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Populations with below replacement fertility: theoretical considerations and scenarioes from the italian laboratory

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Populations with below replacement fertility and immigration: theoretical considerations and scenarioes from the Italian laboratory*

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ABSTRACT

The need to prevent the aging and decline of populations with below replacement fertility has recently become a first concern for population policy makers. This paper discusses, both at a theoretical and practical level, the possible role played by international immigration. On the theoretical side a thorough review is provided of the so called Stable Population Model with Immigration (SPI), which represents the natural tool for the investigation of the impact of exogenous immigration on age structured populations. Several new arguments are used to clarify the working of the SPI model and of its underlying demographic mechanisms. Then the SPI model is used to derive short and long term scenarioes relative to the future Italian evolution. The present Italian situation represents a very interesting "laboratory" within which one can test the predictions of the SPI model. These scenarioes clearly show how uneffective or costly an anti-aging policy based on immigration can be in a country with high levels of "demographic malaise".

1 Introduction

In July 1996 the "Ragioneria Generale dello Stato" provided a series of medium term scenarioes of future Italian demographic evolution which made clear that a widespread trust exists, at an

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official policy level, in a "migratory solution" for the problems of aging and depopulation caused by prolonged periods of below-replacement fertility, of which the Italian case is emblematic. For the last decade Italy has been experiencing low fertility at the lowest levels in Europe (Golini 1994). If the present state continues, the long term outcome of this process will be that our age composition[1] will be of an unusual and "stressed" shape. This would be actually attained in a few decades. At that point our population would enter a phase of very quick decay (a halving time around 42 years). In addition, the immigration inflow from developing countries, which began relatively slowly at the beginning of the eighties, has become such an important reality that more and more people are seeing the need for migratory solutions.

The present Italian reality therefore represents a practical "laboratory" within which to study to what extent international immigration can represent a tool against the problems of aging and depopulation of a population with below replacement fertility.

On the theoretical side, the effects of a constant immigration flow on a stable population framework were known about long ago. Rogers (1967) and Pollard (1973) showed that Below-Replacement Fertility (BRF) combined with a constant age-structured immigration flow, leads to a stationary population. The recent explosion of the "BRF problem" has renewed the interest of demographers for this topic. This has lead to intensive investigations of the Stable Population model with Immigration (SPI). The paper by Arthur et al. (1982) provided the framework and the basic results of the stable population model with immigration in continuous time. The investigation of its properties has been continued by Mitra (1983 and 1990). Cerone (1987) and Inaba (1988) provided solid mathematical proofs of its main results (Cerone used traditional arguments whereas Inaba used modern function theoretic arguments).[2] Arthur and Espenshade (1988) used the notion of reproductive value to evaluate the impact of different age profiles of immigration on the long term age structure of a population exposed to immigration. The long term rejuvenating role of immigration on aging populations has been attacked by Schmertmann (1992). Most of the previous results are based on the long term predictions of the SPI model. For what concerns the transient dynamics of the SPI model, Espenshade (1986) has investigated via simulation the short-term properties of the model and its time to stability. The question whether immigration may solve the problem of the management of pension systems in the short-medium term, has been attacked by Blanchet (1989), who evidenced the pittfalls of short term regulation of age structure through migration.[3]

The present paper has two main goals. On the one hand we intend to provide a thorough discussion of the SPI model, by critically reviewing its main results on BRF populations, and by focusing on its counteintuitive facts and on the interpretation of the underlying demographic mechanisms. Special emphasis is given to the concept of equilibrium age structure. Several new arguments are introduced with the aim to clarify the discussion, such as i) the systematic distinction between the subpopulations of natives and foreign born, and ii) the investigation of the effects of the persistence of the demographic patterns of their country of origin, in the descendants

[1] This situation has been defined, in an evocative way, as "demographic malaise" by Golini (1994).

[2] Mitra (1983) and Cerone (1987) provided results also for stationary or growing native populations. The BRF case is nonetheless the most interesting from the conceptual point of view, and it is the relevant one for the Italian case. For this reason the present paper is essentially concerned with BRF populations.

[3] All the above mentioned papers are based on the continuous framework. Within the discrete setting we recall the theoreticaly oriented paper by Schmidbauer and Rosch (1995).

of immigrants.

On the other hand we systematically apply the SPI model to the Italian "laboratory", in order to extract all the relevant information for what concerns the complex interaction between age structure of populations with BRF and immigration. To this end a set of long and short term scenarioes of the future Italian evolution under several assumptions concerning i) the age profile of the immigration flows and ii) the fertility schedule of the immigrants, is constructed. These scenarioes will practically illustrate several counterintuitive facts, ranging from i) the rejuvenating effects of immigration (studied via the transient dynamics of the average age of the population), to ii) the absolute amount of the immigration flows needed to preserve in the long term the present population figure, to iii) our heritage of baby-boomers.

The present paper is organised as follows. In the second section the impact of immigration on BRF populations is studied by means of "pedagogical" models not explicitly recognising age structure. The fully age-structured problem is postponed to section three. The properties of stationary-through-immigration (SI) populations are discussed in section four. In the last section the model is used to generate short and long term scenarioes of the Italian population.

2 Populations with BRF: models with constant rates

2.1 A basic model

Some basic effects of the impact of immigration on a BRF population can be understood from the following one-sex model with age-independent rates:

$$\begin{aligned} \dot{P}_H(t) &= (b_H - \mu_H)P_H(t) \\ \dot{P}_I(t) &= I + (b_I - \mu_I)P_I(t) \end{aligned} \tag{1}$$

In (1) P_H and P_I are functions of the time denoting respectively the overall native ("host") population, toward which migration are directed, and the overall immigrated population. Moreover $\dot{P}_j (j = H, I)$ denotes time derivatives, I is the gross immigration flow per unit time, b_H, b_I are the crude birth rates while μ_H, μ_I denote the crude exit rates (defined as the sum of the mortality and emigration rates in the two subpopulations). All the rates are assumed constant. In model (1) the two subpopulations evolve independently, as in the basic "multipopulation" model (Keyfitz 1985). If we explicitly assume $b_H = b_I = b$, $\mu_H = \mu_I = \mu$ (equivalent to the immediate adoption by the immigrants of the demographic patterns of the natives), the total population $P = P_H + P_I$ satisfies:

$$\dot{P} = \dot{P}_H + \dot{P}_I = I + (b - \mu)P \tag{2}$$

(since now on we always suppress the time variable). Hence, if $b - \mu = -r < 0$ (the case of below-replacement fertility), the total population becomes stationary in the long run at the level I/r.[4]

[4] Viceversa the population will grow linearly when $b - \mu = 0$, or exponentially when fertility is above replacement.

The SPI model employed in the recent demographic literature considers the following alternative formulation:

$$\begin{aligned} \dot{P}_H(t) &= (b_H - \mu_H)P_H + b_I P_I = -r_H P_H + b_I P_I \\ \dot{P}_I(t) &= I - \mu_I P_I \end{aligned} \tag{3}$$

The formulation (3) differs from (1) in a substantial point: the births from immigrants ($b_I P_I$) are considered "natives" as well (provided they took place in the country of destination). This implies that the formulations (1) and (3) are by no means equivalent. Let us consider for instance the case in which the native population is BRF while the immigrated one has a positive natural growth rate. Model (1) would predict the extinction of the native population and the explosion of the immigrated population. Viceversa, in model (3) the total population will always reach a steady state independently on the actual fertility of the immigrants. In fact in (3) the foreign-born component P_I reaches, in the long term, its equilibrium level $P_I^* = I/\mu_I$. By using the steady-state of immigrants as an input for the native equation, we obtain for the native population the following "asymptotic" differential equation[5]:

$$\dot{P}_H(t) = -r_H P_H + \frac{b_I}{\mu_I} I \tag{4}$$

Hence, also the native population becomes stationary in the long run, at the equilibrium level:

$$P_H^* = \frac{b_I}{\mu_I}\frac{I}{r_H} = \frac{b_I}{r_H} P_I^* \tag{5}$$

(while in model (1) was going extinct!), and this of course holds for the total population whose equilibrium value is:

$$P^* = P_I^* \left(1 + \frac{b_I}{r_H}\right) = \left(\frac{1}{\mu_I} + \frac{b_I}{\mu_I}\frac{1}{r_H}\right) I = \left(e_I + R_0^I \frac{e_H}{1 - R_0^H}\right) I \tag{6}$$

In (6) $e_j = 1/\mu_j$ (j=I,H) denote the life expectancies at birth in the two subpopulations, and R_0^I, R_0^H are the ratios b_j/μ_j (denoting crude surrogates of the net reproduction rates). The dramatic qualitative change in passing from model (1) to model (3) has been caused by redefining as natives the births from immigrated individuals (i.e. by simply "moving" the $b_I P_I$ term from the equation of the immigrated population to the equation of the natives). The explanation of this result is that, however high the fertility of immigrants might be, it can not be "exploited" at subsequent times. In fact the children of immigrants are "natives" and will therefore, once adult, procreate with the fertility schedule of the natives, which is BRF (coeteris paribus a higher fertility of immigrants only increases the long term equilibrium of the host population).[6]

In the event the expectation of life of natives and immigrants are not very different ($e_I \cong e_H$), (6) gives:

$$P^* \cong e_H \left(1 + \frac{R_0^I}{1 - R_0^H}\right) I \tag{7}$$

[5]We are basically interested in long term results. The derivation and solution of the "transient" equation is anyway elementary.

