Università degli Studi di Pisa
Dipartimento di Statistica e Matematica Applicata all'Economia

Report n. 199

The workers' resistance to wage cuts is not necessarily detrimental for the economy: the case of a Goodwin's growth model with endogenous population

Luciano Fanti – Piero Mafredi

Dicembre 2000

- Stampato in Proprio -

Via Cosimo Ridolfi, 10 – 56124 PISA – Tel. Segr. Amm. 050 945231 Segr. Stud. 050 945317 Fax 050 945375
Cod. Fisc. 80003670504 - P. IVA 00286820501 - Web http://statmat.ec.unipi.it/ - E-mail: dipstat@ec.unipi.it

The workers' resistance to wage cuts is not necessarily detrimental for the economy: the case of a Goodwin's growth model with endogenous population
Lavoro preparato per: XV Convegno AIEL, Ancona 28-29/09/2000

Luciano Fanti* and Piero Manfredi**
*Dipartimento di Scienze Economiche
Via Ridolfi 10, 56124 Pisa - Italy
e-mail: lfanti@ec.unipi.it
(address for correspondence)
**Dipartimento di Statistica e Matematica Applicata all'Economia
Via Ridolfi 10, 56124 Pisa - Italy
e-mail: manfredi@ec.unipi.it

September 20, 2000

ABSTRACT

The dynamical effects of the worker's "resistance" to wage cuts during phases of high unemployment are investigated within a "realistic labour market" Goodwin-type model. The long term predictions are striking, especially if compared with those of the original Goodwin's model. First, contrary to the original Goodwin's model, a higher worker's resistance leads to higher wages. Second, contrary to common sense, the resistance of the workers to the wage flexibility plays a stabilising role on the economy as a whole.

Codici JEL: E3, J0, J23

1 Introduction

In the renewed interest for the growth theory, the most influential non-orthodox model of growth with cycle is still the celebrated Goodwin's model. In this paper we study the role played by the working class in the struggle for the wage determination or in other words by the workers' resistance (WR afterwards) to wage cuts in phases of economic crisis (such as phases characterised by a high unemployment rate), within the framework of a "realistic labour market" Goodwin-type model (Manfredi and Fanti 1999a,1999b,2000). As known, in the original Goodwin's model the dynamics of the population and the supply of labour are exogenously given; but already since the Malthus' time the effects of the economic factors on fertility and mortality have neatly been recognised.

Our "realistic labour market" Goodwin's model aims to start fill this gap by endogenising the formation of the labour supply in a manner which takes into account the effect of age structure as well as the positive relation between fertility and wage according to a classical Malthusian view. Malthusian fertility is certainly appropriate in historical epochs; moreover it is still valid, in several cases, in the developing world, and seems also consistent with some modern western fertility patterns, such as the Sweedish case (Hoem 1999).[1] As showed in Manfredi and Fanti (1999b,2000) our Goodwin-type framework is capable of generating sustained oscillations and complex dynamics as well. In sum, our framework appears to be the "demographically appropriate" environment in which to investigate the role played by selected labour market hypotheses and parameters, such as the role of the workers' resistance. So far the investigation of the effects of the intensity of the class struggle on the economy has been done by Balducci and Candela (1982). Their model is very detailed in what concerns the wage determination, but does not take seriously into account the process of formation of the supply of labour. Moreover, their parametrisation of the wage dynamics is different from the original Goodwin's one, which on the contrary, is fully mantained in the present work. This paper is specifically aimed to investigate the static and dynamical role of the workers' resistance, as measured through one of the two parameters on the Phillips curve, which is a basic ingredient of the original Goodwin's model. We could show that the autonomous resistance of the workers may be succesfull in increasing the long term wage, and, surprisingly, in favouring the stability of the economy as a whole.

The present paper is organised as follows. Section two recalls the main features of the classical Goodwin's model, and the role of the WR. Section three introduces first our "realistic labour market" model, and subsequently studies the main static and dynamical effects due to the workers' resistance with special reference to stability and generation of persistent oscillations. Conclusive remarks follow in section four.

2 The classical Goodwin's growth cycle: static and dynamical role of worker's resistance

The Goodwin's model (Goodwin 1967), sometimes known as the Lotka-Volterra-Goodwin (LVG) model, is described by the system :

$$\begin{aligned} \frac{\dot V}{V} &= -(\alpha+\gamma)+\rho U \\ \frac{\dot U}{U} &= m(1-V)-(\alpha+n_s) \end{aligned} \tag{1}$$

where $U = U(t)$ is the employment rate at time t, defined as the ratio between the labour force actually employed $L(t)$ and the supply of labour $N_s(t)$, while $V = V(t)$ is the distributive share of labour, given by the ratio $w(t)L(t)/Q(t)$, where w is the real wage and Q the total product. V can be expressed as: $V = w/A$ where A is the average productivity of labour. Moreover $\gamma > 0, \rho > 0$ are characteristic parameters of the labour market, $m > 0$ is the output-capital ratio, and $\alpha > 0, n_s > 0$ respectively denote the rate of change of the productivity of labour and of the labour supply. The assumptions underlying (1) are: i)the labour market is driven by the Phillips relation: $\dot w = w(-\gamma+\rho U)$ $(0 < \gamma < \rho)$; ii) the accumulation rules are such that: a)the wage earners do not save, b)profits are entirely reinvested, c)the technology is Leontief-type, d)the capital output ratio $K/Q = m^{-1}$ is constant. Thanks to assumptions a),..,d) the rate of growth of the output $g = \dot Q/Q$ obeys: $g = m(1-V) > 0$; iii) the supply of labour and the productivity of labour grow exogenously at the constant rates $n_s > 0$ and $\alpha > 0$.

Model (1) describes a typical Lotka-Volterra system in which the labour share acts as the predator of the employment (the prey). In particular, when $m < \alpha + n_s$ the system has as a unique equilibrium the zero equilibrium $E_0 = (0,0)$ which is globally asymptotically stable (GAS)[2]. Viceversa, provided $m > \alpha+n_s$, system (1) exhibits the traditional Lotka-Volterra conservative oscillations around the positive equilibrium E_1 of coordinates: $U_1 = (\alpha+\gamma)/\rho$; $V_1 = (m-\alpha-n_s)/m$ (notice that E_1 is economically meaningful provided $\rho > \alpha+\gamma$). The

[1] We are aware of the fact that malthusian fertility is just half of the story. In a forthcoming paper (Fanti and Manfredi 2000) we have also considered the role played by the so called post-classical fertility-income relationship (Day et al. 1989).

