Università degli Studi di Pisa
Dipartimento di Statistica e Matematica
Applicata all'Economia

Report n. 248

Bayesian Nash Equilibrium for Insider Trading in Continuous Time

E. Barucci - R. Monte - B. Trivellato

Pisa, Marzo 2004

- Stampato in Proprio –

Via Cosimo Ridolfi, 10 - 56124 PISA - Tel. Segr. Amm. 050 2216231 Segr. Stud. 050 2216317 Fax 050 2216375
Cod. Fisc. 80003670504 - P. IVA 00286820501 - Web http://statmat.ec.unipi.it/ - E-mail: dipstat@ec.unipi.it

Bayesian Nash Equilibrium for Insider Trading in Continuous Time *

Emilio Barucci
Dipartimento di Statistica e Matematica Applicata all'Economia,
Università di Pisa.
Via Cosimo Ridolfi 10 - 56124 Pisa, ITALY
e-mail: ebarucci@ec.unipi.it

Roberto Monte
Dipartimento di Studi Economici, Finanziari e Metodi Quantitativi,
Università di Roma "Tor Vergata".
Via Columbia 2 - 00133 Roma, ITALY
e-mail: monte@sefemeq.uniroma2.it

Barbara Trivellato
Dipartimento di Matematica,
Politecnico di Torino.
Corso Duca degli Abruzzi 24 - 10129 Torino, ITALY
e-mail:barbara.trivellato@polito.it

*This paper is greatly in debt with the unpublished paper [8]. Actually, we consider a similar model and use the same methodology. The main difference is that the drift of the dividend process is not given by a stochastic process but it is a constant. In [8], the authors were not able to characterize the equilibrium explicitly. They addressed the market maker's filtering problem, and the insider's optimal consumption-investment problem and they suggested that numerical methods allow to characterize the equilibrium.

Abstract

We consider the Kyle model [12] and we provide a no trade theorem with no transmission of information in a market with an insider trader, noise traders, and a market maker. The main difference between our setting and Kyle's one is that the insider trader is risk averse and that the risky asset pays a continuous time dividend stream, which is described by a Brownian motion with drift. The dividend realization is observed by all agents, but the insider trader knows the dividend growth rate while the market maker does not. We define a Bayesian-Nash equilibrium by addressing the market maker's optimal filtering problem, and the insider trader's optimal consumption-trading strategy. In equilibrium the insider trader does not trade and the market maker does not update his beliefs. Therefore, the equilibrium does not transmit information.

1 Introduction

A well known result on financial markets with privately informed agents establishes that agents do not trade on the basis of private information, because either the others do not want to trade, or are able to identify their private information by observing market prices.

Two pieces of theory make the point. Provided an ex ante Pareto optimal allocation is reached, the no trade Theorem states that no trade occurs after information becomes heterogeneous among agents, because there is no recontracting leading to a Pareto improvement (see [15]). On the other hand, provided that all agents know the features of the economy and trade in order to maximize their utility, under some general assumptions, it can be shown that the rational expectations equilibrium with heterogeneous-private information coincides with the equilibrium of the artificial economy, i.e., the economy where all pieces of private information are of common knowledge (see for example [9]).

To overcome the no trade theorem and to render non fully revealing the rational expectations equilibrium, two main perspectives have been pursued in the literature: the introduction of noise-liquidity traders in the market, i.e., agents who do not trade according to the rationality paradigm, but only for liquidity reasons; the analysis of non perfectly competitive markets. The Kyle model represents an illuminating example (see [12]). In a two-period economy a risky asset is traded at $t = 0$ and there are two classes of agents: a risk neutral insider trader and a set of noise traders. The risky asset demand by noise traders is described by a random variable which is independent of the asset dividend. The latter is a random variable whose realization is observed by all agents at $t = 1$, while the insider trader knows its value since $t = 0$, before trading. The price at $t = 0$ is determined by the market maker as the conditional expected value of the dividend, given the market order flow (the cumulative insider and noise traders' demand). In this setting, it is shown that there exists an equilibrium in which the insider trades. The higher the variance of the noise traders' demand is, the higher the sensitivity of the insider trader's demand to the dividend becomes. The rationale of this result is very simple: if the noise traders' demand can be very large (high variance), then the insider trader can trade in the market profitably, by knowing that his private information is more easily hidden to the eyes of the market maker, and therefore not reflected in the market price. Also the fully revealing property breaks down: after trading, noise traders' precision is the half of the insider trader's one. The main properties of the model are confirmed in a multiperiod setting, assuming that the insider trades in many intermediate time instants by knowing the dividend delivered at $t = 1$. The insider trader acts gradually in order to strategically exploit his information and market prices do not reflect the insider's private information immediately. When there are multiple insider traders (each insider knows the liquidation value of the asset before starting to trade) we observe that competition among insiders pushes them to trade aggressively and causes most of their private information to be revealed very rapidly (see [10]).

In this paper we consider the Kyle model and we provide a no trade theorem with no transmission of information in a market with an insider, noise traders, and a market maker. The result represents a novelty in the Kyle setting. The main point that differentiates our setting from the Kyle one is that the insider trader is risk averse and the risky asset pays a continuous time dividend stream, which is described by a Brownian motion with drift, an assumption similar to that of [4, 17, 8]. The dividend realization is observed by all agents, but the insider trader knows the dividend growth rate while the market maker does not. The market maker is risk neutral, he updates his beliefs on the dividend growth rate continuously and sets the price of the risky asset as the conditional expectation of the discounted future dividend stream. As in [12, 2], the market maker observes directly only the aggregate order flow, which is a noisy indicator, owing to the presence of noise traders, and he cannot distinguish the insider trader's component. His attempt of detecting the insider trader's demand, and therefore the dividend growth rate, gives rise to a filtering problem: market maker's learning about the insider trader's information from the dividend time series and the aggregate order flow. On the other hand, the insider trader defines his trading strategy by maximizing the expected utility taking into account that the price is set by the market maker.

We define a Bayesian-Nash equilibrium by addressing the market maker's optimal filtering problem and the insider trader's optimal consumption-trading strategy. It is a Bayesian-Nash equilibrium, in a sense that the market maker postulates the insider trader's strategy when he updates his beliefs, and the insider trader conjectures the market maker's updating rule when he addresses his optimal consumption-trading problem. In equilibrium, we impose that conjectures are self-confirming. We restrict our attention to stationary linear equilibria, i.e., both the the market maker's updating rule and the insider trader's trading strategy are linear in the state variables with constant coefficients.

The main result of the paper is that there exists a unique Bayesian-Nash equilibrium which is a No Trade Theorem: in equilibrium the insider trader does not trade and the market maker does not update his beliefs. So the equilibrium does not transmit information. Noise traders are not enough: if the insider trades in the market, then the market maker immediately recovers the drift of the dividend process and sets the price as the expected value of the discounted dividend stream, thereby rendering insider's trading non profitable.

The paper is organized as follows. In Section 2 we present the model. Section 3 addresses the optimal filtering problem of the market maker. In Section 4 we address the optimal investment-consumption problem of the insider trader. In Section 5 we close the model and we characterize the Bayesian-Nash equilibrium. All proofs of the established propositions are given in the Appendix.