[6]Provided the asymptotic ODE (4) holds, the conclusions of the model are unaffected when the immigrant birth rate is variable over time, ideally corresponding to different demographic behaviours of the several generations of immigrants.

2.2 Persistence of the fertility behaviour of immigrants

The assumption that the immigrants need exactly one generation to assume the demographic patterns of the natives is easily removed. Let us consider an intermediate case in which the first generation of descendants (denoted as P_D) of the immigrants preserves a distinct demographic pattern[7]. We can write down the basic model as:

$$\begin{aligned} \dot{P}_I(t) &= I - \mu_I P_I \\ \dot{P}_D(t) &= b_I P_I - \mu_D P_D \\ \dot{P}_H(t) &= b_D P_D + b_H P_H - \mu_H P_H \end{aligned} \tag{8}$$

If the natives are BRF (at least in the long term), then the overall population will become stationary following the same mechanism of model (3). The equilibrium values of the three subpopulations are (by the same reasoning of the previous section):

$$P_I^{**} = \frac{I}{\mu_I} \; ; \; P_D^{**} = \frac{b_I}{\mu_D} P_I^{**} \; ; \; P_H^{**} = \frac{b_D}{r_H} P_D^{**}$$

and:

$$P^{**} = P_I^{**} + P_D^{**} + P_H^{**} = \left(e_I + R_0^I e_D + R_0^I R_0^D \frac{e_H}{1 - R_0^H} \right) I$$

Hence, if $e_I \cong e_D \cong e_H$:

$$P^{**} \cong e_H \left(1 + R_0^I + R_0^D \frac{R_0^I}{1 - R_0^H} \right) I \tag{9}$$

By comparing (9) with (6):

$$P^{**} - P^{*} \cong e_H I \left(\frac{R_0^I}{1 - R_0^H} \left(R_0^D - 1 \right) + R_0^I \right) \tag{10}$$

In words: provided that $R_0^D > 1$, the persistence of heterogeneous fertility behaviour of immigrants, their descendants, and the natives, causes a "multiplier" effect on the long term equilibrium population. This suggests a need for a simple measure against depopulation: to defend this persistence, by preserving to some extent the fertility behaviour of the immigrated.

3 The age structured model

The present section is divided into several subsections. In the first one we formulate the SPI model by resorting to the Von Foerster formalism of the aging process. We then introduce its remarkable result for BRF populations: all BRF populations subjected to a constant migration stream become stationary in the long term. A detailed characterisation of the equilibrium state is provided. The relation with traditional formulations in terms of the Lotka's births equation is then reported. Finally we consider the case in which the fertility of the descendants of the immigrants takes several generations to "stabilize".

[7] There is of course no difficulty in treating the more general case in which the descendants of immigrants need several generations to stabilize their patterns.

3.1 Formulation of the SPI model

The SPI model developed in the recent demographical literature is, along traditional mathematical demography, a one-sex model which investigates the long term effects of a constant age structured immigration inflow on a population subjected to constant vital rates. A crucial assumption is that the births of the immigrants behave as natives. This suggests, as was done in the previous section, to split the overall population into two components: the immigrated one, composed by the survivors of immigrants, and the native ("host") one, composed by the autochtones plus the children and further descendants of the immigrants. The SPI model may therefore be represented via the following pair of McKendrick-Von Foerster PDE's:

$$\begin{aligned} \Delta_{a,t} n_H(a,t) &= -\mu_H(a) n_H(a,t) \\ \Delta_{a,t} n_I(a,t) &= I(a) - \mu_I(a) n_I(a,t) \end{aligned} \tag{11}$$

plus the boundary conditions:

$$\begin{aligned} n_H(0,t) &= B_H(t) + B_I(t) = B(t) \\ n_I(0,t) &= 0 \end{aligned} \tag{12}$$

The equations (11) ($\Delta_{a,t} n_i(a,t)$ is a shortcut for: $\partial n_i(a,t)/\partial a + \partial n_i(a,t)/\partial t$, $i = H, I$) describe the aging process within the two subpopulations. The $n_H(a,t), n_I(a,t)$ functions represent the absolute age densities at age a and time t in the two subpopulations; $\mu_H(a), \mu_I(a)$ are the age dependent mortality schedules (we assume absence of emigration); $I(a)$ is the age-dependent immigration inflow (typically assumed time invariant). Moreover, the functions $B_H(t)$, $B_I(t)$ denote the number of births per unit time produced by the (female) individuals of the two subpopulations. The boundary conditions (12) therefore simply say that all births are natives, irrespective of the origin of their parents. In particular:

$$B_H(t) = \int_0^\infty n_H(a,t) m_H(a) da \ ; \ B_I(t) = \int_0^\infty n_I(a,t) m_I(a) da$$

where $m_H(a), m_I(a)$ denote the age specific birth rates.[8] According with the recent literature (Arthur et. 1982, Arthur and Espenshade 1988, Schmertmann 1992) let us assume that the native population be BRF:

$$R_{0,H} = \int_0^\infty m_H(a) p_H(a) da < 1$$

where $R_{0,H}$ is the net reproduction rate (NRR) of the native population, and $p_H(a)$ is the survival function of the natives: $p_H(a) = \exp\left\{-\int_0^a \mu_H(u) du\right\}$. If we further assume identical vital rates in the two subpopulations ($\mu_H(a) = \mu_I(a) = \mu(a)$; $m_H(a) = m_I(a) = m(a)$), we can add both (11) and (12) obtaining:

$$\Delta_{a,t} n(a,t) = I(a) - \mu(a) n(a,t) \ ; \ n(0,t) = \int_0^\infty m(a) n(a,t) da \tag{13}$$

[8] To formally close the problem, we have to add to (11)-(12) initial conditions of the type:

$$n_H(a,0) = K_H(a); \ n_I(a,0) = K_I(a);$$

where $K_H(a), K_I(a)$ are arbitrarily prescribed functions of age.

where $n(a,t)$ is the total population. The simplified formulation (13), proposed in a classical paper by Langhaar (1972), is the one used in the recent demographic debate. Its formal properties can be investigated by reducing it to an integral equation for the births, as in traditional stable population theory, as we will briefly see later (this has been done by Arthur et al. 1982 (only for the BRF case), Mitra (1983), Cerone (1987), Inaba (1988)[9]). But for the understanding of the long term properties of the model under the BRF assumption, we believe it is of much help the subsequent equilibrium analysis of the general model (11)-(12).

3.2 Equilibrium analysis of the SPI model for BRF populations: stationary-through-immigrations populations

As noticed in the previous section, a BRF population exposed to a stationary immigration inflow becomes stationary in the long term, whatever the age structure of the entries be. This notion of stationarity generalizes the traditional textbook notion of stationary population, which is based on the coupling of a constant number of entries at age zero with a constant mortality schedule.

An important question is: which are the dynamical mechanisms leading to stationarity? To answer let us consider a large native population which is stably decaying due to BRF, and which, starting from a given instant of time, experiences a stationary immigration inflow. Neglecting the disturbances due to age structure (which may be responsible of transient oscillatory behaviours), we understand that[10] the overall population will continue its decay, due to the excess of deaths over births, until the gap between deaths and births is exactly filled by the immigratory input. This leads to the stationarity of both the native and immigrated populations. The argument is the following. The immigrant population reaches in the long term a stationary age distribution. This is the consequence of the fact that the number of entries is constant (and of the fact that the mortality of immigrants does not vary over time). This stationary age distribution will in the long term provide a constant input of extra-births to the native population. If the native population is BRF (at least in the long term), it will become stationary as well, independently from the vital rates of the immigrating population, since at some stage the extra-births by immigrants will be able to exactly fill the gap between births and deaths of the natives. The fact that stationarity will eventually emerge whatever the fertility of immigrants be, is again the consequence of the fact that the children of immigrants are natives: the (possibly) higher fertility of the fathers is not transmitted to their children.

The equilibrium analysis of the SPI model enlights the features of the stationary-through-immigrations (SI) populations. It is easy to check that the SPI model (11)-(12) has one and only one equilibrium age structure that can be calculated by solving the following ordinary differential equations "equilibrium" system[11]:

---

[9] Inaba's analysis concerns the full multiregional/multistate model.

[10] We obviously rule out by assumption the case of a "huge" immigration stream capable of suddenly stopping the decay.

