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Transition into adulthoold: its macro-demographic consequences in a multistate stable population framework

Francesco Billari, Piero Manfredi
Alessandro Valentini, Alberto Bonaguidi

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by Francesco Billari*, Piero Manfredi**, Alessandro Valentini** and Alberto Bonaguidi**

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Abstract

A multistate generalisation of classical one-sex stable population theory is used in order to evaluate structural and long-term effects of changes in the pace of adulthood attainment. The demographic framework that inspires the paper is the Italian case, where a strong delay in the transition to adulthood and union formation has been observed during the last decades. Italy is also one of the countries where fertility has reach very low levels. The mathematics of the model, that generalises the stable population with marriage model developed by Inaba (1996) is discussed. Then, using two distinct approaches, and mainly starting from individual-level FFS data, we present two preliminary empirical applications. The applications aim at evaluating the impact of delay in adulthood attainment on fertility and population reproduction, and on the age structure of the population.

*IMQ, Università Bocconi, Milan; Istituto di Statistica, Università Cattolica, Milan;
Dipartimento di Scienze Statistiche, Università di Padova.
e-mail: billari@stat.unipd.it

**Dipartimento di Statistica e Matematica Applicata all'Economia
Via Ridofi 10, 56124 Pisa
e-mail: manfredi@ec.unipi.it, valentini@ec.unipi.it

1. Introduction

The sparkling ideas for this paper arise from two different standpoints. The first one is a theoretical need, namely trying to advance towards a wider use of classical mathematical demography models, such as stable population in a multistate framework, in order to evaluate the macro-demographic impact of changes in micro-level behaviour. The second starting point is an explanatory necessity, which arises from the observation of demographic behaviour in Italy during the last twenty years, and tries to answer to one of the open research questions delineated by Marini (1984): to outline the consequences of changes in the process of transition into adulthood for societies as units of study.

We shall briefly go through the two points we mentioned in this introduction. In the second paragraph we will outline the mathematics of the model. In the third paragraph, two preliminary applications to Italy, using mainly individual FFS data, but also official aggregate data, will be presented. Finally, some conclusions are given and directions for future research will be traced.

1.1. Is there still any room left for stable population theory in the demography of Western countries?

The use of event history analysis, which has gained an impressive diffusion in population studies during the last years, put the foundations for the study of demographic behaviour at the micro level. When one needs to evaluate the impact of aggregate factors and of individual demographic behaviour, on the experience of an individual (say, how the *macro* and the *micro* affects the *micro*), such methods are certainly the best solution. On the contrary, it is difficult to use these approaches when one wants to outline the emergency of macro-effects of changes in micro-level behaviour (which should "close the bathtub" when dealing of social behaviours, see Coleman, 1990, ch. 1). Moreover, the micro-oriented approach also leaves out something that was at the heart of demography some decades ago: the study of population dynamics. To sum up, only using event history analysis it is difficult to evaluate how the micro (in case influenced by the micro or the macro as we said before) has an impact on the macro-level, and the latter level was the central one in classical stable population modelling. Let us give an example of a research question, that we will develop later in the paper: what is the consequence of a postponement in the process of transition to adulthood on the overall fertility level? And on the long-term structure of the population, and indirectly on the equilibrium of a pay-as-you-go pension system?

It has been underlined elsewhere (see for instance Gilbert and Troitzsch, 1999) that simulation is one of the solutions for the problem we traced. We agree upon that point, and approaches such as microsimulation, macrosimulation or artificial societies may give fundamental keys to answering questions such as the ones just mentioned. However, we feel that simulation could be much more informative when accompanied by some more analytically oriented approach such as stable population theory in its multistate generalisation. So, while the parameters of the dynamics of stable population theory may represent the *micro* (synthesising the behaviour of the individuals belonging to the population we are interested in), the outcomes of the models may throw important lights on the *macro*. This is the perspective we adopt in this paper, the theoretical side of which takes much inspiration from Inaba (1996). It is also necessary to precise that, when it comes to empirical application, stable population theory has no substantial borders with macrosimulation. In fact, one of the straightforward empirical implementation of the mathematical model we are developing is via macrosimulation, as we shall see.

1.2. The Italian scenario: delayed adulthood attainment and union formation, lower fertility

Let us now sketch the specific behavioural situation we have in mind. In some Western European countries of the Mediterranean Area, Italy is a peculiar example, the process of family formation, which takes course generally within a legal marriage, has significantly slowed during the last decades. The transition to adulthood is there also characterised by a strong synchronisation between marriage[1] and leaving the parental home. Moreover, the length of full-time education for the young adults has been steadily increasing, especially for women. Some studies on the process of transition into adulthood suggest that there are typical sequences between events (e.g. Corijn, 1996), even if it is unclear what is the impact of aggregation on such idea (Billari, 1999). However, having left full-time education, or at least having left the parental home seems to be a necessary condition in order to enter a steady (married or unmarried) cohabiting partnership. Then, being in a steady cohabiting partnership seems to be an almost necessary condition in order to become a parent.

In the present work we adopt the hypothesis that there is a main "marker" in the adulthood attainment process that gives rise to a passage to a state that we may call the "marriageable" one, following the seminal ideas of Coale and McNeil (1972), or, less specifically, the "adult" state. In general, we might say that there is a main marker distinguishing young people (we mean, people that do not consider entering a union as an option) from adult people (we mean, people that consider entering a union as an option or that may presently be or might have been in a union). Coale and McNeil state that "in contemporary populations of Western European origin (...) we may conjecture that the age of becoming marriageable is the age at which serious dating, or going steady begins". In this paper as the marker of the transition to adulthood we take an event, namely the first occurring event between the end of formal education and the leaving of the parental home. We shall spend some words here in order to give some justifications for such choice.

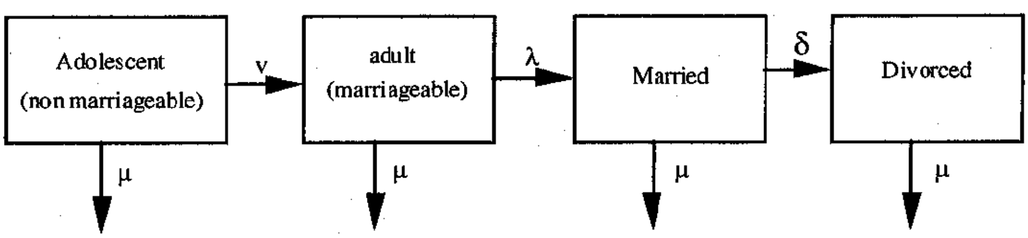
As for the end of formal education, this seems to us a sensible marker because 1) sooner or later education will be completed, and this means that we may assume a unity quantum and just study the tempo of the event; 2) as Blossfeld and De Rose state, about Italy, "finishing

[1] During the paper, we will be using the word "marriage" and "union" as interchangeable, and "marriageable" will be also considered as "willing to enter any union". The low cohabitation intensity in Italy justifies this approach. However, the focus of the model is in fact on unions.

education is expected to count as one of the important prerequisites for entering into adulthood status, and thereby entering into marriage and parenthood"[2]. In addition, when people leave the parental home before the completion of full-time education, we may say that in a sense they may become more ready for unions (and we mainly think here mostly of informal unions).

