\xi(\tau)d\xi d\eta d\tau
\end{aligned} \tag{5.1}$$

By making the change of variable from the "old" set (η,ξ,τ) to the "new" one (η,ξ,a=ξ+τ), we obtain the following alternative definition:

$$R_0 = \int_0^\beta l(a)\left[\int_0^a V(\eta)v(\eta)\left(\int_\eta^a \Lambda_\eta(\xi)\lambda(\xi,\eta)\Delta_\xi(\tau)m(\tau;\xi,\eta)d\xi\right)d\eta\right]da \tag{5.2}$$

which is the form (4.8) and shows that the age-specific fertility rates are of the form:

$$m^*(a) = \int_0^a V(\eta)v(\eta)\left(\int_\eta^a \Lambda_\eta(\xi)\lambda(\xi,\eta)\Delta_\xi(\tau)m(\tau;\xi,\eta)d\xi\right)d\eta \qquad (5.3)$$

specifying the actual relation between the traditional age specific fertility rate m*(a) and the duration specific fertility rates.

The latter formulas embody, via the factorization through the several involved survival functions, the delay due to transition to adulthood and/or marriage on reproduction: in order to be able to make child with the fertility law $m(\tau;\xi,\eta) = m(a-\xi;\xi,\eta)$ typical of her population at a given age, a female individual not only has to survive until that age, as in traditional stable populations, but she has also to:

a)enter the adult state, otherwise she would not be marriageable, at a previous age η, with probability (density) $V(\eta)v(\eta)$

a)get marry at a subsequent age ξ, with probability $\Lambda(\xi)\lambda(\xi)$

b)survive to the risk of divorce until the given age a, with probability $\Delta_\xi(\tau) = \Delta_\xi(a-\xi)$

If we explicitly neglect mortality, the following definition for the TFR (in absence of mortality but in presence of marriage and divorce) arises[6]:

$$TFR = \int_0^\beta \int_\eta^\beta \int_0^{\beta-\xi} m(\tau;\xi,\eta)V(\eta)v(\eta)\Lambda_\eta(\xi)\lambda(\xi,\eta)\Delta_\xi(\tau)d\tau d\xi d\eta \quad (5.4)$$

The meaning of TFR is that of (expected) total number of offsprings produced by a single woman during her effective fertile age span, where the "effective" fertile age span is given by that portion of the fertile age span actually spent within the married state.

We now decompose the TFR by stressing the sequentiality of the several stages of life. We can write:

$$\begin{aligned} TFR &= \int_0^\beta \int_\eta^\beta \int_0^{\beta-\xi} m(\tau;\xi,\eta)V(\eta)v(\eta)\Lambda_\eta(\xi)\lambda(\xi,\eta)\Delta_\xi(\tau)d\tau d\xi d\eta \\ &= \int_0^\beta V(\eta)v(\eta)\left[\int_\eta^\beta \Lambda_\eta(\xi)\lambda(\xi,\eta)\left(\int_0^{\beta-\xi} m(\tau;\xi,\eta)\Delta_\xi(\tau)d\tau\right)d\xi\right]d\eta = \\ &= \int_0^\beta V(\eta)v(\eta)\left[\int_\eta^\beta \Lambda_\eta(\xi)\lambda(\xi,\eta)T(\xi,\eta)d\xi\right]d\eta = \\ &= \int_0^\beta T^*(\eta)V(\eta)v(\eta)d\eta \end{aligned} \qquad (5.5)$$

where:

$$\begin{aligned} T(\xi,\eta) &= \int_0^{\beta-\xi} m(\tau;\xi,\eta)\Delta_\xi(\tau)d\tau \\ T^*(\eta) &= \int_\eta^\beta \Lambda_\eta(\xi)\lambda(\xi,\eta)T(\xi,\eta)d\xi \end{aligned} \qquad (5.6a,b)$$

The quantity $T(\xi,\eta)$ is the conditional TFR of women who became adult at the age of η and married at the age of ξ, in presence of the risk of separation. $T^*(\eta)$ has to be interpreted as the average (averaged on the non normalised conditional density of the age at marriage

[6] It has been chosen the first definition used in (5.1) for the region of integration.

