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Becoming Adult and its Macro-Demographic Impact: Multistate Stable Population Theory and an Application to Italy

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Abstract

A multistate generalisation of classical one-sex stable population theory is used in the paper, in order to evaluate structural and long-term effects of changes in the pace of adulthood attainment. The demographic framework that inspires the paper is the Italian case, where a strong delay in the transition to adulthood and union formation has been observed during the last decades. Moreover, Italy has experienced very low fertility levels in the last fifteen years and the subsequent ageing problems have become of primary concern. We first discuss a theoretical framework, based on a generalisation of the stable population model via first marriage developed by Inaba (1996) to include the process of transition to adulthood as well. Then, we present some interesting specifications of the general model. Two distinct empirical applications based on the model, one via macrosimulation and the other one via linear approximation are also presented. The applications principally aim at evaluating the impact of delay in adulthood attainment on fertility and population reproduction, and on the age structure of the population. Finally, we give some concluding remarks.

Keywords: stable population theory, multistate demography, transition to adulthood, population ageing, macrosimulation.

1. Introduction[1]

The sparkling ideas for this paper arise from two different starting points. The first one is a theoretical need: we want to assess the use of classical mathematical demography models, such as stable population in a multistate framework, in order to evaluate the macro-demographic impact of changes in micro-level behaviour. The second starting point is an explanatory necessity, which arises from the observation of demographic behaviour in Italy during the last twenty years. With respect to that, we try to answer one of the open research questions in the transition to adulthood and population dynamics: to outline the 'macro' consequences of changes in the process of transition into adulthood for the overall evolution of a population[2].

We shall discuss a little bit more in depth the two points we mentioned in the following part of this introduction. In section 2, we will set up the mathematical framework of a multistate model embedding the process of transition to adulthood. The model will be used to attack the questions outlined in the introduction. In section 3, some selected subcases of our model will be developed analytically. Section 4 will introduce to the empirical applications of the model: we discuss the interesting case of Italy, using survey and official data. A macrosimulation perspective will be adopted for the application in section 5: there, we will evaluate the effect of delayed adulthood attainment on the whole population dynamics. A simple linear relationship will be used in section six, in order to evaluate the impact of delayed transition into adulthood on the overall fertility level. Finally, some conclusions are given and directions for future research are traced.

1.1. Is there still any room left for stable population theory in the demography of Western countries?

Let us give an example of a research question central to our purposes, that we will develop later in the paper: what are the consequences of a postponement in the process of transition to adulthood on the overall fertility level? And what are the consequences on the long-term age structure of the population, and indirectly on the equilibrium of a pay-as-you-go pension system? How can we organise a technical toolkit to answer such questions?

The use of event history analysis, which has gained an impressive diffusion in population studies during the last years, put the foundations for an effective empirical study of

[1] Earlier versions of this paper were presented at the workshop on Synthetic Biographies, S. Miniato (Italy), 6-9 June 1999, and at the session 'Explaining population dynamics: the use of theories' at the European Population Conference, The Hague (The Netherlands), 30 August-4 September 1999. The authors warmly thank the participants for several useful comments and questions, and especially Herwig Birg, Jan Hoem, Evert Van Imhoff and Zeng Yi. Usual caveats apply.

[2] The importance of the issue is emphasised – with reference to the American case – by Marini (1984).

demographic behaviour at the micro level. When one needs to evaluate the impact of aggregate factors and of individual demographic behaviour, on the experience of an individual (say, how the *macro* and the *micro* affects the *micro*), such techniques are certainly the best solution that can be adopted. On the contrary, it is difficult to use such approaches when one wants to outline the emergency of macro-effects of changes in micro-level behaviour, say, when one wants to "close the bathtub" of social theory (see Coleman, 1990, Ch. 1). Moreover, new micro approaches seem to have almost no connections with something that was at the heart of demography some decades ago: the study of population dynamics. In other words, by using event history analysis only, it is difficult to evaluate how individual behaviours have an impact on the macro-level, whereas the latter level was the central one in classical stable population modelling.

It has been underlined elsewhere (see e.g. Gilbert and Troitzsch, 1999) that simulation is the main tool to provide answers to complex questions on social systems, especially when one is interested on the effects of the behaviour of single interacting agents. We agree upon that point, and approaches such as microsimulation, macrosimulation or artificial societies may give fundamental keys to answering questions such as the ones just mentioned. However, we feel that simulation could be much more informative when accompanied by some more analytically oriented approach. In our case, the mathematical toolkit we have is the well-known stable population theory, in its multistate generalisation. Here, while the parameters of the dynamics of stable population theory may – though in a very simple way – represent the *micro* (synthesising the behaviour of the individuals belonging to the population we are interested in), the outcomes of the models may throw important lights on the *macro*. This is the perspective we adopt in this paper and the theoretical side owes much inspiration to Inaba (1996). It is also necessary to precise that, when it comes to empirical applications based on real data, stable population theory has no substantial borders with macrosimulation. In fact, one of the straightforward empirical implementation of the mathematical model we are developing is via macrosimulation.

1.2. The Italian demographic scenario: delayed adulthood attainment and union formation, lower fertility

Let us now sketch the specific demographic setting we have in mind. In some Western European countries of the Mediterranean Area, of which Italy is a peculiar example, the process of family formation, which takes course generally within a legal marriage, has significantly slowed during the last decades. The transition to adulthood is there also characterised by a strong synchronisation between marriage[3] and leaving the parental home. Moreover, the length of full-time education for the young adults has been steadily increasing, especially for women. Some studies on the process of transition into adulthood suggest that there are typical sequences between events in early adulthood (e.g. Corijn, 1996), even if it is unclear what is the impact of aggregation on such ideas (Billari, 1999). In any case, having left full-time education, or at least having left the parental home seems to be a necessary condition in order to enter a steady (married or unmarried) cohabiting partnership. Then, being in a steady cohabiting partnership seems to be an almost necessary condition in order to become a parent, and this is particularly true in Southern European countries. It is thus of high interest to evaluate the long-term and macro impacts of changes in adulthood attainment in this context.

Here, we adopt the hypothesis that there is a main 'marker' in the adulthood attainment process that gives rise to a passage to a state that we may call the 'marriageable' one – following the seminal ideas of Coale and McNeil (1972) – or, less specifically, the 'adult' state. In general, we might say that there is a main marker distinguishing young people (we mean, people that do not consider entering a union as an option) from adult people (we mean, people that consider entering a union as an option, or that may have ever been in a union). Coale and McNeil state that "in contemporary populations of Western European origin (...) we may conjecture that the age of becoming marriageable is the age at which serious dating, or going steady begins".

In this paper, as empirical marker of the transition to adulthood, we take a specific event, namely the first occurring event between the end of formal education and the leaving of the parental home. Adulthood attainment is considered as irreversible: this is in the spirit of what has been called the Southern European pattern (Reher, 1998). We shall spend some words here in order to give some justifications for the choice of markers. As for the end of formal education, this seems to us a sensible marker because:

1) sooner or later education will be completed, and this means that we may assume a unity quantum and just study the tempo of the event;

2) as Blossfeld and De Rose state, about Italy, "finishing education is expected to count as one of the important prerequisites for entering into adulthood status, and thereby entering into marriage and parenthood"[4].

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[3] During the paper, we will be using the word "marriage" and "union" as interchangeable, and "marriageable" will be also considered as "willing to enter any union". The low cohabitation intensity in Italy justifies this approach. However, the focus of the model is in fact on unions.

[4] A framework where the end of formal education is a necessary condition for entering a 'marriageable' state may also well apply to developing countries.

In addition, when people leave the parental home before the completion of full-time education, we may say that in a sense they may become more ready for unions (and we mainly here refer to informal unions).

2. The theoretical framework: a multistate stable population model with adult state

The theoretical framework considered here is represented by a one-sex four states stable population model with irreversible transitions[5], which generalizes Inaba's (1996) model with reproduction via first marriage. Multistate stable population models arose in demography as developments of the classical Rogers' multiregional model (see Rogers, 1975). Instances of previous applications of such models in demography are Schoen (1988), Inaba (1988), Ledent (1988), Rogers (1990). As represented in the flow diagram (fig. 1), young individuals can not marry, since they do not become 'marriageable' until they become adult. Moreover only married individuals do reproduce. This scheme, as we mentioned, is apt to represent the patterns of Southern European countries.

