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0 Introduction

The problem of valuing the payments provided by unit linked life insurance policies with minimal return guarantees in case of death (european style guarantees), is described in a large literature of which Brennan and Schwartz (1976), Bacinello and Ortu (1993) and Aase and Persson (1994) are only some of the best known examples.

On the contrary, the same problem with minimal return guarantees also in case of surrender before maturity, that implies the valuation of american style derivatives with random maturity, since it depends on further duration of insured life, is rarely considered.

From the insurer point of view, this problem involves the consideration of three sources of risk:
1) the dynamics of the unit price of the reference fund,
2) the insured survival,
3) the policy surrender strategy adopted by the insured.

In Grosen and Jorgensen (1997), (2000) it is assumed that the insured chooses the policy surrender strategy on the basis of his expectations only on unit price dynamics and not on his own survival.

In Vannucci (1999), (2002) and in Bacinello (2002), this gap is filled and it is assumed that the insured chooses the policy surrender strategy on the basis of his expectations also on his own survival.

The present paper analyzes this problem focusing the aspects deriving from the consideration that insured expectations can be coincident or not coincident with insurer ones.

In particular, it is considered the hypotesis of asymmetric information and, hence, of asymmetric expectations, on insured survival. This hypotesis is crucial in many actuarial problems among which, one of the best known concerns the valuation of pension funds liabilities in case that, arrived at retirement age, each insured can choose between receveing or a capital or the equivalent life annuity. On the contrary, the aspects deriving from hypoteses of asymmetric expectations on unit price dynamics are not considered.

In Vannucci (1999) it is assumed the hypotesis of asymmetric expectations on insured survival that implies the highest valuation from the insurer point of view, namely the hypotesis that each insured acts if he has a deterministic knowledge of his own survival which allows him to choose a policy surrender strategy actually optimal with respect to further duration of his own life. This model was inspired by the exigence of a prudencial management of this kind of life insurance policies which was widely felt some years ago but, nowadays, the competition in this market makes necessary for insurance companies to image new models that allow to obtain more fair valuations.

The main purpose of this paper is to compare the results obtained with the model proposed in Vannucci (1999), with ones obtained with the model proposed in Bacinello (2002) and in Vannucci (2002), that assumes the hypotesis of symmetric information on insured survival persisting up to maturity.

Another purpose of this paper is to show by how much the expected value of the payments is lower if insured chooses optimal strategies on the basis of personal goals as, for example, the decision of not surrendering the policy anyhow after vel before certain dates.

This paper is organized as follows. In the first section the demographic-financial model and the corresponding set of admissible system trajectories are described. The second one is dedicated to the definition of policy surrender strategies. In the third and in the fourth sections, after the definition of optimal surrender strategy is given, valuation models with hypotesis of symmetric and asymmetric information on the insured survival are described. In the last two sections some numerical results and some conclusions are presented.

1 The demographic-financial model

Let consider a unit linked policy, starting at date 0, with maturity T and suppose that the reference time interval $[0, T]$ is divided in n intervals all of equal lenght Δ, so $n\Delta = T$. Let $v = \left(\frac{1}{1+r}\right)^{\Delta}$ the discount factor for a period of lenght Δ, where r is the corresponding yearly valuation rate. Given $S_0 = 1$, for the random value of the unit price at time $h\Delta$, S_h, with $h = 1, 2, ..., n$, it is assumed the classical hypotesis of the so-called recombining binomial tree, i.e. a binomial-geometric distribution with parameters p, h, u, d, with $u > d$ and $p \in (0, 1)$,

$$P\left(S_h = d^{h-i}u^i\right) = \binom{h}{i} p^i (1-p)^{h-i}, \text{ with } i = 0, 1, ...h$$

Let $o(h, i)$, $h = 0, 1, ..., n$ and $i = 0, 1, ...h$, the state of the system associated to the couple of events $S_h = d^{h-i}u^i$ and the insured is living at date $h\Delta$.

The insured can surrender the policy at each date $h\Delta$, with $h = 0, 1, ..., n$ (at $n\Delta$ it is a "compulsory surrender") receveing, if the system is in state $o(h, i)$, the payment

$$a(h, i) = \max\left(d^{h-i}u^i, (1+g_1)^h\right)$$

where $g_1 = (1 + G_1)^{\Delta} - 1$ is the minimal return rate guaranteed in a period of lenght Δ in case of surrender and G_1 is the corresponding yearly rate.