$\Lambda_\eta(\xi)\lambda(\xi,\eta)$) TFR of all the women who entered the adult state at the age of η, independently on the age of their marriage.

By a suitable algebra we can write the general TFR in a more interpretable way. Let us first of all write:

$$TFR = (1 - V(\beta))\int_0^\beta T^*(\eta)W(\eta)d\eta \qquad (5.7)$$

where:

$$W(\eta) = \frac{V(\eta)v(\eta)}{1 - V(\beta)} \qquad (5.8)$$

is the normalised probability density of transition into adulthood (truncated at the end of the fertile age span), and:

$$\int_0^\beta W(\eta)d\eta = 1 - V(\beta) = PEA(\beta) \qquad (5.9)$$

is the "proportion ever adult" (PEA) at the end of the fertile period. Let us then define:

$$\Phi_\eta(\xi) = \frac{\Lambda_\eta(\xi)\lambda(\xi,\eta)}{\int_\eta^\beta \Lambda_\eta(\xi)\lambda(\xi,\eta)d\xi} \quad 0 \le \eta \le \beta;\ \eta \le \xi \le \beta \qquad (5.10)$$

The quantity (5.10) represents the conditional (normalized) density function of marriage at age ξ for the women who entered the adult state at age η and married before the end of the fertile age span. In particular the quantity:

$$\int_\eta^\beta \Lambda_\eta(\xi)\lambda(\xi,\eta)d\xi = 1 - \Lambda_\eta(\beta) = PEM(\eta) \qquad (5.11)$$

is the (conditional) proportion of ever married (PEM) women (to be precise: who married before age β) among those women who entered the adult state at the age of η: let us call it the "conditional" PEM. We can write therefore:

$$T^*(\eta) = PEM(\eta) \cdot \int_\eta^\beta T(\xi,\eta)\Phi_\eta(\xi)d\xi \qquad (5.12)$$

By introducing (5.12) into (5.7), and assuming that, quite reasonably, PEA(β)=1, we obtain:

$$TFR = \int_0^\beta PEM(\eta)W(\eta)\left[\int_\eta^\beta T(\xi,\eta)\Phi_\eta(\xi)d\xi\right]d\eta \qquad (5.13)$$

The inner integral in (5.13) defines the average TFR with respect to the age at marriage of women entered into the adult state at the age of η and married before the end of the fertile age span. It is clearly a conditional average TFR (conditional on being entered the adult state at η). Further elaborations are possible but it seems useful to distinguish two main cases:

a)A simpler one in which the fertility rates do not actually depend on the age of entrance into the adult state, ie: $m = m(\tau;\xi)$. We can call the present case the "pure delayer" case, in the sense that the role of the intermediate state of adult, or marriageable or whatever, does not influences the specific fertility behaviours existing in the (married) population: its possible effects are essentially those of delaying such fertility behaviours by delaying marriage.

b)A more general case in which $m = m(\tau; \xi, \eta)$. In this case in addition to the "pure delayer" effect we would have true fertility effects due to the process of transition into adulthood. This for instance allows for differential fertility behaviours by educational attainment.

In the present paper we will limit our analysis to the "pure delayer" case. We obviously have:

$$T(\xi,\eta) = \int_0^{\beta-\xi} m(\tau;\xi)\Delta_\xi(\tau)d\tau = T(\xi) \qquad (5.14)$$

If now, following again Inaba (1996), we assume a Henry type (Henry 1976) approximated linear relationship between the conditional TFR of women married at age ξ, $T(\xi)$, and the age at marriage:

$$T(\xi) = U - V\xi + R(\xi) \quad U>0, V>0 \qquad (5.15)$$

where R is a reminder, we get the approximated relationship:

$$\begin{aligned} TFR &\cong \int_0^\beta PEM(\eta)W(\eta)\left[\int_\eta^\beta (U - V\xi)\Phi_\eta(\xi)d\xi\right]d\eta = \\ &= U\int_0^\beta PEM(\eta)W(\eta)d\eta - V\int_0^\beta PEM(\eta)W(\eta)\left[\int_\eta^\beta \xi\Phi_\eta(\xi)d\xi\right]d\eta = \qquad (5.16) \\ &= U\int_0^\beta PEM(\eta)W(\eta)d\eta - V\int_0^\beta PEM(\eta)W(\eta)E_\eta(X)d\eta \end{aligned}$$

