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Abstract

The existence of an efficiency wage mechanism in Goodwin-type models may lead to the unexpected appearance of an economically meaningful equilibrium with zero labour share, which is globally stable for some parameter constellation and allows the system to attain its "maximal growth". A subsequent "normative" comparisons among the possible long term regimes of the economy shows that: 1)the zero labour share equilibrium can be the "preferred" equilibrium in terms of welfare: 2)in all the long term regimes the welfare is higher than in the original Goodwin's model; 3) a point of maximal welfare exists. Moreover, the effects of rational behaviour of firms are compared with the "traditional" situation in which rationality is not explicitly assumed. A striking result appears: myopic rationality can have deleterious effects on the profit of firms and on the overall welfare of the economy.

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1. Introduction

A well known theory, among the dozen aiming to explain the working of the labour market, is the one based on the notion of efficiency wage (EW), which emphasises the role of the wage as a primary determinant of the productivity of labour (Akerlof and Yellen 1986). There are several avenues, well acknowledged in the economic literature, through which higher wages may stimulate productivity. Among these the more relevant are (Layard et al. 1991): a)the "gift exchange" mechanism (Akerlof 1982), based on the sociological principle by which higher wages can, within the system of labour market relationships, increase the self-identification of the worker for its firm, and so increase his will to cooperate; b)the "shirking" principle (Shapiro and Stiglitz 1984), by which in a regime of unemployment higher wages could imply, for the worker, a very high loss in case he should be fired, so preventing him from being "lazy"; c)the "threat effect model", by which higher wages can reduce the "fighting" attitude of the working class, and therefore prevent the formation of trade unions.

This paper aims to investigate the dynamical consequences of the efficiency wage hypothesis within the frame of Goodwin-type business cycle models (Goodwin 1967), which are still broadly unexplored. As in Goodwin-type models wages are determined according to the Phillips-curve, the term "efficiency wage" used in this paper only refers to the relationship between the productivity of labour and the wage, which, although it is, formally, of the same type of that used in Akerlof (1982), doesn't involve the effort function in an optimization procedure of wage setting. This terminology was the one adopted in our previous work (Fanti and Manfredi (1995) and also in Chooi (1995).

Chooi (1995) has studied, in the same Goodwinian framework, the effects of the introduction of a general wage dependent effort function

e=e(w). He was able to provide a local picture of the dynamics, in particular to show that when the elasticity of the effort with respect to the wage is negative in a neighborhood of the positive equilibrium, then persistent periodic dynamics can appear through Hopf bifurcations. Fanti and Manfredi (1995) have studied the effects of a "diminishing returns" relationship between the effort and the wage within what they called the GEW model ("Goodwin and the Efficiency Wage"). Their assumption, which excludes for sake of realism the case of a negative effort-wage elasticity permitted by Chooi, modifies the conservative Goodwin's model into a dissipative predator-prey system, therefore permitting to achieve a global (rather than only local) picture of the effects of the efficiency wage.
They also tried to deep the possible consequences of the EW hypothesis, by assuming that the capitalists know their diminishing returns effort function (and try to behave optimally), and investigated the consequences of this assumption as well. We remark that also in this "optimising" extension of the Goodwin's model wages are still set according to the Phillips-curve (differently from the standard efficiency wage literature where the optimising itself is the crucial wage setting tool) and therefore the optimal wage rate found by "optimising" firms only constitutes, in our model, essentially a lower boundary to the wage rate resulting by the Phillips-curve.
Moreover, again in a Goodwin-type framework, Fanti and Manfredi (1999) have studied the interaction between the efficiency wage assumption and gestation lags and found persistent oscillations through Hopf bifurcations.
In this paper we have distinguished two basic situations, depending on whether an "optimising" behaviour of firms is assumed ("rational" case) or not ("traditional" case).
Our aims are twofold. First, we investigate the long term consequences of the introduction of the efficiency wage mechanism within the Goodwin's

model, as they emerge from the study of the long term regimes of the GEW system both in the rational case and the traditional case.

Second, starting from these long term properties, a series of normative implications within the GEW model are found. Since the Goodwin's model is a special case of the Gew system, our analysis provides, furthermore, a comparison between the two models.

Normative aspects have often be neglected even in a huge literature such as the one on the extensions of the Goodwin's model[1]. We have used as a measure of welfare in terms of the consumption of both the working class and the capitalists, the following indicators: the rate of employment and the rate of wage growth for the working class, the rate of profit for the capitalistic class. In the spirit of "goodwinian" economies, which are composed by two classes (capitalists and workers), we say that one long-term regime is better than another one in terms of "social" welfare when the rate of growth of per-capita income is higher and the welfare of at least one class is bettered, ceteris paribus for what concerns the welfare of the other class. Essentially we compare the welfare indicators resulting either by the different models (the original Goodwin's model versus the GEW model) and by different cases of the present GEW model ("rational" versus "traditional"), obtaining very clear-cut results.

The main results are as follows. For what concerns the long term consequences of the EW hypothesis in the traditional case, these are: i)the appearance, together with the "old" positive equilibrium of the Goodwin's model, of a further equilibrium with "zero" labour share which can be fully meaningful (despite of its undesirability at a first sight, as it is shown in the section 3) and with quite appealing "welfare properties" (as it is shown in section 4), ii) a generalised positive effect on growth: the GEW model always grows faster than the corresponding Goodwin's model, iii) the fact that the EW mechanism can push the rate of growth of the economy to its technological upper bound, or in other words it permits the "maximal growth".

In the case in which rationality of the entrepreneurs is explicitly assumed, then, when the parameter measuring the reaction of the productivity to wage changes (RPW since now on) is large enough (otherwise rational and traditional cases obtain the same results), a barrier in the dynamics of the labour share appears which causes the emergence of a new attractor which 'crowds out' the other equilibria of the system. The main long-term effect of this new attractor is that of "protecting" the labour share.

For what concerns the normative aspects, again we distinguish between the rational and traditional cases of the GEW model. In the latter case we remark that: 1) in the "zero labour share" equilibrium the welfare of the capitalists is always better than in the positive equilibrium, while that of the workers almost always does, although not necessarily; hence very often the "zero labour share" equilibrium is, surprisingly, better in terms of welfare; 2) in the Gew economy both the long term regimes exhibit a welfare improvements with respect to the original Goodwin's model. Hence, the welfare of a Goodwin-type economy is always increased when there is an EW effect. Moreover, 3)it exists a unique point of maximal welfare. Hence if the intensity of the RPW could be considered as a control parameter, maximal welfare could be a target of policy. Furthermore we conjecture that also in a strategic context, the choices of capitalists and workers would necessarily lead to such a point, as a non-cooperative equilibrium.

Another outstanding result appears when rational behaviour of firms is explicitly introduced. It is possible to show that in this case, when the EW effect is strong enough, then the optimal behaviour of firms can only reduce.

[i] An exception is Mainwaring (1993).

compared to the situation in which rationality is not assumed, the welfare of the capitalists and of the economic system as a whole. This is a clear example of the fact that if in a realistic world the (real) wages are essentially dependent on the relative strength of the classes the optimising behaviour can result non-optimal.

The present paper is organized as follows. In section two we briefly recall the Goodwin's model and introduce the GEW model. The main results of this latter are reported in the third section. In the fourth section the long term and welfare effects of the efficiency wage hypothesis are discussed. The main results, with their economic interpretation, are summarised in the conclusive section.

