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Demographic transition and balanced growth

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PART 2

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ABSTRACT

The main stylised fact of the demographic transition, a major concerns of modern growth theory, are embedded within the classical Solow's model. The analysis gives considerable insight on two important aspects: i) the existence of poverty traps, and the historically observed mechanisms of escape; ii) a more flexible notion of convergence, which appears a powerful explanatory tool for observed dynamics.

1 Introduction

One of the most challenging problem faced by scholars in demo-economics is the endogenous explanation of the transitions between the great historical regimes, in particular of the so called demographic transition (DT), see for instance Galor and Weil (1999). A quite generally accepted view is that Solow's (1956) model is totally unable to provide an economic explanation of the demographic transition. This is clearly expressed for instance by Chu (1998,133), who, enphasizing the gain permitted by the new growth theory, as exemplified for instance by the Becker-Murphy-Tamura model (Becker et al. 1990), states "... the explanation of the DT *has not been succesful under the neoclassical model of Solow, for it typically predicts a converging steady-state growth rate of per capita income, which is incompatible with the diverging development paths among countries observed over the past 50 years. Moreover, Solow's model is also weak in predicting* *the relationship between income growth rate and population growth rate. It is well known that in Solows's model the steady state rate of percapita income is a decreasing function of the population growth rate. Though this prediction gets some empirical support in areas such China and India, such reasoning is narrowly restricted to the dilution of physical capital, and is not broad enough to be compatible with general development experience, which may be related to the accumulation of human capital.* (Chu 1998, 133).

Though generally accepted the previous reasoning is, to our mind, exhaggerated. In our view the relevant matter can not be whether the original Solows's model with a constant rate of change of the supply of labour can predict the demographic transition, or explain its timing and forms, obviously does'nt. Rather the point is whether, adequately equipped, i.e. by postulating some "transitional" assumption, can the Solow's model offer insight on fundamental aspects of modern growth. In very recent times Strulik (1999 and 1997) has offered new light on the point, showing that by embedding transitional hypotheses in growth models one can get a great deal of insight on the dynamical forms of economic growth.

Despite the interest of Strulik contribution we believe that the mathematical frameworks he considered, i.e. a neoclassical (non optimising) 3-dimensional model with learning by doing (Strulik 1997) and a Solow-type model for the developing world embedding the keynesian feature of a fixed minimum wage (Strulik1999) are even too rich with the potential risk of hidding basic facts of economic growth. For this reason we feel that even sharper insight can be gained when the typical assumptions on population dynamics during the transitional phase are simply cast within the original Solow's model, without any further assumption. This strategy leads to a still highly simplified model, which nonetheless provides a quite rich mess of results. First, multiple equilibria appear, with in particular a stable "poor equilibrium", i.a a poverty ("malthusian") trap, coexisting with a rich, "modern", equilibrium, which is also stable. This allows an interesting discussion of the mechanisms that could have been set in motion in history in order to provide the escape from the poverty trap. Moreover, the well known "convergence" statement arising in Solow's model is replaced by a more general result pleasantly showing that countries that escaped the poverty trap with different endowments of capital would possibly experiment an initial divergent, rather than convergent, phase, which should convert to a convergent regime only in the long (perhaps very long) run. This result thereby provides an answer to the critical difficulty of Solow's model toward observed facts, as previously quoted from Chu.

This paper is organised as follows. In the second section our Solow-type model with "demographic transition" mortality and fertility is presented. In section three its main results are reported. Conclusive remarks follow.

2 The model

We consider here the following extension of the classical Solow's model

$$\dot{k} = sf(k) - (\delta + n(y))k \tag{1}$$

In (1) a standard production function $Y = f(K, L)$ with constant returns to scale is assumed, which may be written as $y = f(k)$ in per capita terms, where k=the capital/labour ratio. $f(k)$ is assumed to satify the Inada's condition. Moreover δ=the rate of capital depreciation, s=the saving rate, $n(y)$ =the growth rate of the population. The growth rate of the population is assumed to be a function of the level of per capita income $y = f(k)$. This motivates

$$\dot{k} = sf(k) - (\delta + n(f(k)))k \tag{2}$$

which is a well acknowledged formulation in the literature on population growth (Nerlove and Raut 1998). We further assume that $n(y) = b(y) - \mu(y)$ where $b(.)$, $\mu(.)$ respectively denote the fertility and mortality rates observed in the population. The functions $b(.)$, $\mu(.)$ are assumed to be decreasing in y according to the typical asynchronous declining form which have been observed almost universally as the offprint of the demographic transition (Strulik 1999, Chesnais 1992).[1] This gives to the $n(y)$ function the traditionally observed humped form, starting from a zero or slightly positive values at the beginning of the transition, raising up to a maximum, and finally going down again up to a slightly positive asymptotic value.[2] For simplicity we write

$$\dot{k} = sf(k) - (\delta + n(k))k \tag{3}$$

which is a well acknowledged formulation in the literature on population growth (Nerlove and Raut 1998).