2. The theoretical framework: a multistate stable population model with adult state

The theoretical framework considered here is represented by a one-sex four states stable population model with "irreversible" transitions (fig. 1)[3], which generalizes Inaba's (1996) population model with reproduction via first marriage. As represented in the flow diagram below, young individuals can not marry, since they do not become marriageable until they become adult. Moreover only married individuals do reproduce.



2.1 A preliminary case without age

Let us consider first a preparatory special case in which age is deliberately ignored. Although oversimplified, this case is useful to clarify the role of "sequential" stages within classical

[2] A framework where the end of formal education is a necessary condition for entering a "marriageable" state may also well apply to developing countries.

stable population frameworks. Let us denote by $p_0(t)$ the number of young individuals at time t, $p_1(t)$ the number of adults, $p_2(t)$ the number of married individuals and finally $p_3(t)$ the number of individuals in the residual state (widowed, divorced or remarried). Moreover, let μ_i's (i=0,1,2,3) be the state specific death rates per unit time (p.u.t. since now on), v the rate of transition into adulthood p.u.t., λ the marriage rate p.u.t of adult individuals, δ the total rate of dissolution of marriages, and m the fertility rate of the married women. By assuming that all the rates in the flow diagram above are constant, we arrive to the following system of linear ordinary differential equations (ODE since now on):

$$\begin{aligned} \dot{p}_0(t) &= mp_2(t) - (\mu_0 + v)p_0(t) \\ \dot{p}_1(t) &= vp_0(t) - (\mu_1 + \lambda)p_1(t) \\ \dot{p}_2(t) &= \lambda p_1(t) - (\mu_2 + \delta)p_2(t) \\ \dot{p}_3(t) &= \delta p_2(t) - \mu_3 p_3(t) \end{aligned} \qquad (1)$$

The system (1)[4] may be represented compactly as $\dot{P}(t) = MP(t)$ where M is the matrix:

$$M = \begin{pmatrix} -(\mu_0 + v) & 0 & m & 0 \\ v & -(\mu_1 + \lambda) & 0 & 0 \\ 0 & \lambda & -(\mu_2 + \delta) & 0 \\ 0 & 0 & \delta & -\mu_3 \end{pmatrix} \qquad (2)$$

The demographically relevant features of system (1) are easily inferred from those of the matrix *M*, which is a Metzler matrix. Metzler matrices are positivity preserving operators in continuous time, playing the same role of positive matrices for discrete dynamical systems and having their specific version of the Pérron-Frobenius theorems (Luenberger 1979). So, in particular, the matrix *M* has a unique dominant eigenvalue K_0, to which it belongs a demographically meaningful (i.e.: non negative) eigenvector, and furthermore all remaining eigenvalues of M have real part which is less than K_0.

[3] Inaba's has considered both the irreversible case (young→married→divorced), with reproduction via first marriage (Inaba 1996) and a reversible case, with iterative marriage (1993).
[4] Systems as (1) are very common in population biology and population dynamics of infectious diseases, see for instance Anderson and May (1991).

It is easy to see that the sign of the dominant eigenvalue K_0, which corresponds to the Lotka's intrinsic rate of growth of the population and hence determines its long-term behaviour, only depends on the sign of the coefficient of the known term of the characteristic polynomial P(K):

$$P(K) = (K + \mu_3)\left(K^3 + aK^2 + bK + c\right) = 0$$

where:

$$\begin{aligned} a &= (\mu_0 + v) + (\mu_1 + \lambda) + (\mu_2 + \delta) > 0 \\ b &= (\mu_0 + v)(\mu_1 + \lambda) + (\mu_0 + v)(\mu_2 + \delta) + (\mu_1 + \lambda)(\mu_2 + \delta) > 0 \\ c &= (\mu_0 + v)(\mu_1 + \lambda)(\mu_2 + \delta) - mv\lambda = ABC - D \end{aligned} \qquad (3)$$

Hence, K_0 will be positive or negative (i.e. we will have stable exponential growth rather than stable exponential decay) depending on whether:

$$mv\lambda - (\mu_0 + v)(\mu_1 + \lambda)(\mu_2 + \delta) \gtreqless 0 \qquad (4)$$

The threshold condition (4) can be represented as:

$$R_0 = \frac{m}{\mu_2 + \delta} \cdot \frac{v}{\mu_0 + v} \cdot \frac{\lambda}{\mu_1 + \lambda} > 1 \qquad (6)$$

where R_0 defines the appropriate net reproductive rate (NRR) of the population for this unstructured case (see Manfredi and Billari 1997). This last result clearly shows that in presence of below replacement fertility no policy aimed to take fertility up to the "zero growth level" which be based on a reduction of the age at marriage and/or of age to adulthood can be successful. Vice-versa, by suitably acting on such parameters can reveal to be an effective policy for taking down to stationarity a population experiencing stable growth. An important example could be for instance a policy of systematically raising alphabetisation and education in developing countries.

2.2 The age-structured model: stable distribution with respect to age and stages of life

Here we introduce our age-structured model which generalises the three states Inaba's (1996) model of reproduction via first marriage by adding the adult state. Following Inaba the model recognises both chronological age and duration of permanence in each state. Let $p_0(a,t)$ denote the density of "never adult" (female) individuals aged a at time t, $p_1(c,t;\eta)$ the density at time t of individuals who entered into adulthood since c years, at the age of η, $p_2(\tau,t;\eta,\xi)$, the density of individuals married since τ years, who became adult at the age of η and married at the age of ξ, $p_3(a,t)$ the density of individuals who are not in the first three states (widowed, divorced or remarried). Moreover, let $\mu(a)$ denote the age dependent mortality rate (or force of mortality), $v(a)$ the rate of transition into adulthood, $\lambda(a,\eta)=\lambda(c+\eta,\eta)$ the force of first marriage, which is assumed to be influenced by the age of entrance into adulthood, and, $\delta(\tau,\xi)$ the force of dissolution of pairs married τ years before when the female was aged ξ..