2. The GEW model

2. 1 The Goodwin's model

The well known Goodwin's growth cycle model (Goodwin 1967) is defined by the following system of two ordinary differential equations:

$$\begin{cases} \dfrac{\dot{V}}{V} = -(\alpha+\gamma)+\rho U \\ \dfrac{\dot{U}}{U} = (m-\alpha-n)-mV \end{cases} \qquad (2.1)$$

where $U=U(t)$ is the employment rate at time t, defined as the ratio between the current employment $L(t)$ and the supply of labour $N(t)$, and $V(t)$ is the wage share, given by the ratio wL/Q, where w is the real wage rate and Q is the product per unit of time. Notice that V can be expressed also as: $V=w/A$, where A is the average productivity of labour : $A=Q/L$. The model (2.1) is obtained by introducing in the "dynamical identities":

$$\frac{\dot{V}}{V}=\frac{\dot{w}}{w}-\frac{\dot{A}}{A};\frac{\dot{U}}{U}=\frac{\dot{Q}}{Q}-\left(\frac{\dot{A}}{A}+\frac{\dot{N}}{N}\right) \qquad (2.2)$$

(expressing the relations $V=w/A$ and $U=L/N$ in terms of their rates of growth over time) the following basic assumptions:

i) the labour market is governed by the linear Phillips relation:

$$\frac{\dot{w}}{w}=-\gamma+\rho U \qquad (2.3)$$

where the parameters γ,ρ reflect respectively the ability of the working class in the defense of its wage in the situation of "zero" employment (γ) and the speed of reaction of the growth rate of the wage to changes in the employment rate (ρ).

ii) the accumulation"rules" are such that: a) the wage earners do not save; b) the profits are entirely reinvested; c) the technology is Leontief-type; the capital/output ratio $1/m$ ($m>0$) is constant over time. The previous accumulation rules lead to the following equation for the rate of growth of the output:

$$\frac{\dot{Q}}{Q}=m(1-V) \qquad (2.4)$$

iii) a constant exogenous growth of the supply of labour N (at the rate $n>0$) and of the average productivity of labour A (at the rate $\alpha>0$).

By assuming that the coefficients of the system (2.1): $A=\alpha+\gamma, B=\rho, C=m-\alpha-n, D=m$, are all positive (and constant), and provided that $\rho>\alpha+\gamma$, the model (2.1) becomes a classical conservative Lotka-Volterra predator-prey model, which has, by definition of the U,V variables, a dynamics bounded in the feasible set $T=[0,1]x[0,1]$. Its a unique economically meaningful non-zero equilibrium has coordinates

$$(U^*,V^*)=(\frac{\alpha+\gamma}{\rho},\frac{m-\alpha-n}{m}) \quad (2.5)$$

2.2 *The GEW model*

The GEW model (Fanti and Manfredi 1995) studies the effects of the introduction, within the classical Goodwin's model, of the following relationship the average productivity of labour a, to the level of the wage:

$$a=k(t)\left[\frac{w(t)}{w_0(t)}\right]^b \qquad 0<b<1 \qquad (2.6)$$

In (2.6) k is a function related to the exogenous component of the growth of the average productivity of labour, w the unit wage, w_0 an exogenous[2] reference wage and b a constant reflecting the diminishing returns of the productivity to wage changes[3] (in the sequel b is often denoted as the "RPW" parameter).

The (2.6) can be rewritten as:

$$a=k^*(t)w(t)^b, \quad where: k^*(t)=\frac{k(t)}{w_0^{\,b}} \qquad (2.6bis)$$

[2] Of course the reference wage could be endogeneised ; a possible formulation. according to Akerlof (1982) could be the following: $w_0 = w^U z^{1-U}$, where z represents an exogenous level of unemployment benefits. However this would rise the dimension of the system and lead to several new results discussed in Fanti-Manfredi (1999).

[3] Obvioulsy other formulations of the effort-wage relationship are possible, see Palley (1994). Nonetheless our choice seems a reasonable starting point in that it permits to capture the more essential feature of the relationship between effort and wage, i.e. diminishing returns. We recall that, moreover, more general assumptions have been made in the literature, concerning the shape of the efficiency wage relationship, see for instance the pioneeristic work of Solow (1979). We also considered the case b>1 which is a quite reasonable assumption in economies with vey low wage levels. In this last case the traditional prey-predator structure of the resulting models is lost (it becomes a competition problem).

In (2.6bis) the function k^* is assumed to be steadily growing at the constant rate $\alpha>0$ which is the difference between the constant rates of growth of the exogeneous components, k and w_0.

The assumption of a constant b is just a starting point and implies an exogenously determined EW effect.[4] Other details concerning the foundation of the equation (2.6) are postponed to appendix 1. The (2.6) defines the productivity as an increasing and concave function of the wage. Our formulation (2.6) is a special case of Akerlof (1982), who used:

$$a=-d+k(w/w_0)^b$$

where d is a further constant. The formulation (2.6) appears to be a first step toward more reasonable formulations of the problem, and, once embodied within Goodwin's frame, at least as long compared with other formulations, as Chooi (1995), seems to be able to offer more clear-cut dynamical results and easier economic interpretations.

Once we explicitly introduce (2.6) within (2.1) we obtain the following Lotka-Volterra dissipative system:

$$\begin{cases} \dfrac{\dot{V}}{V} = -B_1 + B_2 U \\ \\ \dfrac{\dot{U}}{U} = C_1 - C_2 U - C_3 V \end{cases} \qquad (2.7)$$

where the structural parameters are given by:

[4] More reasonably the b parameter should be probably determined as the outcome of an optimal choice of the worker. We remark that if the RPW parameter could be endogenously determined, then GEW-type models would exhibit a typical feature of the "endogenous growth models", via the still unexplored mechanism of the efficiency wage.

$$B_1 = \alpha + (1-b)\gamma > 0; \quad B_2 = (1-b)\rho > 0$$
$$C_1 = m + b\gamma - \alpha - n > 0; \quad C_2 = b\rho > 0; \quad C_3 = m > 0 \qquad (2.8)$$

Clearly the Goodwin's model is obtained from of the GEW model (2.7), by letting *b* tend to zero. Remark that combining (2.6) with (2.3) it follows that:

$$\frac{\dot{a}(t)}{a(t)} = \frac{\dot{k}(t)}{k(t)} + b\frac{\dot{w}(t)}{w(t)} = \alpha + b(-\gamma + \rho U) \qquad (2.9)$$

which is a direct relationship between the rate of change of productivity and the level of employment. The (2.9) is unusual in the standard efficiency wage context. It appears in fact as a consequence of the fact that the wage is determined in a Phillips-type labour market, and not as the outcome of a profit-maximising process.