[11] The system (14) is obtained from (11) by ruling out time-dependencies. Hence, by $n_H(a)$ we denote the equilibrium age structure of the natives.

$$
\begin{aligned}
\frac{dn_H(a)}{da} &= -\mu_H(a)n_H(a) \\
\frac{dn_I(a)}{da} &= I(a) - \mu_I(a)n_I(a)
\end{aligned}
\tag{14}
$$

which inherits the initial conditions:

$$
\begin{aligned}
n_H(0) &= B_H + B_I = B \\
n_I(0) &= 0
\end{aligned}
\tag{15}
$$

In (15) B_H and B_I denote the equilibrium levels of the births in the two subpopulations, given by:

$$
B_H = \int_0^{\infty} n_H(a)m_H(a)da \ ; \ B_I = \int_0^{\infty} n_I(a)m_I(a)da \tag{16}
$$

From the second of (14) we get the (equilibrium) age structure of the foreign-born:

$$
n_I(a) = n_I(0)p_I(a) + \int_0^a I(x)\frac{p_I(a)}{p_I(x)}dx = \int_0^a I(x)\frac{p_I(a)}{p_I(x)}dx \tag{17}
$$

where $p_I(a)$ is the survival function of the immigrant population and we used the assumption $n_I(0) = 0$. The native population is given by: $n_H(a) = n_H(0)p_H(a) = Bp_H(a)$, so that the total population aged a is:

$$
n(a) = n_H(a) + n_I(a) = Bp_H(a) + \int_0^a I(x)\frac{p_I(a)}{p_I(x)}dx \tag{18}
$$

By substituting (18) into the initial condition (15) we get the total births at equilibrium:

$$
B = B_H + B_I = \int_0^{\infty} m_H(a)Bp_H(a)da + B_I = BR_{0,H} + B_I \tag{19}
$$

and hence:

$$
B = \frac{B_I}{1 - R_{0,H}} \tag{20}
$$

which is always meaningful as $R_{0,H} < 1$ by assumption. The last expression shows that the equilibrium value of births depends on the number of births from the immigrants, and on the NRR of the native population. In particular:

$$
B_I = \int_0^{\infty} n_I(a)m_I(a)da = \int_0^{\infty} \left[\int_0^a I(x)\frac{p_I(a)}{p_I(x)}dx\right] m_I(a)da \tag{21}
$$

As in all stable population-type models, the birth relation (20) is the central one, as it permits to derive all the relevant equilibrium relations in our population. In particular the total population aged a at equilibrium will be:

$$
n(a) = \frac{B_I}{1 - R_{0,H}}p_H(a) + \int_0^a I(x)\frac{p_I(a)}{p_I(x)}dx \tag{22}
$$

while the equilibrium size of the overall population will be[12]:

$$P = \int_0^\infty n(a)da = \frac{B_I}{1 - R_{0,H}} e_{0,H} + P_I \tag{23}$$

where $e_{0,H}$ denotes the expectation at birth of the natives, and:

$$P_I = \int_0^\infty n_I(a)da = \int_0^\infty \left(\int_0^a I(x) \frac{p_I(a)}{p_I(x)} dx \right) da \tag{24}$$

is the total foreign-born population.

By writing $I(a) = I \cdot i(a)$ where I is the total number of entries and $i(a)$ the corresponding age profile ($\int_0^w i(a)da = 1$), P_I and B_I may be written as (Arthur and Espenshade (1988)):

$$P_I = I \int_0^\infty i(x) \left(\int_x^\infty \frac{p_I(a)}{p_I(x)} da \right) dx = I \int_0^\infty i(x) e_I(x) dx = I\, \bar{e}_I \tag{25}$$

$$\begin{aligned} B_I &= \int_0^\infty n_I(a) m_I(a) da = \int_0^\infty I(x) \int_x^\infty \frac{p_I(a)}{p_I(x)} m_I(a) dx da = \\ &= I \int_0^\infty i(x) V_I(x) dx = I\, \bar{V}_I \end{aligned} \tag{26}$$

where:

$$V_I(x) = \int_x^\infty \frac{p_I(a)}{p_I(x)} m_I(a) da$$

is the reproductive value of an immigrated woman aged a in "her" stationary population and:

$$e_I(x) = \int_x^\infty \frac{p_I(a)}{p_I(x)} da$$

is the expectation of life at age x in the immigrated population. The quantities $\bar{V}_I$ and $\bar{e}_I$ respectively denote the average content of fertility and survival "potential" embodied in the immigration profile. We hence get for (23):

$$P = \left(\bar{e}_I + \frac{\bar{V}_I\, e_0}{1 - R_0^H} \right) I \tag{27}$$

which is the age-structured extension of (7).

Several features of the equilibrium age structure of a stationary-through-immigrations population will be discussed in the next section.

[12]Contrary to the second section we suppressed * in the definition of the overall equilibrium population P.

3.3 Stability of the equilibrium age structure: the integral equation for births

From the mathematical point of view a central question is whether the equilibrium age structure is also a stable one, i.e. if it is observable as the long term outcome of a population subjected to the given evolution rules. The answer to this question is yes, i.e. the equilibrium age structure is globally (i.e. whatever be the initial age structures) stable. This fact can be proven by converting the basic formulation (11)-(12) into a Volterra integral equation for the births, as in the traditional Lotka's model (Arthur (1982), Cerone (1987), Inaba (1988)[13]). In this subsection we show how the conversion to the birth integral equation is performed, and briefly summarise the main mathematical result.

Let us assume for simplicity that natives and immigrants have identical vital rates[14], i.e.: $p_H(a) = p_I(a) = p(a)$; $m_H(a) = m_I(a) = m(a)$. By formally integrating the VonFoerster PDE's (11) we get:

$$n_H(a,t) = \begin{array}{ll} n_H(a-t;0)\frac{p(a)}{p(a-t)} & t \leq a \\ n_H(0;t-a)p(a) = B(t-a)p(a) & t > a \end{array} \tag{28}$$

and:

$$n_I(a,t) = \begin{array}{ll} n_I(a-t;0)\frac{p(a)}{p(a-t)} + \int_0^a I(x)\frac{p(a)}{p(x)}dx & t \leq a \\ \int_0^a I(x)\frac{p(a)}{p(x)}dx & t > a \end{array} \tag{29}$$

By writing:

$$\int_0^a I(x)\frac{p(a)}{p(x)}dx = p(a)\int_0^a \frac{I(x)}{p(x)}dx = p(a)S(a) \tag{30}$$

we get:

$$n_I(a,t) = \begin{array}{ll} n_I(a-t;0)\frac{p(a)}{p(a-t)} + p(a)S(a) & t \leq a \\ p(a)S(a) & t > a \end{array} \tag{31}$$

If the initial contingent $n_I(a-t;0)$ of foreign born population derives from individuals immigrated before time zero, but following the same migration schedule $I(a)$, we have:

$$n_I(a-t;0) = \left(\int_0^{a-t} I(x)\frac{p(a-t)}{p(x)}dx\right) = p(a-t)S(a-t) \tag{32}$$

From the BC (12) we obtain, by splitting the domain of integration into $(0,t)$ and $(t,+\infty)$ and using (30):

$$B(t) = \int_0^t \left(B(t-a)p(a) + p(a)S(a)\right)m(a)da + G(t) \tag{33}$$

where:

$$G(t) = \int_t^{+\infty}\left[K_H(a-t)\frac{p(a)}{p(a-t)} + (p(a)S(a-t) + p(a)S(a))\right]m(a)da$$

Hence:

$$B(t) = \int_0^t B(t-a)p(a)m(a)da + b_I(t) + G(t) \tag{34}$$

[13]Inaba (1988) has showed, via a semigroup approach, that most of the results valid for the basic single-population model still hold in the general case of a multiregional-multistate model à la Rogers (Rogers 1975) forced by an "international immigration" term.

[14]There are no difficulties in extending to the general case.

where:

$$b_I(t) = \int_0^t S(a)p(a)m(a)da$$

which is the desired (Volterra) integral equation for births.

The first term in (34) is the contribution to births by natives born since time zero; the second one is the contribution to births by immigrated individuals born since time zero; the G function is the contribution of both immigrants and natives born before zero. When $t \geq \beta$ (where β is the upper limit of the reproductive ages) the G function vanishes (as in classical stable population theory), while the term: $b_I(t)$ becomes constant.[15] Hence the long term equation for births has the form:

$$B(t) = \int_0^\beta B(t-a)p(a)m(a)da + b_I \tag{35}$$

which is the one studied in Arthur et al. (1982). The equation (34) defines the corrected renewal problem of the SPI model (while (35) defines the corresponding long term description), which can be analysed via Laplace transforms. In this way Cerone (1987) showed that, given the immigration inflow and the vital rates of the two subpopulations, whatever be the initial distributions, the long term births in the overall population is always given by: $B_\infty = B_I/(1 - R_{0,H})$ which is the equilibrium level of births found in the previous discussion (see (20)). This means that the unique equilibrium solution of the model is globally (asymptotically) stable.

3.4 The model with generational structure

What happens if the immigrants need several generations to stabilize their demographic patterns? Our equilibrium analysis permits to deepen the analysis (begun in the second section) of the long term effects of the distinct generations of immigrants. Let us consider the age-structured extension of system (8)[16]:

$$\begin{array}{l} \Delta_{a,t} n_I(a,t) = I(a) - \mu_I(a) n_I(a,t) \\ \Delta_{a,t} n_D(a,t) = -\mu_D(a) n_D(a,t) \\ \Delta_{a,t} n_H(a,t) = -\mu_H(a) n_H(a,t) \end{array} \tag{36}$$

where $n_D(a,t)$ is the age-time density of the first generation of descendants. The related BC are given by:

$$\begin{aligned} n_I(0,t) &= 0 \ ; \ n_D(0,t) = B_I(t) = \int_0^\infty m_I(a) n_I(a,t) da \\ n_H(0,t) &= B_H(t) + B_D(t) = \int_0^\infty m_H(a) n_H(a,t) da + \int_0^\infty m_D(a) n_D(a,t) da \end{aligned}$$

[15] Even if the immigration input is actually constant at all times, the $b_I(t)$ function is time dependent for definition.
[16] There are no difficulties in considering n distinct generations of descendants.