The backbone of the model is given by the following system of Von Foerster equations[5]:

$$\begin{cases} \Delta_{a,t} p_0(a,t) = -[\mu(a) + v(a)]p_0(a,t) \\ \Delta_{c,t} p_1(c,t;\eta) = -[\mu(c+\eta) + \lambda(c+\eta,\eta)]p_1(c,t,\eta) \\ \Delta_{\tau,t} p_2(\tau,t;\eta,\xi) = -[\mu(\xi+\tau) + \delta(\tau,\xi)]p_2(\tau,t;\eta,\xi) \\ \Delta_{a,t} p_3(a,t) = \iint\limits_{\tau+\xi=a} \delta(\tau,\xi)p_2(\tau,t;\eta,\xi)d\tau d\eta - \mu(a)p_3(a,t) \end{cases} \tag{6}$$

where $\Delta_{a,t}$ is a shortcut for the aging operator $\frac{\partial}{\partial a}+\frac{\partial}{\partial t}$ and similarly $\Delta_{c,t}, \Delta_{\tau,t}$. The system (6) has to be completed with the boundary conditions:

$$\begin{cases} p_0(0,t) = B(t) = \int_0^\beta \int_\eta^\beta \int_0^{\beta-\xi} p_2(\tau,t;\xi,\eta)m(\tau;\xi,\eta)d\tau d\xi d\eta \\ p_1(0,t;\eta) = v(\eta)p_0(\eta,t) \\ p_2(0,t;\eta,\xi) = \lambda(\xi,\eta)p_1(\xi-\eta,t;\eta) \\ p_3(0,t) = 0 \end{cases} \tag{7}$$

[5] The system (4.1) collapses into (3.1) when all the rates are assumed constant.

saying respectively that i)the number of individuals aged zero in the first state at time t is simply the number $B(t)$ of births at time t, where β is the upper bound of the fertile age span and $m(\tau;\xi,\eta)$ the marital fertility rate at marriage duration τ for a married women who entered the adult state at age η and then married at age ξ ($\xi>\eta$); ii)the number of adult women with duration zero of permanence in the adult state and chronological age at adulthood η at time t, is given by the number of transitions into adulthood of individuals aged η at time t; iii)the number of individuals with marriage duration zero, who entered the adult state at age η and married at ξ is given by the number of marriages of $p_1(c,t;\eta)$ individuals at the age $\xi=\eta+c$. Finally a set of prescribed initial distribution, has to be assigned:

$$p_0(a,0)=H_0(a) \quad p_1(c,0;\eta)=H_1(c,\eta) \quad p_1(\tau,0;\eta,\xi)=H_2(\tau;\eta,\xi) \quad p_3(a,0)=H_3(a) \quad (8)$$

Remark. We postulated that the fertility rates depend not only on the duration of marriage, but also on the age at marriage (as in Inaba, 1996) and on the age of transition into adulthood. This appears to be a reasonable point: other things being equal, married individuals characterized by the same marriage duration and by the same age at marriage may be expected to have different fertility if they entered sooner the adult state, as they probably experienced different life histories in terms of job experiences, compared to whose who entered later.

Remark. The following relations which connect our population densities with the traditional age distributions in the four states hold:

$$p_3(a,t)=n(a,t)-\sum_{i=0}^{2} p_i(a,t)$$
$$p_1(a,t)=\int_0^a p_1(a-\eta,t;\eta)d\eta$$
$$p_2(a,t)=\int_0^a\int_0^\xi p_2(a-\xi,t;\eta,\xi)d\xi d\eta$$

The mathematical treatment of the model (6)-(7)-(8) is a tedious but straightforward task as shown in Inaba (1996) for the three states model. Since we are not interested here in the mathematical aspects, we omit most of the technical details (see Manfredi and Billari 1997), and simply recall the most relevant steps of the solution process. It is possible to show that the overall age distribution: $n(a,t) = \sum_{i=0}^{3} p_i(a,t)$ satisfies a traditional Von Foerster PDE. This makes it possible to reformulate the problem into a Lotka-type renewal model in the (chronological) age-time domain, governed by a traditional births integral equation. This permits to show the existence of an asymptotic stable behaviour in Lotka's sense, characterised by stable exponential growth over time of $B(t)$ when the NRR (R_0) of the population is greater than one and, vice-versa, by stable exponential decay when $R_0<1$. The existence of a stable distribution with respect to age and stages of life is easily proved by expressing the formal solutions of the equations (4.1) for $t>a$ in terms of the overall density $n(a,t)$. It holds:

$$
\begin{aligned}
&p_0(a,t) = B(t-a)l(a)V(a) = n(a,t)V(a) \\
&p_1(c,t;\eta) = B(t-c-\eta)l(\eta+c)V(\eta)v(\eta)\Lambda_\eta(c) = n(a,t)V(\eta)v(\eta)\Lambda_\eta(c) \qquad a=\eta+c \\
&p_2(\tau,t;\eta,\xi) = B(t-\tau-\xi)l(\eta)V(\eta)v(\eta)\frac{l(\xi)}{l(\eta)}\Lambda_\eta(\xi)\lambda(\xi,\eta)\frac{l(\xi+\tau)}{l(\xi)}\Delta_\xi(\tau) = \\
&= n(a,t)\left[V(\eta)v(\eta)\Lambda_\eta(\xi)\lambda_\eta(\xi,\eta)\Delta_\xi(\tau)\right] \qquad a=\xi+\tau
\end{aligned} \tag{9}
$$

The relations (9) express the populations densities in the states *0,1,2* in terms of the total population $n(a,t)$[6] and of the following generalised survival functions:

$$
\begin{aligned}
&l(a) = exp\left(-\int_0^a \mu(u)du\right) \\
&V(a) = exp\left(-\int_0^a v(u)du\right) \\
&\Lambda_\eta(c) = exp\left(-\int_\eta^{\eta+c} \lambda(u;\eta)du\right) = exp\left(-\int_0^c \lambda(\eta+s;\eta)ds\right) \\
&\Delta_\xi(\tau) = exp\left(-\int_0^\tau \delta(u;\xi)du\right)
\end{aligned} \tag{10}
$$

where i)$l(a)$ is the "survival to natural mortality" function (the probability that a new-born woman survives to age a); ii)$V(a)$ is the "survival to adulthood" function (the probability that a

[6] They represent the formal solutions to the corresponding PDE's (6) for $t>a$. Such solutions are "formally" derived by integration along characteristics of the given PDE's, but they can be derived in a totally direct way by sequentially applying the survival functions (10).

newborn woman does not enter adulthood before age a in absence of mortality), iii) $\Lambda_\eta(c)$ is the (conditional) "survival to marriage" function (the probability that a woman who entered the adult state at the age η be still unmarried at age $a=\eta+c$), iv) $\Delta_\eta(\tau)$ the survival function to divorce until age $a=\tau+\xi$ for a woman married at age ξ). Suitable integrations on the quantities (10) lead to:

$$\begin{aligned}
&p_0(a,t) = n(a,t)V(a) \\
&p_1(a,t) = n(a,t)\Psi(a); \quad \Psi(a) = \int_0^a V(\eta)v(\eta)\Lambda_\eta(a-\eta)d\eta \\
&p_2(a,t) = n(a,t)\Gamma(a); \quad \Gamma(a) = \int_0^a\int_0^\xi \left[V(\eta)v(\eta)\Lambda_\eta(\xi)\lambda_\eta(\xi,\eta)\Delta_\xi(a-\xi)\right]d\eta d\xi
\end{aligned} \tag{11}$$

The (11) show that when the total population $n(a,t)$ reaches its stable age distribution, this will automatically cause a stable distribution by stages of life as well.