Nerlove and Raut (1998) express some skepticism about this type of approach: "it is clear that merely endogeneizing population growth at the macro-level does not shed light on the shape of $n(k)$ and thus on the nature of the dynamics: a utility-maximising model should be used to elucidate the nature of the function $n(k)$. Though Nerlove and Raut's observation is in principle correct, we believe that an indisputable exception is represented exactly by the previous formulation, which is supported by one of the most massive amount of empirical evidence available in the social sciences. We nonetheless point out that the aggregate relations of demographic transitions can, however, be fully micro-funded. This has been done for instance by Jones (2000), who has determined the fertility function from the maximisation of a utility function under the assumptions that the elasticity of substitution between consumption and children is greater than

[1] A further effect of the demographic transition is the change in the patterns of per-capita saving. Theoretical and empirical evidence (Strulik 1999) suggests that the saving rate is an increasing function of y during the transition. We will add a remark on the point at the end of this paper. The problem will be developed more in detail within a micro-funded approach in a forthcoming paper.

[2] There are several instances in the last decades (Italy and Spain), of the fact that in the post-modern world the rate of change of the population could even become negative (the so-called second transition). In this case economic growth of per capita income would be recovered though in a trivial manner.

one, given a simple "technology" for producing children. Moreover we feel that this could also be done by adopting a fertility function derived from a household optimization with substitution of quantity versus quality of child, i.e. within the Becker and Barro (1988) framework, provided one postulates a mortality function which is declining with income (as postulated by the health revolution). One expects that the delayed (compared to mortality) predicted decline of fertility should then straightforwardly follow, therefore leading to a humped rate of growth of the population. A further avenue is suggested by Strulik (1999) who again use the decline in mortality as the motor of the transition. In the appendix we report some possible avenues, suggested in the literature, for micro-funding the aggregate relation of demographic transition.

3 Results

The analysis of (3) is quickly performed by resorting to usual graphical tools, i.e. the growth diagrams (Barro and Sala-i-Martin, 1995) for standard one dimensional growth models. As the b, μ functions are all monotonic functions of y, and $y = f(k)$ is in its turn a monotonically increasing function of k, it is straightforward to draw the curves $m_1(k) = s(k) f(k)/k$ and $m_2(k) = \delta + n(k)$. In particular $m_1(k)$ is monotonically decreasing and approaches zero as k tends to infinity, whereas $m_2(k)$ inherits the humped shape of $n(k)$. The vertical distance between the two curves is the rate of growth of k, γ_k. A direct graphical inspection of fig.1 and fig. 2, where the curves $m_1(k)$ and $m_2(k)$are reported reveals that *multiple equilibria* may exist.[3] The behaviour of model (3) is summarised by the following

PROPOSITION 1. *Apart the zero equilibrium ($k = 0$), system (3) admits one or three possible equilibria. In the former case call k_1 the unique positive equilibrium. Then k_1 is always (globally) stable. In the latter case call the three equilibria as $k_{low} < k_{med} < k_{large}$. A staightforward phase diagram analysis reveals that k_{med} is unstable, whereas k_{low} and k_{large} are locally stable, with respective basins of attraction: $Bas(k_{low}) = (0, k_{med})$, $Bas(k_{large}) = (k_{med}, +\infty)$. Dynamical considerations are then straightforward.*[4]

The proof of the proposition straightforwardly follows from the inspection of the sign of the

[3]The existence of multiple equilibria is important also in that it can potentially resolve the empirical controversy involving the relation between RPG and economic growth: in fact many empirical works find negative causal effect of the former on the latter as well as absence of any effect. This empirical controversy may be reconciled by the existence of multiple equilibria, which could be the cause of the conflicting empirical findings (Yip-Zhang, 1997).

[4]The effects on endogenous population on Solow's model has been already the object of several efforts, but with results different from the present ones. Solow's himself noticed that the introduction of endogenous population has no relevant effects on the main results of his basic model. Subsequently it has been acknowledge that such endogenization can produce multiple equilibria, some of which are stable and some not (Nerlove-Raut 1998, pp.1127). Nonetheless most of the examples reported up to now (Solow 1956, Nerlove-Raut 1998, fig.1), though based on complicated relationship between population growth and percapita income do not produce interesting results: at most a second, but unstable, equilibrium appears, but all the GAS of the system is unaffected. On the other hand multiple equilibria with several stable equilibria with possible ranking in terms of economic desirability, may be obtained by resorting to special assumptions on the saving function (Blanchet 1994, 27).

rate of change γ_k (the arrows on the horizontal axis in fig. 1 indicate the direction of motion of k). The discussion of the economic consequences of the previous proposition leads to the following substantive results.