[2] The stability of the "zero" equilibrium corresponds to a situation in which accumulation is too weak to permit the birth of a "structured" economic activity, as stated by a labour market plus a production structure.

equilibrium values U_1, V_1 are the average values of (U, V) during the fluctuation period.

2.1 Consequences of worker's resistance

This work is concerned with the effects of the worker's resistance, as measured by the γ parameter in the Phillips' relation. The correct interpretation of γ may be given by considering the special case in which employment is zero ($U = 0$). In this case: $\dot{w} = -\gamma w$, implying that the real wage goes to zero with a speed given by γ. Hence the reciprocal of γ is a true measure of the worker's resistance, which may be taken as a proxy of the level of the power of the trade unions in the economy. The idea implicit in the equation $\dot{w} = -\gamma w$ is that the wage is an independent variable in the economy, as it was the case in Italy during the seventies and at the beginning of the eighties. In our opinion the traditional "purely wage targeting" trade union is faithfully represented by the former equation, which represents the "wage-norm" theory of the sociological literature of the labour market (and which is parametrised by γ in the linear Phillips equation). The second characteristic parameter of the Phillips curve (ρ) measures, coeteris paribus, the sensitiveness of the rate of change of the wage to changes in the employment rate. In some sense it summarises the effect that the conflict between capitalists and workers has on the growth of wages. In other words, a prescribed γ implies a certain level of the unions' power, whereas a prescribed ρ implies a certain level of (reciprocal) aggressiveness of the workers as well of the capitalists.[3]

Let us consider both the static and dynamical effects of worker's resistance. The inspection of the (positive) equilibrium values of the employment level and the labour share shows that γ positively affects the equilibrium value of the employment, whereas it does not affect neither the long term wage nor the the labour share. The dynamical role of γ may be evidenced by considering the small linearised fluctuations around E_1. Even more general information is obtained by studying the effects of γ on the "ovoidals" which define the phase-plane solution of the LVG system. As well known, such a phase plane solution is found by dividing the U equation by the V equation and suitably rearranging terms, obtaining:

$$\rho\,(U - U_1)\,\frac{\dot{U}}{U} = -m\,(V - V_1)\,\frac{\dot{V}}{V}$$

Afetr integration we get

$$\rho\,(U - U_1 \log U) + m\,(V - V_1 \log V) = 0$$

It is so easy to draw the effects of the worker's resistance, which are depicted in fig.1. When the worker's resistance increases (i.e. γ is decreased), the amplitude of the oscillations dramatically increases (a fact that is usually considered socially undesirable). This effect may be classified as a "destabilising" effect of the WR (and in fact it is usually, though not always so, even in broader classes of LVG-type models).

Fig. 1. Sensitivity of the amplitude of the LVG oscillations to the WR parameter

[3] Balducci and Candela (1982) used a different parameter (σ) that they called "degree of conflict". In many cases σ is identical to ρ.

We believe that, contrary to Balducci and Candela, neither σ nor ρ actually define true measures of the degree of aggressiveness of the trade unions. Rather they measure the degree of the social conflict. The possible destabilising effect played by this parameter cannot therefore be ascribed only to the trade unions, but, rather to the overall conflict between the two social parts.

who studied in depth the dynamical role played by ρ. Vice-versa they did not paid much attention to the role of γ, which in their model was hidden by a quite more complex formulation of the wage dynamics equation.

3 Counterintuitive effects of worker's resistance: results from a "realistic labour market" Goodwin-type model

We now further investigate the role of the workers' resistance by studying its effects within a "realistic labour market" Goodwin-type model (Manfredi and Fanti 1999a,1999b,2000). In the original Goodwin's model (1) the rate of change of the supply of labour n_s is fully exogenous. Our "realistic labour market" framework considered here enriches the original formulation by considering: a) age structure effects, b) wage related fertility, along a classical malthusian relation.

3.1 The "realistic labour market" Goodwin's model

As a first step toward more realism the total supply of labour at time t, N_s, may be defined as the product between the total number of individuals in the working age span $N(t)$, and the participation rate $s(t)$: $N_s(t) = s(t)N(t)$. In this paper we disregard participation effects (considered in Manfredi and Fanti 1999b), by putting $s = 1$ (everybody in the work age span offers his/her labour force), and concentrate on the rate of change of $N(t)$,which will be defined as $n(t)$. Following Manfredi and Fanti (1999a,b) the overall partial differential equation describing the population process is collapsed in a three-stage structure aimed to represent the most relevant stages of the individual life-cycle: pre-work ages ("young"), working age span ("adult"), retirement (let us call them stage 1, stage 2 and stage 3 for short). We assume that only adult individuals contribute to natality. Table 1 reports the description for the involved demographic parameters.

Parameter Name	Description
μ_1, μ_2, μ_3	mortality rates in stages 1,2,3
v_1	rate of transition from stage 1 to stage 2
	(rate of entrance in the labour force)
v_2	rate of transition from stage 2 to stage 3
	(rate of retirement)
b	rate of fertility of adults

Table 1 Demographic parameters employed in the model

Under the assumption that all the demographic rates of table 1 are age-independent, the rate of change $n(t)$ of the population in the working age span (details in Manfredi and Fanti 1999b) follows the ordinary differential equation[4]:

$$\dot{n} = (-1)\left[n^2 + Pn - B(w, U)\right] \tag{2}$$

where:

$$P = (\mu_1 + v_1) + (\mu_2 + v_2) > 0\,;\;\; B(w,U) = v_1 b(w,U) - Q\;\;;\;\; Q = (\mu_1 + v_1)(\mu_2 + v_2) > 0 \tag{3}$$

In particular $b(w, U)$ denotes the fertility rate in the overall population. The following form has been chosen:

$$b = b(w,U) = b_e(w)U + b_u(1-U) \tag{4}$$

where $b_e(w)$ denotes the fertility of the employed individuals and b_u the fertility of those who are unemployed. The previous relation defines the overall fertility rate as the weighted average of the fertility of employed and

[4] A similar approach is followed in Feichtinger and Sorger (1989), though they dod not make any explicit reference to the underlying population PDE, whereas our model has been derived via suitable assumptions on the basic PDE. Moreover they did not derive an equation resembling to our equation (2).