2.3. Some remarkable relations involving the reproductivity indices

In what follows we provide some formulas for the Net Reproduction Rate (NRR) and the Total Fertility Rate (TFR), by generalising Inaba's work (1996) to the present four state analysis. For what concerns the NRR, the following equivalent definitions hold

$$\begin{aligned}
R_0 &= \int_0^\beta\int_\eta^\beta\int_0^{\beta-\xi} m(\tau;\xi,\eta)l(\xi+\tau)V(\eta)v(\eta)\Lambda_\eta(\xi)\lambda(\xi,\eta)\Delta_\xi(\tau)d\tau d\xi d\eta \\
&= \int_0^\beta\int_0^{\beta-\tau}\int_0^\xi m(\tau;\xi,\eta)l(\xi+\tau)V(\eta)v(\eta)\Lambda_\eta(\xi)\lambda(\xi,\eta)\Delta_\xi(\tau)d\eta d\xi d\tau \\
&= \int_0^\beta\int_0^\xi\int_0^{\beta-\xi} m(\tau;\xi,\eta)l(\xi+\tau)V(\eta)v(\eta)\Lambda_\eta(\xi)\lambda(\xi,\eta)\Delta_\xi(\tau)d\xi d\eta d\tau
\end{aligned} \tag{12}$$

The change of variable from the "old" set (η,ξ,τ) to the "new" one $(\eta,\xi,a=\xi+\tau)$, leads to the following alternative definition:

$$R_0 = \int_0^\beta l(a)\left[\int_0^a V(\eta)v(\eta)\left(\int_\eta^a \Lambda_\eta(\xi)\lambda(\xi,\eta)\Delta_\xi(\tau)m(\tau;\xi,\eta)d\xi\right)d\eta\right]da \tag{13}$$

The (13) suggests the following definition for the age-specific fertility rates:

$$m^*(a)=\int_0^a V(\eta)v(\eta)\left(\int_\eta^a \Lambda_\eta(\xi)\lambda(\xi,\eta)\Delta_\xi(\tau)m(\tau;\xi,\eta)d\xi\right)d\eta \qquad (14)$$

relating the traditional age specific fertility rate with the duration specific fertility rates.

The last formulas specify that, in order to be able to make child with the fertility schedule $m(\tau;\xi,\eta)=m(a-\xi;\xi,\eta)$ typical of age a, a female not only has to survive until that age, as in traditional stable population model, but she has to: a)enter the adult state, otherwise she would not be marriageable, at a previous age η, with probability (density) $V(\eta)v(\eta)$; b) marry at a subsequent age ξ, with probability $\Lambda(\xi)\lambda(\xi)$; c)survive to the risk of divorce until the given age a, with probability $\Delta_\xi(\tau)=\Delta_\xi(a-\xi)$. If we explicitly neglect mortality the following definition for the TFR (total number of offsprings produced by a woman during her "effective" fertile age span, given by the portion of the her fertile age span actually spent within the married state) arise:

$$TFR=\int_0^\beta\int_\eta^\beta\int_0^{\beta-\xi} m(\tau;\xi,\eta)V(\eta)v(\eta)\Lambda_\eta(\xi)\lambda(\xi,\eta)\Delta_\xi(\tau)d\tau d\xi d\eta \qquad (15)$$

By stressing the sequentiality of the stages of life, we can write:

$$\begin{aligned}TFR&=\int_0^\beta V(\eta)v(\eta)\left[\int_\eta^\beta \Lambda_\eta(\xi)\lambda(\xi,\eta)\left(\int_0^{\beta-\xi} m(\tau;\xi,\eta)\Delta_\xi(\tau)d\tau\right)d\xi\right]d\eta=\\&=\int_0^\beta V(\eta)v(\eta)\left[\int_\eta^\beta \Lambda_\eta(\xi)\lambda(\xi,\eta)T(\xi,\eta)d\xi\right]d\eta=\\&=\int_0^\beta T^*(\eta)V(\eta)v(\eta)d\eta\end{aligned} \qquad (16)$$

where:

$$T(\xi,\eta)=\int_0^{\beta-\xi} m(\tau;\xi,\eta)\Delta_\xi(\tau)d\tau \quad ; \quad T^*(\eta)=\int_\eta^\beta \Lambda_\eta(\xi)\lambda(\xi,\eta)T(\xi,\eta)d\xi \qquad (17a,b)$$

The quantity $T(\xi,\eta)$ is the conditional TFR of those women who became adult at the age of η and married at the age of ξ in presence of the risk of marriage dissolution. For what concerns the quantity $T^*(\eta)$, we can write:

$$T^*(\eta)=PEM(\eta)\cdot\int_\eta^\beta T(\xi,\eta)\Phi_\eta(\xi)d\xi \qquad (18)$$

where:

$$\Phi_\eta(\xi) = \frac{\Lambda_\eta(\xi)\lambda(\xi,\eta)}{\int_\eta^\beta \Lambda_\eta(\xi)\lambda(\xi,\eta)d\xi} \quad 0 \le \eta \le \beta;\ \eta \le \xi \le \beta \qquad (19)$$

represents the conditional (normalized) density function of marriage at age ξ for the women who entered the adult state at age η and married before the end of the fertile age span, and:

$$PEM(\eta) = \int_\eta^\beta \Lambda_\eta(\xi)\lambda(\xi,\eta)d\xi = 1 - \Lambda_\eta(\beta) \qquad (20)$$

is the (conditional) proportion of ever married (PEM) women (to be precise: who married before age β) among those women who entered the adult state at the age of η. By furtherly defining:

$$W(\eta) = \frac{V(\eta)v(\eta)}{1 - V(\beta)} \quad ; \qquad \int_0^\beta V(\eta)v(\eta)d\eta = 1 - V(\beta) = PEA(\beta) \quad (21a,b)$$

where $W(\eta)$ is the normalised probability density of transition into adulthood (truncated at the end of the fertile age span) and $PEA(\beta)$ is the "proportion ever adult" (PEA) at the end of the fertile period, we can write (16) as:

$$TFR = PEA(\beta)\int_0^\beta T^*(\eta)W(\eta)d\eta \qquad (22)$$

By introducing (18) into (22), and assuming that, quite reasonably, $PEA(\beta)=1$, we obtain:

$$TFR = \int_0^\beta PEM(\eta)W(\eta)\left[\int_\eta^\beta T(\xi,\eta)\Phi_\eta(\xi)d\xi\right]d\eta = \int_0^\beta PEM(\eta)W(\eta)\bar{T}(\eta)d\eta \qquad (23)$$

where $\bar{T}(\eta)$ is the average TFR (averaged with respect to the age at marriage) of women entered into adulthood at age η and married before the end of the fertile age span. It is clearly a conditional average TFR (conditional on being entered into adulthood at age η). It seems useful, at this stage, to distinguish two main subcases:

a) the fertility rates do not depend on the age of entrance into the adult state, ie: $m = m(\tau;\xi)$. We call this case the "pure delayer" case, in that the role of the intermediate state of adult does not affect the fertility behaviour of the (married) population: its effects are essentially those of delaying fertility by delaying marriage.

b) a more general case in which $m = m(\tau;\xi,\eta)$, so that in addition to the "delayer" effect, true fertility effects arise due to the process of transition into adulthood.