3. The Gew model: main results

The GEW model (2.7) is well known in basic dynamical system theory: it is a classical Lotka-Volterra's prey-predator system with a "carrying capacity" in the prey equation. Its dynamical properties are well known (see for instance Freedman, 1980). Fanti and Manfredi (1995) have provided a basic analysis of the economic implications of the efficiency wage hypothesis on Goodwin's model. This is done for two basically distinct situations. The first one, called "traditional", is based on the typical "non optimising" setting of all Goodwin-type models (Chooi (1995) is another instance of this approach). This case leads to the study of model (2.7) without any further modifications. The second case, on the contrary, explicitly assumes a "rational behaviour" of firms, and leads to some relevant news. Let us call this case the "rational" one. The main results are as follows.

a) the "traditional" case

By discussing the properties of model (2.7) by using the RPW as the pivotal parameter, it is possible to identify a number of relevant thresholds, all expressed in terms of b, call them S_0, S_1, S_2, S_3 which permit to identify all the relevant dynamical "windows" of the GEW system. For what concerns the steady state analysis, it is so easy to show that the GEW system always has the zero equilibrium $E_0=(0,0)$. Moreover, provided that b is not too large ($0<b<S_1$) the system has a strictly positive equilibrium $E_1=(U_1,V_1)$ (inherited from the underlying Goodwin's model). E_1 disappears when b is very large ($S_1 \leq b<1$). Finally, the system has a further equilibrium $E_2=(U_2,V_2)$, which is characterised by a strictly positive employment but with a "zero" labour share (i.e.: at E_2 the product is completely distributed to profit). Some features of these equilibria are reported later (see table 1). A standard dynamical analysis shows that:

a)when the positive equilibrium E_1 exists ($0<b<S_1$), it is always globally asymptotically stable (GAS). In particular, provided that b is sufficiently small ($0<b<S_3$, $S_3<S_1$), the convergence to equilibrium occurs through damped oscillations, while for $S_3<b<S_1$ the convergence is non-oscillatory. We recall that when E_1 exists all the other equilibria are locally unstable.

b)when $S_1 \leq b<1$ the E_1 is lost and the "zero labour share" equilibrium E_2 becomes GAS. Convergence to E_2 is of a non-oscillatory type.

In words: until the RPW is not too high ($0<b<S_1$) a positive equilibrium of the employment rate U and the labour share V is preserved, and the efficiency wage mechanism is capable of stabilising Goodwin's conservative oscillations ($0<b<S_3$), or even of sterilising them ($S_3<b<S_1$). Vice-versa, when the RPW is very high ($S_1<b<1$), the positive equilibrium E_1 disappears, and the profits "crowds out " the labour share in the long run, by driving the system to the "zero" labour share equilibrium. The explanation of this fact is that when the EW mechanism is strong, it induces a sort of indexation of the productivity, which protects the profit in a very efficient manner from the attack of the wage during the unfavourable phase of the business cycles (namely: when the employment is high). The E_2 equilibrium at a first sight could seem economically meaningless, due to its zero labour share. However, this does not imply at all a zero asymptotic wage. Instead it simply means an annihilation of the labour share in the long-term growth "regime". But the story is not ended here: when E_2 is stable, it is always reached (asymptotically) as the outcome of a long term phase of exponential growth of both wages and productivity. In fact, the zero labour-share equilibrium is located (the reader can convince himself by resorting to a simple arrow diagram) in a region of the phase space characterised by strict growth of the wage.[5] But, since in this case the productivity grows faster than the wage, the labour share necessarily goes to zero in the long run. This result shows therefore that a zero labour share equilibrium, rather than being a theoretical "curiosity", can be a fully relevant long term outcome of a Goodwin-type economy. This will permit us to make meaningful welfare comparisons among the possible long-term states of the economy.

Fig. 1. Dynamics of the GEW model in absence of rational behaviour: a)convergence to the positive equilibrium b)convergence to the zero labour share equilibrium

[5] This feature was already present in the Goodwin's model, although not sufficiently emphasized in the literature. In fact in Goodwin's model two sharply different regimes of cyclical growth are possible, depending on whether the conservative orbit actually followed by the economy, lies completely on the right of the line $U=\gamma/\rho$, or not. Along this line the

b) the "rational" case

By postulating, as we did, a wage-related effort function, the wage should become a control variable for the firm, with respect to its "rational" objectives (maximisation of profits or minimisation of costs), provided that the firm itself knows the structure of the effort function of workers. This makes it of interest to study what happens when we explicitly assume that the firm behaves "rationally", an aspect usually neglected in Goodwin-type models. As we will see in the sequel, profit maximizer firms in an "efficiency wage economy" would not allow wages to freely fall down during the unfavourable phase of the business cycle, since this, by reducing the effort of workers and hence their productivity, could not be rational from the entrepreneur's point of view. The main consequence of this state of affairs is the appearance of a strictly positive lower bound in the labour share V. To check this let us assume that firms perform a static maximisation of the profit[6], given the restraint of the effort function due to the EW hypothesis. It can be shown that the static problem:

$$\max\{P\} = \max\{Q - wL\} = a(w)L - wL$$

where P is the total profit at time t, leads to the optimal wage rate:

$$w^{o} = [bk(t)]^{1/(1-b)} \qquad (3.1)$$

The (3.1) defines at every time a lower bound in the wage. Notice that the optimal wage (3.1) is not constant over time due to the existence of an exogenous source of growth of productivity (the $k(t)$ function). Its rate of growth over time is given by $\alpha/(1-b)$. Clearly, as the rate of growth of the

wages are in equilibrium. Hence, in the former case the cyclical growth of the economy will always experience growing wages, whereas in the latter it will not.

"optimal productivity" (the productivity corresponding to the optimal wage level) is identical to that of the optimal wage, the labour share is necessarily constant in correspondence of the optimal wage. It is easy to check that this constant level of the wage share is exactly given by b. Hence the straight line at the level $V=V^o=b$ defines a "barrier" in the state-space of the variables (U,V).

The economic explanation of the "barrier" is the following. On the basis of their "myopic" rationality entrepreneurs minimise costs subject to the knowledge of their concave effort function. This procedure identifies an optimal level of the wage (which steadily grows over time). Now, it seems reasonable to assume that, as the business cycles dynamics goes on, and so forces of other nature as well act upon the wage dynamics, the optimal wage will constitute a completely effective lower bound of the wage dynamics itself. In fact, even if the economic forces acting during those phases of the business cycles which are unfavourable to wages could, at a certain moment, push wages below the optimal wage level, this decrease would not take place thanks to the rationality of entrepreneurs who know that, when an efficiency wage mechanism is operating, a decrease in wage could imply deleterious effects on labour productivity, and so on their own profit. On the other hand it seems also reasonable as well to believe that entrepreneurs will not be in condition to preserve the optimal wage during those phases of the business cycle which are favourable to the workers (the phases of high employment namely, via the Phillips mechanism). In such phases the wages tend to grow faster than productivity (so raising the labour share) and the capitalists have no tools to prevent this. The lower bound in wages in turn

6 We rule out the possibility of a dynamical optimization rule because we explicitly assume that in the Goodwin's model the entrepreneurs are unable to make a perfect foresight of the future dynamical paths of the relevant state variables.

leads to a "barrier" (located exactly at the actual level of the RPW. i.e. at b) in the labour share. as previously seen.

The protective action played by the barrier in favour of the labour share is evident: the barrier prevents the labour share from going to zero when the efficiency wage effect is very strong. In other terms the E_2 equilibrium ceases to be relevant as the long term outcome of the GEW system in presence of a very high RPW. There are further dynamical consequences of the rational behaviour, depending on whether the RPW is large enough to allow the barrier to be above or not to the value of the positive equilibrium of the labour share. A further relevant threshold S_4, given by the only meaningful positive solution of the quadratic equation in b:

$$m(1-b)^2 - n(1-b) - \alpha = 0$$

appears (it always holds $S_4<S_1$). It can be showed that when $b>S_4$ the barrier exceeds the level V_I, and vice-versa when $b<S_4$. In the latter case E_I remains a meaningful equilibrium for the GEW model, whereas in the former case E_I is no more "attainable" due to the presence of the barrier, which "physically" prevents the system to move below it. The new dynamics "with barrier"[7] is confined in the "restricted" phase space $T^*=(0<U<1)x(b<V<1)$, and shows:

a)when $b<S_4$ (ie: $b<S_4<S_1$), the E_I equilibrium and its local dynamical properties are preserved. In such a case. the dynamics of the restricted system will be completely similar (fig. 3a) to that of the unrestricted system. as long as the initial conditions are such that the barrier will not be "met" as dynamics goes on. In this case the actual dynamics toward E_I is identical to that of the unrestricted GEW system. Vice-versa if the initial conditions are such that at some stage the system approaches the