Let q_h insured death probability into h-th period, $((h-1)\Delta, h\Delta]$, with $h = 1, 2, ..., n$, and $s_h = (1-q_1)(1-q_2)...(1-q_h)$ insured survival probability up to date $h\Delta$, obtained from the data of a certain survival table and assuming the point of view of date 0.

If an insured is living at date $(h-1)\Delta$, with $h = 1, 2, ..., n$, then his death probability into h-th period, $((h-1)\Delta, h\Delta]$ has to be "updated" assuming the point of view of date $(h-1)\Delta$ and it becomes $q_h^* = \dfrac{q_h}{s_{h-1}}$.

Obviously, death and survival probabilities depend on the insured age at date 0, that is not considered in the already introduced notation, for the sake of simplifying it.

1.1 The trinomial tree

The trinomial tree that describes the demographic-financial model considered in this paper, provides a starting state $o(0, 0)$, relative to date 0, $n+1$ terminal states $o(n, i)$, $i = 0, 1, ..., n$, relative to date $n\Delta$, and it is defined as follows.

From the generic state $o(h, i)$, $h = 0, 1, ..., n-1$ and $i = 0, 1, ..., h$, there are three possibile evolutions of the system:

1) insured survives in the interval $(h\Delta, (h+1)\Delta]$, unit price increases from $d^{h-i}u^i$ to $d^{h-i}u^{i+1}$, i.e. the system reaches the state $o(h+1, i+1)$,

2) insured survives in the interval $(h\Delta, (h+1)\Delta]$, unit price decreases from $d^{h-i}u^i$ to $d^{h-i+1}u^i$, i.e. the system reaches the state $o(h+1, i)$,

3) insured dies in the interval $(h\Delta, (h+1)\Delta]$, it is supposed that death happens at time $\left(h + \frac{1}{2}\right)\Delta$, the system definetively ends in the state $o^*(h, i)$ and the beneficiary of the policy receives the payment

$a^*(h,i) = \max\left(d^{h-i}u^i, (1+g_2)^{h+\frac{1}{2}}\right)$
where $g_2 = (1+G_2)^{\Delta} - 1$ is the minimal return rate guaranteed in a period of lenght Δ in case of death and G_2 is the corresponding yearly rate.

So, with this demographic-financial model, the admissible system trajectories are 2^{h-1} if insured dies in the interval $((h-1)\,\Delta, h\Delta]$, for $h = 1, 2, ..., n$, and 2^n if he survives up to maturity. So, Ω, the set of admissible system trajectories has cardinality $2^0 + 2^1 + ... + 2^n = 2^{n+1} - 1$. Let ω the generic element of this set and $p(\omega)$ its probability.

2 Policy surrender strategies

A policy surrender strategy is a specification of a subset of states of the system, the achievent of one of them implies the surrender. Considering that, for each date $(h-1)\,\Delta$, with $h = 1, 2, ..., n$, there are h nodes and hence 2^h admissible subsets, an upper limit of the number of policy surrender strategies is $2^{1+2+3+...+n}$.

But, in this paper, the following assumption of minimal economic rationality of the insured is introduced:

if an insured considers convenient surrendering the policy at date $h\Delta$ if unit price is d^iu^{h-i}, then he would consider not less convenient surrendering the policy at the same date if unit price is lower, i.e. $d^{h-j}u^j$ with $j = i-1, i-2, ..., 0$.

So, with this assumption, a policy surrender strategy can be defined by n-plas $\lambda = (l_0, l_1, ..., l_{n-1})$, in which $l_h \in \{0, 1, ..., h+1\}$, $h = 0, 1, ..., n-1$, represents the surrender threshold relative to date $h\Delta$ such that, if at date $h\Delta$ the system is in the state $o(h,i)$, then the insured decides to surrender the policy if $i < l_h$ and he decides not to surrender otherwise. Hence, with this assumption, Λ, the set of policy surrender strategies has cardinality $n!$.