3.1 Poverty trap

Following Barro and Sala y Martin (1995,49) ".. a poverty trap is a stable steady state with low levels of per capita output and capital stock". Such an equilibrium is termed poverty trap exactly because a) it is an equilibrium with "poverty" (at least in relative sense), and: b) it is stable (implying that any attempts by the economic agents to break out of it will be usuccesfull and the economy will return to this low level steady state). Barro and Sala y Martin (1995) argue that poverty traps typically arise as the result of the coexistence of regions of decreasing returns with regions of increasing returns. In (3) the poverty trap arises not as the consequence of the hypothetical existence of areas of diminishing/increasing returns, but as the consequence of the indisputable shape of the rate of growth of the population during the transitional regime. In the case of multiple equilibria in (3), there is a necessary ranking of equilibria in terms of all the percapita variables, so that the locally stable "low" equilibrium k_{low} is a poverty trap (or "malthusian trap" in the language of demo-economists). There is a second interesting case, occurring in the case of the unique equilibrium, namely when the saving curve $m_1(k)$ is so low (m_1^* in fig.1) that the existence of the richer equilibria k_{med}, k_{large} is prevented. In this case k_1 is a very "poor" equilibrium ($k_1 < k_{low}$ coeteris paribus), and is stable: therefore k_1 may nonetheless termed a poverty trap. Therefore also the case of the unique equilibrium is of theoretical interest, as it represents the case of the "ancient regime" malthusian stagnation (Galor and Weil 1999) during which accumulation forces were so low that the existence itself of a richer regime was impossible. Since now on we also define the poverty trap equilibrium k_{low} as the malthusian equilibrium, and the rich equilibrium k_{rich} as the modern, or post transitional, regime. Notice that there is an opposite case, when the saving curve $m_1(k)$ is so high (m_1^{**}) that the existence of the poorest equilibria k_{low}, k_{med} is prevented, in this case k_{large} becomes a virtuous equilibrium.

It is of interest to look, following Barro and Sala y Martin (1995), at the effects of policies and external shoks, as possible "escape" mechanisms from the malthusian trap. In what follows s standard Cobb-Douglas production functions $Y = BK^{\alpha}L^{1-\alpha}$ with constant returns to scale is assumed. We assume that at some initial stage the economy is emprisoned into the malthusian stagnation one equilibrium regime (the curve m_1^*).

i) *a domestic policy aimed at increasing the saving rate*: this causes an upward shift of the $m_1(k) = sf(k)/k$ curve(Fig. 1). Three distinct situations arise: a) the upward shift is so low that the new $m_1(k)$ curve, termed m_2^{**}, still intersect the $n(k)$ curve only once. In this case no breaking away from poverty trap occurs (though the economy experiences an increase of the per-capita income at equilibrium); b) the increase in the rate of saving is large enough so that three intersections occur. In this case the escape from the poverty trap occurs if history and "luck" allows k to leapfrog k_{med}; 3) the upward shift is large enough so that the hump in the m_2

curve does not meet the m_1 curve. In this case the poverty trap is literally lost and the economy approaches in the long term the virtuous steady state k_{large}. Notice that, still following Barro-Sala-i-Martin, that escape from the trap could also be allowed by a sufficiently long *temporary policy* of raising the saving rate. This aspect may not be evident because the time dimension is not visible on the phase diagram of fig.1,2. A temporary shift of the saving schedule implies a double switch from an "old" model, with low saving rate, to a new one with higher saving rate, and then back again to the old one. These switches take time to be put in practice. It may well happen that, after the last switch has occurred, the economy has inherited a value of the capital endowment k outside the basin of attraction of the trap.[5]

REMARK 1. *(changes in saving patterns as a realistic way of escaping the poverty trap). It is generally ackowledged tha one of true motors of the demographic transition has been the process of mortality decline (observed since 1700-1750 in some western countries). In his micro-foundation (see the appendix) to the transition mechanism, Strulik (1999) gives theoretical support to the fact that mortality decline not only explains the subsequent decline in fertility, but also an increase in the saving rate. Again his too complicated Solow-type model does not make transparent that an increase of the saving rate synchronized with the decline in mortality and fertility may have been the ultimate responsible of the escape from the poverty trap, because it could have sufficiently raisen the* $m_1(k)$ *therefore preventing the birth of the poverty-trap equilibrium. In other words, the transitional process seems to have been endowed by an endogenous escape mechanism, which seems very well suited in order to explain why a subset of countries of the world, the rich ones namely, seemed to have never been involved in the marsh of the poverty trap. This endogenous mechanism has the mortality decline as his ultimate responsible. These considerations seem to suggest a nice research direction on the empirical side.*