barrier (during a phase of decreasing labour share), the system will experience (fig. 3b) a temporary dynamics along the barrier (check this fact by looking at the directions of motion in the phase space) until the point of intersection between the barrier and the vertical U isocline. Since this point is not an equilibrium one for the system, the horizontal movement along the barrier will of course continue, so pushing the system in a region in which the directions of motion are such that the system will immediately move above the barrier. In this case the system will start again moving toward E_1, which appears to the long term outcome of the system in the present case as well.[8]

b)when $b>S_4$ the E_1 equilibrium is "lost" and a new equilibrium, E_3, located exactly on the barrier appears (fig. 3b). The analysis of the directions of motion in the phase space shows that such a new equilibrium is a global attractor in the "restricted phase space" (and this happens in both possible situations $S_4<b<S_1$, and $b>S_1$). Hence, when rational behaviour is assumed, depending on whether b exceeds or not the S_1 threshold, E_3 will "crowd out" in turn E_1 or E_2. This implies that E_3 fully describes the long term dynamics of the GEW system in presence of "rationality".

Fig. 2. a)The case $b<S_4$: the barrier in the wage-share is located below V_1 and E_1 still "globally" attracts b)The case $b>S_4$: the barrier in the wage-share is located above V_1 and prevents the system from approaching E_1: the system goes in E_3.

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[7] The appearance of "barriers" in one or more state variable is typical of non-walrasian macroeconomic dynamic models. An instance of such an approach is Simonovits (1982).

[8] It is possible to show that even if a trajectory should meet the barrier at a certain time, it will not meet it again at subsequent times: this prevents the possibility of periodic behaviours induced by the barrier.

4. Comparison between the different long-term regimes in terms of welfare

The GEW system can assume, depending on its parameter configurations and/or the existence of a rational behaviour of firms, quite different long term regimes. These regimes correspond to the three different equilibria E_1, E_2, E_3. We emphasise the fact that these equilibria are all "interesting" in that they are GAS in some parameters constellations. This raises the point of the mutual desirability of such regimes, and hence of their comparability (in view for instance of policies aimed to shift from one regime to another one). We claim that in a very large set of situations these equilibria can be fully compared among them, and that they can also be ordered in terms of "long run welfare".

In effect the GEW model is a growth model, which is expressed (as the original Goodwin's one) in terms of two pivotal variables, U and V. The equilibrium levels of these variables induce a "growth regime", in the sense that a full set of growth indicators can be computed from the given equilibrium values.

In the present section a discussion of the long-term welfare implications of the GEW hypothesis is made, by using the RPW (b) as the pivotal parameter. In other terms we provide a "one-parameter" discussion of the long-term welfare consequences of the EW assumption, by looking at the effects of varying b while keeping fixed all the other relevant economic parameters.[9]

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[9] In effect two distinct cases arise depending on whether $\rho-\gamma>m-n$ or vice-versa. We discuss here in great detail only the case $\rho-\gamma>m-n$. We point out that the dynamical results in the two cases are identical. The only difference is in the mutual positions of the relevant thresholds which appear with a different order in the cases ($S_0>S_1>S_2$ in the former case and $S_0<S_1<S_2$ when $\rho-\gamma<m-n$). The case $\rho-\gamma<m-n$ also leads to some interesting result (for instance a regime of full employment forced by the efficiency wage). For this reason a more detailed analysis is postponed to a subsequent work.

To measure the welfare in a "goodwinian" economy, which is composed by two classes (capitalists and workers), we avoid the use of aggregate welfare indicators, such as a social welfare function expressing a certain weighted average of the welfare of the two classes. We rather we claim that a given long term regime experiences a "social" welfare improvement with respect to another one if a)the growth of per-capita income is higher[10] and b) the welfare of at least one class is bettered, ceteris paribus for what concerns the welfare of the other class.

The chosen welfare indicators are: the rate of employment and the rate of growth of the wage for the working class, the rate of profit for the capitalistic class. In fact the two first indicators fully express the welfare of the workers in terms of their consumption in a long-term growth regime[11]. Moreover, the rate of profit fully represents the welfare of the capitalists in terms of their accumulation; in fact, since in the present Goodwinian model the capitalists do not consume, we have assumed that they welfare is measured by accumulation, according to the Marxian spirit of Goodwin's work: "Accumulate, accumulate. That is Moses and the Prophets "(Marx, 1976, vol. 1, ch. 24, quoted by Mehrling, 1986, pp. 1283)[12].

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[10] Since the rate of growth of the supply of labour is constant by assumption, a higher rate of growth of the per-capita income faithfully mirrors a higher rate of growth of the total product. Moreover the rate of profit in Goodwin-type models is identical to the rate of growth of the total product, via the identity between profits and investments and the constancy of the output-capital ratio.

[11] For what concerns workers we have neglected the "utility" that they can derive from the wage share as a measure of their "social power". We also neglected the possible disutilities deriving from the additional stress caused by increased level of effort.

[12] We could explicitly introduce the consumption of capitalists without substantial complications, but it is easy to check that the ensuing consumption-based welfare analysis again leads to the rate of profit as a welfare indicator, therefore resulting completely equivalent to the present accumulation-based welfare approach.

In the sequel we compare the welfare indicators resulting either by different models (the original Goodwin's model versus the GEW model), by the different equilibria of the GEW model (the positive equilibrium E_1 versus the axis equilibrium E_2) and by different cases of the GEW model (the "rational" versus the "traditional" case), obtaining very clear-cut results.

a) the "traditional" case

Table 1 summarises the main features of the long term dynamics which would be experienced by the GEW economy depending on whether the system converges to one of the two possible equilibria E_1,E_2. We added, just for a comparison reason, the corresponding features of the relevant equilibrium of the basic Goodwin's model (LVG); in particular we denoted by $g=\alpha+n$ the average long term growth rate of the product in Goodwin's model.

Table 1. Features of the equilibria[13] of the GEW and LVG system

	V	U	$\frac{\dot{Q}}{Q}=\frac{P}{K}$	$\dot{w}/w$	$\dot{a}/a$	$\dot{q}/q$
LVG	$1-\frac{n}{m}-\frac{\alpha}{m}$	$\frac{\gamma}{\rho}+\frac{\alpha}{\rho}$	$g=n+\alpha$	α	α	α
E_1	$1-\frac{n}{m}-\frac{\alpha}{m(1-b)}$	$\frac{\gamma}{\rho}+\frac{\alpha}{\rho(1-b)}$	$n+\frac{\alpha}{1-b}$	$\frac{\alpha}{1-b}$	$\frac{\alpha}{1-b}$	$\frac{\alpha}{1-b}$
E_2	0	$\frac{\gamma}{\rho}+\frac{m-g}{b\rho}$	m	$\frac{m-g}{b}$	$m-n$	$m-n$

In the traditional case the equilibria relevant for the long term regimes of the system are E_1 and E_2. The system converges to E_1 or E_2 depending on

[13] Among these equilibria we did not report the E_0 equilibria of both the GEW and LVG models, in view of their lack of economic interest.

whether the RPW parameter is below or above the threshold S_1. The behaviour of the various quantities relevant for the long-term welfare of the system in E_1 and E_2, as functions of the RPW parameter b in the feasible set 0<b<1, is reported below. Let us first consider the function $U=U_\infty(b)$ describing the steady state value of the employment rate as an explicit function of b. We have:

$$U_\infty(b)=\begin{cases}\dfrac{\gamma}{\rho}+\dfrac{\alpha}{\rho(1-b)} & b<S_1\ (E_1\ regime)\\ \dfrac{\gamma}{\rho}+\dfrac{m-\alpha-n}{b\rho} & b\geq S_1\ (E_2\ regime)\end{cases}$$

Hence, as long as $0<b<S_1$, then the long term employment rate monotonically grows from its minimum $(\alpha+\gamma/\rho)$ up to an upper bound, given by $Sup(U)=(\gamma+m-n)/\rho$. In S_1 the switch between the two long term regimes takes place: this gives rise to a cusp in the U_∞ function, which, by still increasing b, starts decreasing until the value $(\gamma+m-n-\alpha)/\rho$. The explanation of this decrease is simple: since in the E_2 regime exogenous increases of the parameters α,b have no effects on the rate of growth of the product, their effects must necessarily be "concentrate" on the employment rate, by reducing it. The $U_\infty(b)$ function is represented in fig. 3.