unemployed individuals weighted with the employment and unemployment weights. Such a relation, which postulates heterogeneous fertility behaviour of employed and unemployed individuals, is theoretically well motivated in Goodwin-type frameworks, where structural unemployment is the rule. In particular we postulate that only the fertility rate of the employed is wage-related, along a "malthusian" relation (i.e.: fertility is a non decreasing function of the wage), whereas the fertility of the unemployed is exogenous. Our relation (4) is certainly appropriate in historical epochs. Since the start of the demographic transition this relation has been often reversed, for example explained on the basis of the substitution of "quantity" of children for their "quality" (Barro and Becker 1989). Nonetheless it is certainly still valid, in several cases, for the developing world, and seems also consistent with modern western observed fertility patterns, such as the Sweedish case (Hoem 1999). For simplicity here we will limit our analysis to the case in which $b_e(0) = b_u$.[5]

By embedding (2)-(3)-(4) within the model (1) we arrive to our final model, which is defined by the three state equations in the variables w (the real wage), U, and n:

$$\begin{aligned} \dot{w} &= w(-\gamma + \rho U) \\ \dot{U} &= U(m(1-w) - n) \\ \dot{n} &= (-1)\left[n^2 + Pn - B(w,U)\right] \end{aligned} \tag{5}$$

plus (4) and (3). The choice to use the real wage rather than the labour share has been motivated by a need for simplicity: as growth was not our central concern here we put for simplicity $\alpha = 0$, and rescale to one the average productivity of labour (so that in this case w also represents the labour share of the economy).

The system (5) defines a flexible demo-economic framework, the potential applications which go beyond Goodwin-type economies: for instance it is directly applicable to the Solow's neoclassical growth model.

3.2 Properties of the "realistic labour market" model: the equilibria

We sketch the main properties of the model (5). For other details see Manfredi and Fanti (2000). The analysis of equilibria shows that the system (5) always has an equilibrium $E_0 = (0, 0, n_0)$, where

$$n_0 = \frac{1}{2}\left(-P + \sqrt{P^2 + 4B(0,0)}\right)$$

It holds $n_0 \gtrless 0$ depending on whether $R_0^u \gtrless 1$, where R_0^u is the Net Reproduction Ratio of the unemployed part of the population. A local stability analysis about E_0 shows that as in the basic case without age structure, stability and instability of E_0 essentially depend on the mutual magnitude of the accumulation rate m and the rate of growth of the population n_0 at zero wage and employment. When n_0 is negative (i.e. when: $R_0^u < 1$) then E_0 is always (trivially) unstable as a consequence of population decay (as in Goodwin's model). When n_0 is positive (i.e. when: $R_0^u > 1$) nontrivial instability appears when $m - n_0 > 0$.

Moreover, it is possible to show that as long as $b_e(0) = b_u$ the system (5) admits at most one strictly nonzero equilibrium $E_1 = (w_1, U_1, n_1)$, where $U = U_1 = \gamma/\rho$, whereas w_1, n_1 are the positive solutions of:

$$m(1-w) - n = 0 \quad ; \quad n^2 + Pn - B(w, U_1) = 0$$

The following proposition on the existence of equilibria holds (Manfredi and Fanti 2000):

[5] The equality between the fertility of employed and unemployed when the wage is very "low" implies a strong "symmetry". When this symmetry is broken down (Manfredi and Fanti 2000) more complex dynamical facts, namely chaotic oscillations, appear, in particular by allowing $b_e(0) < b_u$. This latter situation (fertility rates of the unemployed which are higher than those of the employed at low wage levels) can well be justified in developed countries according to a microfounded "view" à la Becker (1960) in which the opportunity-cost of the wage and preference for child quality play an unfavourable role for the fertility both of the wage earners and of the middle-class (that is the employed individuals). This view is obviously opposite to the malthusian one adopted in this paper.

PROPOSITION 1. *When the fertility of employed individuals at very low wages is identical to the fertility of unemployed, the system (5) always admits the zero equilibrium $E_0 = (0,0)$, which is a saddle point or a stable node depending on whether: $m \gtrless n_0$. Hence, unless the fertility of the unemployed is implausibly high, the zero equilibrium is unstable and the condition for the economic take-off is met. Moreover as long as E_0 is stable the system can not have positive equilibria. Vice-versa, when $m > n_0$, E_0 becomes unstable. At the same time a positive equilibrium E_1 appears, which is economically meaningful provided that $n(1, U_1) > 0$, i.e. provided that influence of wage on fertility is capable to generate population growth at least when the wage is set up to its maximal value.).*

3.3 Properties of the "realistic labour market" model: stability analysis of the positive equilibrium under linear fertility

For simplicity, the fertility schedule of the employed is now taken as linear: $b_e(w) = c_0 + c_1 w \; c_0 \geq 0, \; c_1 > 0$. We point out that our conclusions are unaffected if we choose a general monotonically increasing (possibly saturating) function of the wage. As in this paper we assume $c_0 = b_u$, we have

$$b_e(w) = b_u + c_1 w \quad b_u \geq 0, \; c_1 > 0 \tag{6}$$

The positive equilibrium E_1, when it exists, has the explicit coordinates:

$$U_1 = \frac{\gamma}{\rho} \; ; \; w_1 = \frac{m - n_1}{m} \; ; \; n_1 = \frac{1}{2}\left(-\left(P + \frac{v_1 c_1}{m} U_1\right) + \sqrt{\left(P + \frac{v_1 c_1}{m} U_1\right)^2 + 4\left(v_1 b\left(1, U_1\right) - Q\right)}\right) \tag{7}$$

The following facts hold:

$$\frac{\partial n_1}{\partial b_2} > 0 \quad \frac{\partial n_1}{\partial c_1} > 0 \quad \frac{\partial n_1}{\partial U} > 0 \quad \frac{\partial n_1}{\partial \gamma} > 0 \quad \frac{\partial n_1}{\partial v_1} > 0 \quad \frac{\partial n_1}{\partial \rho} < 0 \tag{8}$$

The local stability analysis of E_1 leads to the following stability condition

$$2n_1 + P - \gamma > 0 \tag{9}$$

In our previous work (Manfredi and Fanti 2000) the stability condition (9), and the involved bifurcation problem, has been studied by using the age of entrance into the labour force as a pivotal bifurcation parameter. Here we discuss the remarkable role played by the worker's resistance.

3.4 Effects of the worker's resistance

Here we consider the static and dynamical role played by γ.