The corresponding steady-state system is given by:

$$\begin{array}{l} \frac{dn_I(a)}{da} = I(a) - \mu_I(a) n_I(a) \\ \frac{dn_D(a)}{da} = -\mu_D(a) n_D(a) \\ \frac{dn_H(a)}{da} = -\mu_H(a) n_H(a) \end{array} \tag{37}$$

with the initial conditions:

$$\begin{aligned} n_I(0) &= 0 \\ n_D(0) &= B_I = \int_0^\infty m_I(a) n_I(a) da \\ n_H(0) &= B_H + B_D = \int_0^\infty m_H(a) n_H(a) da + \int_0^\infty m_D(a) n_D(a) da \end{aligned} \tag{38}$$

From (37) and (38) we find:

$$\begin{array}{l} n_I(a) = \int_0^a I(x) \frac{p_I(a)}{p_I(x)} dx \\ n_D(a) = n_D(0) p_D(a) = B_I p_D(a) \\ n_H(a) = n_H(0) p_H(a) = (B_H + B_D)\, p_H(a) \end{array} \tag{39}$$

By introducing (39) into the initial conditions (38) we find:

$$n_H(0) = B_H + B_D = n_H(0) R_0^H + n_D(0) R_0^D \tag{40}$$

leading to:

$$n_H(0) = \frac{n_D(0) R_0^D}{1 - R_0^H} = \frac{R_0^D}{1 - R_0^H} B_I \tag{41}$$

The total number of births at equilibrium is therefore:

$$\begin{aligned} n(0) &= B = n_H(0) + n_D(0) = B_I + (B_H + B_D) = \\ &= B_I + \frac{R_0^D}{1 - R_0^H} B_I = \left(1 + \frac{R_0^D}{1 - R_0^H}\right) B_I \end{aligned} \tag{42}$$

The total population aged a at equilibrium is:

$$\begin{aligned} n(a) &= n_I(a) + B_I p_D(a) + n_H(0) p_H(a) = \\ &= n_I(a) + \left(p_D(a) + \frac{R_0^D}{1 - R_0^H} p_H(a)\right) B_I \end{aligned}$$

so that the total population is:

$$P = P_I + \left(e_{0,D} + \frac{R_0^D}{1 - R_0^H} e_{0,H}\right) B_I \tag{43}$$

In the special case $e_{0,D} = e_{0,H} = e_0$, and by using (25) and (26) we get:

$$P = \left(\bar{e}_I + \frac{\bar{V}_I\, e_0}{1 - R_0^H} \left(1 + R_0^D - R_0^H\right)\right) I \tag{44}$$

which is the age-structured extension of (9). The present approach hence shows that the formulas (25)-(26) can straightforwardly be extended to the general case in which the descendants need n generations to stabilize, as, in any case, what really matters is the demographic potential embedded in the age profile of immigrants (the only true forcing term of the model), plus the NRR of the several generations of immigrants.

4 Distinctive features of the stationary-through-immigrations populations

The equilibrium relations derived in the previous section permit the detailed characterisation of the long term properties of the stationary through immigration (SI) population. The present discussion critically reviews results from Arthur et al. (1982), Arthur and Espenshade (1988), Mitra (1990), and Schmertmann (1992).

Let us assume that natives and immigrants have the same survival function $p_H(a) = p_I(a) = p(a)$, implying: $e_0 = e_{0,H} = e_{0,I}$. Following Schmertmann (1992) we assume moreover that the age specific fertility schedule of the immigrants differs from that of the natives only by a proportionality factor s (s>0)[17]: $m_I(a) = s \cdot m_H(a)$ (implying $R_{0,I} = s \cdot R_{0,H}$).

4.1 The long-term in-migrated component

The long term number of immigrants aged a (given by (17)), can be written as:

$$n_I(a) = Ip(a) \int_0^a \frac{i(x)}{p(x)} dx = Ip(a)Z(a) \tag{45}$$

where $Z(a) = \int_0^a \frac{i(x)}{p(x)} dx$ (and $S(a) = IZ(a)$) . The total size of the in-migrated component is then:

$$P_I = I \int_0^\infty p(a)Z(a)da \tag{46}$$

At equilibrium, as the size of the immigrant population (P_I) is constant, then the annual number of deaths among the immigrants (D_I) is necessarily equal to the annual number I of foreign-born entering the host country (remember that the children of immigrants are natives). Formally:

$$\begin{aligned} D_I &= \int_0^\infty n_I(a)\mu(a)da = -I \int_0^\infty p'(a)Z(a)da = \\ &= -I\left[p(a)Z(a)\right]_0^\infty + I \int_0^\infty p(a)\frac{i(a)}{p(a)}da = I \end{aligned}$$

[17] Our working assumption is $s > 1$. The assumption $s \leqslant 1$ does not affect the main features of the model.

having used (45) and $\mu(a) = -p'(a)/p(a)$. This remark is the basis of the definition of generalised stationarity (Schmertmann 1992).

The shape of the age profile $\pi_I(a)$ of the long term immigrated population follows from (45)-(46):

$$\pi_I(a) = p(a)Z(a)/\int_0^\infty p(a)Z(a)da \tag{47}$$

and depends on the age profile of the entries $i(a)$. In many reasonable cases we may expect that the younger $i(a)$ is, the younger $\pi_I(a)$ will be; but this is not always true. Mitra (1990) noticed that $\pi_I(a)$ must have at least one mode because it has zero values at both the extremes of the age range [18]. Of course it can have more than one mode, depending essentially on the structure of the modes of the age profile of entries. Obviously neither the size of annual entries nor the fertility of the immigrants affect the shape of $\pi_I(a)$.

4.2 The native component

By writing:

$$B_I = I\int_0^\infty p(a)\left[\int_0^a \frac{i(x)}{p(x)}dx\right] m_I(a)da = I\int_0^\infty p(a)Z(a)m_I(a)da = Isb_1$$

where:

$$b_1 = \int_0^\infty p(a)Z(a)m_H(a)da$$

the number of natives aged a and the total number of natives can be written respectively as:

$$n_H(a) = I\frac{s\cdot b_1}{1-R_{0,H}}p(a) \;\; ; \;\; P_H = I\frac{s\cdot b_1}{1-R_{0,H}}e_0 \tag{48}$$

The stationarity of the natives, who have a net reproduction ratio less than one, comes about via the exact counterbalancing of the excess deaths over births among natives permitted by the births to immigrants. Formally:

$$D_H = \int_0^\infty n_H(a)\mu(a)da = \int_0^\infty n_H(0)p(a)\mu(a)da = B\int_0^\infty p(a)\mu(a)da = B$$

Hence, the fact that births from immigrants are natives makes the whole native population stationary in the same way as the usual textbook stationary population. For this reason the age structure of the natives $\pi_H(a)$ is proportional to their survival function. This may be formally seen by dividing $n_H(a)$ by P_H (see (48)), obtaining the well-known relation: $\pi_H(a) = p(a)/e_0$.

[18]From equation (47) we have that $\pi_I(0) = 0$ as $Z(0) = 0$ and $\pi_I(\infty) = 0$ as $p(\infty) = 0$

Hence, changes in the fertility of the immigrants (i.e. changes in s) affect the size of the native population, by changing the equilibrium level of B_I and therefore of B, but do not affect the age structure of the natives (this is just another way to say that the annual number of births is only a scale factor for the (textbook) stationary population).

Finally notice that the age structure of the immigrants, given in equation (47), is always older than the age structure of the natives, whatever the age structure of entries be (see the appendix for a proof).

4.3 The total population

By adding the two components, native and immigrated, we get:

$$n(a) = I\left(p(a)\frac{sb_1}{1-R_{0,H}} + p(a)Z(a)\right) \quad (49)$$

$$P = I\left(e_0\frac{sb_1}{1-R_{0,H}} + \int_0^w p(a)Z(a)da\right) \quad (50)$$

The long-term age structure of the total population $\pi_T(a)$ can be calculated by dividing equation (49) by (50). It is, as expected, the weighted mean of the quantities $\pi_I(a)$ and $\pi_H(a)$, the weights being given by the proportions of the native and immigrated components on the total population:

$$\pi_T(a) = \pi_H(a)\frac{P_H}{P_H + P_I} + \pi_I(a)\frac{P_I}{P_H + P_I} \quad (51)$$

This last fact clarifies that the age structure of the total population must always fall between the age structure of the foreign-born and that of the natives (Schmertmann, 1992). Useful information hence follows from the aggregate ratio:

$$\frac{P_H}{P_I} = \frac{se_0b_1}{(1-R_{0,H})\int_0^\infty p(a)Z(a)da} \quad (52)$$

From (52) we see that the overall ratio between natives and the immigrated (and hence the weight of the natives) increases as the fertility of the immigrants (as tuned by s) increases, while it is not affected by changes in the yearly number of entries (I). Both these facts are critical features of the model. To justify the former notice that any increase of the age specific fertility rates of the immigrants has the only effect of increasing the total number of births of immigrants, and hence the native population (children born to immigrants are "natives"). Notice moreover that any increase in the total number of immigrants I increases the number of births of immigrants, thereby increasing the number of natives in the same proportions.

It is to be noticed finally that, when considered at all ages, the ratio between natives and foreign-born is not constant, but rather tends to decline with age[19] (Schmertmann, 1992).

[19]From (48) and (46) we have $\frac{n_H(a)}{n_I(a)} = \frac{sB_1}{(1-R_{0,H})Z(a)}$, which is a nonincreasing function of age as $\frac{dZ(a)}{da} \geq 0$.