In this paper we limit our analysis to the "pure delayer" case. In this case we have:

$$T(\xi,\eta) = \int_0^{\beta-\xi} m(\tau;\xi)\Delta_\xi(\tau)d\tau = T(\xi) \quad (24)$$

If now, following Inaba (1996), we assume a Henry-type (Henry 1976) linear relation between the conditional TFR of women married at age ξ, *T(ξ)*, and the age at marriage:

$$T(\xi) = U - V\xi + R(\xi) \quad U > 0, V > 0 \qquad (25)$$

where R is a reminder, from (23) we get, by neglecting the reminder, the approximated relation:

$$\begin{aligned} TFR &\cong \int_0^\beta PEM(\eta)W(\eta)\left[\int_\eta^\beta (U - V\xi)\Phi_\eta(\xi)d\xi\right]d\eta = \\ &= U\int_0^\beta PEM(\eta)W(\eta)d\eta - V\int_0^\beta PEM(\eta)W(\eta)\left[\int_\eta^\beta \xi\Phi_\eta(\xi)d\xi\right]d\eta = \\ &= U\int_0^\beta PEM(\eta)W(\eta)d\eta - V\int_0^\beta PEM(\eta)W(\eta)\bar{\xi}(\eta)d\eta \end{aligned} \qquad (26)$$

where $\bar{\xi}(\eta)$ is the average age at marriage of the women who entered the adult state at the age of η and married before the end of the fertile age span. The quantities:

$$B = \int_0^\beta PEM(\eta)W(\eta)d\eta \qquad W^*(\eta) = \frac{PEM(\eta)W(\eta)}{B} \quad 0 \le \eta \le \beta \qquad (27a,b)$$

respectively define the "average PEM", i.e. the average value of the conditional (on the age of entrance into adulthood) proportion ever married, and the probability density function of transition into adulthood at age η for the women who married before the end of the fertile age span. Thanks to (27a,b) we write (26) as:

$$TFR \cong B\left\{U - V\int_0^\beta \bar{\xi}(\eta)W^*(\eta)d\eta\right\} = B\left\{U - V\bar{\xi}\right\} \qquad (28)$$

The (28) factors the overall TFR as the product of the average PEM times the value of the Henry relationship evaluated in correspondence of the average age at marriage $\bar{\xi}$ (averaged on the density of transition to adulthood) of the women married before the end of the fertile age span. The (28) makes it of interest to investigate at the empirical level: i)the relation between the age of entrance into adulthood and the corresponding proportions ever married and; ii)the relation between the age of entrance into adulthood and the corresponding average age at marriage. If we assume $\bar{\xi}(\eta)$ to be linear, i.e.:

$$\bar{\xi}(\eta) = \bar{\xi}(\eta_A) + q(\eta - \eta_A) = p + q\eta \quad \eta_A \leq \eta \leq \beta \qquad (29)$$

where η_A is the lower bound in the possible ages of transition into and $\bar{\xi}(\eta_A)$ the corresponding average age at marriage, (28) leads to (see Manfredi and Billari (1997) for further details):

$$TFR = B\left\{U - V\int_0^\beta [p + q\eta] W^*(\eta) d\eta\right\} = B\{U - pV - qV\bar{\eta}\} \quad (30)$$

The formula (30) relates the TFR with the average PEM and the average age at adulthood and could therefore be used to roughly estimate the "pure delayer" effects of an increase of the average age $\bar{\eta}$ of transition to adulthood of women married before the end of their fertile age span, on TFR. This operation needs to estimate the parameters V and q. This will be done, by relying on survey data, in the first illustration of the next section.

3. Two preliminary applications to the Italian case

In this section, we shall discuss two distinct and rather simplified applications sparkling from our mathematical model. First, we use the linear approximations introduced in the previous section in order to give a simple evaluation of the relationship between the TFR and the average age at adulthood attainment. Then, we use a simplified discrete-time version of the model, in order to evaluate the long term impact of changes in the shape of the transition to adulthood curve in Italy.

The individual data we use mainly come from the 1995/96 Italian Fertility and Family Survey (De Sandre *et al.*, 1996), a retrospective survey on Italian men and women born between 1946 and 1975. The survey contains event history information on the month and the year in which people left their parental home, left full time education, entered a union, bore children and other demographic events. As we are dealing with a one-sex model, we chosen to use only female data.

3.1. The impact of increases in the mean age at adulthood: a simple formula

In this section the mathematical relationships previously developed are used in order to get insights on the effect of changing tempo in adulthood attainment on the reproduction of the population. We only refer here to women aged 40 and over (born 1946-1955) in order to approximately avoid censoring problems. Some further assumptions need to be introduced, concerning both the connections between unions and fertility on the one side, and between attaining adulthood and entering a union on the other side.

As for the first problem, we follow the approach of Inaba (1996) and use the linear approximation $T(\xi) = U - V\xi$ first proposed by Henry (1976), between the number of children ever born and the age at first union. Though in principle very rough, this is essentially a local approximation, has the merit of being both surprisingly accurate from the empirical point of view and mathematically manageable. It is thus possible to estimate via linear regression the relationship $T(\xi) = U - V\xi$, using our individual-level data. The results are reported in table 1 (see also fig. 1a). Notice that the R^2 values are computed on the conditional averages values of the TFR computed for each age at union and not on the individual data, to be comparable with Inaba (1996).

Relation to be Estimated	Parameter Estimates (Standard Errors in parentheses)	R^2	p-values
$T(\xi) = U - V\xi$	U=3.8068 (0.02880) V=0.07695 (0.00120)	0.7544	0.0001
$\bar{\xi}(\eta) = p + q\eta$	p=13.710 (0.08266) q=0.5373 (0.00458)	0.9088	0.0001

Tab. 1 Results from the linear regressions between a)TFR and age at marriage, and b)age at marriage and age at adulthood

In order to obtain the approximate TFR, the mean age at union and the proportion ever in union have been computed from our data. With a reproductive behaviour such as the one exhibited by the cohort we considered, a one year increase in the age at marriage would bring down the number of female children ever born[7] by 0.07695.

To evaluate the effect of an increase in the age of attaining adulthood we adopt the linear approximation (29). This assumption permits to remain close to the spirit of the Henry-type relationship, i.e. an approximate but possibly useful one[8]. As women who attained adulthood later tend to have a shorter interval between adulthood attainment and entering a union, a regression coefficient less than the unity is expected[9]. This is confirmed by our results (table 1). Although very heuristic, the linear approximation seems to work quite well, as witnessed by fig. 1b. By combining the two linear relations, the expected decrease in the TFR for women ever entering a union as a consequence of a one-year delay in adulthood attainment, given by the product *(BVq)* (see formula (30)), is definitively equal to 0.039.[10].

3.2. A stable population experiment: evaluating the macro-impact of a delayed transition into adulthood

In this section a preliminary set of long-term (stable) macro-simulations is performed, for the Italian case, on a simplified version of the four state model taking into account only the chronological age, but not other durations.[11]. The aim of this section is to evaluate the long-term consequences arising from a pure delay effect in the transition to adulthood, other things being constant. At this scope we split our sample into two broad cohorts (the 'old', born 1946-1960

[7] Using Japanese data, Inaba (1996) computed an expected increase for the TFR of 0.11. Using data for younger Italian cohorts, Manfredi and Billari (1997) computed an expected increase of the TFR of 0.08-0.09.