3.4.1 Steady state effects

For what concerns equilibrium properties, in our general model (5), γ continues to determine the equilibrium level of employment ($U_1 = \gamma/\rho$), as in the original Goodwin's model. The important difference is that now γ affects the equilibrium level of the rate of growth of the supply of labour, and hence of the real wage as well, as clear from (7):

$$w_1(\gamma) = \frac{m - n_1(\gamma)}{m}$$

As $\partial n_1 / \partial \gamma > 0$ from (8), a larger worker's resistance (i.e. a smaller value of γ) implies a larger (equilibrium value of the) wage. The following proposition is a direct consequence of a comparison of (7) with the equilibrium results in the original Goodwin's model:

PROPOSITION 2. *Whereas in the Goodwin's model the WR only affects the rate of employment, in our "realistic labour market" model the WR has also an effect, via its influence on the rate of growth of the supply of labour, on the wage (and hence on the labour share). More specifically, whereas in the Goodwin's model a high WR is not capable of defending the equilibrium wage, we find that the WR is successful in order to raise the wage.*

The interest of this counterintuive result lies in the fact that provides completely new results compared to previous works on the wage struggle in Goodwin-type economies, such as Balducci and Candela (1982). Balducci and Candela found a very negative result for the working class: "*Il risultato della nostra analisi è assai limitativo per la classe dei lavoratori. Infatti, una maggiore conflittualità sociale si risolve alla lunga in una magggiore dis-occupazione senza alcun vantaggio distributivo...*" (Balducci and Candela 1982, p.115).[6] But their interpretation of the wage conflict was purely based on the role of the ρ parameter of the Phillips curve. The analysis of our realistic labour market model shows that the other way round may be true: a working class having the wage as its only target, is not only successful in raising its own wage, but can, as we shall later see, increase the stability of the economy as a whole. In addition, the two following facts seem to be worth to be noticed:

REMARK 1. *The fact that, as previously showed, $dw_1(\gamma)/d\gamma < 0$, agrees with the known empirical fact that countries with strong trade unions have higher wages. On the contrary, the original Goodwin's model and several of its variants have no predictive ability on this stylised fact.*

REMARK 2. *The fact that $dn_1(\gamma)/d\gamma > 0$ suggests that countries with a very strong working class tend to be the first in experiencing a reduction in the rate of growth of population. This has been observed in the case of England and other western countries.*

3.4.2 Dynamical effects

Let us now consider the effects of γ on the stability condition, i.e. let us look at the dynamical action of the WR. Let us write the stability condition as

$$H(\gamma) > \gamma \tag{10}$$

where $H(\gamma) = 2n_1(\gamma) + P$. The (8) implies that the function $H(\gamma;..)$ is a strictly increasing function of γ. Moreover $H(0) > 0$. The form of the stability region in terms of γ therefore depends on the shape of $H''(\gamma)$. It is possible to show that H is a concave function of γ, therefore implying a unique stability switch.[7]

The following proposition therefore holds (we omit the proof for brevity):

PROPOSITION 3. *Given the shape of the $H(\gamma)$ function, then a threshold value γ^* exists such that the E_1 equilibrium is LAS for $\gamma < \gamma^*$ and, vice-versa, is unstable for $\gamma > \gamma^*$. The threshold value γ^* (which is the unique solution of the equation $H(\gamma) - \gamma = 0$) is a Hopf bifurcation point for E_1.*

The content of the proposition is illustrated in fig. 2. The function $H(\gamma)$ lies above the 45° line in the interval $(0, \gamma^*)$, and below it for $\gamma > \gamma^*$. Therefore, the WR has a sharp stabilising effect on the economy: when γ is small, stability tends to prevail. A larger WR implies a smaller γ, and hence that the system will lie more far apart from the instability region.

Fig. 2. Stability regions and bifurcation in the realistic labour market LVG model

as functions of the worker's resistance.

[6] We do want enphasise the difference between Balducci and Candela's results and our ones, in that the parameter they use to measure the strenght of the unions is different from our one.

[7] For sake of brevity we omit the proof, which is available on request.

4 Numerical results

The fact to know that a Hopf bifurcation exists nothing says about the stability properties of the involved periodic orbits, i.e. it does not say whether the bifurcation is supercritical or subcritical (i.e. whether the periodic orbit is locally stable or unstable). Unfortunately the investigation of the stability properties of periodic orbits emerged via Hopf bifurcation at dimensions greater than dimension two is a hard task (Marsden and MacCracken 1976). Moreover the predictions of the Hopf theorem are local in nature: they nothing say about global behaviour. We therefore resorted to numerical simulation to clarify the stability nature of the Hopf bifurcation occurred at the point $\gamma = \gamma^*$. The simulation shows that the Hopf bifurcations is supercritical for $\gamma^* < \gamma < \gamma_{SUPER}$ where γ_{SUPER} is the upper bound of the set in which stable limit cycles are observed. Moreover, it does exists a subcritical window $\gamma_{SUB} < \gamma < \gamma^*$ where unstable limit cycles emerge. This is at all coherent with the stabilising effect of the WR: starting from a stable situation, when the worker's resistance decreases, a subcritical bifurcation emerges, generating a so-called corridor stability situation (Leijonhufvud 1981), in which the local stability of the equilibrium is guaranteed, though only in a neighbouring corridor of unspecified size. Further decrease of the WR of course generates stable sustained fluctuations of the economy. Finally, when the WR is decreased below a further threshold, global instability arises.

We illustrate the actual working of our model by resorting to a concrete example, in which, just to reduce complexity, we concentrate only on the dynamical effects of the WR (γ) and keep constant all the remaining parameters. In the following experiments we set $\rho = 0.6$, $m = 0.2$, $v_1 = 0.08$ (corresponding to an average age of entrance into the labour force $T = 1/0.08 = 12.5$ years), $v_2 = 0.02$, $\mu_1 = \mu_2 = 0.01$, $c_0 = b_u = 0.05$, $c_1 = 0.1$. As initial conditions we set: $w_0 = 0.88$, $U_0 = 0.175$, $n_0 = 0.02$. For a large WR ($\gamma = 0.04$ in our simulation) the system converges to a stable equilibrium with damped oscillations (fig. 3,4). The observed equilibrium values are $U_1 = 0.06$, $w_1 = 0.90$, $n_1 = 0.021$. For a lower WR ($\gamma = 0.17$ in our simulation), the system is destabilised and a stable limit cycle appears around the observed equilibrium values $U_1 = 0.27$, $w_1 = 0.87$, $n_1 = 0.03$. As expected a lower WR destabilises the system; the long term wage share is reduced, and the rate of growth of the population is higher.