4.4 Impact of the age profile of immigration on the long term size: deeping our understanding of the complex metabolism of age structures

A remarkable problem is the evaluation of the impact of distinct age profiles of immigration on the long term equilibrium size of the population. The formula (27):

$$P = \left(\bar{e}_I + \frac{\bar{V}_I \, e_0}{1 - R_0^H} \right) I \tag{53}$$

where: $\bar{V}_I = \int_0^\omega i(x) \cdot V_I(x)dx$, $\bar{e}_I = \int_0^\infty i(x)e_I(x)dx$, answers the question. In particular if we assume, following Arthur and Espenshade (1988), that the migration flow is concentrated at only one age a_0, (ie: $I(a_0) = I$ immigrants aged exactly a_0) we can approximate (53) as:

$$P = I \left(\frac{e_0}{1 - R_0} V_I(a_0) + e(a_0) \right) \tag{54}$$

(the extension to the more general case with persistence of the fertility of the immigrants in subsequent generations is immediate). The (54) traces the long term impact of a highly concentrated immigration input and witnesses of the nonlinear relation existing between the long term size of the total population and the age of admission of immigrants.

The fig. 1 illustrates this nice nonlinear relation for the present italian demographic state, both under the standard assumption of section 3.2 (children of immigrants subsume the fertility of the natives, fig. 1a) and under the assumption of section 3.4 (the childeen of immigrants preserve the fertility of their fathers, fig. 1b). More in detail, fig. 1a reports the long-term size of the total italian female population which would be attained if the present fertility of the natives ($R_{0,H} = 0.62$) would be mantained in the future, under three different assumptions on the fertility of the immigrants, i.e. that i)the fertility of immigrants is identical to the fertility of the natives ($s = 1$), ii)the fertility of immigrants is twice the fertility of the natives ($s = 2$), iii)the fertility of immigrants is three times the fertility of the natives ($s = 3$). Fig. 1b shows the long term size that would follow when the fertility of the immigrants is twice the fertility of the natives, but the children of the immigrants preserve a distinct fertility pattern. Comparing fig. 1a with fig. 1b we see that if the individuals of the first generation of descendants of immigrants could preserve the fertility of their parents, a dramatic increase (about 50%) of the long term size would follow. This illustrates our previous findings on the "virtuous" effects of the persistence of the fertility of immigrants (at least when they have "high" fertility) beyond the first generation.

Fig. 1. Long term final size of the 1991 italian population ($R_0 = 0.62$) corresponding to different assumptions on the age of admission of the immigrants (total hypothetical flow $I =$ 25.000; $s = 1, 2, 3$). a)The children of immigrants assume the fertility of the natives. b)The children of immigrants preserve the fertility of their parents (only their children become natives)

4.5 Rejuvenating role of immigrations on the long term age structure

It is of interest to investigate the sensitivity of the long term age structure predicted by the SPI model, under changes in three specific parameters: i)the ratio between the fertility schedules of natives and immigrants (s), ii) the annual number of entries(I) and iii) the age profile of the entries, $i(a)$. Precise display of these effects would be of much help in clarifying the Italian political debate, presently divided.

The effects of the first two factors are easily evaluated. Let us consider the effects of an increase in s (all other things being equal). The total population increases because of the increase of the native population. As a result the long term age structure of the total population will be closer to the age structure of the natives. This is a true rejuvenation effect, as the age profile of the natives is younger than that of the immigrants.

On the other hand an increase in the annual number of entries increases the long term size of both the immigrated and the native populations by the same percentage. As there are no changes in the long term age structure we have no rejuvenation effects.

The evaluation of the impact of two different age profiles of immigration on the overall age structure is, vice-versa, a hard task, as the age profile of immigration affects all the terms in the mean (51). The answer thus seems strictly dependent on how the differences between the two profiles of entries are distributed over ages. Hence, although heuristically a younger age profile of entries causes a younger age profile of the total population, full understanding of the impact of different profiles of entries on the overall age structure in concrete situations, makes it unavoidable to resort to simulation. In fact, even in the simplest case, i.e. the case of dominance (the first age profile is younger than the second, i.e. it dominates it) we have not been able to find out clear-cut results.

Note, however, that there is at least one remarkable case in which the effect on the overall (long term) age structure is not ambiguous. This happens when the differences between two profiles occur after the end of the reproductive period (age β). In this case we have a counterintuitive aging effect. Let us consider two profiles $i(a)$ and $i^*(a)$, identical at all ages before the end of the reproductive period, the first of which is younger (i.e. "dominant"). From formula (45) we see that the younger profile $i(a)$ will cause a larger foreign born population at equilibrium, characterised by, compared to $i^*(a)$, the same number of individuals at young ages, but a larger number of individuals at the ages beyond β. Hence the immigrated population becomes, under this younger profile, older and larger (while nothing happens to the native one). This implies in turn that the overall (natives and foreign born) population becomes older, thanks to (51).

5 Long-term and short-term scenarioes of the italian population

The present state of Italian fertility, one of the lowest in the world since the beginning of the nineties, is progressively leading to a "stressed" shape of the age structure of our population. This unusual state of affairs (called "demographic malaise" by Golini (1994)), makes it the present italian situation as the practical "laboratory" in which to study the complex underpinning between age structure and immigration. In this section the SPI model is used to build short and long term scenarioes of the future italian population under several assumptions concerning the age structure of immigrants and their fertility. Most of the results presented here rely on the standard SPI assumptions of section 3.2 (children of foreign born assume the fertility of natives) but there are no difficulties in considering more complex patterns of fertility by the descendants of immigrants, such as those of section 3.4.

Our simulations are based on the italian female population at 1991 (Istat 1993). Its total size was 29.220.000, with an observed NRR ($R_{0,H}$) of 0.621 (Lotka's $r = -0.0166$), and a life expectancy at birth $e_{0,H} = 80.85$ (Istat (1995)). The average age ($\bar{a}$) was of about 40.5 years, and the aging index (AI) given by the ratio between the population aged 65 and over, and the population below 20 years, was $AI = 0.74$.

If the present state should be mantained in the long term in absence of immigration, our population would experience a somewhat fast aging process, partially slowed, at the beginning, by our historical heritage of a young population due to the baby-boom in the sixties. This would lead to a stable long term age structure in about 80-100 years[20]. At 2091, the total size of our female population would be of about 9 million (9.012.700). Once achieved its stable age structure our population would start decay with a halving-time of about 42 years. The mean age would increase to more than 51 years around 2060 and remain constant thereafter due to the stability of the age structure, while the AI would increase to the dramatic level of about 2.5. Figure 2 reports the present and the long term age-structures of our (female) population under the assumption of absence of immigration.

Fig. 2. Present and long-term age structures of the italian

(female) population in absence of immigrations. The initial population is that observed at 21/10/1991; the vital rates are those of 1991.

To what extent can international immigration affect the "demographic malaise", i.e. how would our scenarioes change under the action of immigration? To answer these questions within the framework of the SPI model we have to make some specific assumptions about: a) the yearly number and age structure of entries; b) the vital rates of immigrants.

For what concerns the fertility rates of the immigrants, we follow Schmertmann (1992) by making three basic (and purely indicative) assumptions in terms of their net reproductive rate: $R_{0,I} = s \cdot R_{0,H}$ with $s = 1, 2, 3$ (corresponding to a "lower", an "intermediate", and an "extreme" fertility scenario, as done in the previous section). Moreover we assume that immigrants and natives have the same survival function. For what concerns the yearly number of entries we assumed $I = 25.000$, i.e. a number of female entries equal to about a half of the average number

[20]Such a long time to stability is actually due to the fact that the present age structure (at 1991) is quite far from its long term age structure.

of net entries observed in the last decade (an assumption largely employed in other projection works, for instance Golini (1994, 1995), Maffioli (1996), ISTAT (1997)). Finally we made four basic assumptions on the age structure of entries, denoted as $A1, A2, A3, A4$. These assumptions (summarised[21] in table 1) correspond to a sufficiently general typology of the possible immigration profiles. The first one ($A1$) corresponds to a very young age profile, with a negative exponential shape, in which 67.5% of the entries are represented by individuals below 15 years. The remaining profiles ($A2$, $A3$, $A4$) have the typical double exponential shape, and differ essentially for what concerns their modal age, which is given by 15 years under $A2$, 30 years under $A3$, and 45 under $A4$. We point out that the assumptions $A1$ and $A4$ are surely unrealistic: they are essentially "boundary assumptions".

Table 1. A simplified representation of the four assumptions $A1, A2, A3, A4$ on the age profile of immigrants (percentages).

AGE	A1	A2	A3	A4
0-15	67,5	40,5	10,2	10,2
15-30	21,2	43,8	33,5	3,2
30-45	7,4	3,9	43,8	33,5
45-60	2,6	3,2	3,9	43,8
60 and more	1,3	8,6	8,6	9,3
	——	——	——	——
Total	100	100	100	100

By combining the four assumptions on the age structure of entries with the three assumptions on the fertility rates of the immigrants, we have in total twelve different "scenario assumptions", denoted in the sequel as Ai, j (for instance $A1, 3$ denotes the first assumption on the age profile of entries combined with the third assumption on immigrants fertility). Let us now examine in detail what type of scenarioes arise from the given assumptions.

5.1 Long term scenarioes

Under all the scenarioes considered the "time to stationarity" is always very long (close to 400-500 years), hence much longer than the times to stability of the traditional stable model for closed populations (the time-scales of which usually constitute the basis for the demographer's definitions of short and long term). Table 2 reports the "final size" (i.e. the steady state levels) of the italian population under the several assumptions considered.