[8] The same assumption, though in a different framework, that of a structural model, was used by Marini (1985).

[9] Using data on earlier Italian cohorts in Italy, and using the average age at attaining the highest educational level as a marker of attaining adulthood, Manfredi and Billari (1997) estimated that an increase of one year in women's age at adulthood would lead to an increase of about 0.4/0.5 years in the age at marriage.

[10] This result confirms the decrease of 0.03-0.04 in the TFR computed in paper by Manfredi and Billari (1997).

[11] Wolf (1988) outlines the application of a multistate life table which in fact contains all the necessary tools to fully exploit the mathematical model. He also emphasise the demanding data needs of his approach.

and the 'young', born 1961-1975):[12]: the first one will provide a "benchmark" stable scenario, whereas the second, being characterised by a somewhat different estimated shape of the overall curve of the rates of transition into adulthood (fig. 2), is used to provide an "alternative" scenario. The only difference between the benchmark and the alternative scenario is represented by the patterns of transition into adulthood (for instance the average age at adulthood is 17.77 years in the benchmark and 21.34 in the alternative one). All the other schedules (rates of transition from the adult state to the married state, fertility rates of the married women, rates of transition from the married state to the residual one, and the mortality rates) are kept equal in both scenarioes. In particular the rates of transition from the adult state to the married state, and the fertility rates of married women, are estimated[13] from the old cohort (fig. 3). The remaining data, not available from the FFS (mortality rates, separation and divorce rates, and rates of transition to widowhood) come from other sources (Istat (1996)).

The event of becoming adult has been defined here as the first occurring event among the following three: i)leaving formal education, ii)leaving the parental home, or iii)attaining the age of 35. We however assume that it is not possible to become adult before the age of 15, also because the lowest legal age at marriage is 16. Hence, transitions occurring before the 15th birthday or later than the 35th have been assigned to the corresponding birthday.

Fig. 4 represents the distribution of the population across the four states (young, adult, married, residual) in the stationary population, corresponding to the overall set of transition rates estimated for the older cohort. The average ages in the four groups in the stationary population are respectively 10.6 years for the young, 37.2 for the adult, 50 years for married and 64.7 for widows and divorced. Moreover, of the 80.84 years lived on average by a woman, 18.98 are lived in the young state, 11.4 in the adult one, 35.73 in the married one and 14.73 in the residual state. Let us now move to the application of the our multistate stable model.

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[12] The same cohort definition was used for instance by Geremei (1999).
[13] Our estimates use persons-year, in order to avoid censoring problems, and the linear integration hypothesis.

As initial age structure in our simulation we used the age distribution observed at the 1991 Census of the italian population. The 1991 observed figures were then subdivided with respect to the four states according to the corresponding figures observed in the stationary population. The results of our macrosimulations run are reported below. The *age specific fertility rates* corresponding to the two scenarios are represented in fig. 5. They give TFR of 1.841 for the benchmark, and of 1.412 for the alternative scenario. Thus, the fertility effect of an about three and a half years delay in the transition to adulthood results in the long run in a 0.429 reduction of the TFR, *coeteris paribus*. Fig. 6 illustrates the convergence to the stable state in the two scenarioes in terms of the respective births (fig. 6). The long term effects on age structure, which usually is the main concern of stable simulations, is represented in fig. 7. We may notice, in particular, that the ratio of people aged 65 or over, on people younger than 20 is almost doubled in the alternative scenario (from 1.20 to 2.22). As Italy has a *pay-as-you go* pension system, and many debates presently concern the long-term problem of its sustainability, it is interesting to note that the ratio between people aged 65 or more ("retirement" ages) and people aged 20-64 ("working" ages) shifts from 47% to 64%. As far as we know this is the first (even if simple) evaluation of the impact of the delayed transition into adulthood on the long-term equilibrium of the Italian pension system. Finally, the ratio of the population aged *20-64* on the remaining population decrease from *1.16* to *1.09*.

4. Conclusions and directions for future research

This paper has been concerned with the macro-demographic consequences of delays in the transition into adulthood within a multistate stable population framework. We believe that for what concerns the field side on transition into adulthood, a lot of work has already been done (see Billari 1999 for an up-t-date review of the literature). Vice-versa much can still be done, in our opinion, on the theoretical side, in the spirit of our section 2.3 and of Inaba (1996). Moreover, it seems promising to consider more general multistate models, such as reversible models, embedding the effects of remarriage, which can become highly relevant, and of the existence of different routes to marriage. Finally, it appears desirable in a more systematic manner two-sex frameworks, although this would totally limit the possibility for analytical results. Possible dramatic effects of changing patterns of transition into adulthood for the management of pension systems seem, as outlined in this paper, seem to be a further area of growing interest. In a forthcoming paper we intend to investigate the impact on the italian pension system of changing patterns of transition into adulthood jointly with immigrations.

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Figure 1. Linear relations between age at union and TFR and between age at adulthood and age at union.

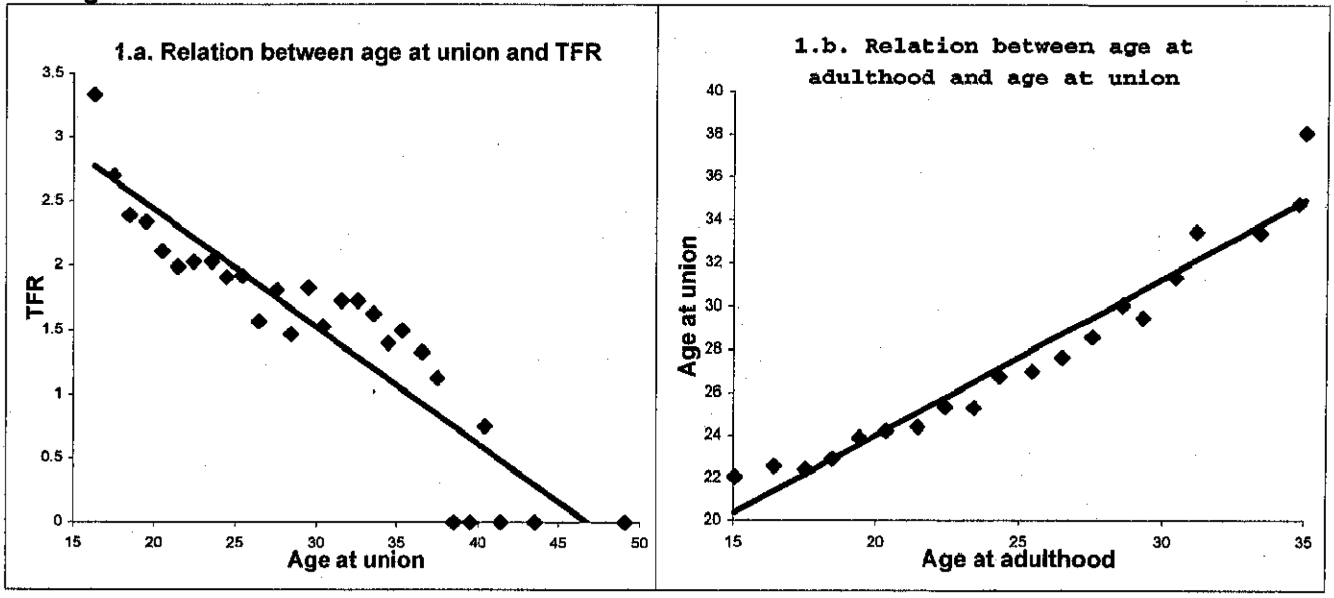

Figure 2. Rates of transition into adulthood by age. Two cohorts.