ii) *aid to developments.* Another way of escaping the trap is through a policy of international assistance by rich countries based for instance on a donation of capital. Since such a policy only affects the initial conditions of the system while leaving unaffected the equilibrium structure, it can be successful only provided that the "assisted" economy is in a parameter constellation for which three equilibria exist. Assume that the assisted economy is stably evolving it its trappy equilibrium k_{low}. In this case, if the donation raises per-capita capital to a level above k_{med}, the economy will approach the high level steady state k_{large}, where it is permanently escaped from the trap. Vice-versa, if the state of the under-developed economy is such that no other equilibria exists apart the poor one, then the task of the international aids policy would be harder, as it should first affects the parameters of the assisted economy in order to allow the appearance of the other equilibria. Only then could the donation be useful!. Be this as it may, a critical threshold exists in the donation size, below which the international aid has no long period effects.

iii) *changes in the index of technology B.* Their effects are analogous to those due to changes in the saving rate. Since B is a usual candidate to random shocks, technological shocks can favour the escape from the poverty trap (positive shocks) as well as to prevent an impending take-off provoking a return to the development trap (negative shocks).

[5] The same result would be attained via international loans, rather than domestic saving. keep temporarily high the domestic investment to GDP ratio.

iv) *changes in the technological parameter* α. The α parameter measures in a certain sense the "capitalistic intensity" of the technique. As shown in fig. 2, more intensively capitalistic technologies may favour the escape from poverty trap. This agrees with the fact that some Western country, which have experienced either an initial or a rapidly occurred larger capital-intensive production, have quickly moved towards the Modern Regime.

To sum up on the properties of our model we recall:

1) Luck in the timing and magnitude of shocks is needed to give a sufficiently big push to investment in capital. In thw words of Becker et al. (1990, S33) " We believe that the West's primacy, which began in the XVII century was partly due to a " lucky" timing of technological and political changes in West " .

2) Even temporary events, if they are strong enough, can permanently move the economy away from underdevelopment. As noticed by Becker et al. (1990) the growth may display path dependence.

3.2 Realistic convergence patterns

Let us consider the rate of change of k on the region (k_{med}, k_{large}). As figs.2 shows, γ_k increases from zero up to a maximum k^*, and then monotonically declines to zero again, as the system is attracted in the k_{large} equilibrium. The implications are far-reaching. In the original Solow's model γ_k monotonically declines to zero as k increases toward its positive equilibrium value. This has led to the quite debated concept of convergence (Barro and Sala y Martin 1995, 26-27): economies with lower capital per person are predicted to grow faster in per-capita terms, a fact which has been denied on the empirical ground in many instances (see the abovementioned quotation by Chu). In our transitional Solow-type model, economies which in the end escaped the malthusian trap and entered (due to external shocks, aids to development, etc) do not exhibit convergent paths as in the Solow's model. Consider two economies A, B which escaped the malthusian trap, i.e. that after some shock entered the (k_{med}, k_{large}) region with respective endowments $k_{poor} < k_{rich}$. These is a whole region of the set (k_{med}, k_{large}) in which the two economies will experience initially *divergence*, i.e. a situation in which the richer economy grows faster than the poorer one (the amplitude of such region depends on the the actual position of $k_{poor} < k_{rich}$): richer countries become even richer, a largely observed fact for instance if one compares developed versus developing countries. Only at a later stage the two economies will enter a phase of convergent dynamics similar to that observed in the textbook Solow's model. Moreover, we remark that the lenght of the divergent phase may be quite large.

Fig. 1. The three equilibria in the Solow's model with transitional dynamics.

Fig. 2. Escape from the poverty trap allowed by the transitional increase in the saving rate

4 Conclusions

This paper has investigated the static and dynamical consequences of perhaps the most important historical process of long term population evolution, the demographic transition, within the framework of the classical Solow's model. The model leads to the coexistence of two stable equilibria, a poor one (i.e. a poverty trap) corresponding to the historical malthusian stagnation regime, and a rich, modern, equilibrium. Escape from poverty and stagnation towards the virtuous modern regime are then major concerns.

We have discussed at lenght policy implications in the model, motivated by the fact that, as opposite to other models with multiple equilibria, most of the assumptions underlying our model are, especially on the demographic side, are based on perhaps the largest body of empirical evidence existing in social sciences (loosely speaking). We have showed that i) a higher rate of saving; b) a positive random shock on technology, iii) a more capital intensive technology may favour the escape from the poverty trap. On the other hand an unexpected result is that policies aimed to restore high fertility may lead to an undesidered reversion toward the poverty trap. Moreover our model predicts that temporary policies, as aids to development in poor countries could allow for the escape from the development trap provided that they are strong enough or mantained for a suffciently long time, otherwise they risk to be wasted, because after some period of time the economy could be condamned to return to the malthusian trap. There is scope for luck and history to determine the fate of the economy.