Table 2. Long term total size of the italian population under

the 12 assumptions

[21] These assumptions were made by starting from a theoretical immigration curve based on a multiple-exponential parametric model (Rogers and Castro (1981)).

	A1	A2	A3	A4
$R_{0,H} = 0,62\,(s=1)$	4.106.000	4.194.000	2.359.000	1.252.000
$R_{0,H} = 1,24\,(s=2)$	6.626.000	6.895.000	3.502.000	1.556.000
$R_{0,H} = 1,86\,(s=3)$	9.146.000	9.597.000	4.645.000	1.861.000

The dramatic differences in the long term sizes in the various scenarioes are to be explained partly with the differences in the age profile of immigrants, a younger profile permits for instance to exploit in a more effective way the fertility of the immigrants, and partly with the assumed differences in the fertility of the immigrants themselves.

The inspection of table 2 reveals that the present flow of about 25.000 female entries per year, is completely insufficient to avoid the long term decline of our population. It is of some interest to compute the yearly number of entries that would lead to a long term size of the italian population equal to the size observed in 1991 (notice that this does not imply to keep the population constant since 1991!). The corresponding figures are reported in Table 3.

Table 3. Yearly number of entries which would restore the 1991 population size in the long term

	A1	A2	A3	A4
$R_{0,H} = 0,62$	178.000	175.000	294.000	589.000
$R_{0,H} = 1,24$	110.000	106.000	194.000	470.000
$R_{0,H} = 1,86$	79.000	76.000	145.000	391.000

The dramatic point is therefore that, even the most "optimistic" assumptions[22], i.e. A13 (first column and third row of table 3) and A23, in order to restore in the long term the 1991 size, would need a yearly number of entries which is about 3 times higher than the present number of 25000![23]

May there be a long term rejuvenating effect of immigration on the age structure of the italian population? As it is known from the paper by Schmertmann (1992), immigration may represent a rejuvenating solution for an aging population (although always a less efficient solution compared to the recovery of the fertility of the natives to the replacement level).

To pose the question on a sound basis, it is important to observe that even if the italian population is presently aging, its age structure at 1991 was still slightly younger with respect to the structure embedded in its survival curve. If, from today on, it would be possible to produce

[22] "Most optimistic" in that they are the assumptions which needs, coeteris paribus, the smallest yearly flow of entries (hopefully leading to the smaller number of problems of social "acceptance" of in-migrated).

[23] Notice how the complex metabolism of the model leads to results which are quite close for both the two assumptions A1 (highly unrealistic) and A2 (much more realistic)!

an abrupt recovery of fertility at the replacement level, then our population would, in absence of immigration, become stationary with an average age of about 41.2 years. This is a slightly higher than the average age at 1991 (40.5 years). The obvious explanation of this difference lies in the inertial effect of our young age structure inherited from the baby-boom period.

As expected from the previous considerations, in presence of immigration, the increase in the mean age of the long term population is systematically higher. The amplitude of this increase depends on the level of fertility of the immigrants, and on the age profile of the entries. Table 4 reports the mean age of the long term total population under our distinct assumptions.

Table 4. Average age of the long term total population

under the 12 different assumptions

	A1	A2	A3	A4
$R_{0,H} = 0,62$	$44,2$	$44,6$	$49,0$	$56,9$
$R_{0,H} = 1,24$	$43,10$	$43,5$	$46,5$	$53,9$
$R_{0,H} = 1,86$	$42,8$	$42,9$	$45,4$	$51,8$

Table 4 shows that, compared to the mean age of the population which would result from the recovery of fertility at the replacement level, the average age of the long term total population subjected to migrations would experience an increase ranging from a minimum of about 1.6 years, under the most optimistic assumptions $(A1,3)$ and $(A2,3)$, up to a maximum of 15.7 years under the most pessimistic assumption of our grid $(A4,1)$. This sharply confirms the theoretical finding of Schmertmann (1992), by which immigrations, although able to stop the decline of a population, may constitute a very inefficient rejuvenating tool.

Figure 3 reports the long term age structure of our population with immigrations under assumption $A2,2$ compared with the long-term age structure which would emerge in absence of immigrations and the assumption of relapse of fertility to the zero growth level ($R_{0,H} = 1$) (this latter age structure is the same of the natives).

Fig.3. Long term age structure of the italian female population under $A2,2$ compared with the long-term age structure which would emerge (in absence of immigrations) under relapse to $R_{0,H} = 1$

What are, finally, the predictions of the model on the immigrants' weight on the overall long term population? This weight was estimated to be around 2.6% in 1991 by Golini (1994). Under the several assumptions the immigrants weight would increase from a minimum of 15% (under $A2,3$) to a maximum of 69% (under $A4,1$). It does not come as a surprise the fact that the minimum weight is not associated to the smaller number of entries of the most optimistic assumption $(A1,3)$, but to $A2,3$. The explanation is that this assumption leads to the greatest number of births from immigrants, as implied by the underlying age structure of the immigrants themselves.

5.2 Short term scenarioes

Given the very long time scale needed to reach stability in the SPI model, it is of course more interesting to assess which is the short term impact of the immigration inflow. Table 5 reports the short term projections of the italian population in the time interval 1991-2041, under some of the assumptions considered in this paper.

Table 5. Short term projections of the italian population

and relative mean age in the next decades under $A2,2$ *and* $A4,2$ *(*$R_0^I = 1.24$*);* $I = 25.000$.

Year	Tot. pop. (A22)	Mean Age (A22)	Tot. pop. (A24)	Mean Age (A24)
1991	29.220.000	40.5	29.220.000	40.5
2001	29.341.120	42.9	29.307.300	43.1
2011	28.540.060	45.1	28.349.500	45.7
2021	27.044.200	46.9	26.606.200	48.1
2031	25.302.700	48.3	24.533.300	50.1
2041	23.211.100	49.5	21.988.600	51.8

Under both assumptions it is possible to observe a weak growth effect (to be explained as a consequence of the initial age structure) in the first few years since the beginning of the immigration process. This trend is then quickly reversed and in the subsequent 10 years the population will start decrease, with a strong increase in the mean age. Figure 4 evidences this short term effect and shows furthermore that in the first decades after 1991 the effects of the immigrations is relatively weak. Fig. 5 reports a "film" sequence representing the way in which the anomalous wave represented in the italian female age structure at 1991 by the heritage of the baby -boom generations is reabsorbed by the aging process which finally leads to the equilibrium curve.

Fig. 6 reports the dynamics of the mean age of the (female) italian population during the entire adjustment process with and without migrations. As it was clear from the previous considerations, the heritage of the baby-boom implies some dramatic short term age structure effects, which are well summarised by the dynamics of the mean age. This latter, in absence of immigrations, would reach its maximum of about 53 years just before the exit of the baby-boomers and will remain more or less constant thereafter, due to the fact that the population has reached a stable state. Viceversa, in presence of immigrations, under our assumption $A2,2$, the mean age would reach a maximum of about 50 years more or less in the same period, but since then would start decrease till the long term level of about 43.5, thanks to the rejuvenating effect played by immigrations. We emphasize again the fact that the dramatic short term increase of the mean age even in presence of immigrations is the direct consequence of the heritage of the baby-boom: in fact the maximum is reached just before the "exit" of the baby-boomers.

Fig. 4. Transient dynamics (after 1991) of the total female italian population without immigrations and with immigrations (under A2,2)

Fig. 5 Shapes of the age structure of the italian female population at 1991, 2011, 2031, 2051 compared with the long term age structure under A2,2

Fig. 6. Evolution of the mean age of the italian female population without and with immigrations (under A2,2)

An important question connected with the short term view concerns the stable oscillations which could be echoed by the perturbation to age structure which is caused by immigrations. Such fluctuations could largely vanify the efforts aimed to "protect" pension systems by resorting to immigration policies (an aspect pointed out, in a different framework, by Blanchet (1989)). This makes it of special interest to investigate the predictions of the SPI model on the amplitudes of these stable echoes with special reference to problems of management of the pension system. Fig. 7 reports the transient oscillations of the dependency index (the ratio between the pre-work and post-work age population $(P(0,20) + P(65,\infty))$ and the work age population $(P(20,65))$ under some of the assumptions considered in this paper. The big initial wave (the peak is located around 2040), leads to the same dramatic increase of the dependency index from 0.7 (at 1991) to about 1.0 in 2040 under three strongly different set of assumptions. This result appears remarkable: assumptions leading to quite different long term behaviours (in terms of aging) give rise to essentially the same short term fluctuations. This gives sharp evidence against those feeling in the possibility to use immigration as a short term policy in the management of pension systems.

Fig. 7. Short term dynamics of the dependency index of the italian (female) population under A1,3; A2,2; A41

6 Conclusions

To prevent, by means of an "immigration" policy, the aging and decline of a population with a highly compromised fertility schedule, as the Italian population (presently the country with the lowest fertility in the world), may result very uneffective and very costly. Long term projections of the Italian population by means of the so called stable population model with immigration show that the decline may be stopped in the long term only by accepting a very high immigration stream per year (see Table 3). This implies a lot of social problems connected with the need of a very broad social policy of integration. At the same time the aging process of the population is only partially slowed even in presence of a relatively young age profile of the entries. This unpleasant state of affairs can be somewhat improved if the higher fertility of immigrants can be extended to their descendants, as evidenced by the comparison between fig. 1a and 1b. This fact seems to deserve a further investigation.