Let observe that $l_h = h+1$ and $l_h = 0$ respectively imply that, at date $h\Delta$, the insured decides to surrender and not to surrender in any state of the system. Obviously, if $l_h = h+1$, then specification of $l_{h+1}, l_{h+2}, ..., l_{n-1}$ results useless.

3 The valuation with hypotesis of symmetric information

In this section, two valuation models with hypotesis of symmetric information on insured survival are described, in particular, in section 3.1, a model proposed in Vannucci (2002) and, in section 3.2, a model proposed in Bacinello (2002) and in Vannucci (2002).

3.1 The valuation fixed the policy surrender strategy

Indicating with $a(\lambda,\omega)$ the deterministic discounted value at date 0 of the payment due to the realization of the trajectory ω, in case strategy λ is adopted, the distribution of the random discounted value fixed the surrender strategy λ, $a(\lambda)$, is known and its generic r-th moment is given by

(1) $E\left[a^r(\lambda)\right] = \sum_{\omega\in\Omega} p\left(\omega\right) a^r(\lambda,\omega)$

Let observe that (1) provides the sum of $2^{n+1}-1$ terms and hence, also for values of n relatively small, it could become necessary to resort to the simulation way that consists in considering a subset of Ω.

3.1.1 Maximization of the expected value as criterium to choose the optimal policy surrender strategy

Since that, fixed a certain policy surrender strategy, the distribution of the random discounted value of the payments is known, it is possible to choose the optimal policy surrender strategy with respect to any kind of criterium as, for example, ones that are based on the maximization of the expected value, on the efficient mean-variance frontier, on the concept of VaR and so on.

In this paper it is considered the criterium based on the maximization of the expected value which implies the following definition of optimal policy surrender strategy

(2) λ^* is the optimal policy surrender strategy $\Longleftrightarrow E\left[a(\lambda^*)\right] = \max_{\{\lambda\in\Lambda\}} E\left[a(\lambda)\right]$

Let observe that to find the optimal policy surrender strategy defined by (2), it could be necessary to calculate (1) up to $n!$ times and hence, also for values of n relatively small, to do it could really require too much time.

3.2 The valuation with the iterative backward algorithm

Since that the problem of valuing the payments provided by unit linked life insurance policies with minimal return guarantees also in case of surrender implies the problem of valuing american style derivatives, then, as proposed in Bacinello (2002) and Vannucci (2002), to solve this problem it can be employed the well known model based on the iterative backward algorithm on bi/trinomial tree.

Let observe that the criterium underlying the valuations obtained with this model is just the maximization of the expected value.

To obtain the expected value of the payments at date 0, $E\left[o(0,0)\right]$ with the iterative backward algorithm on the trinomial tree described before, first it is necessary the computation of the payments relative to terminal states $o(n,i)$ and $o^*(h,i)$ and, after that, it is possible to obtain the expected value of the payments in case the system has reached the generic state $o(h,i)$, $h = 0,1,...,n-1$ and $i = 0,1,...,h$, given by the following recursive formula, which is to be backward applied starting from states relative to date $(n-1)\,\Delta$, up to the state $o(0,0)$,

3) $$E\left[o(h,i)\right] = \max\Big[a(h,i), v^{\frac{1}{2}} q^*_{h+1} a^*(h,i) + \\ + v\left(1-q^*_{h+1}\right)\left(pE\left[o(h+1,i+1)\right] + (1-p)\,E\left[o(h+1,i)\right]\right)\Big]$$

with the position $a(n,i) = E\left[o(n,i)\right]$.

Since the underlying criterium of the two valuation models is the maximation of the expected value, it obviously holds that $E\left[o(0,0)\right] = E\left[a(\lambda^*)\right]$. Moreover, the surrender strategy implicit in the valuation obtained with the iterative backward algorithm is reconstructable by considering 3), and it will be just λ^*.

A crucial remark is the following: since this iterative backward algorithm assumes that the insured has to choose if surrendering or not surrendering at date $h\Delta$ as if he is really at that date, it follows that in 3) the "updated" probabilities q^*_h's have to be employed.

4 The valuation with hypotesis of asymmetric information

In this section it is described the valuation model, proposed in Vannucci (1999), which assumes that each insured acts as if he has a deterministic knowledge of his own survival that allows him to choose the policy surrender strategy actually optimal with respect to further duration of his own life (from a strictly financial point of view).