Finally, our model exhibits a nice property related to one of the most important question of economic growth: does there tend to be convergence across economies? In our model, in the modern regime (i.e. the regime experienced by those economie which escaped the malthusian trap) convergence is not the rule. Rather the rule seems to be that developing countries will, for a perhaps wide range of levels of k and therfore for large periods of time, exhibit rates of growth systematically lower respect to richer countries. Convergence in the Solow's sense returns to be the rule only in the long term. Therefore we expect that, empirical analysis should mainly confirm the absence of convergence as the more likely result (Chu 1998).

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Appendix

We observed in the main text that the "humped" form of the rate of change of the population can be micro-funded. In this appendix we review some of the strategies for its micro-foundation proposed in the literature, and also show how would change our basic Solow type model under the new conditions.

Jones strategy (Jones 1999)

Here we discuss Jones' (1999) formulation. As Jones' model does not consider saving (s=0) we enrich it by adding an (exogenous) non zero saving rate, with the purpose to make Jones' formulation compatible with the accumulation side in Solow's model. Each individual has, in each period of time, an endowment of one unit of labour which can be used to obtain consumption or children. Let h= fraction of time spent working ((1-h)=time spent producing children), w= earnings per unit of time worked, s= constant rate of saving of earnings, c°=the (constant) subsistence consumption, b° =the constant number of children independent from the time spent on childrearing. The utility of individuals depends on consumption *c* and number of children *b*, and has the form

$$u(c,b)=\frac{(1-m)(c-c^{o})^{1-\gamma}}{1-\gamma}+\frac{m(b-b^{o})^{1-\eta}}{1-\eta} \qquad 0<m<1\ ;\ 0<\gamma,\eta\le 1 \qquad (1)$$

They solve the problem $\max\limits_{c,b,h}\ u(c-c^{o},b-b^{o})$ (2)

s.t.

$$i)\ \ c=(1-s)wh; \qquad ii)\ b=f(1-h) \qquad (3)$$

Individuals take w as given; ii) is the technology for producing children: each unit of time spent producing children produces f births with $f>b^{o}$. Let $\hat{c}=c-c^{o}$; $\hat{b}=b-b^{o}$. It is easy to show (Jones 1999) that the "net" optimal quantity of children and the optimal consumption respectively are

$$\hat{b}=\left[\frac{fm\hat{c}^{\gamma}}{(1-s)w(1-m)}\right]^{\frac{1}{\eta}} \qquad c=(1-s)w(1-\frac{b}{f}) \qquad (4a,b)$$

Remembering that $\hat{c}=c-c^{o}$; $\hat{b}=b-b^{o}$, one can write the relation (4a) as

$$b-b^{o}-\left\{\frac{mf}{(1-m)(1-s)w}\left[w(1-\frac{b}{f})-c^{o}\right]^{\gamma}\right\}^{\frac{1}{\eta}}=0 \qquad (5)$$

which defines the relation between the optimal " net" number of children and the wage w in implicit terms. By the implicit function theorem it is possible to show (Jones 1999) that (5) actually defines b as function of w, i.e. b=b(w), which has a humped shape, therefore mirroring the most important stylised facts of the demographic transition process.[1] Finally by using (3), we obtain the optimal value of the time spent working:

[1] Jones (1999) remarks: "The traditional income and substitution effects are reflected in the second term. As the wage goes up, the income effect leads individuals to increase both consumption and fertility. The substitution effect, on the other hand, leads people to substitute toward consumption and away from fertility: the discovery of new ideas raises the productivity of labour at producing consumption, but the technology for producing children is unchanged. If $\gamma<1$, then the substitution effect dominates, while if $\gamma>1$,the income effect dominates. As usual, if $\gamma=1$,i.e.with log utility, these two effects offset. A third effect not traditionally present is reflected in the first term: as the wage rises, the subsistence consumption level which the consumer i required to purchase gets cheaper, leading consumers to have more after-subsistence income to spend on both more children and more consumption. This effect disappears as the wage gets large. The assumption that $0<\gamma<1$,then, leads the subsistence effect to dominate for small values of the wage and the substitution effect to dominate for large values of the wage, producing one component of the demographic transition: fertility rises and then falls as the wage rate rises."(Jones 2000, p. 9).