2 The Model

The model considered in this paper is that in [8], the main difference is that the drift of the dividend process is constant and it is not stochastic. There are a risk free and a risky asset, the risk free asset pays an instantaneous interest rate $r > 0$, while the risky asset, with price $P(t)$, yields a dividend stream $D(t)$. We assume that the history of $D(t)$, up to the current instant t, is observed by all agents, that is to say the filtration $(\mathfrak{F}_s^D)_{s \leq t}$ generated by the dividend process is part of the publicly available information. In addition, we assume that the dynamics of $D(t)$ is driven by the stochastic differential equation

$$dD(t) = \pi\, dt + \sigma_D\, dw_D(t), \tag{1}$$

where π and σ_D are constant parameters, with $\sigma_D > 0$, and $w_D(t)$ is a Wiener process.

There are three types of agents: a representative noise trader, an insider trader, and a market maker.

The representative noise trader trades smoothly. Namely, the value $\Theta(t)$ of the representative noise trader's order flow is the "derivative" of his inventory. As a consequence $\Theta(t)$ follows an Ornstein-Uhlenbeck process, which is independent of the information supplied by the dividend history. More precisely, we have

$$d\Theta(t) = -\alpha_\Theta \Theta(t)\, dt + \sigma_\Theta\, dw_\Theta(t), \tag{2}$$

where α_Θ and σ_Θ are constant positive parameters, and $w_\Theta(t)$ is a Wiener process, which is independent of $w_D(t)$.

Also the insider trader trades smoothly. More precisely, writing $\Psi(t)$ for the insider trader's inventory and $\Phi(t)$ for his order flow, we have that

$$d\Psi(t) = \Phi(t)\,dt. \tag{3}$$

The insider trader knows the true value of the parameter π in (1) and he can observe the process $\Theta(t)$, because he observes the market order flow. The insider trader is risk averse. He maximizes the expected value of his exponential utility over an infinite time horizon, by controlling the variation of his order flow, $d\Phi(t)$, and his consumption, $c(t)$. Therefore, the insider trader's objective function becomes

$$V(t,m,Y) \stackrel{\text{def}}{=} \sup_{d\Phi(\cdot),c(\cdot)} \{\mathbf{E}_{t,m,Y}[\int_t^{+\infty} -e^{-(\rho s+\psi c(s))}\,ds]\}, \tag{4}$$

where $\mathbf{E}_{t,m,Y}[\cdot]$ is the conditional expectation operator given the instant t, the state vector Y of the economy, and the informed trader's wealth m, and where ρ is the discount factor, and ψ is the coefficient of absolute risk-aversion.

The market maker ignores the true value of the parameter π in (1). When making his estimates, he assumes that π is normally distributed. The market maker is risk neutral. He updates his beliefs continuously and sets the price of the risky asset equal to the conditional expected present value of the future dividend stream, given the history of the dividend process and the aggregate order flow information:

$$P(t) \stackrel{\text{def}}{=} \mathbf{E}[\int_t^{+\infty} e^{-r(s-t)}D(s)\,ds|\mathfrak{F}_t], \tag{5}$$

where $\mathfrak{F}_t$ is the σ-field generated by the dividend process and the total order flow up to t. Note that the market maker observes directly only the aggregate order flow

$$O(t) \stackrel{\text{def}}{=} \Theta(t) + \Phi(t). \tag{6}$$

3 Optimal Filtering by the Market Maker

Our first step is to adopt the market maker's point of view. As above discussed, the market maker only observes the dividend flow $D(t)$ and the aggregate order flow $O(t)$. On the basis of this information, he aims to compute the estimates $\pi_e(t)$ and $\Phi_e(t)$ of the parameter π and of the informed trader's order flow $\Phi(t)$, respectively. The model is described by the system of stochastic differential equations

$$\begin{cases} dD(t) = \pi\, dt + \sigma_D\, dw_D(t), \\ d\Theta(t) = -\alpha_\Theta \Theta(t)\, dt + \sigma_\Theta\, dw_\Theta(t), \\ d\Psi(t) = \Phi(t)\, dt, \end{cases} \tag{7}$$

which can be managed more easily by introducing a suitable matrix notation. Actually, setting $y^\top(t) \equiv (D(t), \pi, \Theta(t), \Phi(t), \Psi(t))$, we can write

$$dy(t) = Ay(t)\, dt + Q^{1/2}\, dw(t) + k_\Phi\, d\Phi(t), \tag{8}$$

where

$$A \equiv \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\alpha_\Theta & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix}, \quad Q^{1/2} \equiv \begin{pmatrix} \sigma_D & 0 \\ 0 & 0 \\ 0 & \sigma_\Theta \\ 0 & 0 \\ 0 & 0 \end{pmatrix}, \quad k_\Phi \equiv \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \tag{9}$$

and

$$w(t) \equiv \begin{pmatrix} w_D(t) \\ w_\Theta(t) \end{pmatrix}. \tag{10}$$

In addition, we write $y_o^\top(t) \equiv (D(t), O(t))$ for the observation vector, and $y_e^\top(t) \equiv (D_e(t), \pi_e(t), \Theta_e(t), \Phi_e(t), \Psi_e(t))$ for the market maker's estimate vector of the state vector $y(t)$, given the dividend and the aggregate order flow information. Then we have

$$y_o(t) = M^\top y(t), \tag{11}$$

where

$$M^\top \equiv \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \end{pmatrix},$$

and

$$y_e(t) = \mathbf{E}\left[y(t) | \mathfrak{F}_t^{y_o}\right], \tag{12}$$

where $\mathfrak{F}_t^{y_o}$ is the σ-field generated by $y_o(s)$, for $s \leq t$.

In order to perform his estimates, the market maker postulates that the informed trader adjusts his trading strategy linearly. Then we are able to show that also the market maker can adjust his beliefs linearly. The following Proposition can be established, see the Appendix for the proof.

Proposition 1 *Let us assume that the market maker postulates the informed trader's order flow* $\Phi(t)$ *satisfies the linear stochastic differential equation*

$$d\Phi(t) = a^\top y(t)\, dt + b^\top y_e(t)\, dt + q^\top dw(t), \tag{13}$$

for suitable vectors

$$a^\top \equiv (a_1, a_2, a_3, a_4, a_5)\,, \quad b^\top \equiv (b_1, b_2, b_3, b_4, b_5)\,, \quad q^\top \equiv (q_1, q_2)\,. \tag{14}$$

Then the market maker can write an evolution equation for $y_e(t)$ *in the form*

$$dy_e(t) = G_e(t) y_e(t)\, dt + H_e(t)\, dy_o(t), \tag{15}$$

where, setting

$$\hat{A} \equiv A + k_\Phi a^\top, \quad \hat{B} \equiv k_\Phi b^\top, \quad \hat{Q}^{1/2} \equiv Q^{1/2} + k_\Phi q^\top, \tag{16}$$

the matrices $G_e(t)$ *and* $H_e(t)$ *are given by*

$$H_e(t) = (\hat{Q} + \Sigma(t)\hat{A}^\top) M R^{-1} \tag{17}$$

and

$$G_e(t) = \left(I - H_e(t) M^\top\right)(\hat{A} + \hat{B}), \tag{18}$$

for

$$\hat{Q} \equiv \hat{Q}^{1/2}(\hat{Q}^{1/2})^\top, \qquad R \equiv M^\top \hat{Q} M,$$

and $\Sigma(t)$ *is a positive solution of the Riccati equation*

$$\begin{aligned} d\Sigma(t) = {} & (\hat{A}\Sigma(t) + \Sigma(t)\hat{A}^\top + \hat{Q})\, dt \\ & - (\Sigma(t)\hat{A}^\top + \hat{Q}) M R^{-1} M^\top (\Sigma(t)\hat{A})^\top + \hat{Q}^\top)\, dt. \end{aligned} \tag{19}$$