We finally recognise that the SPI model has some drawbacks, the absence of regional structure and of a true two-sex structure, suggesting that the robustness of the conclusions reached in this paper has to be checked by also using more realistic models. This will be the aim of subsequent work.

7 Appendix

We provide here a proof of two of the results presented in the sections 4.2 and 4.3. Let us first of all prove that the age structure of the immigrant population given by (47) is always older than the corresponding age structure of the host population, given by: $\pi_H(a) = p(a)/e_0$. To prove the statement we use a dominance argument. We first notice that, as $\pi_I(0) = 0$, then necessarily $\pi_H(0) > \pi_I(0)$. Moreover the ratio:

$$\frac{\pi_I(a)}{\pi_H(a)} = \frac{e_0 Z(a)}{\int_0^\infty p(a)Z(a)da} = \Lambda Z(a) \qquad (55)$$

(where Λ is a constant), is a non-decreasing function of a. Indeed its behaviour only depends on the function $Z(a)$, which satisfies:

$$\frac{dZ(a)}{da} = \frac{i(a)}{p(a)} \geq 0 \qquad (56)$$

Finally, as both the functions $\pi_I(a), \pi_H(a)$ are probability density functions, then they have unit area on their whole support. By combining the last observation with (55) and (56) we conclude that, independently on its actual shape, the $\pi_I(a)$ density must cross $\pi_H(a)$ once and only once. Let a^* the intersection point: clearly $\pi_I(a)$ will lie above $\pi_H(a)$, for all $a > a^*$. As a consequence it always holds:

$$\int_0^x \pi_H(a)da \geq \int_0^x \pi_I(a)da$$

for every age a, having therefore proved that that the age structure of the immigrant population is always older than the corresponding age structure of the host population. By finally remembering that the overall age composition $\pi(a)$ is the weighted mean of the two densities $\pi_H(a)$ and $\pi_I(a)$, it follows imediately that the overall population is older than the native one. These results were only outlined in Schmertmann (1992).

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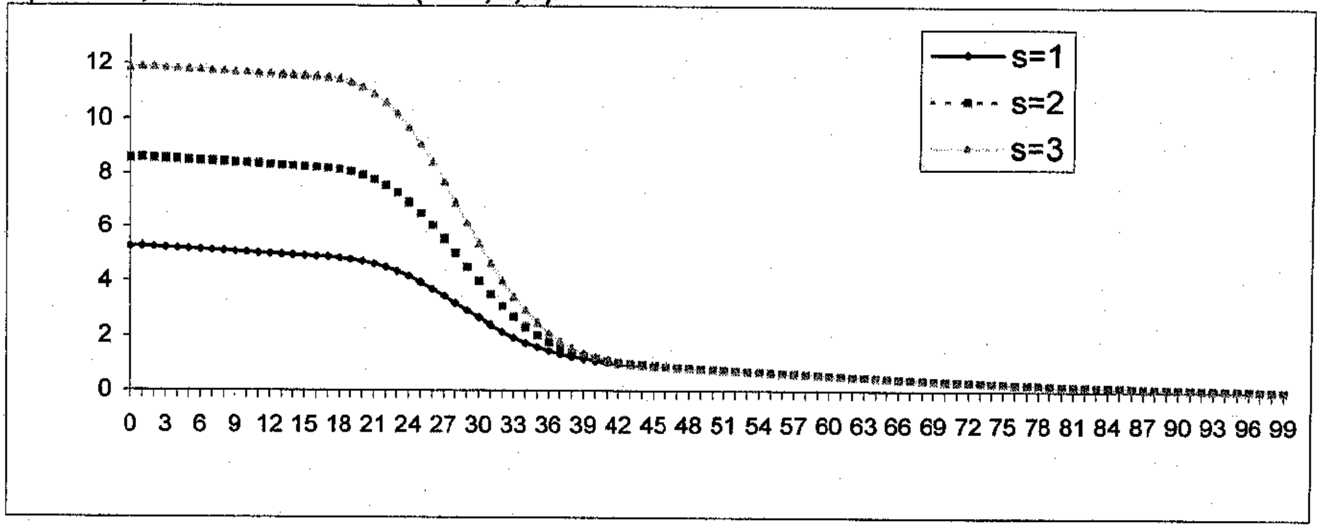
Figure 1. Long term final size (in millions) of the 1991 italian population corresponding to different assumptions on the age of admission of the immigrants

a) RoH = 0,62 and RoI = s*RoH (s = 1, 2, 3).



b) RoH = 0,62; RoI = 1,24 (s = 2) and RoD = 0,62; 0,93; 1,24.

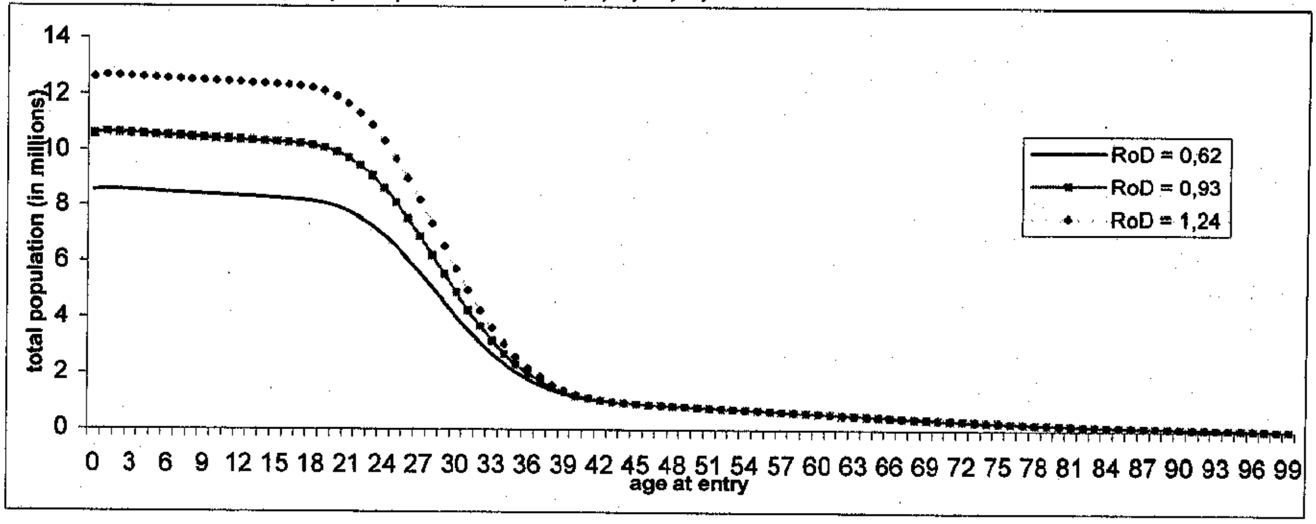


Figure 2. Present and long-term age structures of the italian (female) population in absence of immigrations. The initial population is that observed at 21/10/1991; the vital rates are those of 1991.

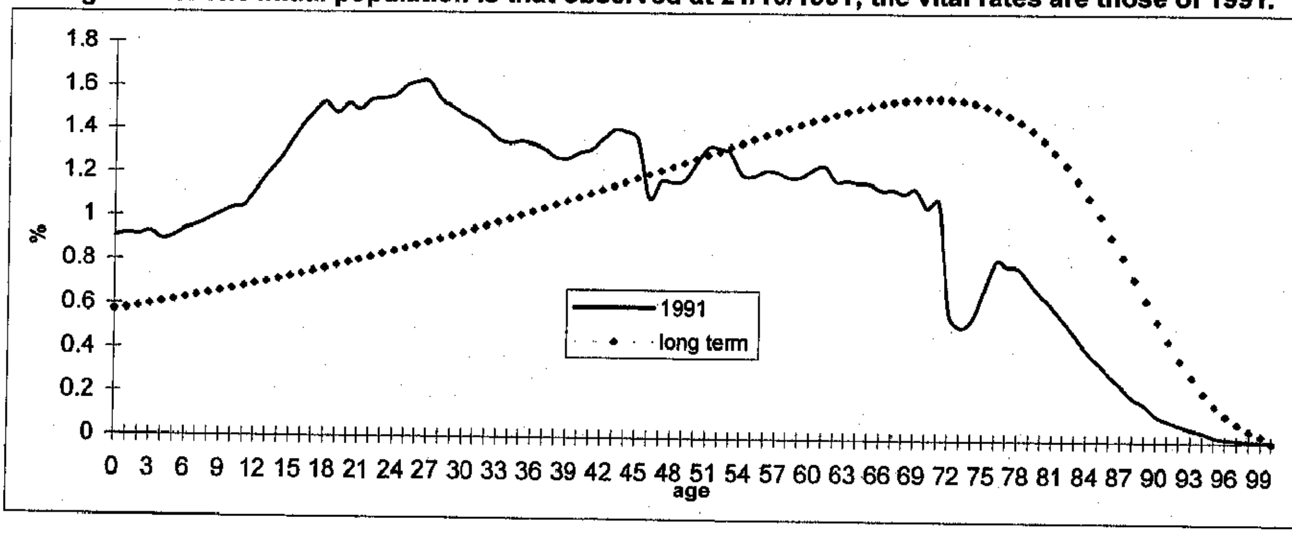


Figure 3. Long term age structure of our population under A2,2 and the long run age structure of the same population in absence of immigrations but under $R_{0,H} = 1$