$$h = 1 - \frac{b(w)}{f} \qquad (6)$$

where b is evaluated at the optimal point. Let us now explicitly assume that w, however given, be a function of time with some specific patterns. It holds

$$\frac{\dot{h}}{h} = (-1)\frac{b'(w)\dot{w}(t)}{f - b(w)} \qquad (7)$$

Let us now cast the aforementioned theory within the basic Solow's model. Remember first that i) the quantity of labour supplied for production (L) is related to the total population by $L = hN$ (so that $\frac{\dot{k}}{k} = \frac{\dot{K}}{K} - \frac{\dot{L}}{L} = \frac{\dot{K}}{K} - \frac{\dot{h}}{h} - \frac{\dot{N}}{N}$), and ii) $w = f(k) = k^{\alpha}$. The latter implies

$$\frac{\dot{h}}{h} = (-1)\frac{b'(k^{\alpha})\alpha k^{\alpha-1}}{f - b(k^{\alpha})}\dot{k} \qquad (8)$$

We therefore obtain the following extended Solow-type model (micro-founded with optimal fertility behaviour) under Cobb-Douglas technology

$$\dot{k} = sBk^{a} - (\delta + b(k^{\alpha}) - \mu)k + \alpha\frac{b'(k^{\alpha})k^{\alpha-1}}{(f - b(k^{\alpha}))}\dot{k} \qquad (9)$$

which once reduced looks as

$$\dot{k} = g(k)(sBk^{a} - (\delta + b(k^{\alpha}) - d)k) \qquad (10)$$

where

$$g(k) = \frac{f - b(k^{\alpha})}{(f - b(k^{\alpha})) - \alpha b'(k^{\alpha})k^{\alpha-1}} \qquad (11)$$

Model (10) differs from our "transitional" Solow-type model reported in the main text just for the g(k) function. If one can ensure that g(k)>0 then the main results reported in the paper are unaffected, with the advantage of having being proved to be the outcome of a fully micro-founded approach.

Strulik's strategy (Strulik 1999)

The strategy followed by Strulik is completely different. More coherently with the Solow's framework not only fertility but also saving are endogenously determined within a simple intertemporal scheme. An individual's life is separated into three periods: childhood (=period 0), ii) adulthood (=period 1), iii) old age (=period 2). At the beginning of period 1 he/she is endowed with his/her working income (w), and plans consumption during periods 1,2 (denoted as C_1 and C_2), and therefore saving, and the number of offsprings, denoted by *n*. Rearing a child requires a fixed fraction of income, denoted by *e*. Moreover, decisions are made under uncertain duration of life (the consequence is that consumptions is discounted not only for time preference but also for uncertainty in the duration of life). The chosen intertemporal utility function is the Cobb-Douglas function $V = C_1^{\beta_1} C_2^{\beta_2} n^{\beta_3}$. Passing to log, the consumer maximises:

$$U = \beta_1 \log C_1 + \pi\beta_2 \log C_2 + \beta_3 \log n \qquad (1)$$

under

$$w - \left(c_1 + \frac{c_2}{1+r} + ewn\right) = 0 \qquad (2)$$

where π is the probability $\pi_{1,2}$ of surviving from period 2 to period 3 (whereas for simplicity $\pi_{1,2}$=1), and r is the period interest rate. One then easily gets the optimal saving rate and the optimal demand for children

$$s=\frac{\pi\beta_2}{\beta_1+\beta_3+\pi\beta_2} \quad ; \quad n=\frac{\beta_3}{e(\beta_1+\beta_3+\pi\beta_2)} \tag{3}$$

It is therefore immediate to conclude that, as π increases (which is universally recognised as the motor of the demographic transition), the saving rate cannot but increase, whereas the number of children planned for a given amount of income cannot but decrease. If the time dimension would be explicitly introduced this state of affairs implies that the decrease in mortality observed as a main stylised fact of the demographic transition leads, as main consequences, to a subsequent decrease in the fertility rate, and to an increase in the saving rate. Compared to Jones (1999) this reasoning appears to be based on simpler assumprions, and moreover i) it simply captures the pure facts of the transition. In fact the asynchronous decrease in fertility and mortality is sufficient to generate the humped form in the rate of growth of the population, i.e.: the initial increase in fertility predicted by Jones is totally un-necessary ;) it shows where the explanation for the increase in the rate of change of the population lies, namely in the increase in reproductivity allowed by the reduction in mortality; iii) it has an endogenous explanation of saving, which is well taylored with Solow-type models.

Once these facts are introduced within the classical Solow's model, by postulating, as done in the main text, that the decreasing over time pattern of mortality during the transition goes along the growth of per capita income, i.e. that $\pi=\pi(w)=\pi(k^{\alpha})$, where π is an increasing function of w, (we recall that π is a survival probability and therefore the mortality rate μ, which is negatively related to π, will be a decreasing function of k^{α}), one easily gets the following extended Solow's model

$$\dot{k}=s(\pi(k^{\alpha}))Bk^{a}-(\delta+b(k^{\alpha})-\mu(k^{\alpha}))k$$

or simply

$$\dot{k}=s(k^{\alpha})Bk^{a}-(\delta+b(k^{\alpha})-\mu(k^{\alpha}))k \tag{4}$$

The latter model suggests the nice idea reported in the main text, i.e. that the decrease in mortality has not simply been the motor of the demographic transition, but also the responsible, for those countries in which the saving rate started increasing as a consequence of the reduction in mortality, of the definitive escape from the poverty trap.