From the insurer point of view, the expected value of the payments is a weighted average of as many expected values as the possible insured life durations are and, the weight relative to each of these expected values is given by the probability of the corresponding insured life duration.

So, the expected value of the payments provided by the policy for an insured that will die in the interval $((h-1)\,\Delta, h\Delta]$, with $h = 1,2,...,n$, $E\left[o^h(0,0)\right]$ is obtainable starting from the computation of the payments relative to the states at date $(h-1)\,\Delta$ (that are terminal states if the insured will die in the interval $((h-1)\,\Delta, h\Delta])$ given by

$$E\left[o^h(h-1,i)\right] = \max\left(a(h-1,i), v^{\frac{1}{2}} a^*(h-1,i)\right)$$

After that, it is possible to obtain the expected value of the payments in case the system has reached the generic state $o^h(j,i)$, $j = 0,1,...,h-2$

and $i = 0, 1, ..., j$, given by the following recursive formula, which is to be backward applied starting from states relative to date $(h-2)\Delta$, up to state $o(0,0)$,

$$E\left[o^h(j,i)\right] = \max\left[a(j,i), v\left(pE\left[o^h(j+1,i+1)\right] + (1-p)E\left[o^h(j+1,i)\right]\right)\right]$$

Moreover, the expected value of the payments provided by the policy for an insured that will survive up to maturity $n\Delta$, $E[o^s(0,0)]$, is obtainable in the same way, unless it is necessary to start from the computation of the payments relative to the terminal states at date $n\Delta$, that are given by $a(n,i)$.

Finally, the expected value of the payments provided by the policy at date 0, $E[o^{asym}(0,0)]$, is given by the following weighted average of the expected values $E\left[o^h(0,0)\right]$'s and $E[o^s(0,0)]$,

$$E[o^{asym}(0,0)] = \sum_{h=1}^{n} q_h E\left[o^h(0,0)\right] + s_n E[o^s(0,0)]$$

where the weight associated to each of these expected values is just the probability, from the point of view of date 0, of the corresponding insured life duration.

5 Numerical results

For the numerical examples proposed in this section, it is assumed that the stochastic process of unit price is the standard geometric brownian motion with i.i.d. increments of Black-Scholes model. So, the random variable, $\log\left(\frac{S_{h+1}}{S_h}\right)$, for $h = 0, 1, ..., n-1$, is normally distributed with mean $\mu\Delta$ and standard deviation $\sigma\sqrt{\Delta}$ and the first two moments of the random variable $\frac{S_{h+1}}{S_h}$, lognormally distributed, are $m_1 = e^{(\mu+\frac{\sigma^2}{2})\Delta}$ and $m_2 = e^{2(\mu+\sigma^2)\Delta}$.

To value the parameters p, u and d, that characterize the binomial-geometric distribution of S_h assumed in the previous sections, as functions of μ, σ and Δ, it is employed one of the best known set of conditions that is expressed by the following system

$$\begin{cases} up + d(1-p) = m_1 \\ u^2 p + d^2(1-p) = m_2 \\ p = 0.5 \\ u > d \end{cases}$$

that has the unique solution

$$\begin{cases} u = m_1 + \sqrt{m_2 - m_1^2} \\ d = m_1 - \sqrt{m_2 - m_1^2} \\ p = 0.5 \end{cases}$$

Notice that with this set of conditions, the binomial-geometric distribution of S_h used in the previous sections has the same first two moments of the lognormal distribution of S_h employed in Black-Scholes model, for each $h = 1, 2, ..., n$.

In the following numerical examples, financial scenarios will be described in terms of yearly rates μ, σ, r, G_1 and G_2.

For the demographic aspects, insured age at date 0 is indicated by x and it is assumed that his death and survival probabilities are obtained from the data of the most recent survival table for italian males, the so called RG 48.