Fig. 3. Convergence to the equilibrium in the 2-dim phase space (U, w) $(\gamma = 0.17)$.

Fig. 4. Convergence to the equilibrium in the 2-dim phase space (U, n).$(\gamma = 0.17)$.

Fig. 5. Convergence to a stable limit cycle in the 2-dim phase space (U, w).$(\gamma = 0.04)$.

Fig. 6. Convergence to a stable limit cycle in the 2-dim phase space (U, n).$(\gamma = 0.04)$.

5 Conclusive remarks

This paper shows some unexpected effects of the workers resistance in a "realistic Goodwin-type labour market model". The main long term result is that while in the original LVG model the WR only affects the rate of employment, in our extended model has also effect, via its influence on the population growth, on the wage. In fact, contrary to the original Goodwin's model in our formulation the WR is successful in achieving long term wage increases. Moreover the results of our model agree with two main stylised facts: countries with a higher

WR (and possibly with strong trade unions) exhibit (England is the most important example): a) higher wages, b) lower rates of growth of the population.

Even more interesting appears the role played by the WR on the system's stability: while in the original LVG model and perhaps in the common sense the resistance of the workers to wage flexibility is destabilising, in our model such a resistance has a stabilising effect. Moreover in our model Goodwin's conservative oscillations are lost, and persistent stable oscillations appear via Hopf bifurcations when the WR is low. Furthermore the simulations reveals that unstable limit cycles exist when the WR is high, so that "corridor" stability prevails.

We conclude that the addition of endogenous population growth to the original Goodwin's model may modify the dynamic behaviour of the economy substantially. More specifically the existence of an autonomous resistance of the workers in the determination of the wage can better the wage as well as favour the economic stability. In simple words: a purely wage targeted strategy by the working class, is not only successful in defending the wage, but also has strong stabilisation effects, and therefore is beneficial for the economic system as a whole.

6 REFERENCES

[1] Balducci R., Candela G. (1982), Contrattazione salariale e ciclo economico, Firenze,NIS.

[2] Barro R.J., Becker G.S. (1989), Fertility choice in a model of economic growth, Econometrica.

[3] Becker G.S. (1960), An economic analysis of fertility, in NBER (ed.), Demographic and economic change in developed countries, Princeton Univ. Press.

[4] Day R.H., Kim K.H., Macunovich D. (1989), Complex demoeconomic dynamics, *Journal of Population Economics*, 2, 2,139-159.

[5] Fanti L., Manfredi P. (1998b), A Goodwin-type growth cycle model with profit-sharing, *Economic Notes, 3, 183-214.*

[6] Fanti L., Manfredi P. (2000), Post-classical fertility in a realistic labour market Goodwin-type model, manuscript in preparation.

[7] Feichtinger G., Sorger G. (1989), Self-generating fertility waves in a nonlinear continuous overlapping generations model, *Journal of Population Economics*, 2, 267-280.

[8] Hoem B. (1999), Fertility in Sweeden. Quantum and tempo effects revisited, *Demographic Research.*

[9] Leijonhufvud A. (1981), Information and coordination, Oxford University Press, Oxford.

[10] Manfredi P., Fanti L. (1999a), Embedding population dynamics in macro-economic models. The case of the Goodwin growth cycle, WP 155, Dipartimento di Statistica e Matematica Applicata all'Economia, Pisa.

[11] Manfredi P., Fanti L. (1999b), Population dynamics, labour supply and sustained oscillations in Goodwin-type models, paper presented at CEF 99 (Annual meeting Society for Computation in Economics and Finance), Boston, 24-26 June 1999.

[12] Manfredi P., Fanti L. (2000), Population heterogeneities, nonlinear oscillations and chaos in some Goodwin-type demo-economic models, Paper presented at the Second workshop on "Nonlinear demography", Max Planck Institute for Demographic Research, Rostock, May 31-June 2, 2000.

[13] Marsden J., MacCracken M. (1976), The Hopf bifurcation and its applications, Springer Verlag, New York, Tokio.

[14] Samuelson P.A. (1976), An economist's nonlinear model of self-generated fertility waves, *Population Studies*, 30, 243-247.

[15] Solow R.M. (1956), A model of balanced growth, *Quarterly Journal of Economics*, 70, 65-94.