[Figure 1 about here]

This section is mainly theoretical: we first introduce (subsection 2.1) our general model, which recognises both the chronological age and the duration of permanence in each state, and report some basic results concerning its stable state. Subsequently (section 2.2) we show how the formulas for the reproductivity indices in presence of multiple duration-dependencies, extend to the present case.

2.1 Formulation of the general model with multiple duration-dependencies and its long-term solution: stable distributions with respect to age and stages of life

Let $p_0(a,t)$ denote the density of 'young' (female) individuals aged a at time t, $p_1(c,t;\eta)$ the density at time t of adult individuals who entered into adulthood since c years, at the age of η, $p_2(\tau,t;\eta,\xi)$, the density of individuals married since τ years, who became adult at the age of η and married at the age of ξ, $p_3(a,t)$ the density of individuals who are not in the first three states (widowed, divorced or remarried). Moreover, let $\mu(a)$ denote the age dependent mortality rate (or force of mortality), $\nu(a)$ the rate of transition into adulthood, $\lambda(a,\eta)= \lambda(c+\eta,\eta)$ the force of first marriage, which is assumed to be influenced by the age of entrance into adulthood, and, $\delta(\tau,\xi)$ the force of dissolution of pairs married τ years before when the female was aged ξ.

[5] Inaba's has considered both the irreversible case (young→married→divorced), with reproduction via first marriage (Inaba 1996) and a reversible case, with iterative marriage (1993). We also invite the reader to refer to his paper for the past relevant mathematical literature on multistate stable population theory.

The backbone of the model is given by the following system of Von Foerster equations:

$$\begin{cases} \Delta_{a,t} p_0(a,t) = -[\mu(a) + v(a)] p_0(a,t) \\ \Delta_{c,t} p_1(c,t;\eta) = -[\mu(c+\eta) + \lambda(c+\eta,\eta)] p_1(c,t,\eta) \\ \Delta_{\tau,t} p_2(\tau,t;\eta,\xi) = -[\mu(\xi+\tau) + \delta(\tau,\xi)] p_2(\tau,t;\eta,\xi) \\ \Delta_{a,t} p_3(a,t) = \iint\limits_{\tau+\xi=a} \delta(\tau,\xi) p_2(\tau,t;\eta,\xi) d\tau d\eta - \mu(a) p_3(a,t) \end{cases} \quad (1)$$

where $\Delta_{a,t}$ is a shortcut for the aging operator $\frac{\partial}{\partial a} + \frac{\partial}{\partial t}$ and similarly $\Delta_{c,t}, \Delta_{\tau,t}$. The system (1) has to be completed with the boundary conditions:

$$\begin{cases} p_0(0,t) = B(t) = \int_0^{\beta} \int_{\eta}^{\beta} \int_0^{\beta-\xi} p_2(\tau,t;\xi,\eta) m(\tau;\xi,\eta) d\tau d\xi d\eta \\ p_1(0,t;\eta) = v(\eta) p_0(\eta,t) \\ p_2(0,t;\eta,\xi) = \lambda(\xi,\eta) p_1(\xi-\eta,t;\eta) \\ p_3(0,t) = 0 \end{cases} \quad (2)$$

The conditions (2) say respectively that

i) the number of individuals aged zero in the first state at time t is simply the number $B(t)$ of births at time t, β is the upper bound of the fertile age span, and $m(\tau;\xi,\eta)$ the marital fertility rate at marriage duration τ for a married women who entered the adult state at age η and then married at age ξ ($\xi>\eta$);

ii) the number of adult women with duration zero of permanence in the adult state and chronological age at adulthood η at time t, is given by the number of individuals aged η at time t who became adult;

iii) the number of individuals with zero marriage duration, who entered the adult state at age η and married at ξ is given by the number of marriages of $p_1(c,t;\eta)$ individuals at the age $\xi=\eta+c$.

Finally a set of prescribed initial distributions, has to be assigned:

$$p_0(a,0) = H_0(a) \quad p_1(c,0;\eta) = H_1(c,\eta) \quad p_1(\tau,0;\eta,\xi) = H_2(\tau;\eta,\xi) \quad p_3(a,0) = H_3(a) \quad (3)$$

We postulated that the fertility rates depend not only on the duration of marriage, but also on the age at marriage, as in Inaba (1996), and on the age of transition into adulthood. The latter appears to be a reasonable point: in general, other things being equal, married individuals characterized by the same marriage duration and by the same age at marriage may be expected to have different fertility if they entered sooner the adult state, as they probably experienced different life histories in terms of educational attainment and job experiences, compared to whose who entered later.

The mathematical treatment of the model (1)-(2)-(3) is a tedious but straightforward task, similar to that Inaba (1996) did for the three states model. Since we are not interested here in the mathematical aspects, we omit most of the technical details (see Manfredi and Billari 1997), and simply recall the most relevant steps of the solution process. As a first step it is possible to show that the overall age distribution: $n(a,t)=\sum_{i=0}^{3} p_i(a,t)$ satisfies a traditional Von Foerster partial differential equation (PDE). This makes it possible to reformulate the problem into a Lotka-type renewal model in the (chronological) age-time domain, governed by a traditional integral equation for births. This permits to show the existence of an asymptotic stable behaviour in Lotka's sense, characterised by stable exponential growth over time of $B(t)$ when the net reproduction rate (NRR), R_0,of the population is greater than one and, vice-versa, by stable exponential decay when $R_0<1$. The existence of a stable distribution with respect to age and stages of life is easily proved by expressing the formal solutions of the equations (1) for t large 'enough' (where 'enough' means that the terms depending on initial conditions vanish), in terms of the overall density $n(a,t)$. It holds:

$$\begin{aligned} p_0(a,t) &= B(t-a)l(a)V(a) = n(a,t)V(a) \\ p_1(c,t;\eta) &= n(a,t)V(\eta)v(\eta)\Lambda_\eta(c) \\ p_2(\tau,t;\eta,\xi) &= n(a,t)\left[V(\eta)v(\eta)\Lambda_\eta(\xi)\lambda_\eta(\xi,\eta)\Delta_\xi(\tau)\right] \end{aligned} \tag{4}$$

The relations (4) express the populations densities in the states *0,1,2* in terms of the total population $n(a,t)$[6] and of the following generalised survival functions:

$$\begin{aligned} l(a) &= exp\left(-\int_0^a \mu(u)du\right) \\ V(a) &= exp\left(-\int_0^a v(u)du\right) \\ \Lambda_\eta(c) &= exp\left(-\int_\eta^{\eta+c} \lambda(u;\eta)du\right) = exp\left(-\int_0^c \lambda(\eta+s;\eta)ds\right) \\ \Delta_\xi(\tau) &= exp\left(-\int_0^\tau \delta(u;\xi)du\right) \end{aligned} \tag{5}$$

where

i) $l(a)$ is the 'survival to natural mortality' function (the probability that a new-born woman survives to age a);

ii) $V(a)$ is the 'survival to adulthood' function (the probability that a newborn woman does not enter adulthood before age a in absence of mortality);

iii) $\Lambda_\eta(c)$ is the (conditional) 'survival to marriage' function (the probability that a woman who entered the adult state at the age η is still unmarried at age $a=\eta+c$);

[6] They represent the formal solutions to the corresponding PDE's (6) for t>a. Such solutions are "formally" derived by integration along characteristics of the given PDE's, but they can be derived in a totally direct way by sequentially applying the survival functions (5).

iv) $\Delta_\eta(c)$ the survival function to divorce until age $a=\tau+\xi$ for a woman married at age ξ.
Suitable integrations on the quantities (4) lead to:

$$\begin{aligned}
&p_0(a,t)=n(a,t)V(a)\\
&p_1(a,t)=n(a,t)\Psi(a);\quad \Psi(a)=\int_0^a V(\eta)\nu(\eta)\Lambda_\eta(a-\eta)d\eta \qquad (6)\\
&p_2(a,t)=n(a,t)\Gamma(a);\quad \Gamma(a)=\int_0^a\int_0^\xi\left[V(\eta)\nu(\eta)\Lambda_\eta(\xi)\lambda_\eta(\xi,\eta)\Delta_\xi(a-\xi)\right]d\eta d\xi
\end{aligned}$$

The relations (6) show that when the total population $n(a,t)$ reaches its stable age distribution, it will automatically have a stable distribution by stages of life as well.