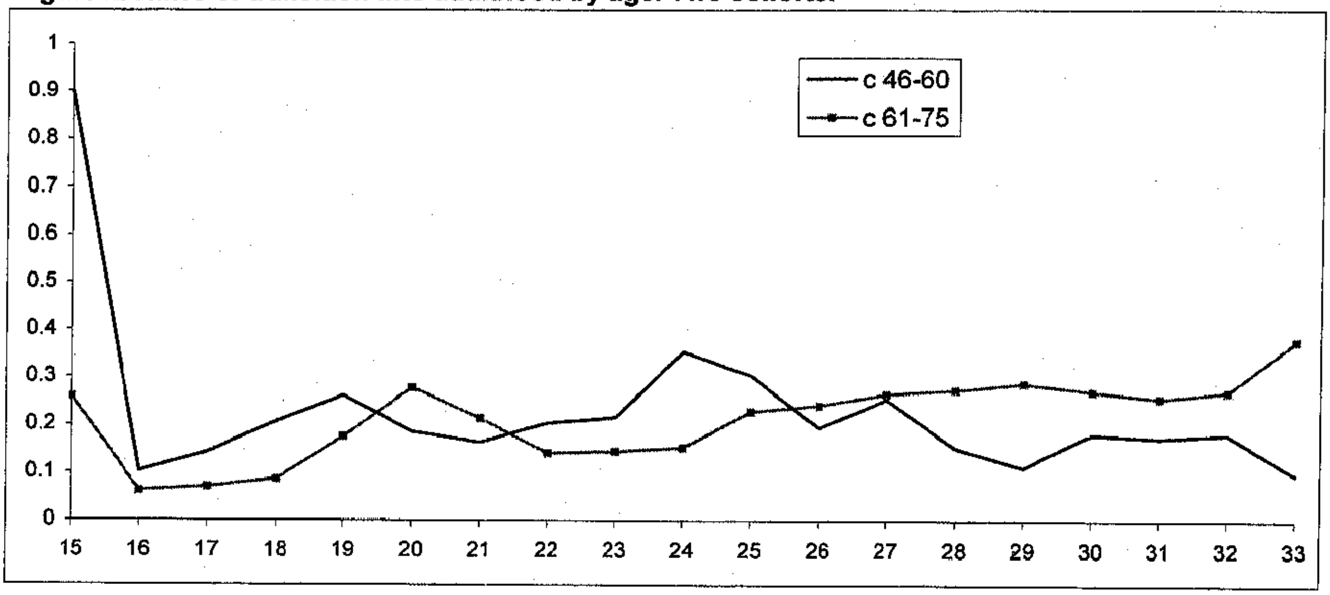

Figure 3. Rates of transition from the adult to the married state and fertility rates of married by age.

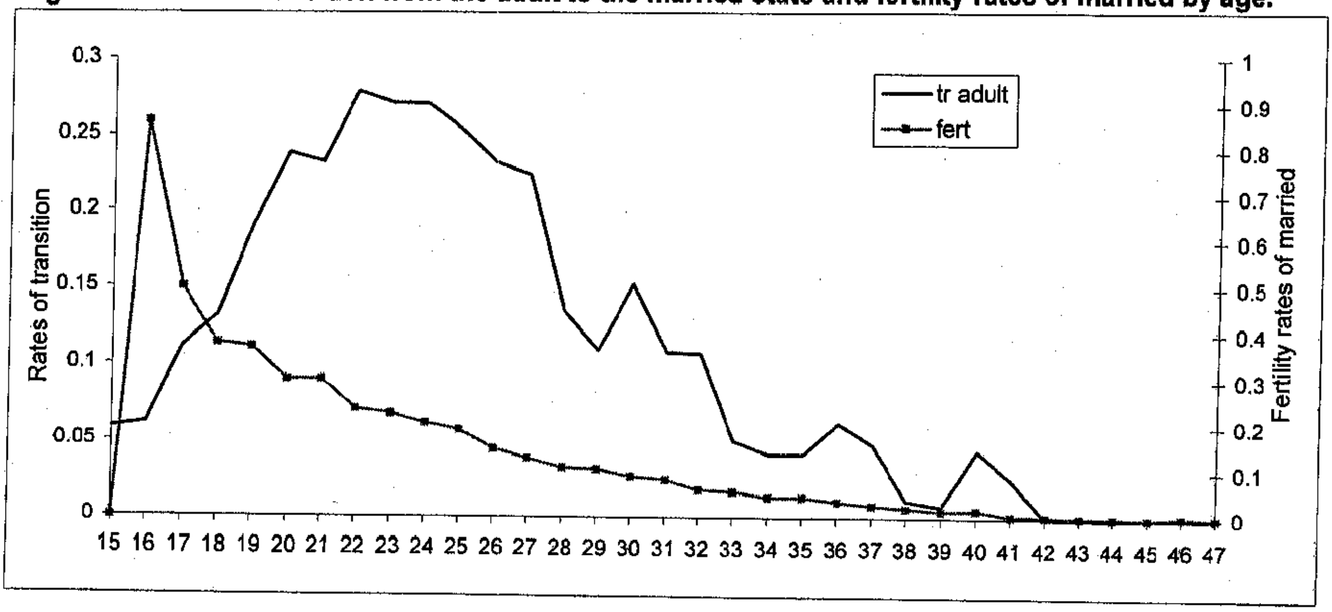

Figure 4. State distribution by age in the stationary population hypothesis.