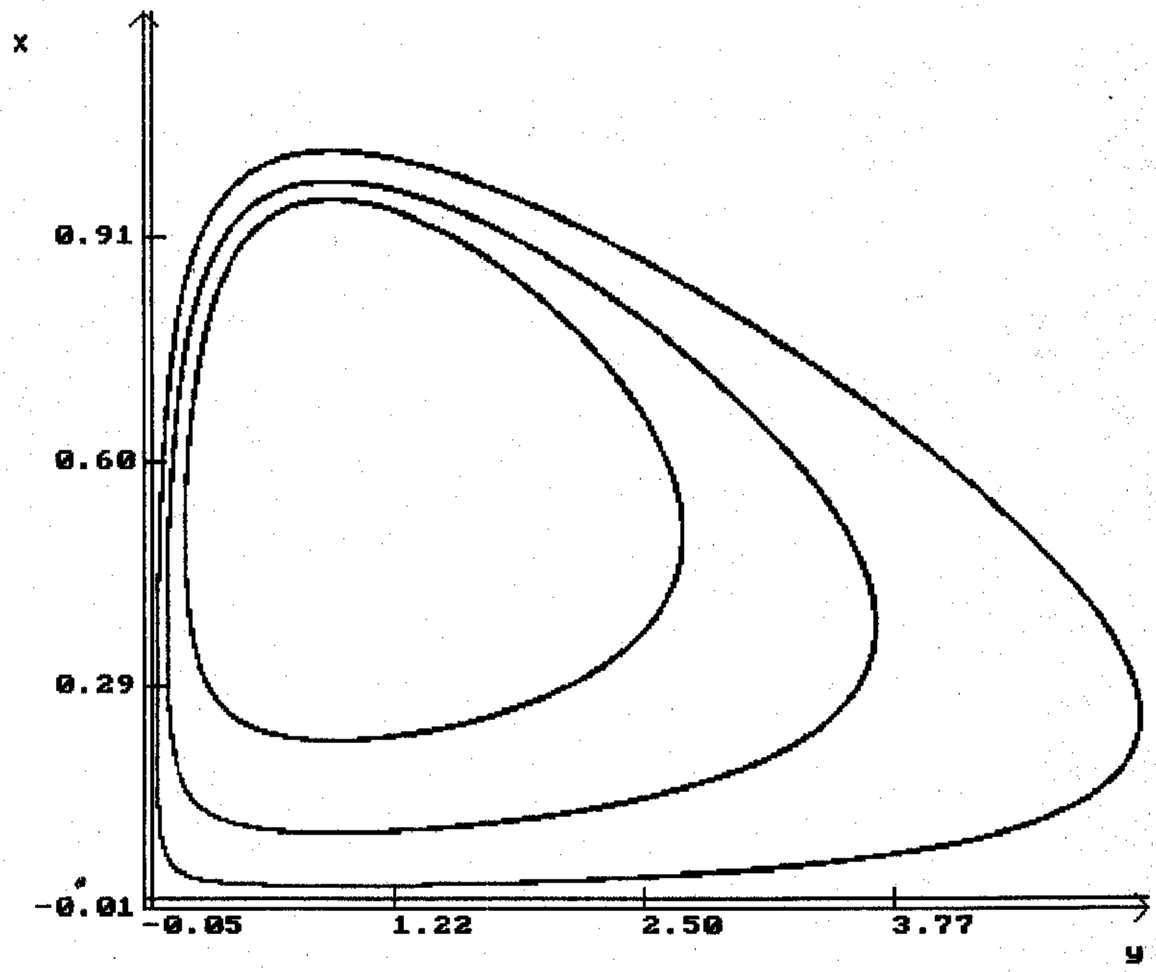

Fig. 1

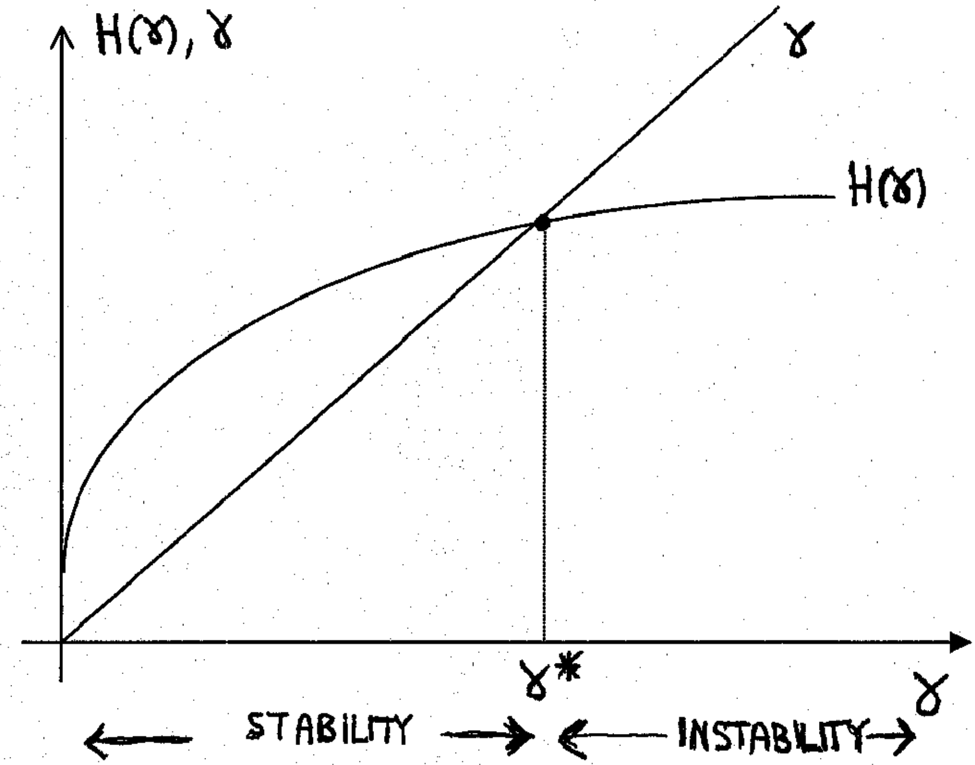

Fig. 2