where $E_\eta(X)$ is the average age at marriage of those women who entered the adult state at the age of η and married before the end of the fertile age span. The last expression in (5.16) can be furtherly transformed by writing:

$$B = \int_0^\beta PEM(\eta)W(\eta)d\eta \qquad (5.17)$$

and:

$$W^*(\eta) = \frac{PEM(\eta)W(\eta)}{\int_0^\beta PEM(\eta)W(\eta)d\eta} = \frac{PEM(\eta)W(\eta)}{B} \quad 0 \le \eta \le \beta \qquad (5.18)$$

The B quantity defines the "average PEM", i.e. the average value of the conditional (at the age of entrance into adulthood) proportion ever married over the density of entering into adulthood, and W* is the probability density function of transition into adulthood at age η for those individuals who married before the end of the fertile age span. Thanks to (5.17) and (5.18) we obtain for (5.16):

$$TFR \cong B\left\{U - V\int_0^\beta E_\eta(X)W^*(\eta)d\eta\right\} \qquad (5.19)$$

Expression (5.19) shows, it is useful to compare it with Inaba 1996, that the TFR of the whole population factors as the product of the average PEM times the value of the Henry relationship evaluated in correspondence of the average age at marriage of those women who married before the end of the fertile age span. The last relationship is remarkable in that explicitly puts in evidence the need to investigate, especially at the empirical level, two relationships which until today do not seem to have yet found so many applications in population studies, i.e. the: i)the *relationship between the age of entrance into adulthood and the corresponding proportions ever married* and; ii)*the relationship between the age of entrance into adulthood and the corresponding average age at marriage.*

By further noting that (by using the CoV: $\xi=\eta+\tau$):

$$\begin{aligned} E_\eta(X) &= \int_\eta^\beta \xi\Phi_\eta(\xi)d\xi = \int_0^{\beta-\eta}(\eta+\tau)\Phi_\eta(\eta+\tau)d\tau = \\ &= \eta + \int_0^{\beta-\eta}\tau\Phi_\eta(\eta+\tau)d\tau = \eta + E_\eta(\Delta) \end{aligned} \qquad (5.20)$$

where $g(\eta)=E_\eta(\Delta)$ is the average difference (ie the average delay) between the average age at marriage and the average age at adulthood, (5.19) becomes:

$$\begin{aligned} TFR &\cong B\left\{U - V\int_0^\beta\left[\eta + E_\eta(\Delta)\right]W^*(\eta)d\eta\right\} = \\ &= B\left\{U - V\left[E(\eta) + \int_0^\beta E_\eta(\Delta)W^*(\eta)d\eta\right]\right\} \end{aligned} \qquad (5.21)$$

where $E(\eta)$ is the average age at entrance into adulthood for the subset of women married before the end of the fertile age span.

It is to be noticed that some information is available about the shape of the average delay function $g(\eta)$ and the conditional age at marriage. In particular since $E_\eta(X)$ is computed over women who married before the end of the fertile age span, it is clear that:

$$\left[E_\eta(X)\right]_{\eta=\beta} = \beta \text{ [7]} \qquad (5.22)$$

which in turn implies $g(\beta)=0$. In the event that $E_\eta(X)$ be linear, ie of the form[8]:

$$E_\eta(X) = E_{\eta_A}(X) + q(\eta - \eta_A) \quad \eta_A \le \eta \le \beta \qquad (5.23)$$

where η_A is the lower bound in the possible ages of transition into adulthood (in many cases it will be fixed by law) and $E_{\eta_A}(X)$ the corresponding average age at marriage, this would imply, if $E_{\eta_A}(X)$ were known, the obvious restriction on q:

---

[7] By definition it holds: $\xi \ge \eta$, which implies: $E_\eta(X) \ge \eta$. At the same time, since we are considering only individual who married before the end of their fertile age span, it is clear that: $E_\eta(X) \le \beta$.