A further model

A logarithmic specification of the utility function leads to explicit solutions in Jones' model. For $\gamma\to 1,\ \ \eta\to 1$ we have

$$\hat{b}=\frac{mf}{(1-m)(1-s)w}\left[(1-s)w(1-\frac{b}{f})-c^{\circ}\right] \tag{16}$$

leading to the explicit expression

$$b=\frac{b^{\circ}+mf}{1+m}+\frac{mfc^{\circ}}{(1-s)w(1+m)} \tag{17}$$

(17) shows that the relation between fertility rate and income per capita is decreasing as in the Beckerian state. The same algebra used for Jones' model gives

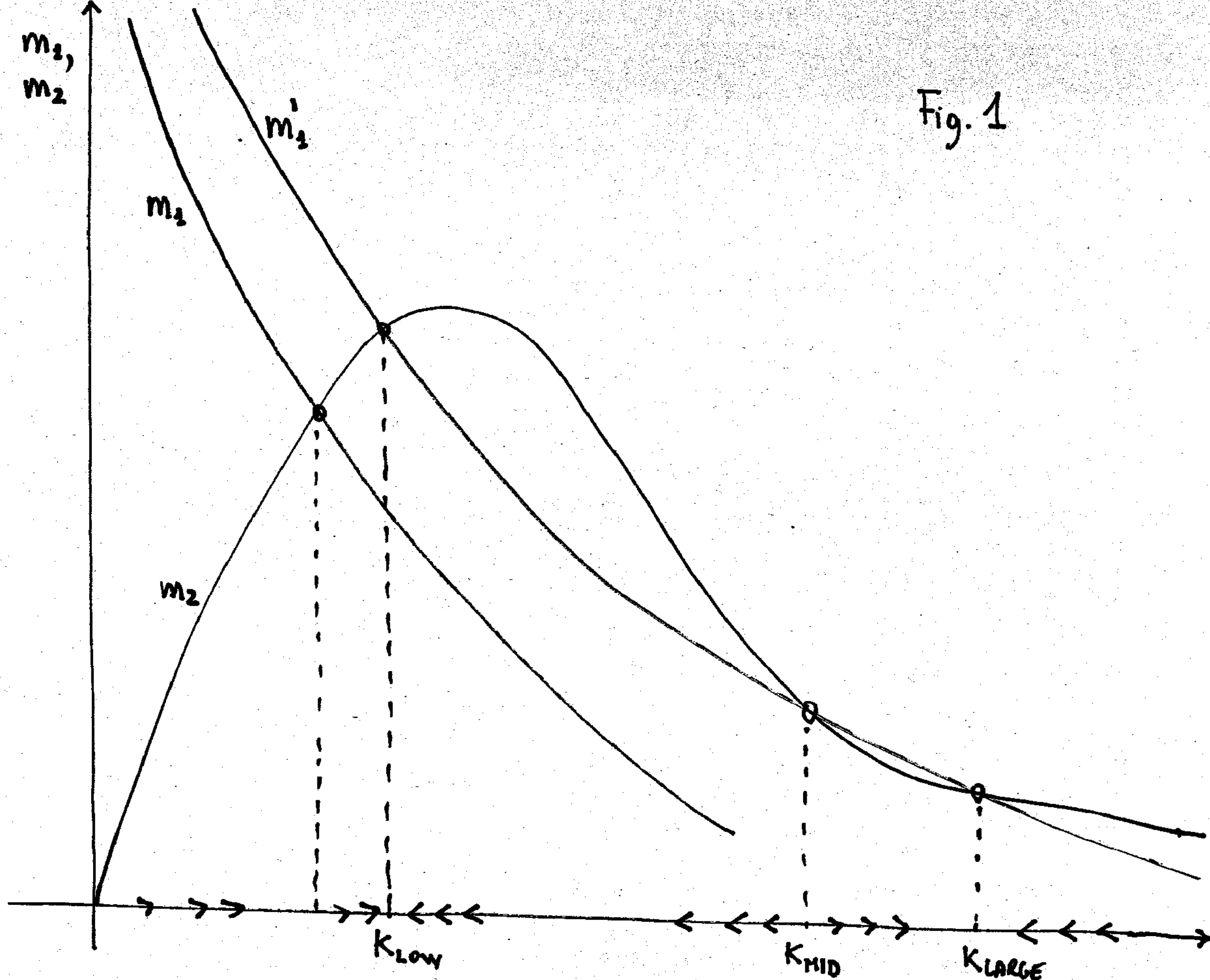
$$\frac{\dot{h}}{h} = \frac{-Qa}{(f-b)Bk^{1+a}}\frac{\dot{k}}{k} \qquad Q = \frac{mfc^{\circ}}{(1-s)(1+m)}$$

The Solow's equation becomes

$$k = \left[\frac{Bk^{1+a}(f-b(k^{\alpha}))}{Bk^{1+a}(f-b(k^{\alpha}))+Qa}\right]\left[sBk^{a}-(\delta+b(k^{\alpha})-d)k\right]$$

which offers interesting possibilities for the study of the post-malthusian phase.