2.2.Indices of reproductivity (and structure)

In what follows, we provide some formulas for the Net Reproduction Rate (NRR) and the Total Fertility Rate (TFR), by generalising Inaba's (1996) work to the present four state framework. For what concerns the NRR, the following equivalent definitions hold

$$\begin{aligned}
R_0 &= \int_0^\beta\int_\eta^\beta\int_0^{\beta-\xi} m(\tau;\xi,\eta)l(\xi+\tau)V(\eta)\nu(\eta)\Lambda_\eta(\xi)\lambda(\xi,\eta)\Delta_\xi(\tau)d\tau d\xi d\eta\\
&=\int_0^\beta\int_0^{\beta-\tau}\int_0^\xi m(\tau;\xi,\eta)l(\xi+\tau)V(\eta)\nu(\eta)\Lambda_\eta(\xi)\lambda(\xi,\eta)\Delta_\xi(\tau)d\eta d\xi d\tau \qquad (7)\\
&=\int_0^\beta\int_0^\xi\int_0^{\beta-\xi} m(\tau;\xi,\eta)l(\xi+\tau)V(\eta)\nu(\eta)\Lambda_\eta(\xi)\lambda(\xi,\eta)\Delta_\xi(\tau)d\xi d\eta d\tau
\end{aligned}$$

The change of variable from the 'old' set (η,ξ,τ) to the 'new' one $(\eta,\xi,a=\xi+\tau)$, leads to the following alternative definition:

$$R_0=\int_0^\beta l(a)\left[\int_0^a V(\eta)\nu(\eta)\left(\int_\eta^a\Lambda_\eta(\xi)\lambda(\xi,\eta)\Delta_\xi(\tau)m(\tau;\xi,\eta)d\xi\right)d\eta\right]da \qquad (8)$$

The (8) suggests the following definition for the age-specific fertility rates:

$$m^*(a)=\int_0^a V(\eta)\nu(\eta)\left(\int_\eta^a\Lambda_\eta(\xi)\lambda(\xi,\eta)\Delta_\xi(\tau)m(\tau;\xi,\eta)d\xi\right)d\eta \qquad (9)$$

The definition (9) relates the traditional age specific fertility rate with the duration-specific fertility rates. If we explicitly neglect mortality, the following definition for the TFR (total number of offsprings produced by a woman during her 'effective' fertile age span, the latter given by the portion of the her fertile age span actually spent within the married state) arises:

$$TFR=\int_0^\beta\int_\eta^\beta\int_0^{\beta-\xi} m(\tau;\xi,\eta)V(\eta)\nu(\eta)\Lambda_\eta(\xi)\lambda(\xi,\eta)\Delta_\xi(\tau)d\tau d\xi d\eta \qquad (10)$$

By stressing the sequential nature of the stages of life, we can write:

$$\begin{aligned}
TFR&=\int_0^\beta V(\eta)\nu(\eta)\left[\int_\eta^\beta\Lambda_\eta(\xi)\lambda(\xi,\eta)T(\xi,\eta)d\xi\right]d\eta=\\
&=\int_0^\beta T^*(\eta)V(\eta)\nu(\eta)d\eta
\end{aligned} \qquad (11)$$

where:

$$T(\xi,\eta)=\int_0^{\beta-\xi} m(\tau;\xi,\eta)\Delta_\xi(\tau)d\tau \quad;\quad T^*(\eta)=\int_\eta^\beta\Lambda_\eta(\xi)\lambda(\xi,\eta)T(\xi,\eta)d\xi \qquad (12a,b)$$

The quantity $T(\xi,\eta)$ is the conditional TFR of those women who became adult at the age of η and married at the age of ξ in presence of the risk of marriage dissolution. For what concerns the quantity $T^*(\eta)$ we can write:

$$T^*(\eta) = PEM(\eta)\cdot\int_\eta^\beta T(\xi,\eta)\Phi_\eta(\xi)d\xi \qquad (13)$$

where:

$$\Phi_\eta(\xi) = \frac{\Lambda_\eta(\xi)\lambda(\xi,\eta)}{\int_\eta^\beta \Lambda_\eta(\xi)\lambda(\xi,\eta)d\xi} \quad 0\le\eta\le\beta;\ \eta\le\xi\le\beta \qquad (14)$$

represents the conditional (normalized) density function of marriage at age ξ for the women who entered the adult state at age η and married before the end of the fertile age span, and:

$$PEM(\eta) = \int_\eta^\beta \Lambda_\eta(\xi)\lambda(\xi,\eta)d\xi = 1-\Lambda_\eta(\beta) \qquad (15)$$

is the (conditional) proportion of ever married (PEM) women (to be precise: who married before age β) among those women who entered the adult state at the age of η. By further defining:

$$W(\eta) = \frac{V(\eta)v(\eta)}{1-V(\beta)} \quad ; \quad \int_0^\beta V(\eta)v(\eta)d\eta = 1-V(\beta) = PEA(\beta) \qquad \text{(16a,b)}$$

where $W(\eta)$ is the normalised probability density of transition into adulthood (truncated at the end of the fertile age span) and $PEA(\beta)$ is the 'proportion ever adult' at the end of the fertile period, we can write the TFR as:

$$TFR = PEA(\beta)\int_0^\beta T^*(\eta)W(\eta)d\eta \qquad (17)$$

By introducing (13) into (17) and assuming that, quite reasonably, $PEA(\beta)=1$, we obtain:

$$TFR = \int_0^\beta PEM(\eta)W(\eta)\left[\int_\eta^\beta T(\xi,\eta)\Phi_\eta(\xi)d\xi\right]d\eta = \int_0^\beta PEM(\eta)W(\eta)\bar{T}(\eta)d\eta \qquad (18)$$

where $\bar{T}(\eta)$ is the TFR (averaged with respect to the age at marriage) of women entered into adulthood at age η and married before the end of the fertile age span. $\bar{T}(\eta)$ is a conditional average TFR (conditional on being entered into adulthood at age η). It seems useful, at this stage, to distinguish two main subcases:

a) the fertility rates do not depend on the age of entrance into the adult state, i.e.: $m = m(\tau;\xi)$. We call this case the 'pure delayer' case, in that the role of the intermediate state of adult does not affect the fertility behaviour of the (married) population: its effects are essentially those of delaying fertility by delaying marriage.

b) a more general case in which $m = m(\tau;\xi,\eta)$, so that in addition to the 'delayer' effect, true fertility effects arise due to the process of transition into adulthood.

This latter distinction will be used in this paper, when dealing with empirical applications.

3. Some noteworthy specifications of the general model

In this part, we analytically develop two interesting specifications of the general model. The first one – with transition rates independent on age – will help us in gaining a better understanding of the model from the mathematical point of view. The second one will provide a linear approximation, which will become useful for the application to real data.

3.1 Transition rates independent on age

Here we consider the special case of age-independent rates. Although oversimplified, this case is useful from several standpoints: it helps to clarify the role of 'sequential' stages within classical stable population theory, and it represents an instance of explicit parameterisation within our general framework. Let us $p_i(t)$ (i=0,1,2,3) respectively denote the total numbers of young, adult, married and 'residual' individuals at time t. Moreover, let μ_i's be the (age-independent) state specific death rates per unit time (p.u.t. since now on), v the (age-independent) rate of transition into adulthood p.u.t., λ the marriage rate p.u.t of adult individuals, δ the total rate of marriage dissolution, and m the marital fertility rate. When all the rates in the general model (1) are constant, the general PDE system (1) reduces to the following system of linear ordinary differential equations (ODE since now on):

$$\begin{aligned} \dot{p}_0(t) &= mp_2(t) - (\mu_0 + v)p_0(t) \\ \dot{p}_1(t) &= vp_0(t) - (\mu_1 + \lambda)p_1(t) \\ \dot{p}_2(t) &= \lambda p_1(t) - (\mu_2 + \delta)p_2(t) \\ \dot{p}_3(t) &= \delta p_2(t) - \mu_3 p_3(t) \end{aligned} \qquad (19)$$

The system (19)[7] may be represented compactly as $\dot{P}(t) = MP(t)$ where M is the matrix:

$$M = \begin{pmatrix} -(\mu_0 + v) & 0 & m & 0 \\ v & -(\mu_1 + \lambda) & 0 & 0 \\ 0 & \lambda & -(\mu_2 + \delta) & 0 \\ 0 & 0 & \delta & -\mu_3 \end{pmatrix} \qquad (20)$$

The demographically relevant features of system (19) are easily inferred from those of the matrix M, which is a Metzler matrix. So, in particular, M has a unique dominant eigenvalue K_0, to which a demographically meaningful (i.e.: non negative) eigenvector belongs. Furthermore, all remaining eigenvalues of M have a real part which is less than K_0.