[8] This appears at best a rough approximation. Anyway we do not yet dispose of an adequate body of data to study the problem.

$$q = \frac{\beta - E_{\eta_A}(X)}{\eta - \eta_A} \qquad (5.24)$$

The (5.24) is consisten with the obvious fact that q is expected to be less than one. By using (5.23) and (5.24):

$$\begin{aligned} TFR &\cong B\left\{U - V\int_0^{\beta}\left[E_{\eta_A}(X) + q(\eta - \eta_A)\right]W^*(\eta)d\eta\right\} = \\ &= B\left\{\left[U - V\left(E_{\eta_A}(X) - q\eta_A\right)\right] - qVE(\eta)\right\} \end{aligned} \qquad (5.25)$$

The relationship (5.25) could be used, for instance, to estimate the "pure delayer" effects of increase of the legal minimal age to work (ie: the lower bound among the possible ages of end of formal education) on TFR.

We do not intend to discuss in this work the more general case, in which true fertility effects due to the transition into adulthood do superimpose to the "pure delay" effects. We want anyway to point out that, given the possibly important roles played by the process of transition into adulthood on reproductivity of a population, serious investigation of the relationships between fertility rates and age of entrance into adulthood appear quite necessary developments. A useful starting point could be the field study of the relationship age at adulthood and the conditional TFR:

$$T(\xi, \eta) = \int_0^{\beta-\xi} m(\tau; \xi, \eta)\Delta_\xi(\tau)d\tau$$

6. Evaluating the role of increasing women's education on the TFR of a population

In this section the relationships developed so far are used in order to answer the specific question of what might be the effect of an increase in women's education on the total fertility rate of a given population?

The data set used here was collected by the Italian National Statistical Institute (ISTAT) during a retrospective survey held in 1988 ("Indagine Multiscopo sulle Famiglie, secondo ciclo"). From the survey individual data about each interviewed woman are available for what concerns in particular the following variables:

- the number of children ever born;
- marital status and age at marriage;
- educational level.

Only data for women aged between 40 and 49 at the moment of the interview are used for the present investigation. The sample was splitten into two five-years cohorts: 2450 women between 40 and 44 and 2321 women between 45 and 49. The average number of children ever born computed from the survey data is respectively 2.04 for the oldest cohort and 1.92 for the youngest one. Comparing with table 1, in which the same statistics are calculated on the same cohorts from the whole italian population data by Santini (1995), it is worth noting that these are exactly the two cohorts in correspondence of which Italy switches to the status of below-replacement fertility country. Survey data seem thus to be underestimated with regard to population data.

Table 1. Children ever born for Italian female cohorts aged 40-49 in 1988 (Santini, 1995).

Cohort	1939	1940	1941	1942	1943	1944	1945	1946	1947	1948
TFR	2.19	2.16	2.14	2.14	2.12	2.10	2.08	2.07	2.01	1.96

Some further assumptions need to be introduced for our investigation, concerning the connections between marriage and fertility on the one side, and between the end of formal education and marriage on the other.

For what concerns the connection between marriage and fertility, the already mentioned linear Henry's approximation (5.15) is used: the number of children ever born is supposed to be a linear function of the age at first marriage. This assumption, introduced for the first time by Henry (1976), was also used by Inaba (1996). Though in principle very rough, it is to be regarded essentially as a local approximation, (5.15) has the merit of being surprisingly accurate from the practical point of view, and moreover is particularly useful for its mathematical manageability. In this paper, estimates of the parameter of the regression line are calculated from individual-level data (excluding first marriages over the age of 40). The estimates of the regression line for the relation (5.17) are reported in table 2.

Table 2. Parameter estimates for the linear relationship between children ever born and age at marriage.

Parameter	*Estimate*	*Std. Error*	*P-value*
Age at the interview: 40-44			
U	+3.85720	0.01261	0.0001
V	-0.07754	0.00539	0.0001
Age at the interview: 45-49			
U	+4.25626	0.14209	0.0001
V	-0.08780	0.00590	0.0001

In order to obtain the approximated TFR, the mean age at marriage and the proportion ever married for the two cohorts were calculated. The mean age at marriage is 23.74 for the oldest cohort and 23.16 for the youngest one; the percentages ever married are given, respectively, by 94.2% and 93.6%.