To establish the existence of a stationary Bayesian-Nash equilibrium, we restrict our attention to $G_e(t)$ and $H_e(t)$ constant over time and to the stationary solutions of (19), which arise as positive solutions of the algebraic Riccati equation

$$\hat{A}\Sigma + \Sigma\hat{A}^\top + \hat{Q} - (\Sigma\hat{A}^\top + \hat{Q}) M R^{-1} M^\top (\Sigma\hat{A}^\top + \hat{Q})^\top = 0. \tag{20}$$

These stationary solutions are steady-state solutions of (19), to which the ordinary solutions converge as the initial time t_0 goes to $-\infty$, under rather mild conditions (see, e.g. [1]).

For computational purposes, it is useful to show that (20) can be reduced to an equivalent 3×3 equation. Actually, we have the following Proposition, see the Appendix for the proof.

Proposition 2 *Let* $\tilde{M} \equiv (\hat{M}, \hat{M}_\perp)$, *where*

$$\hat{M}^\top \equiv \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1/\sqrt{2} & 1/\sqrt{2} & 0 \end{pmatrix}$$

is the matrix whose rows are an orthonormal basis for the linear span of the rows of $M^\top$ *in* $\mathbb{R}^5$, *and*

$$\hat{M}_\perp^\top \equiv \begin{pmatrix} 0 & 0 & 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix},$$

is the matrix whose rows are an orthonormal basis for the subspace of $\mathbb{R}^5$ *which is orthogonal to the above mentioned linear span. Then (19) is equivalent to*

$$\begin{aligned} d(\hat{M}_\perp^\top \Sigma(t) \hat{M}_\perp) =& \hat{M}_\perp^\top (\hat{A}\Sigma(t) + \Sigma(t)\hat{A}^\top + \hat{Q})\hat{M}_\perp \, dt \\ & - \hat{M}_\perp^\top (\Sigma(t)\hat{A}^\top + \hat{Q})\hat{M}\hat{R}^{-1}\hat{M}^\top (\Sigma(t)\hat{A}^\top + \hat{Q})^\top \hat{M}_\perp \, dt, \end{aligned} \tag{21}$$

where

$$\hat{R} \equiv \hat{M}^\top \hat{Q} \hat{M}.$$

Finally, we are left with the task of finding a positive solution of the algebraic Riccati equation

$$\hat{M}_\perp^\top (\hat{A}\Sigma + \Sigma\hat{A}^\top + \hat{Q})\hat{M}_\perp - \hat{M}_\perp^\top (\Sigma\hat{A}^\top + \hat{Q})\hat{M}\hat{R}^{-1}\hat{M}^\top (\Sigma\hat{A}^\top + \hat{Q})^\top \hat{M}_\perp = 0. \tag{22}$$

4 The Optimal Investment-Consumption problem for the insider trader

As already discussed, the insider trader aims to maximize the expected value of his exponential intertemporal utility over an infinite time horizon, by controlling the state vector of the economy, actually the stacked vector $Y(t)$, and his wealth $m(t)$, by means of the variation of his order flow $d\Phi(t)$ and his consumption rate $c(t)$.

Under the assumption that the insider trader postulates a linear adjustment of the market makers' estimates $y_e(t)$, as in (15), we can write

$$dy_e(t) = G_e y_e(t)\, dt + H_e M^\top (Ay(t)\, dt + Q^{1/2}\, dw(t) + k_\Phi \, d\Phi(t)), \tag{23}$$

where

$$H_e = (\hat{Q} + \Sigma \hat{A}^\top) M R^{-1} \tag{24}$$

and

$$G_e = \left(I - H_e M^\top\right)(\hat{A} + \hat{B}), \tag{25}$$

with Σ solution to the algebraic Riccati equation (22). Hence, it is easy to see that the stacked vector $Y(t)$ satisfies

$$dY(t) = \bar{A} Y(t)\, dt + \bar{Q}^{1/2}\, dw(t) + \bar{K}_\Phi\, d\Phi(t), \tag{26}$$

where

$$\bar{A} \equiv \begin{pmatrix} A & 0 \\ H_e M^\top A & G_e \end{pmatrix}, \; \bar{Q}^{1/2} \equiv \begin{pmatrix} Q^{1/2} \\ H_e M^\top Q^{1/2} \end{pmatrix}, \; \bar{K}_\Phi \equiv \begin{pmatrix} k_\Phi \\ H_e M^\top k_\Phi \end{pmatrix}. \tag{27}$$

The informed trader's wealth, $m(t)$, is modelled as a solution of the stochastic differential equation

$$dm(t) = r m(t)\, dt + \Psi(t)\left[(D(t) - rP(t))\, dt + dP(t)\right] - c(t)\, dt, \tag{28}$$

where the risky asset price, $P(t)$, is given by

$$P(t) = \mathbf{E}\left[\int_t^{+\infty} e^{-r(s-t)} D(s)\, ds | \mathfrak{F}_t^{y_o}\right].$$

On the other hand, since for every $s > t$ we have

$$\begin{aligned}
\mathbf{E}\left[D(s)|\mathfrak{F}_t^{y_o}\right] &= \mathbf{E}\left[D(t)|\mathfrak{F}_t^{y_o}\right] + \mathbf{E}\left[D(s) - D(t)|\mathfrak{F}_t^{y_o}\right] \\
&= D(t) + \mathbf{E}\left[\pi(s-t) + \sigma_D\left(w_D(s) - w_D(t)\right)|\mathfrak{F}_t^{y_o}\right] \\
&= D(t) + \pi_e(t)(s-t),
\end{aligned}$$

being $w_D(s) - w_D(t)$ future with respect to $\mathfrak{F}_t$, we can write

$$P(t) = r^{-1} D(t) + r^{-2}\pi_e(t) \equiv p^\top Y(t), \tag{29}$$

where $p^\top \equiv \left(r^{-1}, 0, 0, 0, 0, 0, r^{-2}, 0, 0, 0\right)$. Therefore, combining (28) with (26) through (29), and observing that we can also write

$$\Psi(t) = k_\Psi^\top Y(t) = Y^\top(t) k_\Psi, \quad \text{and} \quad D(t) - rP(t) = -r^{-1} k_{\pi_e}^\top Y(t),$$

where $k_\Psi^\top \equiv (0,0,0,0,1,0,0,0,0,0)$, and $k_{\pi_e}^\top \equiv (0,0,0,0,0,0,1,0,0,0)$, we obtain

$$dm(t) = \left(r m(t) - Y^\top(t) k_\Psi \left(r^{-1} k_{\pi_e}^\top Y(t) - p^\top dY(t)\right) - c(t)\right) dt. \tag{30}$$