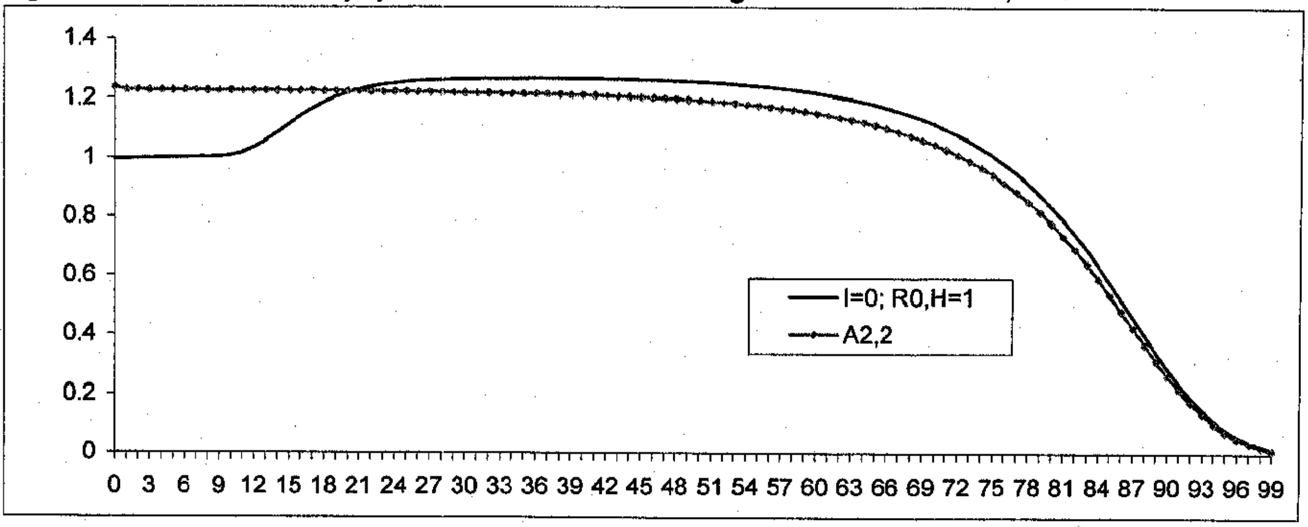


Figure 4. Transient dynamics (after 1991) of the total female italian population (in millions) without immigrations (I=0) and with immigrations (under assumption A2,2)

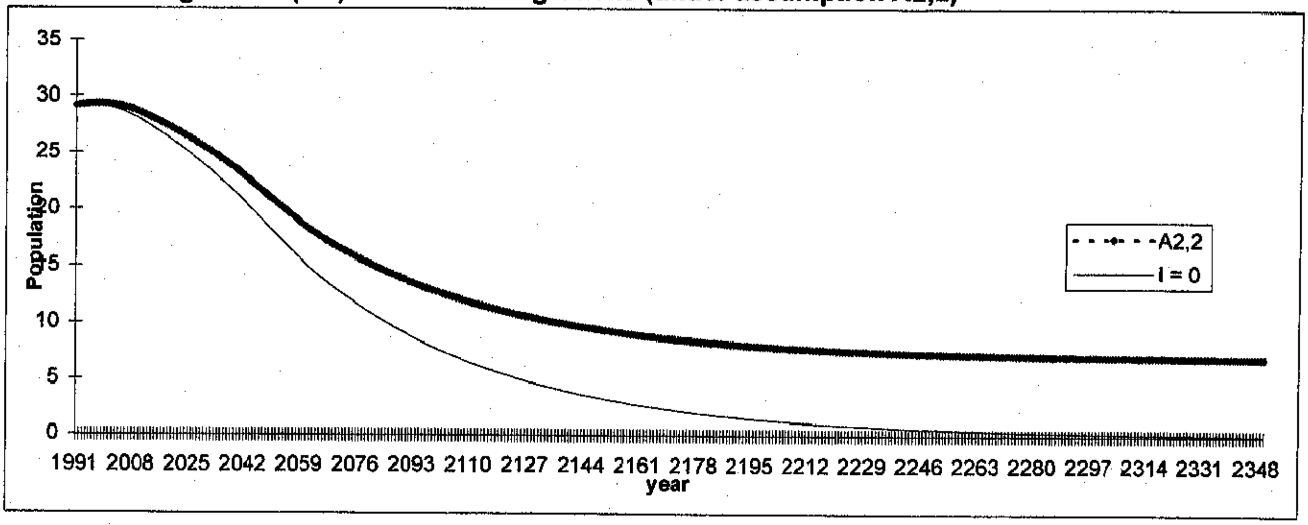


Figure 5. Shapes of the age structure of the italian female population at 1991 (t=0), 2011 (t=20), 2031 (t=40); 2051 (t=60) compared with the long term age structure under assumption A2,2 (t=200)

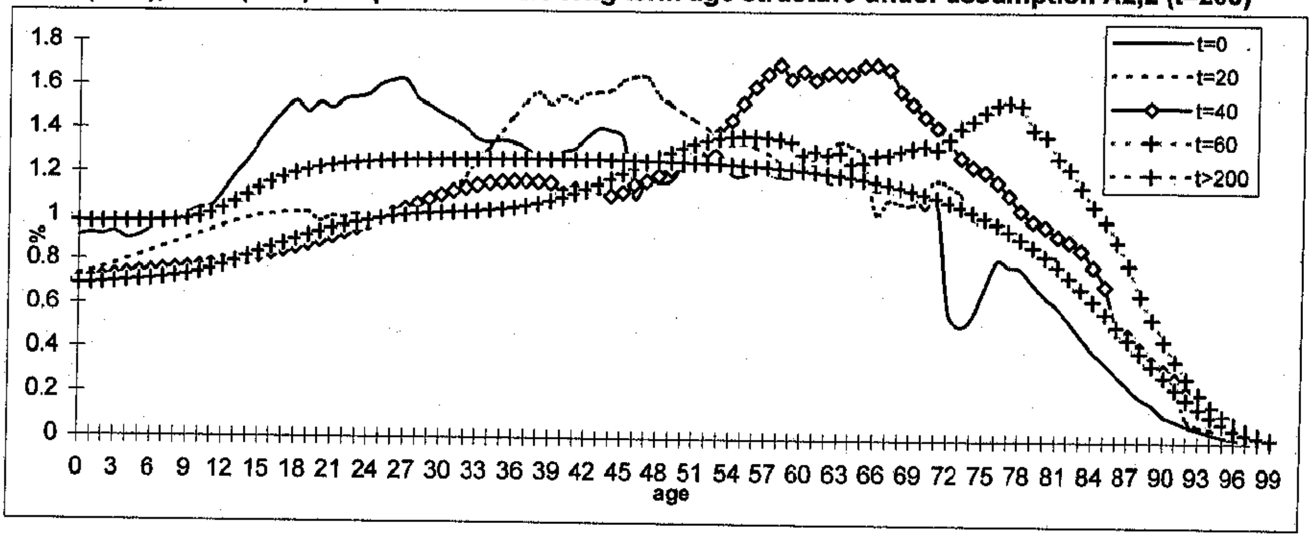


Figure 6. Dynamics over time of the mean age of the italian female population without (I=0) and with immigrations (under assumption A2,2)

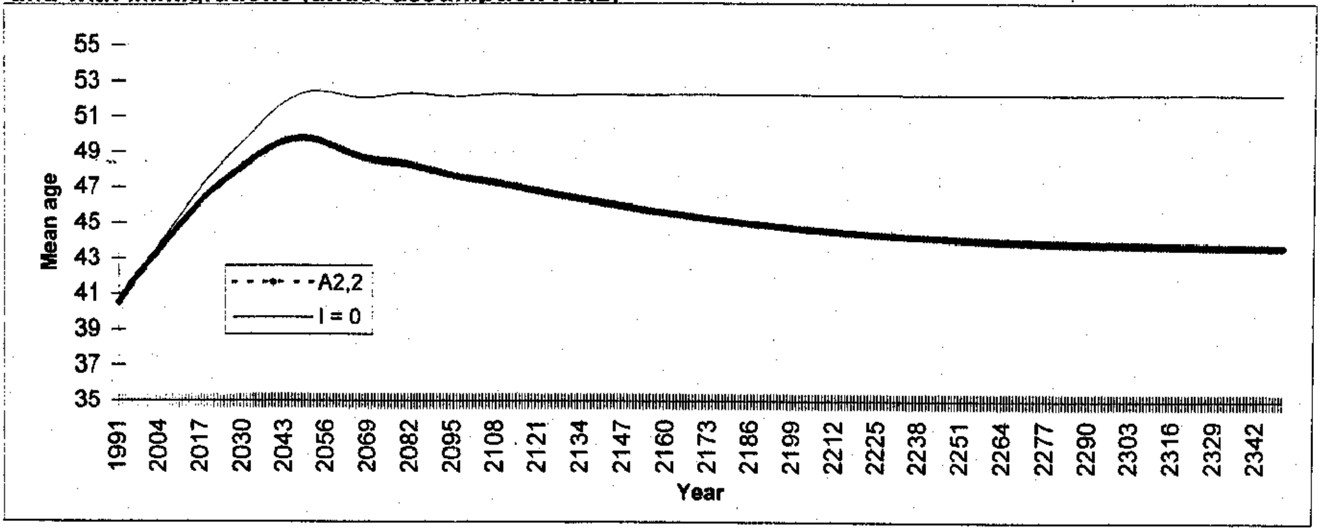


Figure 7. The dependency index of the italian female population under assumptions A13; A22; A41.