The TFR calculated by means of the approximated method are respectively 2.05 and 1.94, giving thus a quite satisfactory approximation for the TFR's of the survey data.

With a reproductive behaviour such as that exhibited by the two cohorts considered, a one-year increase in the age at marriage would bring down the number of children ever born for married women by about 0.08-0.09 (parameter V). Inaba (1996) estimates with the same method, for Japanese women, a decrease of about 0.11; the decrease is obviously lessened by the presence of women never marrying, which enters as a multiplicative factor.

But the main interest of this part is to evaluate the effect of an increase in women's educational level. In order to achieve this goal, the conditional age at marriage is assumed to increase linearly with the age at the end of formal education. Formally:

$$E_{\eta}[X] = A_1 + A_2\eta \qquad (6.1)$$

By assuming (6.1) we deliberately avoided to impose the restrictions found in the previous section and simply looked for a linear relationship between age at adulthood and the mean age at marriage (in some sense remaining more close to the spirit of Henry's relationship, i.e. of an approximate but possibly useful, relationship). The same assumption, though in a different framework, that of a structural model, was formulated for instance by Marini (1985). This assumption is consistent with the model, if a delay in the end of formal education brings a delay in age at marriage. As women who attained a higher level of formal education tend to have a shorter interval between the end of formal education and marriage, regression coefficient less than 1 is expected.

As the age at the end of formal education was not asked by the interviewer, the ordinary age required for obtaining the highest educational level achieved is used as a proxy variable. Parameter estimates of the model are reported in table 3.

Table 3. Parameter estimates for the linear relationship between age at marriage and age at the end of formal education.

Parameter	*Estimate*	*Std. Error*	*P-value*
Age at the interview: 40-44			
A_1	16.64738	0.44447	0.0001
A_2	0.41732	0.02844	0.0001
Age at the interview: 45-49			
A_1	16.87574	0.55466	0.0001
A_2	0.45360	0.03635	0.0001

As expected, the A_2 parameter is always estimated to be less than the unit. The model estimates that an increase of one year in women's education would lead to an increase of about 0.4/0.5 years in the age at marriage.

Both these coefficients are lower than those obtained by Marini (1985) for a cohort of students in Illinois high schools in 1957-1958, followed up in 1973-74, who estimated a coefficient of 0.718.Clearly, her estimate for the slopes are not imediately comparable with ours, not only because of the different historical and geographical context, but also because she uses a structural model, and the parameters measures only the direct estimate (intercepts estimates as well cannot be imediately compared). For our purposes, it is anyway interesting to note that also in Marini's study the estimate of slope coefficient was significantly lower than the unity.

Referring to formula (5.19), we have that the decrease in the TFR for married women expected as a consequence of a one-year increase in the average educational level is to be estimated by the product VA_2. This is 0.03 for the oldest cohort and 0.04 for the youngest one: an increase of one year in women's education should lead to a decrease of 0.03-0.04 of the total fertility rate for married women. The decrease of the TFR is lessened by the presence of women never marrying.

It is clearly to be noticed that the empirical relation used allows marriage to take place before the end of formal education if the latter happens after around the age of 29. This should not be a problem (the approximation has all the faults all linear approximations have: Henry relationships itself predict sooner or later a negative TFR) provided that we are looking for a local approximation. Clearly if the end of formal education should take place after the age of 29, then our whole model should be questioned.