In what follows we show that there exists a linear diffusion evolution for $d\Phi(t)$ and a consumption strategy which allow the informed trader to maximize his intertemporal utility. Note that the insider trader knows $Y(t)$ and $m(t)$ at any t before adjusting his trading strategy. In other words, he enjoys a complete observation. Therefore (see [5], [6]), it makes sense that he adjusts the variation of his order flow in a feedback form. Moreover, since the state vector of the economy follows an autonomous system of stochastic differential equations, he can choose a stationary Markov policy

$$d\Phi(t) = a^\top(t)\,dt + q(t)\,dw(t), \tag{31}$$

where

$$a(t) \equiv a(Y(t), m(t)), \qquad q(t) \equiv q(Y(t), m(t)). \tag{32}$$

The same argument holds for the choice of the insider trader'consumption

$$c(t) \equiv c(Y(t), m(t)).$$

Hence, the insider trader's optimization problem becomes to compute the value function

$$V(t, Y, m) \equiv \max_{a(\cdot),q(\cdot),c(\cdot)} \left\{ \mathbf{E}_{t,Y,m} \left[\int_t^{+\infty} -e^{-(\rho s+\psi c(s))}\,ds \right] \right\}, \tag{33}$$

where $\mathbf{E}_{t,Y,m}[\cdot]$ is the conditional expectation operator given $Y(t) = Y$ and $m(t) = m$, and the state vector $(Y(t), m(t))$ is subject to

$$dY(t) = \left(\bar{A}Y(t) + \bar{K}_\Phi a^\top(t)\right) dt + \left(\bar{Q}^{1/2} + \bar{K}_\Phi q(t)\right) dw(t), \tag{34}$$

which results from the combination of (26) and (31), and

$$\begin{aligned} dm(t) =& \left(rm(t) - Y^\top(t)k_\Psi\left(r^{-1}k_{\pi_e}^\top Y(t) - p^\top\left(\bar{A}Y(t) + \bar{K}_\Phi a^\top(t)\right)\right) - c(t)\right) dt \\ &+ Y^\top(t)k_\Psi p^\top\left(\bar{Q}^{1/2} + \bar{K}_\Phi q(t)\right) dw(t), \end{aligned} \tag{35}$$

which results from the combination of (30) and (34). In addition, note that the entries of $Y(t)$ must satisfy the relationships

$$y_1(t) \equiv D(t) = D_e(t) \equiv y_6(t), \tag{36}$$

$$y_3(t) + y_4(t) \equiv \Theta(t) + \Phi(t) \equiv O(t) = O_e(t) = \Theta_e(t) + \Phi_e(t) \equiv y_8(t) + y_9(t) \tag{37}$$

for every $t \geq 0$. The following Proposition can be established, see the Appendix for the proof.

Proposition 3 *Let us assume that (33) is of the form*

$$V(t,Y,m) = -e^{-(\rho t+\frac{1}{2}Y^\top LY+\psi rm+\lambda)}, \tag{38}$$

where L is a symmetric matrix and λ is a real parameter. Then, L must satisfy the equations

$$\bar{K}_\Phi^\top \left(L+\psi rpk_\Psi^\top\right) Y = 0, \tag{39}$$

and

$$\begin{aligned} Y^\top \Big(rL + \left(L+\psi rpk_\Psi^\top\right)^\top \bar{Q} \left(L+\psi rpk_\Psi^\top\right) \\ - \left(L+\psi rpk_\Psi^\top\right)^\top \bar{A} - \bar{A}^\top \left(L+\psi rpk_\Psi^\top\right) - \psi\left(k_\Psi k_{\pi_e}^\top + k_{\pi_e} k_\Psi^\top\right)\Big) Y = 0, \end{aligned} \tag{40}$$

for every $Y \in \mathbb{R}^{10}$ whose entries are characterized by (36) and (37), and λ satisfies

$$r\left(1+\lambda-\ln(r)\right) - \rho - \frac{1}{2}\mathbf{tr}\left(\Upsilon^\top L\Upsilon\right) = 0, \tag{41}$$

where $\Upsilon \equiv \bar{Q}^{1/2} - (\bar{K}_\Phi^\top L\bar{K}_\Phi)^{-1}\bar{K}_\Phi\bar{K}_\Phi^\top L\bar{Q}^{1/2}$. Moreover, the optimal trading strategy is given by

$$\mathring{a}(t) = -(\bar{K}_\Phi^\top L\bar{K}_\Phi)^{-1}(\bar{K}_\Phi^\top L\bar{A} + \psi r\bar{K}_\Phi^\top pk_\Phi^\top)\mathring{Y}(t) \tag{42}$$

and

$$\mathring{q}(t) = -(\bar{K}_\Phi^\top L\bar{K}_\Phi)^{-1}\bar{K}_\Phi^\top L\bar{Q}^{1/2}, \tag{43}$$

and the optimal consumption is

$$\mathring{c}(t) = \frac{\frac{1}{2}\mathring{Y}^\top(t)L\mathring{Y}(t) + \psi r\mathring{m}(t) + \lambda - \ln(r)}{\psi}, \tag{44}$$

where $(\mathring{Y}(t), \mathring{m}(t))$ is the solution of (34) and (35) corresponding to the choice of the control $(\mathring{a}(t), \mathring{q}(t), \mathring{c}(t))$.

Remark 4 *Note that, by applying a well known verification theorem (see e.g., Fleming & Rishel [5, Thm 4.1, p. 159] (1975), Fleming & Soner [6, Thm 5.1, p. 172] (1993)), it can be shown that (38) is actually the value function (33), for the choice of the symmetric matrix L and the real number λ satisfying (39), (40), and (41).*

5 Bayesian-Nash Equilibrium

To get a Bayesian Nash equilibrium we have to find a positive symmetric solution Σ of the algebraic Riccati Equation

$$\hat{M}_\perp^\top(\hat{A}\Sigma + \Sigma\hat{A}^\top + \hat{Q})\hat{M}_\perp - \hat{M}_\perp^\top(\Sigma\hat{A}^\top + \hat{Q})\hat{M}\hat{R}^{-1}\hat{M}^\top(\Sigma\hat{A}^\top + \hat{Q})^\top \hat{M}_\perp = 0, \tag{45}$$

a symmetric matrix L which fulfills

$$\bar{K}_\Phi^\top \left(L + \psi r p k_\Psi^\top\right) Y = 0, \tag{46}$$

and

$$\begin{aligned} &Y^\top \Big(rL + \left(L + \psi r p k_\Psi^\top\right)^\top \bar{Q} \left(L + \psi r p k_\Psi^\top\right) \\ &\quad - \left(L + \psi r p k_\Psi^\top\right)^\top \bar{A} - \bar{A}^\top \left(L + \psi r p k_\Psi^\top\right) - \psi \left(k_\Psi k_{\pi_e}^\top + k_{\pi_e} k_\Psi^\top\right) \Big) Y = 0, \end{aligned} \tag{47}$$

for every $Y \in \mathbb{R}^{10}$ satisfying (36) and (37), and a set of parameters

$$a^\top \equiv (a_1, a_2, a_3, a_4, a_5)\,, \quad b^\top \equiv (b_1, b_2, b_3, b_4, b_5)\,, \quad q^\top \equiv (q_1, q_2)\,,$$

such that

$$\begin{aligned} &(a^\top, b^\top) Y(t)\, dt + q\, dw(t) \\ &\quad = -(\bar{K}_\Phi^\top L \bar{K}_\Phi)^{-1} \left(\left(\bar{K}_\Phi^\top L \bar{A} + \psi r \bar{K}_\Phi^\top p k_\Phi^\top \right) Y(t)\, dt + \bar{K}_\Phi^\top L \bar{Q}^{1/2}\, dw(t) \right). \end{aligned} \tag{48}$$