5.1 Symmetric vs asymmetric information

This section is dedicated to compare the results obtained with the hypotesis of symmetric information, given by the model described in section 3.2 and reported in the penultimate column of the following table, with the ones, reported in the last column of the following table, obtained with the hypotesis of asymmetric information and given by the model described in section 4.

x	μ	σ	r	G_1	G_2	T	Δ	$sym.$	$asym.$	$\frac{asym.-sym.}{sym.}$
50	.04	.25	.04	.03	.03	20	.5	2.0475	2.0492	0.08%
50	.04	.25	.04	.03	.1	20	.5	2.1184*	2.1305	0.57%
30	.04	.25	.04	.03	.03	20	.5	2.0775	2.0779	0.02%
30	.04	.25	.04	.03	.1	20	.5	2.0888	2.0939	0.24%
50	.02	.25	.04	.03	.03	20	.5	1.5362	1.5375	0.08%
50	0	.25	.04	.03	.03	20	.5	1.2655	1.2663	0.06%
50	.04	.3	.04	.03	.03	20	.5	2.6487	2.6507	0.08%
50	.04	.3	.04	.03	.1	20	.5	2.7172*	2.7295	0.45%

Results emphasized with * are obtained without that to surrender before maturity is never convenient.

Notice that differences are always relatively small (under 1%) and anyhow they are more significative in the scenarios that provide higher minimal return guarantees in case of death.

5.2 Optimal vs personal goals

This section is dedicated to compare the results obtained in case that the insured pursues optimal goals, given by the model described in section 3.2

(so the same results of symmetric information case considered in the previous example) and reported in the penultimate column of the following table, with ones, reported in the last column of the following table, obtained in case that the insured pursues personal (non optimal) goals, such as the decision of not surrendering the policy after vel before certain dates.

The expected value of the payments provided by the policy in case the insured decides not to surrender before a certain date $h^b\Delta$ vel after a certain date $h^a\Delta$, with $h^b, h^a = 0, 1, ..., n$ and $h^b \leq h^a$ is given by the model described in section 3.2 with the set of admissible strategies $\Lambda^{a,b}$ given by $\Lambda^{a,b} = \left\{\lambda \mid l_h = 0 \text{ if } h < h^b \text{ and } l_{h^a} = h^a + 1\right\}$. Let observe that $\Lambda^{n,0}$ coincides with Λ.

x	μ	σ	r	g_1	g_2	T	Δ	h^b	h^a	*opt.*	*pers.*	$\frac{pers.-opt.}{opt.}$
50	.04	.25	.04	.03	.03	20	.5	10	40	2.0475	2.0475	
50	.04	.25	.04	.03	.03	20	.5	20	40	2.0475	2.0469	−0.03%
50	.04	.25	.04	.03	.03	20	.5	0	30	2.0475	1.7851	−12.81%
50	.04	.3	.04	.03	.03	20	.5	10	40	2.6487	2.6487	
50	.04	.3	.04	.03	.03	20	.5	20	40	2.6487	2.6482	−0.01%
50	.04	.3	.04	.03	.03	20	.5	0	30	2.6487	2.1733	−17.94%
50	.04	.25	.04	.03	.1	20	.5	39	40	2.1184	2.1184	
50	.04	.25	.04	.03	.1	20	.5	0	30	2.1184	1.8045	−14.81%

When the two valuations give the same result, it signifies that to surrender before date h^b is never convenient and, in particular, the last scenario provide that to surrender before maturity is never convenient (see the table of section 5.1).

Moreover, these results emphasize that insurance companies could have relevant economic advantages from the choice of non optimal policy surrender strategies, in particular if the insured decides to surrender anyhow before maturity.

6 Conclusions

This paper makes in evidence that to value american style guarantees embedded in insurance life policies as surrender options, it is necessary to face the problem of valuing american style derivatives with random maturities due to the randomness of further duration of insured life.

This peculiarity makes interesting the consideration of the problem with hypoteses of symmetric and asymmetric information on insured survival, between insurer and insured. At this purpose, further research could concern the analysis of intermediate scenarios between ones of symmetric and of complete asymmetric information that are analyzed in this paper.

Notice also that, fixed a policy surrender strategy, the valuation model described in section 3.1, also if in many cases it is too much time consumer, allows to value the distribution of the random discounted value of the payments more widely then the models based on the iterative backward algorithm (that give informations only on its expected value) and it could be necessary if insurance companies have to value the riskyness of their portfolios.

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