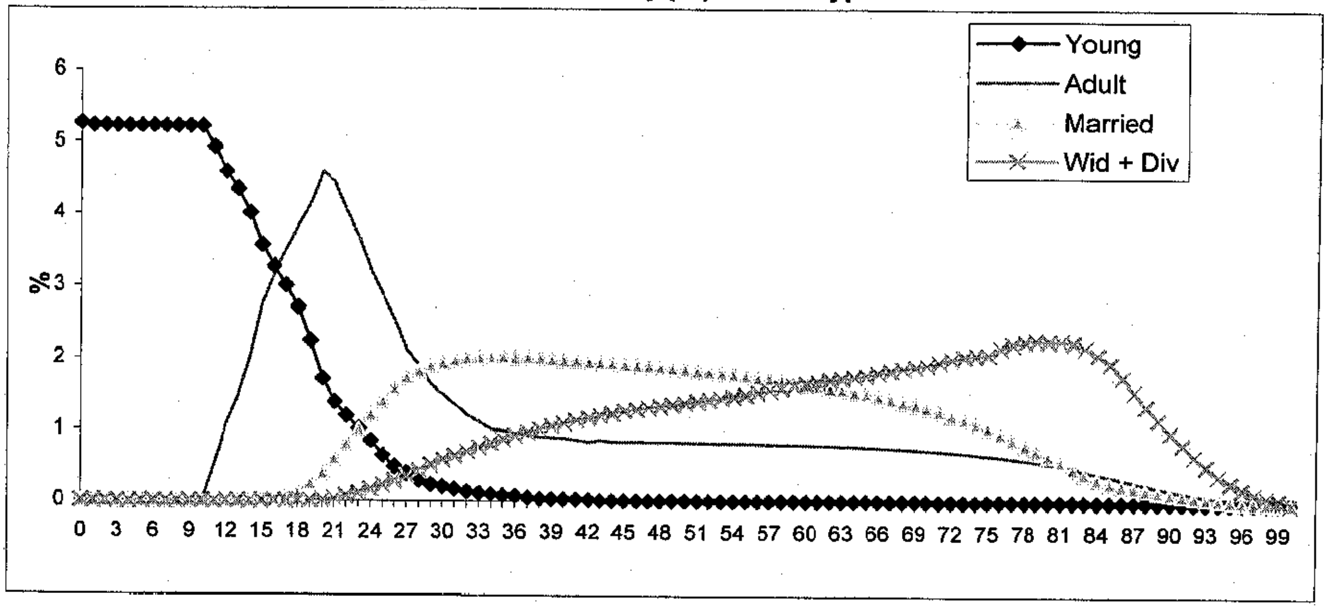

Figure 5. Stable state. Age specific fertility rates in the two scenarios.

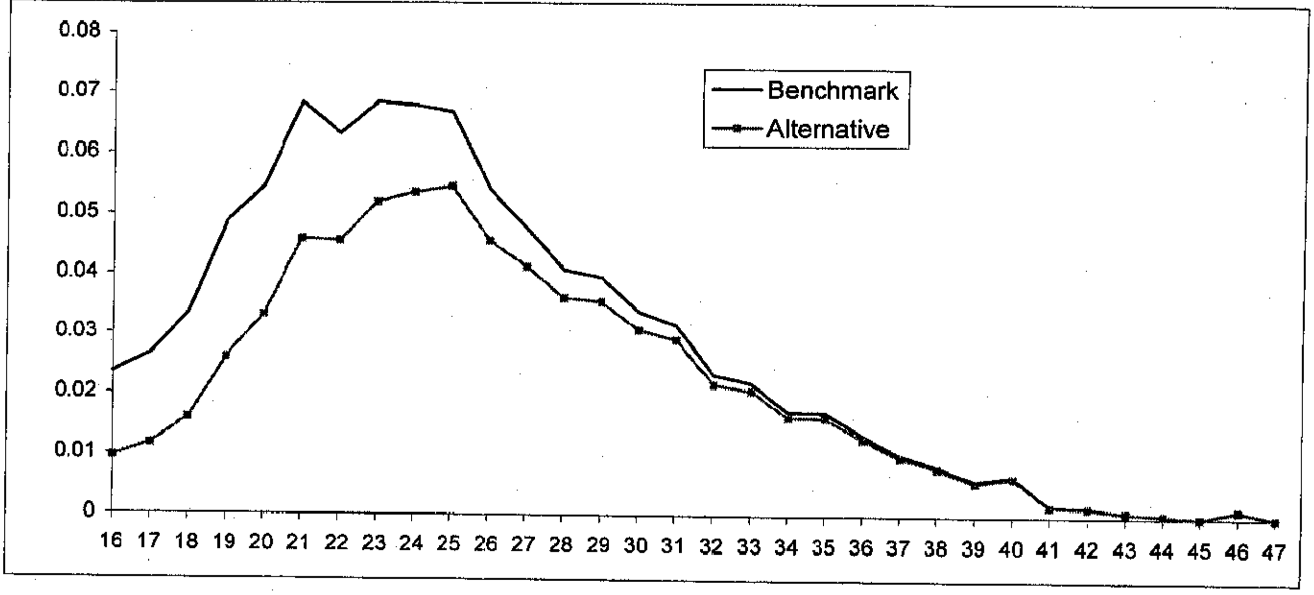

Figure 6. Convergence to the stable state. Yearly births according to the two scenarios

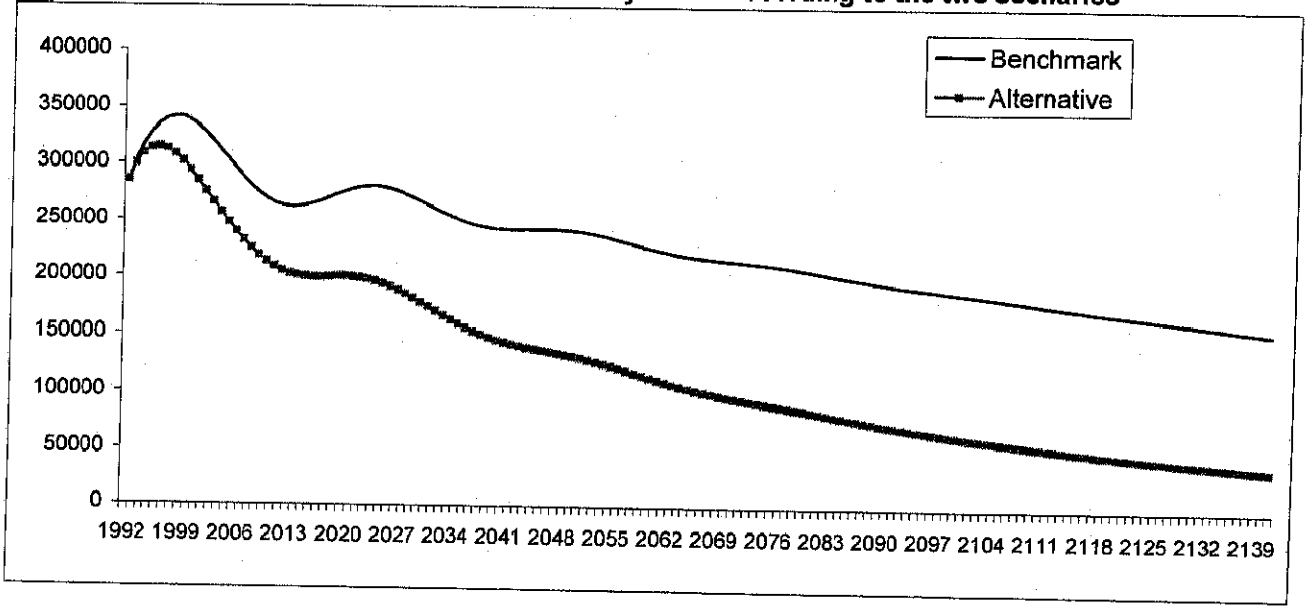

Figure 7. Stable population. Ergodic age structure according to the two scenarios.