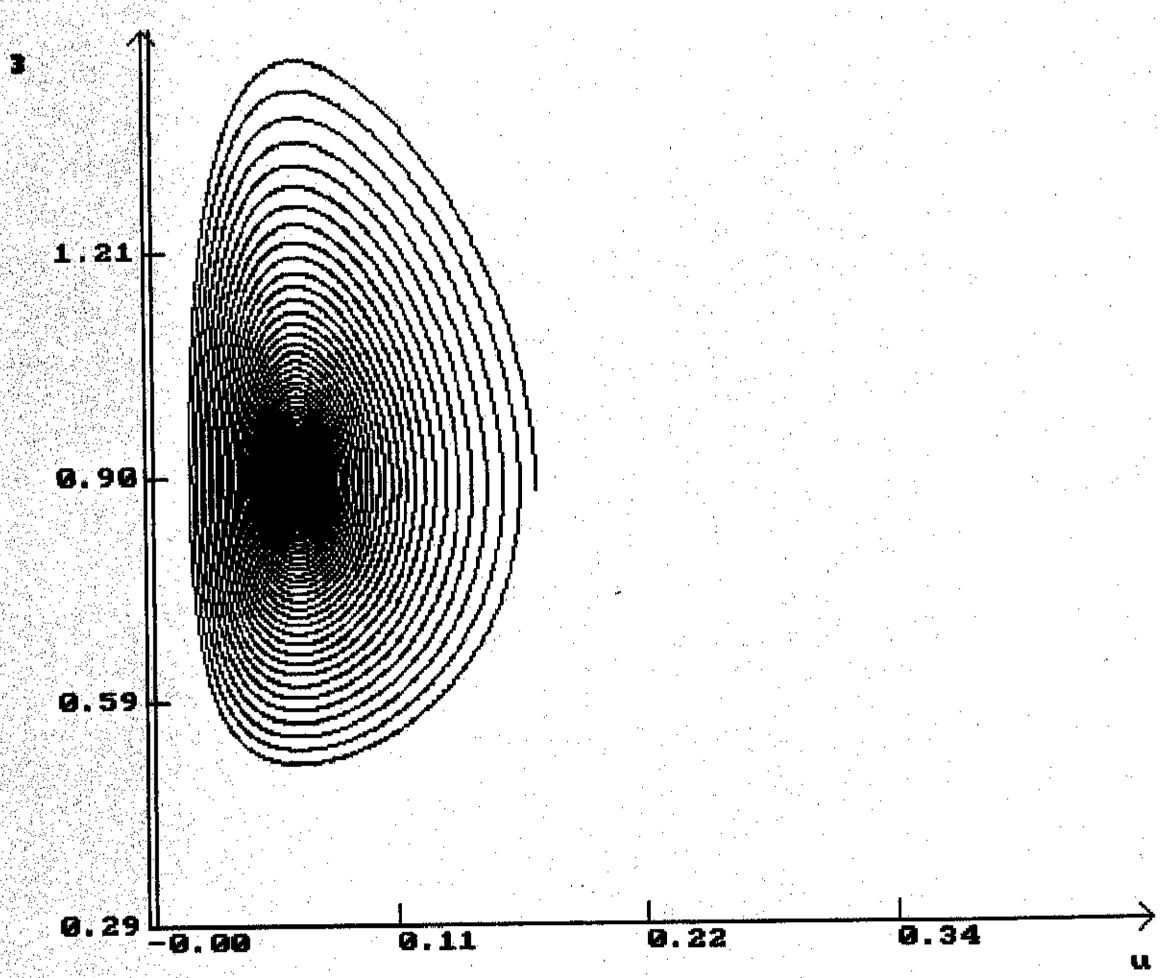

Fig 3

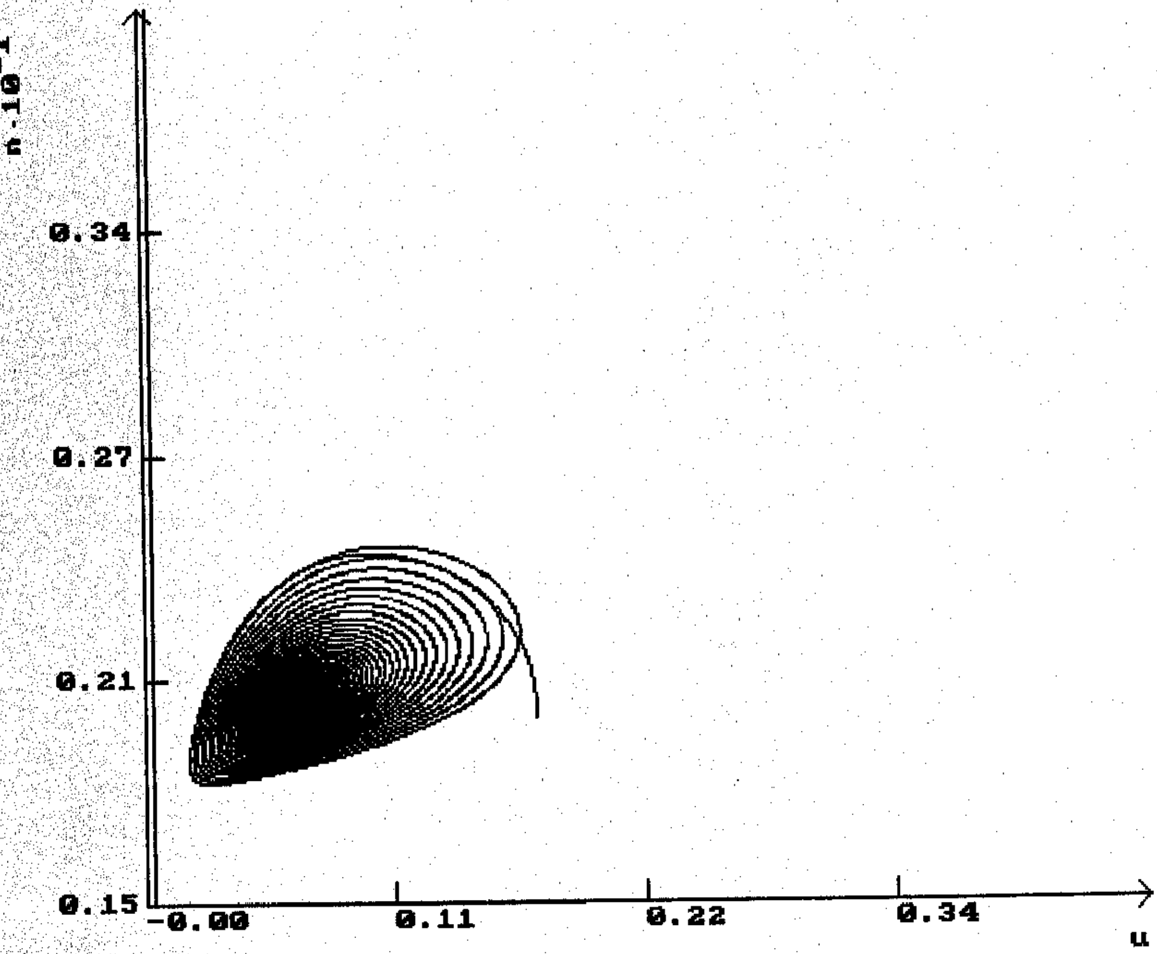

Fig 4