Equation (45) comes from the optimal filtering problem of the market maker; Equation (46) and (47) come from the optimal consumption-investment problem of the insider trader. The first one is needed to eliminate the linear term in the Bellmann equation related to (33) and the second one represents the first-order conditions on the quadratic term of the Bellmann equation. Equation (48) comes from a fixed point argument requiring that the optimal strategy of the insider, conjectured by the market maker, coincides with the optimal strategy of the insider trader and vice versa.

The computations needed to this task are rather heavy. However, we are able to prove that there exists a unique linear Bayesian-Nash equilibrium such that no insider's trade and no market maker's learning occur. Also in this case, see the Appendix for the proof.

Proposition 5 *In the linear Bayesian-Nash equilibrium we have*

$$q_1 = q_2 = a_1 = a_2 = a_3 = a_4 = a_5 = b_1 = b_2 = b_3 = b_4 = b_5 = 0,$$

$$\Sigma = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \sigma_{5,5} \end{pmatrix}, \tag{49}$$

for an unconstrained coefficient $\sigma_{5,5} > 0$, *and*

$$L = \begin{pmatrix} \ell_{1,1} & -\ell_{2,6} & -\ell_{3,6} & -\ell_{4,6} & -\ell_{5,6} & \ell_{1,6} & -\ell_{6,7} & -\ell_{6,8} & -\ell_{6,9} & -\ell_{6,10} \\ -\ell_{2,6} & 0 & 0 & 0 & 0 & \ell_{2,6} & 0 & 0 & 0 & 0 \\ -\ell_{3,6} & 0 & \ell_{9,9} & \ell_{9,9} & 0 & \ell_{3,6} & 0 & -\ell_{9,9} & -\ell_{9,9} & 0 \\ -\ell_{4,6} & 0 & \ell_{9,9} & \ell_{9,9} & 0 & \ell_{4,6} & 0 & -\ell_{9,9} & -\ell_{9,9} & 0 \\ -\ell_{5,6} & 0 & 0 & 0 & -\frac{\sigma_D^2\psi^2}{r} & \ell_{5,6} & \frac{\psi}{r} & 0 & 0 & 0 \\ \ell_{1,6} & \ell_{2,6} & \ell_{3,6} & \ell_{4,6} & \ell_{5,6} & \ell_{6,6} & \ell_{6,7} & \ell_{6,8} & \ell_{6,9} & \ell_{6,10} \\ -\ell_{6,7} & 0 & 0 & 0 & \frac{\psi}{r} & \ell_{6,7} & 0 & 0 & 0 & 0 \\ -\ell_{6,8} & 0 & -\ell_{9,9} & -\ell_{9,9} & 0 & \ell_{6,8} & 0 & \ell_{9,9} & \ell_{9,9} & 0 \\ -\ell_{6,9} & 0 & -\ell_{9,9} & -\ell_{9,9} & 0 & \ell_{6,9} & 0 & \ell_{9,9} & \ell_{9,9} & 0 \\ -\ell_{6,10} & 0 & 0 & 0 & 0 & \ell_{6,10} & 0 & 0 & 0 & 0 \end{pmatrix}, \tag{50}$$

where

$$\ell_{1,1} = -(2\ell_{1,6} + \ell_{6,6}), \qquad \ell_{9,9} \neq 0,$$

and the other entries $\ell_{j,k}$ *can be chosen freely.*

6 Conclusions

This paper provides a striking result. In the Kyle setting with noise traders, when the insider trader knows the drift of the dividend process, no trade occurs. Noise traders are not enough to hide the trading of the insider: as the insider trades, the market maker identifies exactly his private information, sets the market price according to it and this renders the trade of the insider not profitable. As a consequence, in equilibrium the insider trader does not trade, there is no learning by the market maker and no diffusion of information in the market.

Appendix

Proof. of Proposition 1

Substituting (13) into (8), we obtain

$$dy(t) = \hat{A}y(t)\,dt + \hat{B}y_e(t)\,dt + \hat{Q}^{1/2}\,dw(t). \tag{51}$$

Then, combining (51) and (11), we have

$$dy_o(t) = M^\top \hat{A}y(t)\,dt + M^\top \hat{B}y_e(t)\,dt + M^\top \hat{Q}^{1/2}\,dw(t), \tag{52}$$

and, substituting the latter into (15) we can write

$$dy_e(t) = \hat{A}_e(t)y(t)\,dt + \hat{B}_e(t)y_e(t)\,dt + \hat{Q}_e^{1/2}(t)\,dw(t), \tag{53}$$

where

$$\hat{A}_e(t) \equiv H_e(t)M^\top \hat{A},\quad \hat{B}_e(t) \equiv G_e(t) + H_e(t)M^\top \hat{B},\quad \hat{Q}_e^{1/2}(t) \equiv H_e(t)M^\top \hat{Q}^{1/2}.$$

Hence, introducing the stacked vector $Y(t) \equiv (Y_j(t))_{j=1}^{10}$, given by

$$Y(t) \equiv \begin{pmatrix} y(t) \\ y_e(t) \end{pmatrix},$$

on account of (51) and (53), it is easy to see that $Y(t)$ satisfies the stochastic differential equation

$$dY(t) = \tilde{A}(t)Y(t)\,dt + \tilde{Q}^{1/2}(t)\,dw(t),$$

where

$$\tilde{A}(t) = \begin{pmatrix} \hat{A} & \hat{B} \\ \hat{A}_e(t) & \hat{B}_e(t) \end{pmatrix}, \qquad \tilde{Q}^{1/2}(t) = \begin{pmatrix} \hat{Q}^{1/2} \\ \hat{Q}_e^{1/2}(t) \end{pmatrix}. \tag{54}$$

On the other hand, thanks to (52), we can formulate the evolution of $y_o(t)$ in terms of $Y(t)$ by writing

$$dy_o(t) = CY(t)\,dt + R^{1/2}\,dw(t),$$

where

$$C \equiv (M^\top \hat{A}, M^\top \hat{B}), \qquad R^{1/2} \equiv M^\top \hat{Q}^{1/2}. \tag{55}$$

Therefore, the filtering problem becomes

$$\begin{cases} dY(t) = \tilde{A}(t)Y(t)\,dt + \tilde{Q}^{1/2}(t)\,dw(t), \\ dy_o(t) = CY(t)\,dt + R^{1/2}\,dw(t). \end{cases} \tag{56}$$