Fig. 3. a)Steady state behaviour of the employment rate by varying b;

b) Steady state behaviour of the labour share by varying b

As it was expected, when $b=0$ the equilibrium value of the employment rate in the GEW and LVG models is identical. Hence, from figure 3a we notice that the existence of an efficiency wage behaviour automatically "improves" the long term employment of the system with respect to Goodwin's model as long as E_1 is the long term outcome of the system. When the long term outcome is E_2 this fact continues to be true unless the RPW and the exogenous component of the rate of growth of the productivity of labour are extremely large.

The long term labour share $V=V_\infty(b)$ is described by the function:

$$V_\infty(b)=\begin{cases}1-\dfrac{n}{m}-\dfrac{\alpha}{m(1-b)} & b<S_1\\ 0 & b\geq S_1\end{cases}.$$

which monotonically decreases in the range $b<S_1$ from its upper bound of $Sup(V)=(m-\alpha-n)/m$ at $b=0$ (corresponding to the equilibrium labour share in Goodwin's model) to zero (at the level $b=S_1$), and remains constant thereafter at the level of zero. This situation corresponds to a "total protection" of the profit share by the efficiency wage mechanism (fig. 3b).

Correspondingly, the long term rate of growth of the total product is described by (fig. 4):

$$\left(\frac{\dot{Q}}{Q}\right)_\infty(b)=\begin{cases}\dfrac{\alpha}{(1-b)}+n & b<S_1\\ m & b\geq S_1\end{cases}$$

which shows that, by raising b from 0 to S_1, the rate of growth of the product monotonically increases from the Goodwin's value $(\alpha+n)$ at $b=0$, up to its upper bound given by m, in S_1. Further increases of b above S_1 have no further effects on the rate of growth of the product. The explanation stands in the basic Goodwin's assumption:

$$\frac{\dot{Q}}{Q} = m(1 - V) \leq m$$

implying that the growth of the economy depends on two factors: i)the available technology, fixed by assumption. ii)the distribution of the product between capitalists and workers. The output-capital ratio *m* is a theoretical upper bound for the rate of growth of the product, which is attainable when the labour share is driven to zero. The existence of a sufficiently high efficiency wage effect allows the system to attain its maximal rate of growth, by progressively excluding, in the long term, the labour from income distribution.

Fig. 4. Long term rate of growth of the total product by varying b

Other growth indicators are depicted in fig. 5. The form of the *long term rate of growth of the average productivity of labour* (fig. 5a) is easily understood given the form of the rate of growth of the product previously discussed. More interestingly, the long term rate of growth of the real wage directly follows the corresponding behaviour of the employment rate: it increases by varying *b* from zero to S_1, where it has a cusp, and then monotonically decrease.

$\left(\frac{\dot{a}}{a}\right)_\infty (b)$	$\left(\frac{\dot{w}}{w}\right)_\infty (b)$
$\begin{cases} \frac{\alpha}{1-b} & b < S_1 \\ m - n & b \geq S_1 \end{cases}$	$\begin{cases} \frac{\alpha}{(1-b)} & b < S_1 \\ \frac{m - \alpha - n}{b} & b \geq S_1 \end{cases}$

Fig.5. a)long term rate of growth of the productivity of labour;   b) long term rate of growth of the wage rate

Let us collect the main results concerning the possible effects of the RPW (and keeping fixed all the other relevant economic parameters) on the long term dynamics of the GEW system in the traditional approach. We have:

Res. 1. Together with the "old" positive equilibrium of Goodwin's model, there is a further equilibrium with "zero" labour share which can be, however, fully meaningful from the economic point of view.

Res 2. The existence of an EW effect (b>0) in the economy, allows, independently on the fact that the long-term outcome of the dynamics be E_1 or E_2, the economic system to improve its growth performances with respect to the Goodwin's economy. Or in other words, the GEW model always grows faster than the corresponding Goodwin's model.

Res. 3. The presence of an EW effect (b>0) in the Goodwin model would enable, if b could be in some way controllable and kept above S_1, the economic system to "maximise" its growth, in the sense that the rates of growth of the total and per capita product, of the average productivity of labour and of the wage rate may attain their theoretical upper bound, as given by the technology (*m*), in presence of a distribution completely favourable to profits. Hence the EW mechanism can push the rate of growth of the economy to its technological upper bound, or in other words it permits the "maximal growth".

For what regards the normative aspects we remark that:

Res. 4. In the "zero labour share" equilibrium the welfare of the capitalists is always better than in the positive equilibrium, while that of the workers almost always does, although not necessarily; hence very often the "zero labour share" equilibrium is surprisingly better in terms of "social" welfare (to realize this just give a look to fig. 4-6).

Res. 5. In the Gew economy both the long term regimes show a welfare improvements with respect to the original Goodwin's model.

Hence, the welfare is always bettered when there is an EW effect.

Res. 6. By increasing b from 0 to S_l all the chosen welfare indicators monotonically increase up to their maximal values corresponding to the threshold value S_l of the RPW, in correspondence of which the system shifts from the positive equilibrium to the "zero labour share" equilibrium. In correspondence of the value $b= S_l$ the welfare of the economic system is "maximal" and thus such a value is "optimal" ($b=S_l=b_{OPT}$). This can easily be seen on the basis of the shape of the several relevant curves as depicted in fig. 4,5,6. In particular in the present one-parameter discussion $b=S_l$ is the only point of maximal "social" welfare. In fact by further raising b beyond S_l the rates of growth of employment and of the real wage will start decrease, so lowering the welfare of the workers. Hence it exists a unique point of maximal "social" welfare.

Res. 7. If b could be a "manageable" parameter we argue that the choices in a possible strategic context of both capitalists and workers would tend to the optimal welfare solution (b_{OPT}). In fact if b were smaller than b_{OPT} then both classes would agree to raise b, since the welfare of both classes is an increasing function of b for $b< b_{OPT}$. Vice-versa if b were greater than b_{OPT}, then the workers would operate in order to reduce b, since their welfare is a decreasing function of b for $b> b_{OPT}$, while the capitalists are indifferent since their welfare is constant for $b> b_{OPT}$. Therefore, since the capitalists have no economic convenience in preventing the reduction in b, also in this case b would tend to its optimal welfare solution (b_{OPT}). Notice that this process may lead to the optimal solution thanks to the peculiar shape of the welfare curves in terms of b. Hence, despite the fact that the class struggle is still the funding argument of all Goodwin-type models, in the case of the GEW model, the class struggle may lead to a unique maximal social welfare solution.

b) the "rational" case

The introduction in the GEW model of the assumption of a rational behaviour of firms had the main consequence of introducing a barrier at the *b* level in the labour share, so completely excluding the E_2 equilibrium from being a possible long term outcome of the model. Two basically distinct situations were then considered, depending on whether the barrier in the labour share exceeded or not the positive equilibrium of the labour share itself. In the latter case ($b<S_4$) the real impact of the V barrier was simply that of giving rise, for a certain period of time, to a boundary dynamics, but without preventing E_1 to become the long term outcome of the system. Viceversa, in the former case ($b>S_4$) a new global attractor, E_3, emerged.