[7] Systems as (1) are very common in population biology and population dynamics of infectious diseases, see for instance Anderson and May (1991).

It is easy to see that the sign of the dominant eigenvalue K_0, which corresponds to Lotka's intrinsic rate of growth of the population and hence determines its long-term behaviour, only depends on the sign of the coefficient of the known term of the characteristic polynomial P(K):

$$P(K) = (K + \mu_3)\left(K^3 + aK^2 + bK + c\right) = 0$$

where:

$$\begin{aligned} a &= (\mu_0 + v) + (\mu_1 + \lambda) + (\mu_2 + \delta) > 0 \\ b &= (\mu_0 + v)(\mu_1 + \lambda) + (\mu_0 + v)(\mu_2 + \delta) + (\mu_1 + \lambda)(\mu_2 + \delta) > 0 \\ c &= (\mu_0 + v)(\mu_1 + \lambda)(\mu_2 + \delta) - mv\lambda \end{aligned} \tag{21}$$

Hence, K_0 will be positive or negative (i.e. we will have stable exponential growth rather than stable exponential decay) depending on whether:

$$mv\lambda - (\mu_0 + v)(\mu_1 + \lambda)(\mu_2 + \delta) \begin{matrix} \geq \\ = \\ \leq \end{matrix} 0 \tag{22}$$

The threshold condition (22) can be represented as:

$$R_0 = \frac{m}{\mu_2 + \delta} \cdot \frac{v}{\mu_0 + v} \cdot \frac{\lambda}{\mu_1 + \lambda} > 1 \tag{23}$$

where R_0 defines the NRR of the population for this unstructured case (see Manfredi and Billari 1997). This last result clearly shows that in presence of below replacement fertility no policy aimed to take fertility up to the 'zero growth level', which is based on a reduction of the age at marriage and/or of age to adulthood can be completely successful. Vice-versa, by suitably acting on such parameters can reveal to be an effective policy for taking down to stationarity a population experiencing stable growth. An important example could be for instance a policy of systematically raising alphabetisation and education in developing countries.

The long term age structure by state which is implicit in model (19) can be determined analytically (see for details Manfredi and Billari, 1997). In the standard case in which mortality is state-independent (i.e.: $\mu_i = \mu$ for all i), the population weights $w_i(a) = p_i(a,t)/n(a,t)$ which emerge in the long term stable regime are given by

$$\begin{aligned} w_0(a) &= e^{-va} \\ w_1(a) &= \frac{v}{v - \lambda}\left(e^{-\lambda a} - e^{-va}\right) \\ w_2(a) &= \frac{v\lambda}{v - \lambda} e^{-\delta a} \left[\frac{1 - e^{-(\lambda - \delta)a}}{\lambda - \delta} - \frac{1 - e^{-(v - \delta)a}}{v - \delta} \right] \\ w_3(a) &= 1 - \sum_0^2 w_i(a) \end{aligned} \tag{24}$$

A numerical example of the stable population distribution according to (24), on a yearly time scale, with the rates fixed respectively at ν=1/19, λ=1/11, δ=1/36 (corresponding to an average age of transition into adulthood of 19 years, an average duration of permanence in the adult state of 11 years, and an average duration of marriage of 36 years[8]), is given in figure 2.

[Figure 2 about here]

3.2 A linear approximation

By resorting to some linear approximations we derive here some nice formulas relating the impact of changes in the mean age of transition into adulthood and the TFR. Let us consider the 'pure delayer' case (section 2.3), in which the fertility rates of married women do not depend on their age of entry into adulthood. In this case we have, from (12a):

$$T(\xi,\eta)=\int_0^{\beta-\xi} m(\tau;\xi)\Delta_\xi(\tau)d\tau = T(\xi) \quad (25)$$

Let us now assume, following again Inaba (1996), a Henry-type (Henry, 1976) linear relation between the conditional TFR of women married at age ξ, $T(\xi)$, and the age at marriage:

$$T(\xi)=U-V\xi+R(\xi) \quad U>0, V>0 \quad (26)$$

where R is a remainder. From the relation (18) we get, by neglecting the reminder, the approximated relation:

$$TFR = U\int_0^\beta PEM(\eta)W(\eta)d\eta - V\int_0^\beta PEM(\eta)W(\eta)\bar{\xi}(\eta)d\eta \quad (27)$$

where $\bar{\xi}(\eta)$ is the average age at marriage of the women who entered the adult state at the age of η and married before the end of the fertile age span. Let us introduce the quantities:

$$B=\int_0^\beta PEM(\eta)W(\eta)d\eta \qquad W^*(\eta)=\frac{PEM(\eta)W(\eta)}{B} \quad 0\le\eta\le\beta \quad (28a,b)$$

respectively defining the 'average PEM', i.e. the average value of the conditional (on the age of entrance into adulthood) proportion ever married, and the probability density function of transition into adulthood at age η for the women who married before the end of the fertile age span. Thanks to (28a,b) we write (27) as:

$$TFR \cong B\left\{U - V\int_0^\beta \bar{\xi}(\eta)W^*(\eta)d\eta\right\} = B\left\{U - V\bar{\xi}\right\} \quad (29)$$

In (29), the overall TFR is factored as the product of the average PEM times the value of the Henry relationship evaluated in correspondence of the age at marriage $\bar{\xi}$ (averaged on the density of transition to adulthood) of the women married before the end of the fertile age span. If we assume $\bar{\xi}(\eta)$ to be linear, i.e.:

[8] These values are quite reasonable for the italian case.

$$\bar{\xi}(\eta) = \bar{\xi}(\eta_A) + q(\eta - \eta_A) = p + q\eta \quad \eta_A \le \eta \le \beta \qquad (30)$$

where η_A is the lower bound in the possible ages of transition into and $\bar{\xi}(\eta_A)$ the corresponding average age at marriage, (29) leads to[9]:

$$TFR = B\left\{U - V\int_0^{\beta}[p + q\eta]W^*(\eta)d\eta\right\} = B\{U - pV - qV\bar{\eta}\} \qquad (31)$$

The nice formula (31) relates the TFR with the average PEM and the average age at adulthood and could therefore be used to roughly estimate the 'pure delayer' effects of an increase of the average age $\bar{\eta}$ of transition to adulthood of women married before the end of their fertile age span, on the TFR. This operation, when empirically justifiable, needs only that the parameters V and q are estimated.

4. Empirical applications: data from the Italian case

In the following two subsections, we shall discuss some distinct applications based on our mathematical model, both aimed at discussing the impact of changes in adulthood attainment. First, we will perform some macrosimulation on a discrete-time version of the model. Then, we will use the linear approximation. We first give some brief information on the data we use.

To estimate rates of adulthood attainment, union formation and fertility, we use individual data from the 1995/96 Italian Fertility and Family Survey (De Sandre *et al.*, 1996), a retrospective survey on Italian men and women born between 1946 and 1975. The survey contains event history information on the month and the year in which people left their parental home, left full time education, entered the first union, bore children and other demographic events. As we are dealing with a one-sex model, we only use female data. The remaining data, not available from the FFS (mortality rates, separation and divorce rates, and rates of transition to widowhood) come from official statistics at the aggregate level (Istat (1996)).

The event of becoming adult has been defined here as the first occurring event among the following three: i)leaving formal education, ii)leaving the parental home, or iii)attaining the age of 35. We however assume that it is not possible to become adult before the age of 15, also because the lowest legal age at marriage is 16. Hence, transition events occurring before the 15th birthday or later than the 35th have been assigned to the corresponding birthday.