The latter is a generalization of the standard Kalman-Bucy linear filtering problem, because the matrix

$$R \equiv R^{1/2}(R^{1/2})^\top = \begin{pmatrix} \sigma_D^2 & \sigma_D q_1 \\ \sigma_D q_1 & (\sigma_\Theta + q_2)^2 + q_1^2 \end{pmatrix}$$

has positive eigenvalues, and we assume that for some initial time t_0 the prior $Y(t_0) \sim \mathcal{N}(Y_0, \tilde{\Sigma}_0)$ is independent of the Wiener process $w(t)$. Hence, we are in a position to apply [14, Thm 10.3, p. 392] to obtain the equation for the optimal estimate $Y_e(t) \equiv \mathbf{E}\left[Y(t)|\mathfrak{F}_t^{y_o}\right]$ for every $t \geq t_0$. This is given by

$$\begin{aligned} dY_e(t) &= \tilde{A}(t)Y_e(t)\,dt \\ &\quad + (\tilde{Q}^{1/2}(t)(R^{1/2})^\top + \tilde{\Sigma}(t)C)R^{-1}(C\,(Y(t) - Y_e(t))\,dt + R^{1/2}\,dw(t)), \end{aligned} \tag{57}$$

where, setting $\tilde{Q}(t) \equiv \tilde{Q}^{1/2}(t)(\tilde{Q}^{1/2}(t))^\top$, the matrix

$$\tilde{\Sigma}(t) \equiv \mathbf{E}\left[(Y(t) - Y_e(t))\,(Y(t) - Y_e(t))^\top\right]$$

satisfies the Riccati equation

$$\begin{aligned} d\tilde{\Sigma}(t) &= \left(\tilde{A}(t)\tilde{\Sigma}(t) + \tilde{\Sigma}(t)\tilde{A}^\top(t) + \tilde{Q}(t)\right)dt \\ &\quad - \left(\tilde{Q}^{1/2}(t)(R^{1/2})^\top + \tilde{\Sigma}(t)C^\top\right)R^{-1}\left(\tilde{Q}^{1/2}(t)(R^{1/2})^\top + \tilde{\Sigma}(t)C^\top\right)^\top dt. \end{aligned} \tag{58}$$

On the other hand, $y_e(t)$ is the market maker's estimate of $y(t)$, therefore we must clearly have

$$y_e(t) = \mathbf{E}\left[y_e(t)|\mathfrak{F}_t^{y_o}\right].$$

Hence,

$$Y_e(t) = \begin{pmatrix} y_e(t) \\ y_e(t) \end{pmatrix},$$

and it is easily seen that $\tilde{\Sigma}(t)$ can be decomposed into four $5 * 5$ blocks as follows

$$\tilde{\Sigma}(t) = \begin{pmatrix} \Sigma(t) & 0 \\ 0 & 0 \end{pmatrix}, \tag{59}$$

where $\Sigma(t) \equiv \mathbf{E}\left[(y(t) - y_e(t))\,(y(t) - y_e(t))^\top\right]$. Then we have

$$\tilde{\Sigma}(t)C^\top = \begin{pmatrix} \Sigma(t) & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} \hat{A}M \\ \hat{B}M \end{pmatrix} = \begin{pmatrix} \Sigma(t)\hat{A}M \\ 0 \end{pmatrix},$$

and

$$\tilde{Q}^{1/2}(t)(R^{1/2})^\top + \tilde{\Sigma}(t)C^\top = \begin{pmatrix} (\hat{Q} + \Sigma(t)\hat{A}^\top)M \\ H_e(t)R \end{pmatrix}.$$

where $\hat{Q} \equiv \hat{Q}^{1/2}(\hat{Q}^{1/2})^\top$. Hence, (57) becomes

$$\begin{aligned} dY_e(t) = & \begin{pmatrix} \hat{A} & \hat{B} \\ H_e(t)M^\top \hat{A} & G_e(t) + H_e(t)M^\top \hat{B} \end{pmatrix} Y_e(t)\, dt \\ & + \begin{pmatrix} (\hat{Q} + \Sigma(t)\hat{A}^\top)M \\ H_e(t)R \end{pmatrix} R^{-1}M^\top\big(\hat{A}\,(y(t) - y_e(t))\, dt + R^{1/2}\, dw(t)\big). \end{aligned} \tag{60}$$

On the other and, on account of (51),

$$\hat{A}\,(y(t) - y_e(t))\, dt + R^{1/2}\, dw(t) = -(\hat{A} + \hat{B})y_e(t) + dy(t)$$

and thanks to (11), we can rewrite (60) as

$$\begin{aligned} \begin{pmatrix} dy_e(t) \\ dy_e(t) \end{pmatrix} = & \begin{pmatrix} (\hat{A} + \hat{B} - (\hat{Q} + \Sigma(t)\hat{A}^\top)MR^{-1}M^\top(\hat{A} + \hat{B}))y_e(t)\, dt \\ G_e(t)y_e(t)\, dt \end{pmatrix} \\ & + \begin{pmatrix} (\hat{Q} + \Sigma(t)\hat{A}^\top)MR^{-1}\, dy_o(t) \\ H_e(t)\, dy_o(t) \end{pmatrix}, \end{aligned}$$

and the latter clearly implies that (17) and (18) are fulfilled. Now, a straightforward computation gives

$$\tilde{A}\tilde{\Sigma}(t) + \tilde{\Sigma}(t)\tilde{A}^\top + \tilde{Q} = \begin{pmatrix} \hat{A}\Sigma(t) + \Sigma(t)\hat{A}^\top + \hat{Q} & (H_e(t)M^\top(\hat{A}\Sigma(t) + \hat{Q}))^\top \\ H_e(t)M^\top(\hat{A}\Sigma(t) + \hat{Q}) & H_e(t)RH_e^\top(t) \end{pmatrix},$$

and

$$\begin{aligned} & \Big(\tilde{Q}^{1/2}(R^{1/2})^\top + \tilde{\Sigma}(t)C^\top\Big)R^{-1}\Big(\tilde{Q}^{1/2}(R^{1/2})^\top + \tilde{\Sigma}(t)C^\top\Big)^\top \\ & = \begin{pmatrix} (\hat{Q} + \Sigma(t)\hat{A}^\top)MR^{-1}M^\top(\hat{Q} + \Sigma(t)\hat{A}^\top)^\top & (H_e(t)M^\top(\hat{A}\Sigma(t) + \hat{Q}))^\top \\ H_e(t)M^\top(\hat{A}\Sigma(t) + \hat{Q}) & H_e(t)RH_e^\top(t) \end{pmatrix}. \end{aligned}$$

Therefore, on account of (58) and (59), (19) immediately follows. □

Proof. of Proposition 2

It is evident that with respect to the vector $\hat{y}_o(t) \equiv \hat{M}^\top y(t)$, which carries the same information of the observation vector $y_o(t)$, (19) becomes

$$\begin{aligned} d\Sigma(t) = & (\hat{A}\Sigma(t) + \Sigma(t)\hat{A}^\top + \hat{Q})\, dt \\ & - (\Sigma(t)\hat{A}^\top + \hat{Q})\hat{M}\hat{R}^{-1}\hat{M}^\top(\Sigma(t)\hat{A}^\top + \hat{Q}^\top)\, dt. \end{aligned} \tag{61}$$