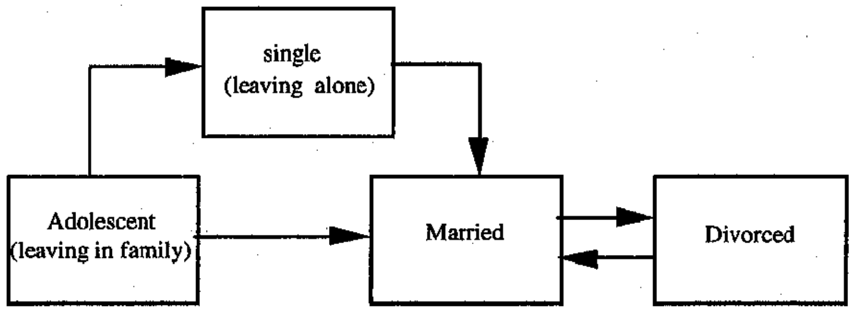
The model suggests that the fall in the Italian TFR may not be only explained by the increase of women's educational level, as the TFR of the 1962 cohort is 1.57, .39 lower than the 1948 cohort, the youngest involved in this example: in order to reach the lower lever just by postponing the end of formal education, an increase of about 10 years should be required. As the mean age at the end of formal education is 15.07 for the oldest cohort and 15.39 for the youngest one, an increase of 10 years of the length of formal education would lead to a mean age at the end of about 25, that is actually more than the ordinary age when the highest level of education is achieved.

May these considerations bring to an explanation calling for the distinction between the end of formal education and the real moment of "transition into adulthood", the application presented here is meant to be an example of the possibilities given by the model presented with real data, answering to questions related to the demographic consequences arising from changes in the process of transition into adulthood. It is the aim of the authors to further explore the field implications of a model as the one presented here, using different data, both for the Italian case and for low development countries, where the importance of studying the effect of increases in women's education on the TFR might be fundamental.

7. Directions for future work

Among the directions for future work suggested by the results of the present paper there are certainly the field investigations of the several demographic functions introduced thanks to the present age structured formulation involving the process of transition into adulhood, namely: i)the proportions ever married and the average age at marriage stratified with respect the age of entrance into adulthood, ii)the fertility rates stratified with respect the age of entrance into adulthood.

Looking more forward it seems useful to introduce other selected markers of transition into adulthood, such as the process of leaving the family, or that of entering the labour market. Furthermore it appears desirable the possibility to consider more general and realistic models for population reproduction. For instance it seems promising to study the effects of the presence of remarriage, which can become highly relevant for reproduction especially in modern societies, and of the existence of different routes to marriage. For instance in the optic of family formation processes, where individuals may or may not experiencing separate living before constituting their own family, it can be useful to assume that marriage can be the outcome of different routes: directly from the birth family, or from the intermediate state of single. The model below is a representation of this "set of innovations":



and appears a promising possible continuation of our work. From this point of view the other developments by Inaba (1993) seem represent a quite powerful tool-box for the problem at hand.

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Appendix. The stable age-stage structure implicit in the basic model (3.1)

As pointed out in the main text, in order to sort out the chronological age structure implicit in the constant rates model (3.1), we consider the enlarged model recognising age structure as well:

$$\begin{aligned}
\Delta_{a,t} p_0(a,t) &= -(\mu_i(a) + v(a)) p_0(a,t) \\
\Delta_{a,t} p_1(a,t) &= v(a) p_0(a,t) - (\mu_i(a) + \lambda(a)) p_1(a,t) \\
\Delta_{a,t} p_2(a,t) &= \lambda(a) p_1(a,t) - (\mu_i(a) + \delta(a)) p_2(a,t) \\
\Delta_{a,t} p_3(a,t) &= \delta(a)) p_0(a,t) - \mu_i(a) p_3(a,t)
\end{aligned} \qquad (1)$$

where:

$$\Delta_{a,t}=\frac{\partial}{\partial a}+\frac{\partial}{\partial t}$$

are Von Foerster aging operators, $p_i(a,t)$ are the age-time densities in the four states, and $v(a),\lambda(a),\mu(a)$ are age-dependent rates mimicking the constant rates of model (3.1). Moreover the following BC holds:

$$p_0(0,t)=B(t) \quad p_1(0,t)=p_2(0,t)=p_3(0,t)=0 \qquad (2)$$

where B(t) are the total births at time t. Finally initial age-distributions are supposed to be given. In the event there is no state-dependent mortality ($\mu_i(a)=\mu(a)$) we simply have:

$$\Delta_{a,t}n(a,t)=-\mu(a)n(a,t) \qquad (3)$$

showing that the total population satisfies a traditional Von Foerster PDE. In case of state-independent mortality, the mortality process does not "select", and the system of the population weights (or profiles) is independent on the force of mortality. Let us study the systems of the weights defined by the variables $w_i(a,t)=p_i(a,t)/n(a,t)$. We quickly have:

$$\left(\frac{\partial}{\partial a}+\frac{\partial}{\partial t}\right)p_i(a,t)=n(a,t)\left(\frac{\partial}{\partial a}+\frac{\partial}{\partial t}\right)w_i(a,t)+w_i(a,t)\left(\frac{\partial}{\partial a}+\frac{\partial}{\partial t}\right)n(a,t) \qquad (4)$$

But both n(a,t) and $p_i(a,t)$ satisfy their Von Foerster-type PDE's (3) and (1). We hence get Von Foerster type PDE's for the weights by solving the previous expression and by using (1) and (3):

$$\begin{aligned}
&\Delta_{a,t}w_0(a,t)=-v(a)w_0(a,t)\\
&\Delta_{a,t}w_1(a,t)=v(a)w_0(a,t)-\lambda(a)w_1(a,t)\\
&\Delta_{a,t}w_2(a,t)=\lambda(a)w_1(a,t)-\delta(a)w_2(a,t)\\
&w_3(a,t)=1-\left(w_0(a,t)+w_1(a,t)+w_2(a,t)\right)
\end{aligned} \qquad (5)$$

plus the boundary conditions:

$$w_0(0,t)=1 \quad w_1(0,t)=w_2(0,t)=w_3(0,t)=0 \qquad (6)$$

By eliminating time-dependencies from (5)-(6) we get the corresponding equilibrium system :

$$\frac{d}{da} w_0(a) = -v(a) w_0(a)$$
$$\frac{d}{da} w_1(a) = v(a) w_0(a) - \lambda(a) w_1(a) \qquad (7)$$
$$\frac{d}{da} w_2(a) = \lambda(a) w_1(a) - \delta(a) w_2(a)$$
$$w_3(a) = 1 - \left(w_0(a) + w_1(a) + w_2(a)\right)$$

Notice that the solution of the system (7) defines the population weights in the asymptotic stable regime of the system (1). This aspect can be checked in several manners. One intuitive manner of understanding the point is the following. If we develop the formal solutions of (1) (still under the assumption of state-independent mortality) we find that the following relations hold for t>a:

$$p_i(a,t) = n(a,t)\omega_i(a) \qquad \text{for all } i$$

where the quantities ω_i are age-dependent but not time dependent. This means that once the total population achieves its long term stable state, also the population in each state is forced to be stable. As the convergence of the total population to its stable form is a global result (whatever be the form of the initial distribution of the problem), this in turn means that the solution of the PDE system (5) of the weights, must converge in the long term to a unique time-independent form $\omega_i(a)$ .which is necessarily described by the unique solution of ODE system (7) (i.e. it holds the identification $w_i(a)=\omega_i(a)$).
In the special case of constant transition rates considered in section three of the main text, the system of the equilibrium weights has the form (again assuming that mortality is state-independent):

$$\frac{d}{da} w_0(a) = -v w_0(a)$$
$$\frac{d}{da} w_1(a) = v w_0(a) - \lambda w_1(a) \qquad (8)$$
$$\frac{d}{da} w_2(a,= \lambda w_1(a) - \delta w_2(a)$$
$$w_3(a) = 1 - \left(w_0(a) + w_1(a) + w_2(a)\right)$$

A direct resolution of the recursive system quickly leads to the formulas (3.10) of the main text:

$$
\begin{aligned}
w_0(a) &= e^{-va} = V(a) \\
w_1(a) &= \frac{v}{v-\lambda}\left(e^{-\lambda a} - e^{-va}\right) \\
w_2(a) &= \frac{v\lambda}{v-\lambda} e^{-\delta a}\left[\frac{1-e^{-(\lambda-\delta)a}}{\lambda-\delta} - \frac{1-e^{-(v-\delta)a}}{v-\delta}\right]
\end{aligned}
\qquad (9)
$$