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[9] See Manfredi and Billari (1997) for further details.

4.1 A multistate stable population experiment: simulating the macro-impact of a delayed transition into adulthood

In this section we illustrate a set of long-term (stable) macro-simulations[10]. The aim of this section is to evaluate the long-term and structural consequences arising from a 'pure delay' effect in the transition to adulthood, other things being constant. That means, delayed adulthood attainment can have an impact only via delaying marriages, and marital fertility rates do not depend on the age at which young people become adult. For this purpose, we split the sample of women from the FFS survey into two broad cohorts (the 'old', born 1946-1960 and the 'young', born 1961-1975)[11]. The first cohort provides a 'benchmark' stable scenario, whereas the second, being characterised by a delayed pattern of transition into adulthood (fig. 3), is used to provide an 'alternative' scenario. The average age at adulthood is respectively 17.8 and 21.3 years. Our general idea is to evaluate the effect of a delayed transition to adulthood with a sort of 'comparative statics' approach, by putting side by side the stable distributions that arise from both scenarios.

The only difference between the benchmark and the alternative scenario is thus represented by the patterns of adulthood attainment. All other schedules (rates of transition from the adult state to the married state, fertility rates of the married women, rates of transition from the married state to the residual one, and the mortality rates) are kept equal. In particular, we estimated from the old cohort[12]: transition rates from the adult to the married states (fig. 4), age-specific marital fertility rates (fig. 4), marriage duration-specific marital fertility rates (fig. 5). We also estimated marital fertility rates depend jointly on age and marriage duration, where the latter has only four categories for reasons of stability in estimates (fig. 6).

[Figures 3-6 about here]

As the initial age structure in our simulations we used the age distribution observed at the 1991 Census of the Italian population. The figures observed for 1991 were then subdivided with respect to the four states according to the corresponding figures observed in the stationary population.

Some information on the benchmark scenario comes from the stationary population, corresponding to the overall set of transition rates estimated for the benchmark scenario. That is, the population distribution in the case of zero growth. In this case, the distribution by state of

[10] Wolf (1988) outlines the application of a multistate life table which in fact contains all the necessary tools to fully exploit the mathematical model. He also emphasise the demanding data needs of his approach.

[11] The same cohort definition was used for instance by Geremei (1999).

[12] Our estimates use persons-year, in order to avoid censoring problems, and the linear integration hypothesis.

the overall population is: 21.4% in the young, 9.9% in the adult, 48.5% in the married and 20.6% in the no longer married state. In terms of life expectancy, a woman lives on average 80 years, of which 17.1 are lived as a young, 7.9 as an adult, 38.5 as a spouse, 16.5 in the residual state.

We now compare the benchmark and the alternative scenario with three different simulations, which have been run until the stable structure was achieved with the desired accuracy[13]. Our comments will be focused on the effect on age structure and on reproductivity.

In the first simulation, marital fertility is dependent on age only, as described in figure 4. Figure 7 compares the resulting stable distributions, whereas some synthetic measures are reported in table 1a. Here the delay in adulthood attainment has an extraordinary effect: for instance the stable TFR goes from 1.6 to 1.23, and the ratio of people aged 65 and over to people aged 20-64 goes from 159% to 293%. The mean age of the population increases by more than five years, and also the dependency index is significantly affected. The reason is that, if marital fertility depends on age only, there can be no recuperation for childbearing purposes of the years 'lost' because of delayed transition to adulthood; this has also obvious consequences on population ageing.

Then, we take the opposite point of view, and simulate the model when marital fertility depends on the duration of marriage only, as it is in figure 5. In this case (figure 8, table 1b), the effect of delayed transition to adulthood is less significant. There are still some differences due to the biological limit of reproduction.

The third simulation is in between, uses more information, and is more useful as a representation of real situations. Marital fertility there depends on both age and duration of marriage, where the latter has been split into four categories (figure 6). The results are displayed in figure 9 and table 1c. The stable TFR in this case goes from 1.74 to 1.56: the effect of a 3 ½ years average delay in adulthood attainment is thus of diminishing fertility of around 0.2 children. The impact on the stable structure is indeed important: the mean age of the population raises from 45.3 to 47.6, and the ratio of people aged 65 and over to people aged 20-64 goes from 131% to 167%. The latter results provide us with an important evaluation of the impact of delayed adulthood attainment.

[Figures 7-9 and Table 1 about here]

[13] The simulations were performed with a program written in Visual Basic for Excel by A. Valentini.

4.2 The impact of increases in the mean age at adulthood on fertility: use of the linear approximation

In this section the mathematical relationships (26), (30), (31), developed in subsection 3.2 are used in order to get insights on the effect of changing tempo in adulthood attainment on the reproduction of the population. We only refer here to women aged 40 and over (born 1946-1955) in order to approximately avoid censoring problems.

First of all we follow the approach of Inaba (1996) and use the linear approximation (26): $T(\xi) = U - V\xi$ first proposed by Henry (1976), between the number of children ever born and the age at first union. Though in principle very rough, this is essentially a local approximation, has the merit of being both surprisingly accurate from the empirical point of view and mathematically manageable. It is thus possible to estimate via linear regression the relationship $T(\xi) = U - V\xi$, using our individual-level data. The results of a simple OLS regression are reported in table 2. Notice that the R^2 values are computed on the conditional averages values of the TFR computed for each age at union and not on the individual data (fig. 10), this to stress that aggregate-level data could be used for this approximation. With a reproductive behaviour such as the one exhibited by the cohort we considered, a one year increase in the age at marriage would bring down the number of female children ever born[14] by 0.0769.

Second, to evaluate the effect of an increase in the age of attaining adulthood on the average age at marriage we adopt the linear approximation (30). This assumption is not common but permits to remain close to the spirit of the Henry-type relationship, i.e. an approximate but possibly useful one[15], and seems to be well justified by the available data (table 2 and fig. 11): although very heuristic, the linear approximation seems to work quite well. It is worthwhile to notice that a one-year increase in adulthood attainment provokes a 0.54 year increase in age at marriage (notice that the estimated regression coefficient is, as expected, less than the unity as women who attained adulthood later tend to have a shorter interval between adulthood attainment and entering a union)[16]. Finally, in order to obtain the approximate TFR, the mean age at union and the proportion ever in union have been computed from our data. By combining the two linear relations, the expected decrease in the TFR for women ever entering a union as a

[14] Using Japanese data, Inaba (1996) computed an expected increase for the TFR of 0.11. Using data for younger Italian cohorts, Manfredi and Billari (1997) computed an expected increase of the TFR of 0.08-0.09.

[15] The same assumption, though in a different framework, that of a structural model, was used by Marini (1985).

[16] Using data on earlier Italian cohorts in Italy, and using the average age at attaining the highest educational level as a marker of attaining adulthood, Manfredi and Billari (1997) estimated that an increase of one year in women's age at adulthood would lead to an increase of about 0.4/0.5 years in the age at marriage.

consequence of a one-year delay in adulthood attainment, given by the product *(BVq)* (see (31)), is definitively equal to 0.041[17].

The comparison between this result and the ones given by the macrosimulation seems of particular interest to us. Using the mean ages at adulthood attainment of the former benchmark and alternative scenario (17.8 and 21.3 respectively), and by assuming constant parameters for the linear relationships estimated, we may evaluate the change in the TFR between ever married women to be 0.041·(17.8–21.3)= –0.14. If the PEM stays as high as in the benchmark scenario – that is, a somewhat strong hypothesis for it is 95% –, the expected decrease in the total TFR is slightly less than –0.14 children per woman. The result is not far from the decrease we obtained in the macrosimulation with marital fertility dependent on both age and marriage duration (–0.18); this difference could be due to the fact that the macrosimulation takes into account the effects on the PEM, at least the one for the reproductive ages, of delayed transition to adulthood. With the other two macrosimulations, the values obtained were –0.08 when age has no influence and –0.37 when marriage duration has no influence.

The general impression is that the two linear approximations, in case with some supplementary hypotheses on the PEM, can be easy to compute and give significant results, with the advantage of limited data need.