Now, since the columns of the matrix $\tilde{M}$ make up an orthonormal basis of $\mathbb{R}^5$, (61) is equivalent to

$$\begin{aligned} d(\tilde{M}^\top \Sigma(t) \tilde{M}) &= \tilde{M}^\top (\hat{A}\Sigma(t) + \Sigma(t)\hat{A}^\top + \hat{Q})\tilde{M}^\top \, dt \\ &\quad - \tilde{M}^\top (\Sigma(t)\hat{A}^\top + \hat{Q})\hat{M}\hat{R}^{-1}\hat{M}^\top (\Sigma(t)\hat{A}^\top + \hat{Q})^\top \tilde{M}^\top \, dt, \end{aligned} \tag{62}$$

which splits in (21) plus the three following

$$\begin{aligned} d(\hat{M}^\top \Sigma(t) \hat{M}) &= \hat{M}^\top (\hat{A}\Sigma(t) + \Sigma(t)\hat{A}^\top + \hat{Q})\hat{M} \, dt \\ &\quad - \hat{M}^\top (\Sigma(t)\hat{A}^\top + \hat{Q})\hat{M}\hat{R}^{-1}\hat{M}^\top (\Sigma(t)\hat{A}^\top + \hat{Q})^\top \hat{M} \, dt, \end{aligned} \tag{63}$$

$$\begin{aligned} d(\hat{M}^\top \Sigma(t) \hat{M}_\perp) &= \hat{M}^\top (\hat{A}\Sigma(t) + \Sigma(t)\hat{A}^\top + \hat{Q})\hat{M}_\perp \, dt \\ &\quad - \hat{M}^\top (\Sigma(t)\hat{A}^\top + \hat{Q})\hat{M}\hat{R}^{-1}\hat{M}^\top (\Sigma(t)\hat{A}^\top + \hat{Q})^\top \hat{M}_\perp \, dt, \end{aligned} \tag{64}$$

$$\begin{aligned} d(\hat{M}_\perp^\top \Sigma(t) \hat{M}) &= \hat{M}_\perp^\top (\hat{A}^\top \Sigma(t) + \Sigma(t)\hat{A}^\top + \hat{Q})\hat{M} \, dt \\ &\quad - \hat{M}_\perp^\top (\Sigma(t)\hat{A}^\top + \hat{Q})\hat{M}\hat{R}^{-1}\hat{M}^\top (\Sigma(t)\hat{A}^\top + \hat{Q})^\top \hat{M} \, dt, \end{aligned} \tag{65}$$

On the other hand, the first and the second column of the matrix $\Sigma(t)\hat{M}$ are given by

$$\begin{pmatrix} \mathbf{E}\left[(D(t) - D_e(t))\,(D(t) - D_e(t))\right] \\ \mathbf{E}\left[(D(t) - D_e(t))\,(\pi(t) - \pi_e(t))\right] \\ \mathbf{E}\left[(D(t) - D_e(t))\,(\Theta(t) - \Theta_e(t))\right] \\ \mathbf{E}\left[(D(t) - D_e(t))\,(\Phi(t) - \Phi_e(t))\right] \\ \mathbf{E}\left[(D(t) - D_e(t))\,(\Psi(t) - \Psi_e(t))\right] \end{pmatrix},$$

and

$$1/\sqrt{2} \begin{pmatrix} \mathbf{E}\left[(O(t) - O_e(t))\,(D(t) - D_e(t))\right] \\ \mathbf{E}\left[(O(t) - O_e(t))\,(\pi(t) - \pi_e(t))\right] \\ \mathbf{E}\left[(O(t) - O_e(t))\,(\Theta(t) - \Theta_e(t))\right] \\ \mathbf{E}\left[(O(t) - O_e(t))\,(\Phi(t) - \Phi_e(t))\right] \\ \mathbf{E}\left[(O(t) - O_e(t))\,(\Psi(t) - \Psi_e(t))\right] \end{pmatrix},$$

respectively. Therefore, since both the processes $D(t)$ and $O(t)$ are observed, the matrix $\Sigma(t)\hat{M}$ vanishes identically. Hence, a straightforward computation shows that also (63)-(65) vanish identically, and the desired result follows. □

Proof of Proposition 3

It is well known that (33) must satisfy both the Bellman equation

$$\partial_t V(t,Y,m) + \max_{a,q,c}\{\mathcal{L}V(t,Y,m) - e^{-(\rho t+\psi c)}\} = 0, \tag{66}$$

where $\mathcal{L}$ is the infinitesimal generator of the diffusion process (34), (35), and the transversality condition

$$\lim_{T\to+\infty} \mathbf{E}_{t,m,Y}\left[V(t+T,\mathring{Y}(t+T),\mathring{m}(t+T))\right] = 0, \tag{67}$$

where $(\mathring{Y}(t),\mathring{m}(t))$ is a solution of (34) and (35) corresponding to the choice of an optimal control $(\mathring{a}(t),\mathring{q}(t),\mathring{c}(t))$. In this case, a feedback optimal control is given by

$$(\mathring{a}(t),\mathring{q}(t),\mathring{c}\,(t)) = \underset{a(t),q(t),c(t)}{\arg\max}\,\{\mathcal{L}V(t,Y(t),m\,(t)) - e^{-(\rho t+\psi c(t))}\}, \tag{68}$$

where $(Y(t),m(t))$ is a solution of (34) and (35) corresponding to the choice of the control $(a(t),q(t),c(t))$.

To prove that the function $V(t,Y,m) = -e^{-(\rho t+\frac{1}{2}Y^\top LY+\psi rm+\lambda)}$ is a solution to (66), let's start by computing the operator $\mathcal{L}$. It is easily seen that

$$\begin{aligned}\mathcal{L} \equiv\; & \frac{1}{2}\sum_{i,j=1}^{10}\left(\left(\bar{Q}^{1/2}+\bar{K}_\Phi q\right)\left(\bar{Q}^{1/2}+\bar{K}_\Phi q\right)^\top\right)_{i,j}\partial^2_{Y_i,Y_j} \\ & + Y^\top k_\Psi \sum_{j=1}^{10}\left(p^\top\left(\bar{Q}^{1/2}+\bar{K}_\Phi q\right)\left(\bar{Q}^{1/2}+\bar{K}_\Phi q\right)^\top\right)_j \partial^2_{m,Y_j} \\ & + \frac{1}{2}Y^\top k_\Psi p^\top\left(\bar{Q}^{1/2}+\bar{K}_\Phi q\right)\left(\bar{Q}^{1/2}+\bar{K}_\Phi q\right)^\top p\,k_\Psi^\top Y\,\partial^2_{m,m} \\ & + \sum_{j=1}^{10}\left(\bar{A}Y+\bar{K}_\Phi a^\top\right)_j \partial_{Y_j} \\ & + \left(rm - Y^\top k_\Psi\left(r^{-1}k_{\pi_e}^\top Y - p^\top\left(\bar{A}Y+\bar{K}_\Phi a^\top\right)\right) - c\right)\partial_m.\end{aligned} \tag{69}$$