Fig. 1



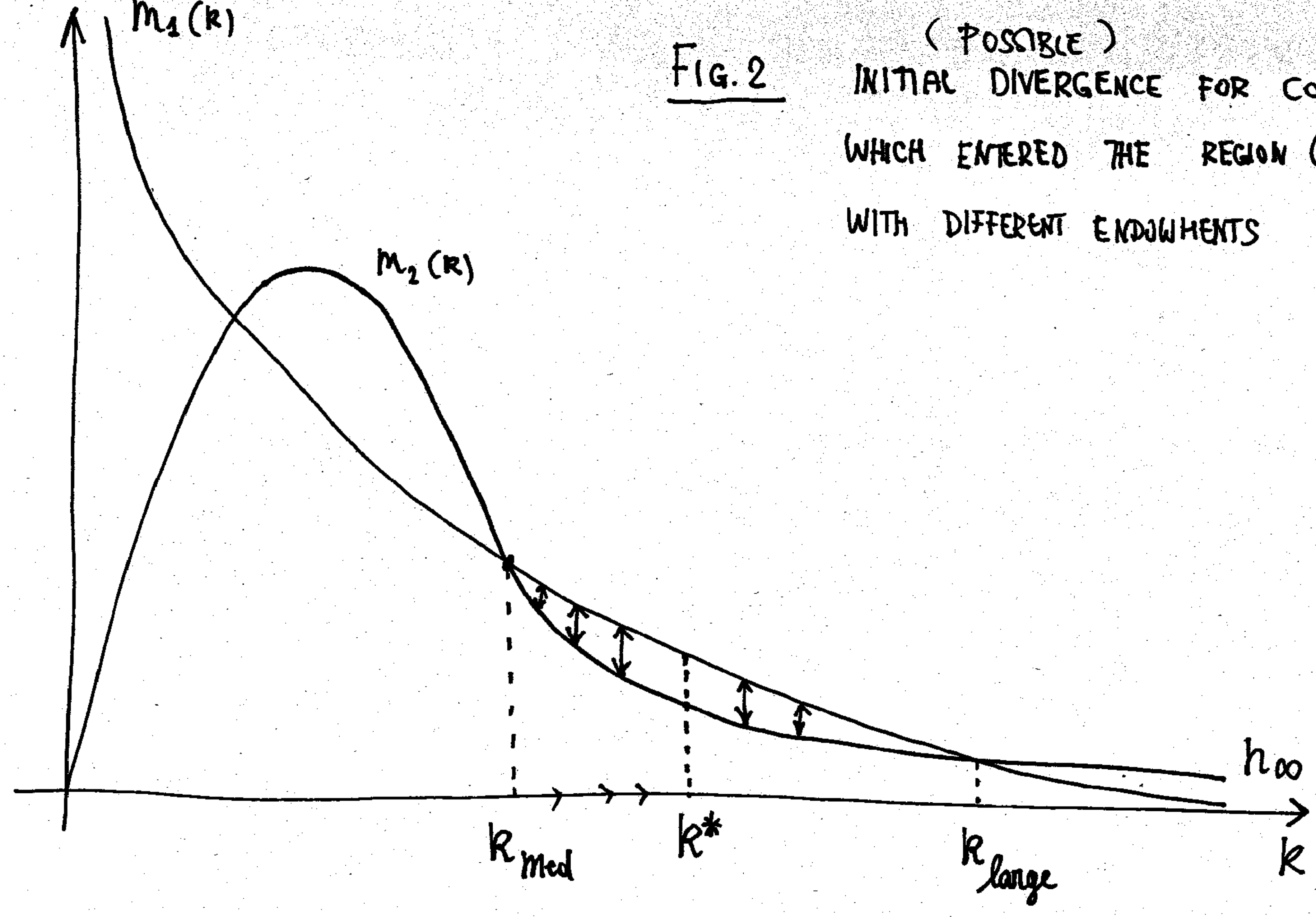


FIG. 2 (POSSIBLE) INITIAL DIVERGENCE FOR COUNTRIES WHICH ENTERED THE REGION (k_{med}, k_{large}) WITH DIFFERENT ENDOWMENTS