[Figures 10-11 and Table 2 about here]

5. Conclusions and directions for future research

This paper has been concerned with the macro-demographic consequences of delays in the transition into adulthood within a multistate stable population framework. We developed a particular multistate stable population model, which is particularly suitable for the study of the 'Mediterranean pattern', and discussed some analytical results such as the stable population structure. We showed that some interesting sub-cases – for instance the one with constant transition rates – can provide useful information on the mechanisms that affect the dynamics of population in such case.

We then applied the model to the Italian case, estimating its parameters either from survey data or from official statistics. Some macrosimulations of the model allowed us to empirically assess the impact of delayed adulthood on both reproductivity and the structure of the population. The application of two simple linear approximations gave an easy-to-use tool to evaluate the impact on reproductivity.

[17] This result substantially confirms the decrease of 0.03-0.04 in the TFR computed with the same approach in paper by Manfredi and Billari (1997).

It has to be taken into account that the comparison of stable states is a sort of parable: it cannot be directly interpreted as a projection, rather as a tool for the assessment of present trends. In the Italian case, for instance, a change in the union formation or fertility pattern or immigration can change the picture we outlined. In general, our idea is that simulation of population models is particularly useful, but it is even more powerful when supplemented by some analytical support.

Much can still be done, in our opinion, on the theoretical side of the model. It seems promising to analytically discuss and apply more general multistate models, such as reversible models, i.e. embedding the effects of remarriage which can become highly relevant, and of the existence of different routes to marriage. It also appears desirable to move to more realistic two-sex frameworks, although this would really limit the possibility for analytical results. Also, migration can be the issue of further generalisations. From the applied side, the possible remarkable effects of changing patterns in adulthood attainment for the management of pay-as-you go pension systems (such as the one common in Europe), seem to be a further area of interest.

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Tables and figures

Figure 1. Flow diagram of the general model.

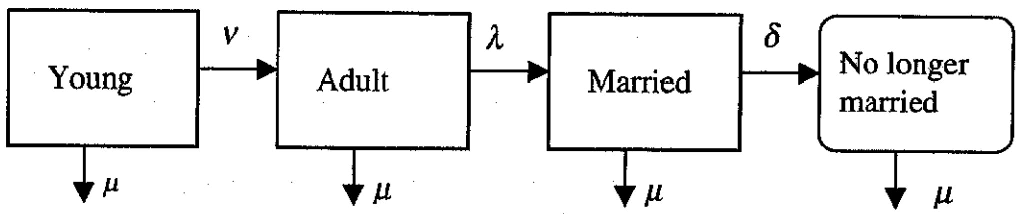
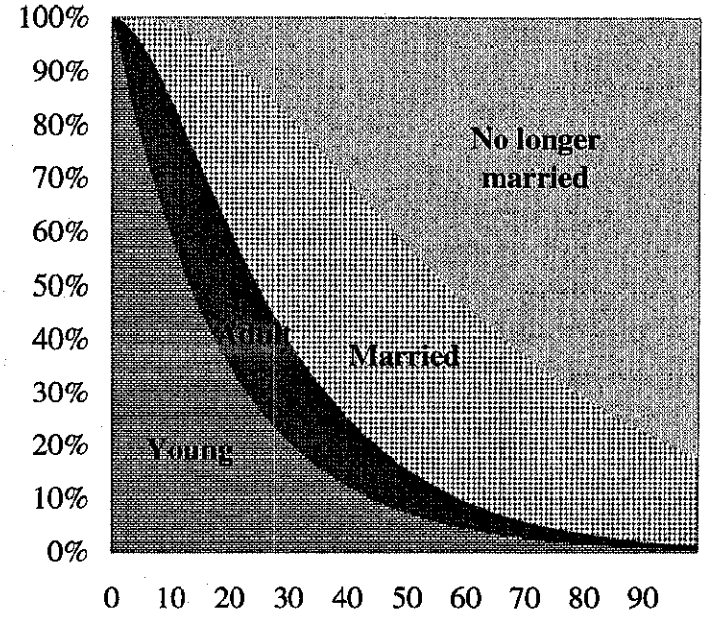


Figure2. State distributions by age in the case of transition rates independent on age (parameters: ν=1/19, λ=1/11, δ=1/36).

2a. Overall composition



2b. Percentage distribution

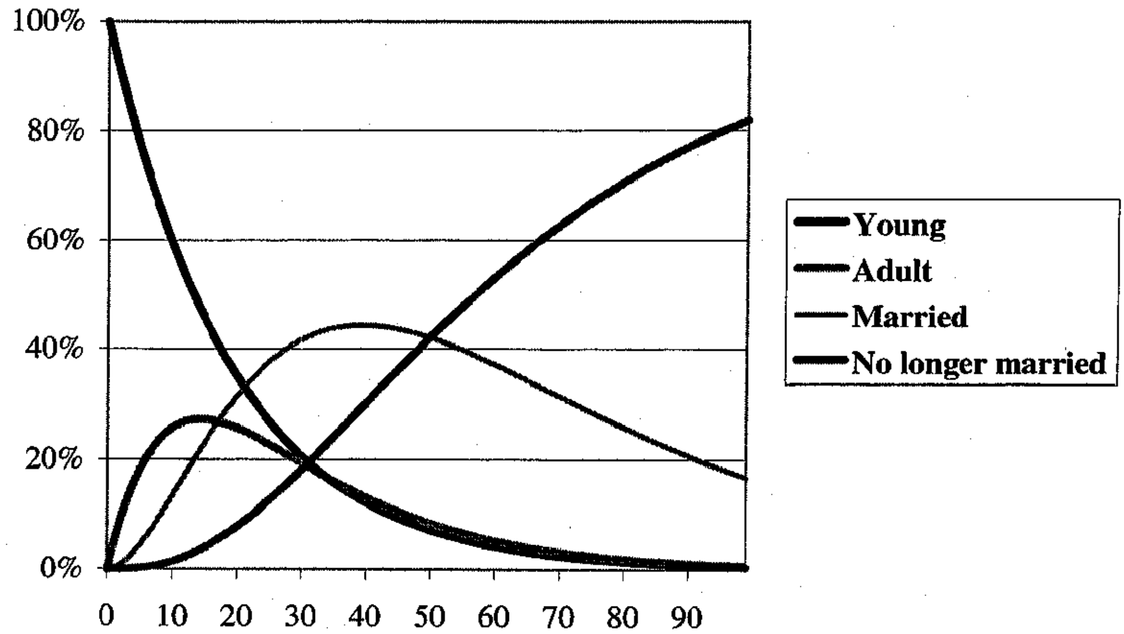


Figure 3. Rates of transition to adulthood by age. Benchmark (c 46-60) and alternative (c 61-75) scenarios.

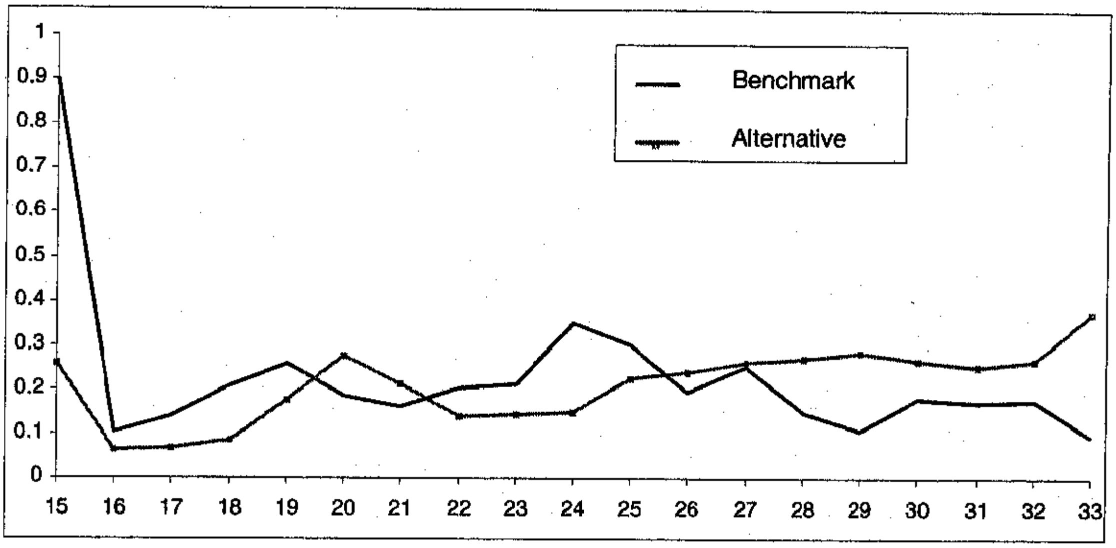


Figure 4. Transition rates from the adult to the married state and marital fertility rates (fertility dependent on age only).