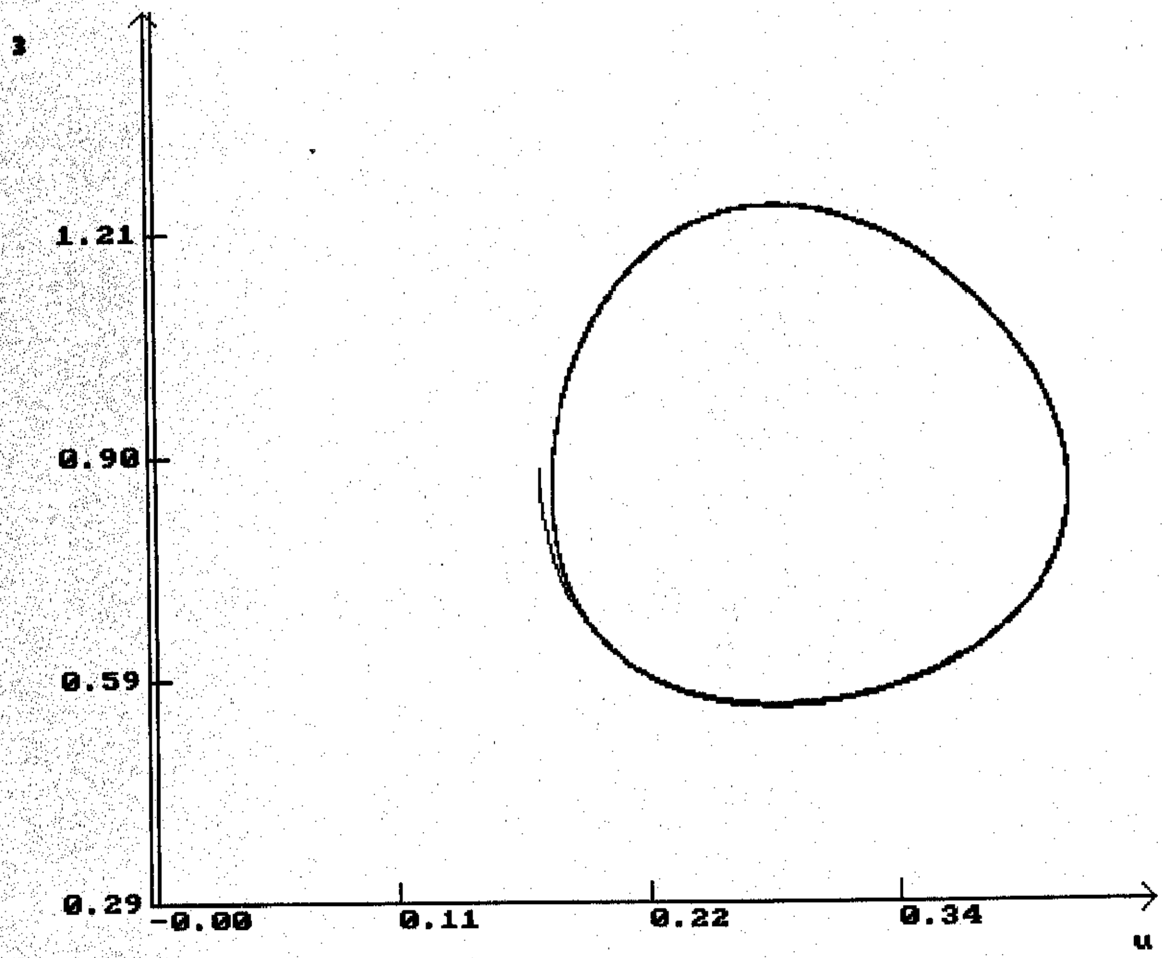

Fig 5

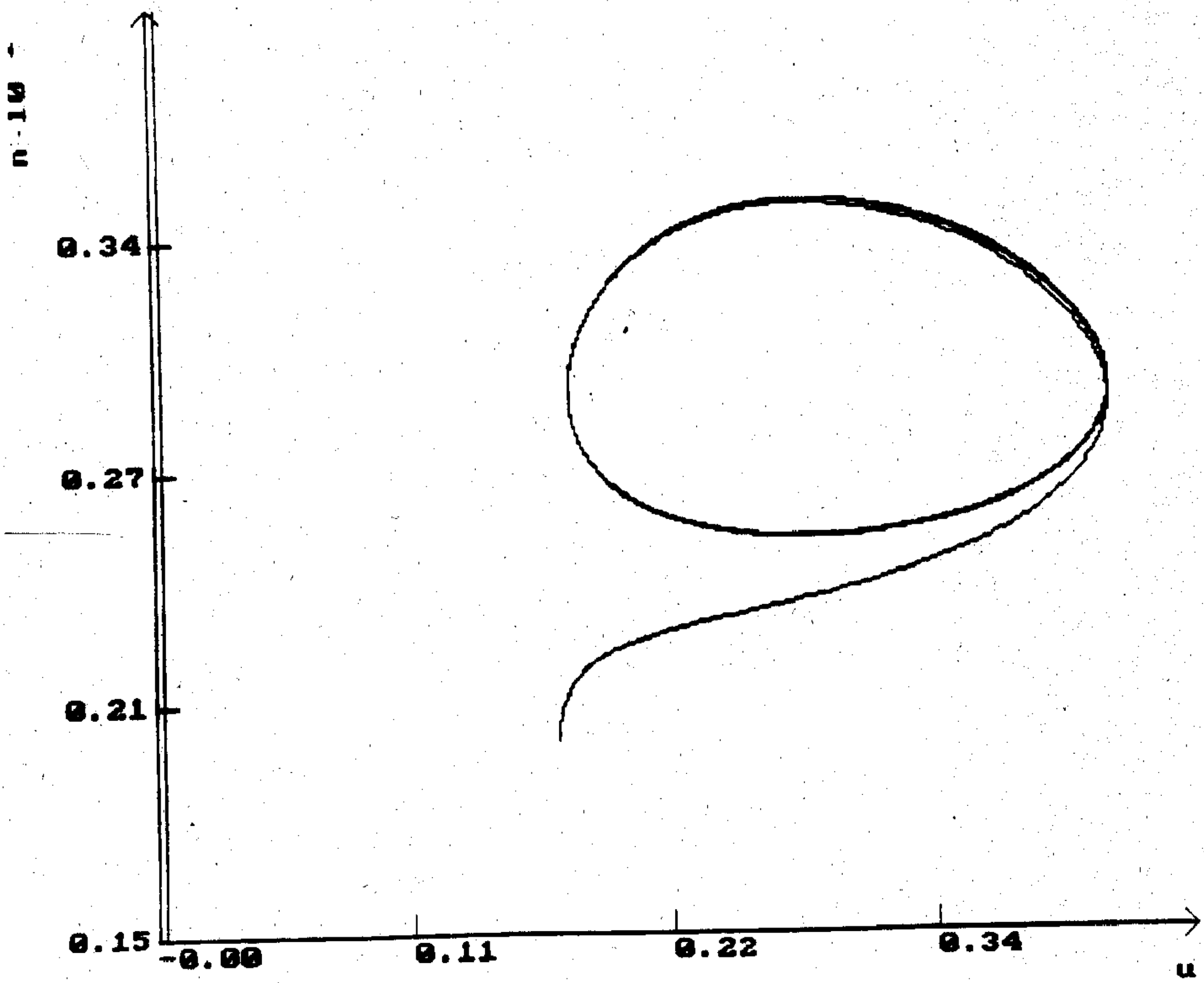

Fig. 6