In other terms: the introduction of rationality has in some cases a pure medium term "protection effect" of wages and productivity during the low phases of the business cycles ($b<S_4$), which is not relevant for the long term dynamics, whereas in other cases, ie for a large RPW ($b>S_4$), rationality has a strong impact on the long term dynamics as well of the GEW system: a new global attractor (E_3) appears which "crowds out" the E_1,E_2 equilibria.

This makes it particularly interesting to ground the "welfare" effects of rationality, by comparing the long term features of the system in the E_3 steady state versus the corresponding features of the two equilibria. E_1,E_2, which are literally crowded out by the assumption of rational behaviour. It is quite easy to verify that the long term welfare features of E_3, when it exists, are systematically worse than the corresponding features of E_1 and E_2 (see the Tables 1 and 2).

Table 2. Features of the E_3 equilibrium of the GEW system in the "rational" case.

	V	U	$\frac{\dot{Q}}{Q}=\frac{P}{K}$	$\dot{w}/w$	$\dot{a}/a$	$\dot{q}/q$
E_3	b	$\frac{\gamma}{\rho}+\frac{m(1-b)-g}{b\rho}$	$m(1-b)$	$\frac{m(1-b)-g}{b}$	$m(1-b)-n$	$m(1-b)-n$

The check is immediate in the comparison between E_3 and E_2, since they are quite similar in many senses: the former suffers of an impossibility to attain the "growth upper bound" since the barrier prevents the profit to absorb the entire product. The question can anyhow be easily solved in the comparison between E_1 and E_3 as well. By systematically comparing the long term welfare features of both the equilibria, we can see that E_3 would be "better" than E_1, in terms of welfare, if and only if $b<S_4$, which is impossible, since E_3 would not exist at all in such an event.[14]

It is easy to check that for all the relevant welfare indicators these "rankings" are systematically the same. To summarise, the main effects of the rational behaviour of firms for large b ($b>S_4$) are as follows:

[14] For instance, in the case of the product growth indicator, by comparing the growth rates of the product Q(t) in E_1, and E_3 we get the contradiction:

$$\left(\frac{\dot{Q}(t)}{Q(t)}\right)_{E_3} = m(1-b) > \frac{\alpha}{1-b}+n = \left(\frac{\dot{Q}(t)}{Q(t)}\right)_{E_1} \rightarrow$$

$$m(1-b)^2 - m(1-b) - \alpha > 0 \rightarrow b < S_4$$

a)when E_3 "crowds out" E_1, then $E_3 << E_1$

b)when E_3 "crowds out" E_2, then $E_3 << E_2$

where we use the symbol << to denote a worsen welfare situation, compared to the basic situation of no rational behaviour.

These considerations lead us to the following outstanding result: the introduction of rational behaviour in a GEW system can have, when the RPW parameter is very large (ie when E_3 is the relevant long term regime), undesired consequences in terms of welfare. The interpretation of this result is clear: the profit-maximiser firm in a context of efficiency wages must prevent strong decreases of wages in order to protect the productivity in those situations in which the wage rate becomes "too low". This behaviour keeps artificially high both the wage and the labour share, so preventing their decrease in those phases of the economic business cycle in which labour is weak. This has negative consequences for the economy as a whole. In fact in this way the firm unwillingly attacks, at the same time, its own profit and the social welfare as well, because it prevents the "natural" decreases of the wage rate and the labour share during the phases in which they are weak due to the "natural dynamics" of the business cycle.

To summarise the main normative result: when the EW effect is strong, then the optimal behaviour of firms can only reduce, in comparison with the traditional case, the welfare of both the capitalists and the economic system as a whole, showing that if the (real) wages are essentially dependent on the relative strength of the classes an optimising behaviour of the firms can result non-optimal.

5. Conclusions

The long run properties and the normative implications of an extension of the Goodwin's model, embedding the efficiency wage hypothesis, have been investigated. The main results can be summarised as follows: the existence of the efficiency wage relationship in a Goodwin-type economy leads to dramatic long term consequences among which: 1) the appearance of a second equilibrium with zero labour share, having appealing growth properties; 2) a sharp increase in welfare with respect to the original Goodwin's model; 3) the existence of a unique optimal welfare solution with respect to the parameter measuring the intensity of the efficiency wage effect. This optimal welfare solution occurs, surprisingly, in correspondence of the "zero labour share" equilibrium. If b could be a "manageable" parameter, we argue that there could be a convergence of both capitalists and workers in order to attain the maximal (long-term) social welfare solution.

Moreover we have investigated the possibility, permitted by the EW hypothesis, that firms behave "optimally" even in a wage determination context à la Phillips. An outstanding result appears in this latter case: a strong efficiency wage effect can unexpectedly be detrimental not only for the economy as a whole, but also for the capitalists themselves, which, while thinking to be optimal, are, on the contrary, unwillingly responsible of a more effective protection of labour in the income distribution. This suggests that in a realistic world where the Phillips-curve is the main determinant of wages, the (myopic) optimising firm could even worse its long-term performances compared to the non-optimising one.

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Appendix 1: foundation of the efficiency wage relation (2.1)

In order to define an "efficiency wage" relation, a reasonable starting point seems to be the relation:

$$a=k \cdot g(w) \qquad \text{(a.1)}$$

(or $a(t)=k(t) \cdot g(w(t))$ by making explicit the time-dependencies). A similar choice was made by Chooi (1995). In (a.1) a is the actual level of the average productivity of labour, k an exogenously determined "baseline" level of the productivity associated to some "normal" level of the input of labour, *w* the real wage, and *g* a nonlinear function capturing the efficiency wage effect. The workers are assumed to provide a "normal" level of effort when the level of the wage is fixed to some "normal" level. The rationale behind (a.1) is that the actual value of the average productivity of labour should depend, in general, on the value of the baseline productivity, but is also influenced by the changes in the wage: from (a.1), the actual productivity *a* will be above or below the baseline productivity *k* depending on whether $g(w)>1$ or $g(w)<1$.

Though (a.1) is apparently reasonable, it suffers of the fact that *a(t)*, *k(t)* and *w(t)* are measured in the same dimensional unit.[15] In order to write (a.1) in a dimensionally consistent form we must use a dimensionless wage, i.e. we must normalise via a suitable constant reference value of the wage (let us call it w_0). In this paper we adopted for simplicity the form:

$$a(t) = k(t)\left(\frac{w}{w_0}\right)^b \qquad (a.2)$$

where we have defined $g=(w(t)/w_0)^b$. Notice that (a.2) defines the effort as an increasing concave function of the ratio (w/w_0) as in Akerlof (1982).