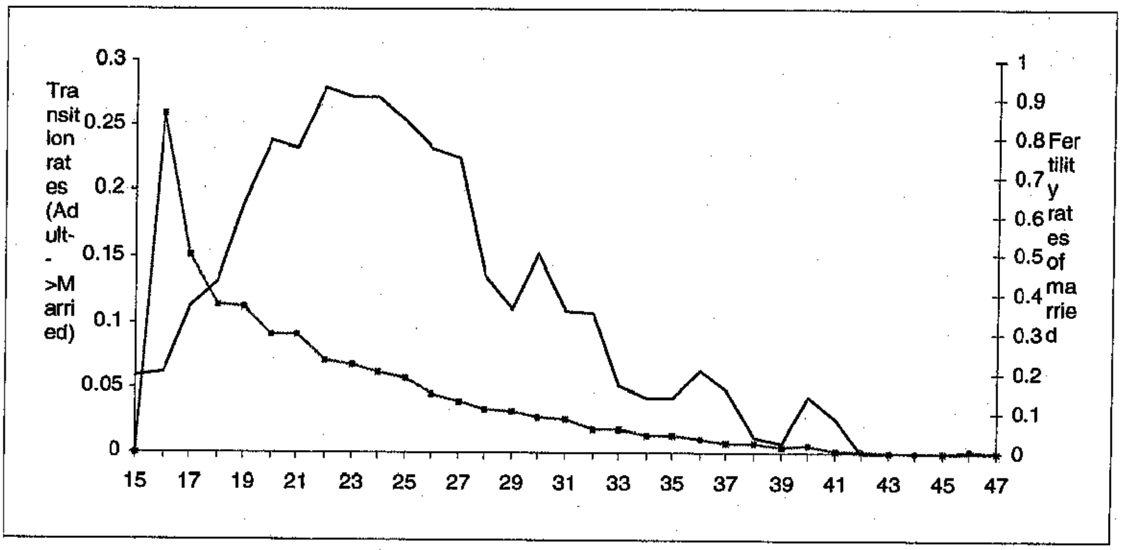


Figure 5. Cumulated marital fertility rates (fertility dependent on marriage duration only).

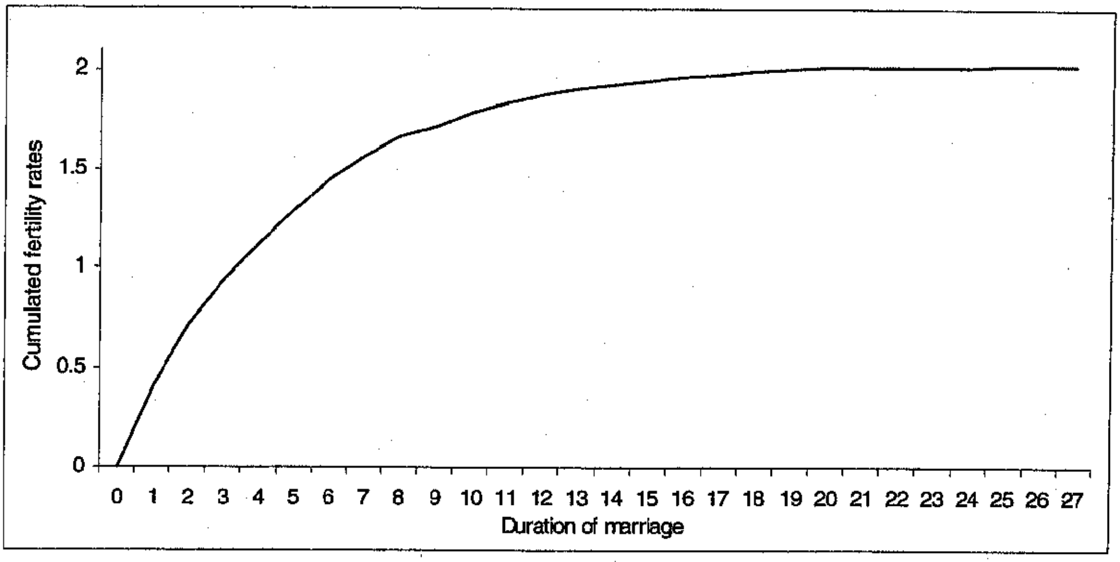


Figure 6. Cumulated marital fertility rates (fertility dependent on age and marriage duration).

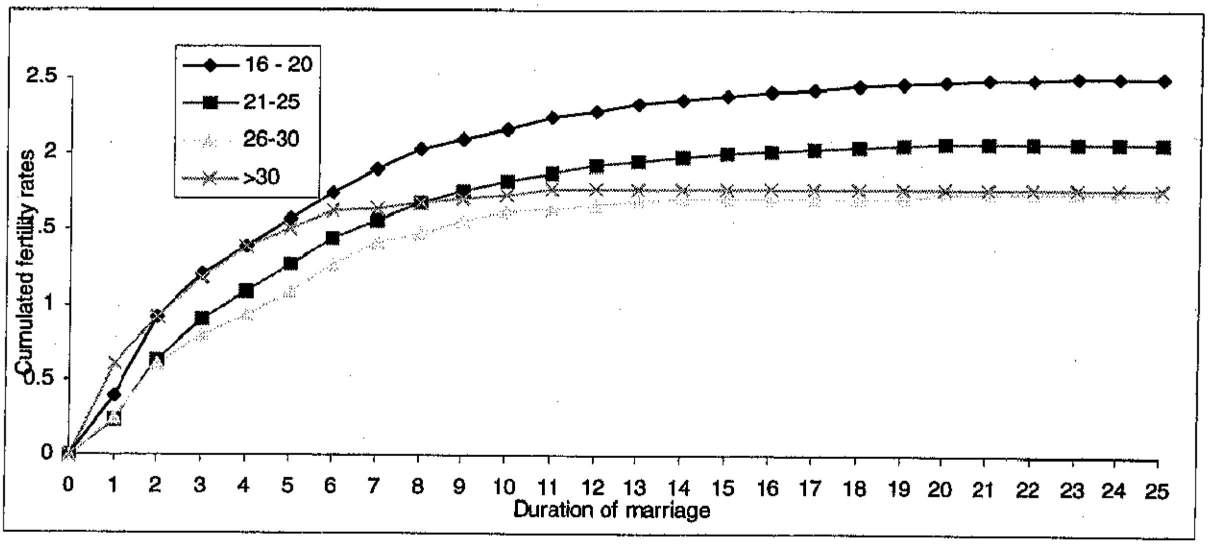


Figure 7. Age structure of the population (million) when marital fertility depends on age only.

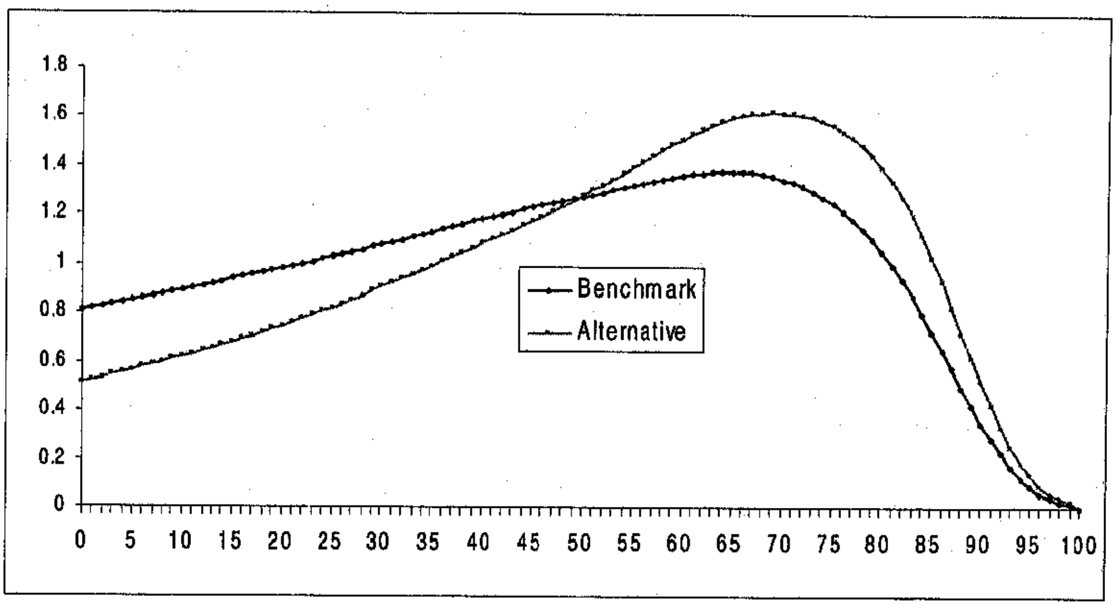


Figure 8. Age structure of the population (million) when marital fertility depends on marriage duration only.

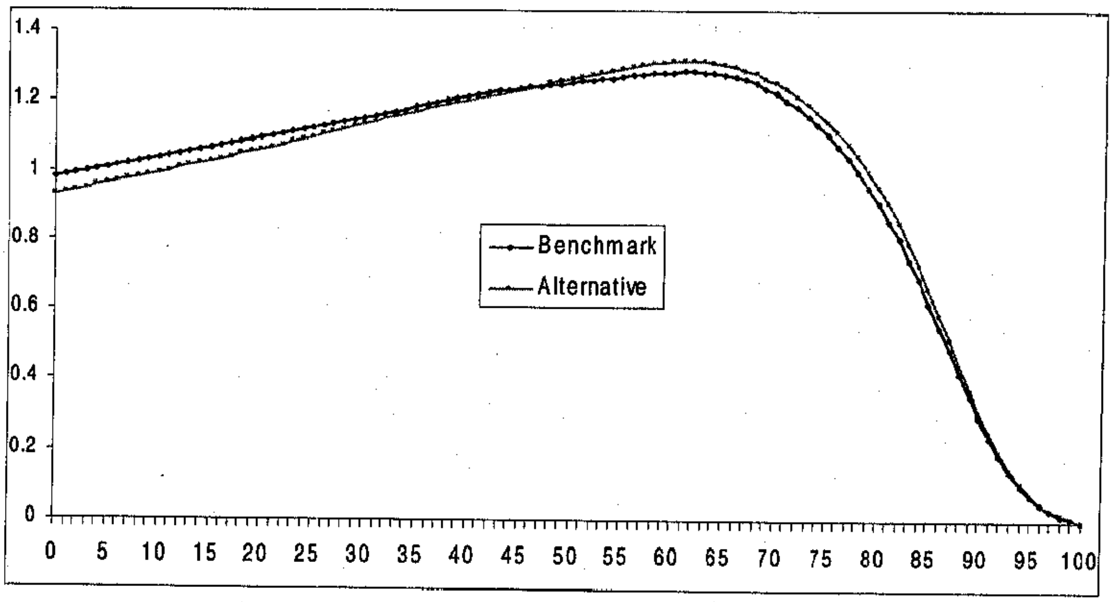


Figure 9. Age structure of the population (million) when marital fertility depends on age and marriage duration.

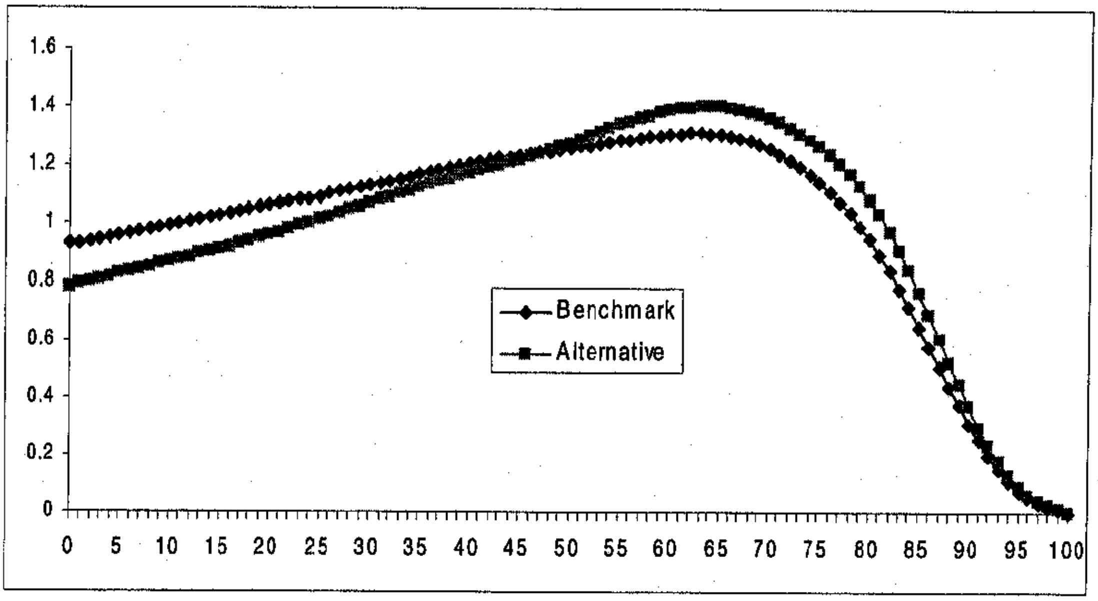


Table 1. Measures of synthesis in the benchmark and reference scenarios according to distinct hypotheses on fertility.

1a.

AGE (Marital fertility dependent on age only)	**Benchmark**	**Alternative**
TFR	1.60	1.23
Mean age	47.12	52.54
Population 65+/Population 0-19	1.59	2.93
(Population 0-19 and 65+)/Population 20-64	0.86	0.94

1b.

DUR (Marital fertility dependent on marriage duration only)	**Benchmark**	**Alternative**
TFT	1.79	1.71
Mean age	44.57	45.31
Population 65+/Population 0-19	1.20	1.31
(Population 0-19 and 65+)/Population 20-64	0.83	0.84

1c.

AGE+DUR (Marital fertility dependent on age and marriage duration)	**Benchmark**	**Alternative**
TFT	1.74	1.56
Mean age	45.32	47.60
Population 65+/Population 0-19	1.31	1.67
(Population 0-19 and 65+)/Population 20-64	0.84	0.86

Table 2 Results from the linear regressions between a) TFR and age at marriage, and b) age at marriage and age at adulthood.

Relation to be Estimated	Parameter Estimates (Standard Errors in parentheses)	R^2
$T(\xi) = U - V\xi$	U=3.8068 (0.02880) V=0.07695 (0.00120)	0.7544
$\bar{\xi}(\eta) = p + q\eta$	p=13.710 (0.08266) q=0.5373 (0.00458)	0.9088

Figure 10. Total Fertility Rates conditional on given ages at first union and the linear approximation.

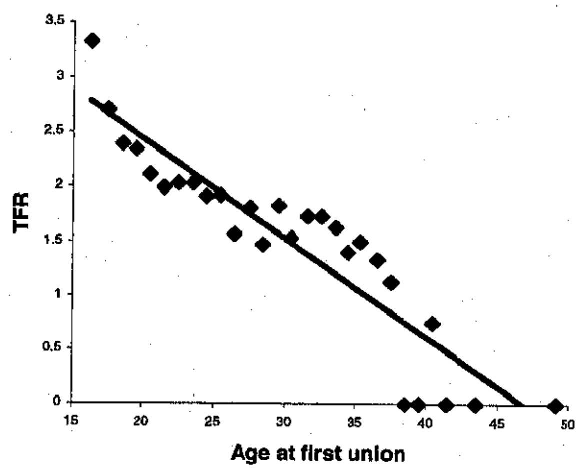


Figure 11. Age at first union conditional on given ages at adulthood attainment and the linear approximation.