Although whatever reference constant value of the wage formally solves the dimensional problem, an important point is of course: how should we choose the normalising constant w_0? In order to answer this question it is useful to distinguish between two different situations: (a) equilibrium situations, (b) steady growth situations. In "equilibrium" situations the workers tend to provide "more (or less) effort than normal" in their work (i.e. they tend to be more or less productive than "normal") only if they realise that they can get a larger wage compared to some standard *constant* reference value w_0 of the wage (w_0 is the "normal" level of the wage). This

[15] This aspect was pointed out to the authors by one of the referee of the Journal. His observation allowed us to clarify our efficiency wage relation.

is reasonable: it is never the absolute level of the wage that matters but rather its relative level compared to some well defined standard reference level. In this case the formulation (a.2) seems to remain valid without modifications. The constant ("normal") value w_0 of the wage could for instance be the equilibrium value of the wage prevailing in the economy. Notice that in this case (a.2) would be dimensionally correct, and fully equivalent to (a.1) as the normalising constant w_0 may be embodied within the term **k(t).**

Let us now suppose that the economy is in a state of steady growth so that the absolute level of the wage and the exogenous baseline productivity k(t) are steady growing as well. In this case it is unreasonable to assume that the reference level w_0 be constant. We rather expect that w_0 experiences steady growth as well. Let us suppose, just to fix the ideas, that: $w_0(t) = w_0(0)e^{qt}$ where q>0, is the time path of the reference wage w_0 expected by the workers on the basis of some forecasting rule (adopted for instance by the trade unions). In this case our efficiency wage relation would rather take the form:

$$a(t) = k(0)e^{\alpha t}\left(\frac{w}{w_0(0)e^{qt}}\right)^b = \frac{k(0)}{\left(w_0(0)\right)^b}e^{(\alpha-q)t}(w(t))^b = k*(t)(w(t))^b$$

which is dimensionally correct.

Appendix 2. Thresholds for the RPW parameter in the GEW model

We report here some details concerning the several thresholds of the GEW model. The one-parameter analysis of the "traditional" case based on the RPW parameter shows several threshold values, called S_0, S_1, S_2, S_3. We briefly recall their "nature". The first threshold $S_0 = (\rho - \gamma - \alpha)/(\rho - \gamma)$ (S_0 is strictly positive thanks to the assumption $\rho > \gamma + \alpha$) governs the existence of a positive equilibrium value of the employment: this holds for $b<S_0$. The thresholds $S_1 = (m-\alpha-n)/(m-n)$ governs the existence of an economically meaningful equilibrium for the labour share: this is possible for $b< S_1$. The S_3 threshold (we do not report it here for brevity; it is a solution of a cubic equation and a has a complicated expression, see Fanti and Manfredi (1995)) discriminates between the two modes of convergence to the positive equilibrium: damped oscillations ($b<S_3$) or non-oscillating behaviour ($S_3<b<S_1$). Finally, below $S_2=(m-\alpha-n)/(\rho-\gamma)$ the E_2 equilibrium looses its economic meaningfulness. As:

$$\frac{S_1}{S_2}=\frac{(\rho-\gamma)}{(m-n)} \quad ; \quad \frac{S_2}{S_0}=\frac{(m-\alpha-n)}{(\rho-\gamma-\alpha)}$$

if follows that we have to distinguish two main cases. For $\rho-\gamma>m-n$ it holds: $1>S_0>S_1>S_2$, whereas in the opposite case: $\rho-\gamma<m-n$ it holds: $1>S_2>S_1>S_0$. The analysis of the "rational" case involves the fourth threshold value S_4, which governs whether the positive equilibrium in the labour share exceeds or not the barrier in the wage share.

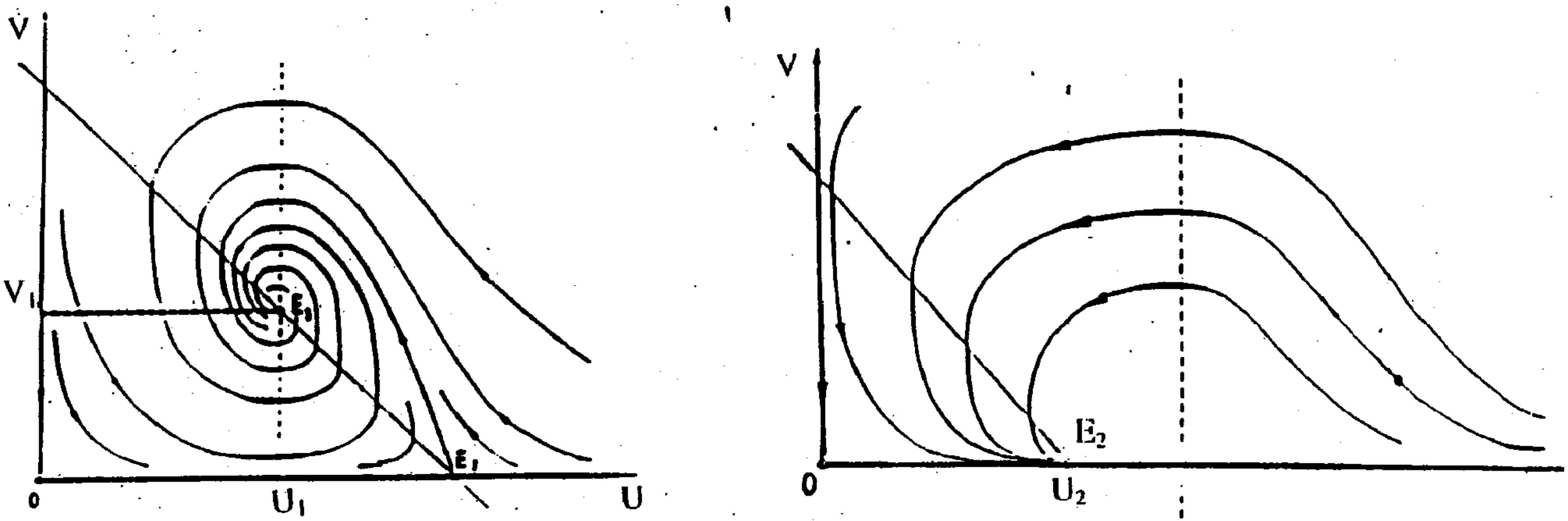

Fig. 1a,b

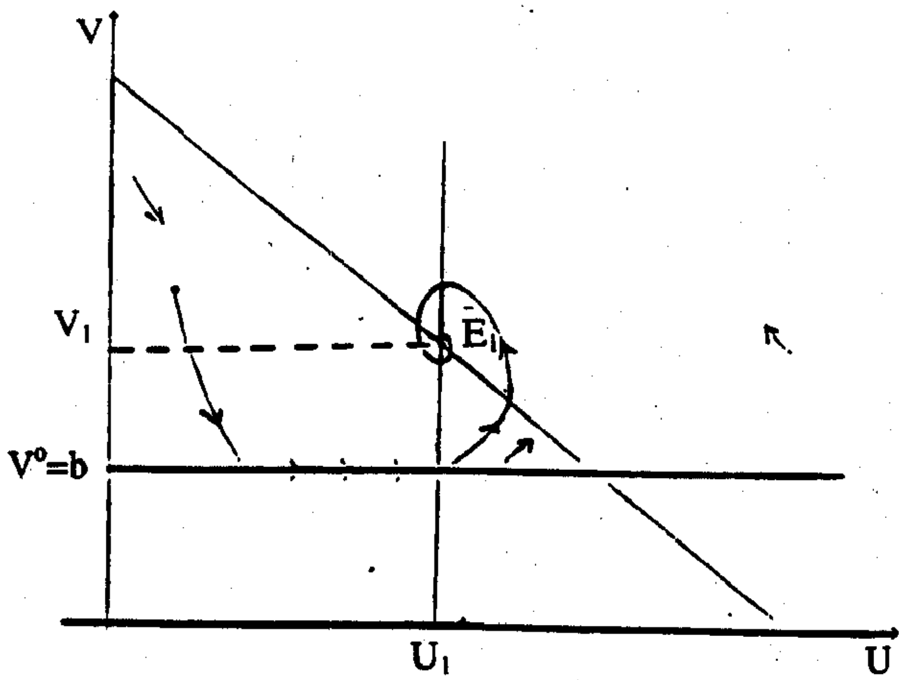

Fig. 2a

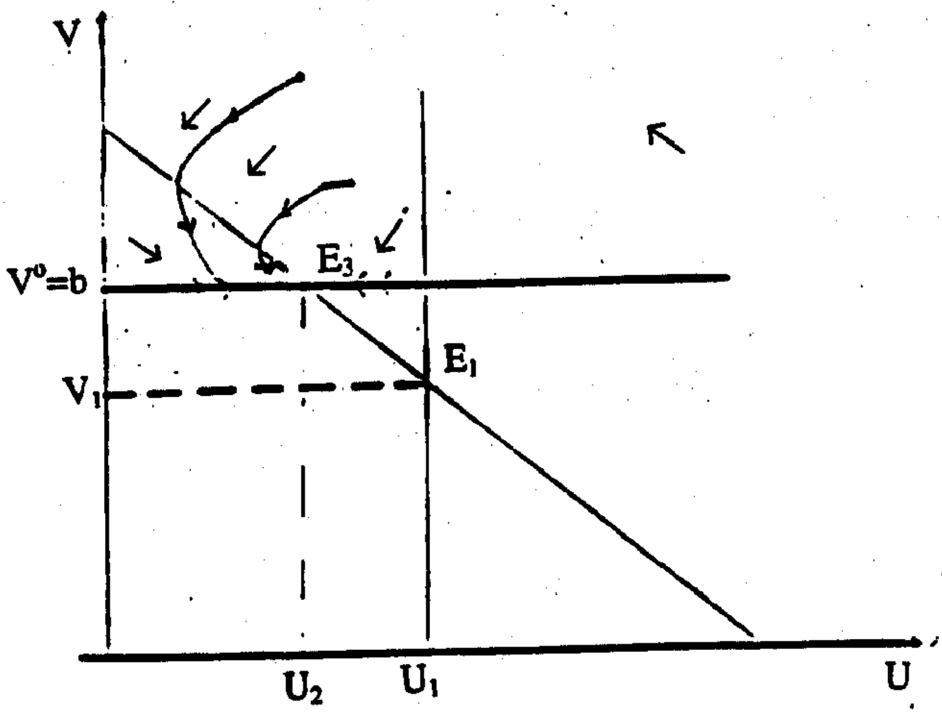

Fig 2b

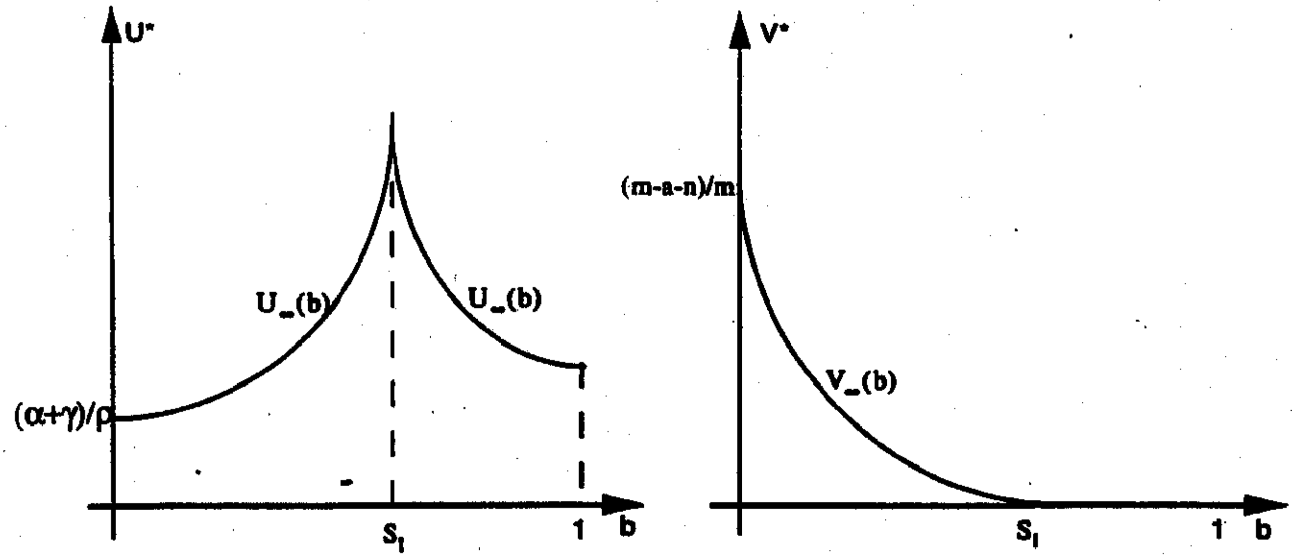

Fig. 3a,b

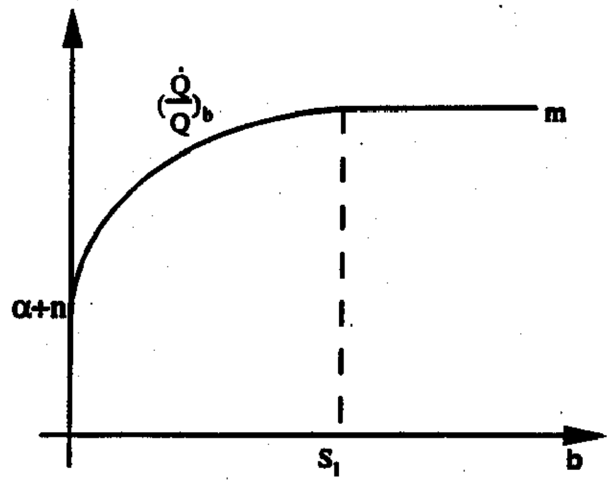

Fig. 4

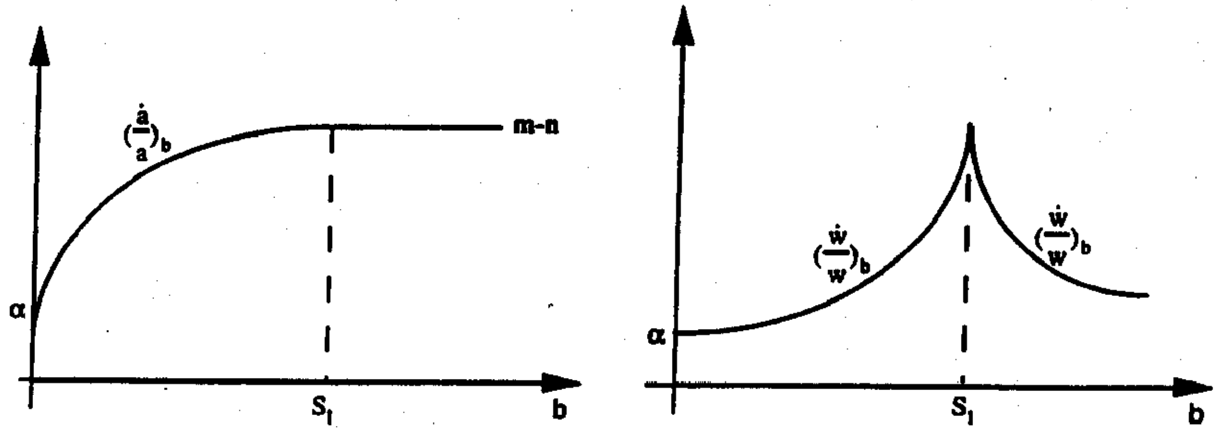

Fig. 5a, b