Hence, on account of

$$\begin{aligned}
\partial_t V &= -\rho V, \\
\partial_{Y_j} V &= -(Y^\top L)_j V, \\
\partial_m V &= -\psi r V, \\
\partial^2_{Y_i,Y_j} V &= \left(LYY^\top L - L\right)_{i,j} V, \\
\partial^2_{Y_j,m} V &= \psi r (Y^\top L)_j V, \\
\partial^2_{m,m} V &= \psi^2 r^2 V,
\end{aligned}$$

where we are using V as a shorthand for $V(t, Y, m)$, we obtain

$$\begin{aligned}
\mathcal{L}V =& \frac{1}{2}\sum_{i,j=1}^{10}\left(\left(\bar{Q}^{1/2} + \bar{K}_\Phi q\right)\left(\bar{Q}^{1/2} + \bar{K}_\Phi q\right)^\top\right)_{i,j}\left(LYY^\top L - L\right)_{i,j} V \qquad (70)\\
&+ \psi r Y^\top k_\Psi \sum_{j=1}^{10} p^\top \left(\bar{Q}^{1/2} + \bar{K}_\Phi q\right)\left(\bar{Q}^{1/2} + \bar{K}_\Phi q\right)^\top_j (Y^\top L)_j V \\
&+ \frac{1}{2}\psi^2 r^2 Y^\top k_\Psi p^\top \left(\bar{Q}^{1/2} + \bar{K}_\Phi q\right)\left(\bar{Q}^{1/2} + \bar{K}_\Phi q\right)^\top p\, k_\Psi^\top Y V \\
&- \sum_{j=1}^{10}\left(\bar{A}Y + \bar{K}_\Phi a^\top\right)_j (Y^\top L)_j V \\
&- \psi r \left(rm - Y^\top k_\Psi \left(r^{-1} k_{\pi_e}^\top Y - p^\top \left(\bar{A}Y + \bar{K}_\Phi a^\top\right)\right) - c\right) V.
\end{aligned}$$

On the other hand, thanks to the properties of the trace functional, we can write

$$\begin{aligned}
&\sum_{i,j=1}^{10}\left(\left(\bar{Q}^{1/2} + \bar{K}_\Phi q\right)\left(\bar{Q}^{1/2} + \bar{K}_\Phi q\right)^\top\right)_{i,j}\left(LYY^\top L - L\right)_{i,j} V \qquad (71)\\
&= \mathbf{tr}\left(\left(\bar{Q}^{1/2} + \bar{K}_\Phi q\right)^\top \left(LYY^\top L - L\right)\left(\bar{Q}^{1/2} + \bar{K}_\Phi q\right)\right) \\
&= Y^\top L \left(\bar{Q}^{1/2} + \bar{K}_\Phi q\right)\left(\bar{Q}^{1/2} + \bar{K}_\Phi q\right)^\top LY \\
&\quad - \mathbf{tr}\left(\left(\bar{Q}^{1/2} + \bar{K}_\Phi q\right)^\top L \left(\bar{Q}^{1/2} + \bar{K}_\Phi q\right)\right).
\end{aligned}$$

Moreover,

$$\sum_{j=1}^{10}\left(p^\top\left(\bar{Q}^{1/2}+\bar{K}_\Phi q\right)\left(\bar{Q}^{1/2}+\bar{K}_\Phi q\right)^\top\right)_j (Y^\top L)_j \tag{72}$$
$$= p^\top\left(\bar{Q}^{1/2}+\bar{K}_\Phi q\right)\left(\bar{Q}^{1/2}+\bar{K}_\Phi q\right)^\top LY,$$

and

$$\sum_{j=1}^{10}\left(\bar{A}Y+\bar{K}_\Phi a^\top\right)_j (Y^\top L)_j = Y^\top L\left(\bar{A}Y+\bar{K}_\Phi a^\top\right). \tag{73}$$

Therefore, combining (70) with (71)-(73), and observing that

$$\begin{aligned}
&\frac{1}{2}Y^\top L\left(\bar{Q}^{1/2}+\bar{K}_\Phi q\right)\left(\bar{Q}^{1/2}+\bar{K}_\Phi q\right)^\top LY\\
&+\psi r Y^\top k_\Psi p^\top\left(\bar{Q}^{1/2}+\bar{K}_\Phi q\right)\left(\bar{Q}^{1/2}+\bar{K}_\Phi q\right)^\top LY\\
&+\frac{1}{2}\psi^2 r^2 Y^\top k_\Psi p^\top\left(\bar{Q}^{1/2}+\bar{K}_\Phi q\right)\left(\bar{Q}^{1/2}+\bar{K}_\Phi q\right)^\top p k_\Psi^\top Y\\
&=\frac{1}{2}Y^\top\left(L+\psi r p k_\Psi^\top\right)^\top\left(\bar{Q}^{1/2}+\bar{K}_\Phi q\right)\left(\bar{Q}^{1/2}+\bar{K}_\Phi q\right)^\top\left(L+\psi r p k_\Psi^\top\right)Y,
\end{aligned}$$

and

$$\begin{aligned}
&Y^\top L\left(\bar{A}Y+\bar{K}_\Phi a^\top\right)+\psi r Y^\top k_\Psi p^\top\left(\bar{A}Y+\bar{K}_\Phi a^\top\right)\\
&= Y^\top\left(L+\psi r p k_\Psi^\top\right)^\top\left(\bar{A}Y+\bar{K}_\Phi a^\top\right)
\end{aligned}$$

we can rewrite

$$\begin{aligned}
\mathcal{L}V = &-\frac{1}{2}\mathbf{tr}\left(\left(\bar{Q}^{1/2}+\bar{K}_\Phi q\right)^\top L\left(\bar{Q}^{1/2}+\bar{K}_\Phi q\right)\right)V\\
&+\frac{1}{2}Y^\top\left(L+\psi r p k_\Psi^\top\right)^\top\left(\bar{Q}^{1/2}+\bar{K}_\Phi q\right)\left(\bar{Q}^{1/2}+\bar{K}_\Phi q\right)^\top\left(L+\psi r p k_\Psi^\top\right)YV\\
&-Y^\top\left(L+\psi r p k_\Psi^\top\right)^\top\left(\bar{A}Y+\bar{K}_\Phi a^\top\right)V\\
&-\psi\left(r^2 m+Y^\top k_\Psi p^\top Y\right)V+\psi r c V.
\end{aligned} \tag{74}$$

From the above computations, it follows that to achieve the maximum in (66) we must get rid of the linear term

$$-Y^\top\left(L+\psi r p k_\Psi^\top\right)^\top\bar{K}_\Phi a^\top V.$$

To this goal, we need to establish (39). As a consequence, (74) becomes

$$
\begin{aligned}
\mathcal{L}V = &-\frac{1}{2}\mathbf{tr}\left(\left(\bar{Q}^{1/2}+\bar{K}_{\Phi}q\right)^{\top}L\left(\bar{Q}^{1/2}+\bar{K}_{\Phi}q\right)\right)V \\
&+\frac{1}{2}Y^{\top}\left(L+\psi rpk_{\Psi}^{\top}\right)^{\top}\bar{Q}\left(L+\psi rpk_{\Psi}^{\top}\right)YV \\
&-Y^{\top}\left(L+\psi rpk_{\Psi}^{\top}\right)^{\top}\bar{A}YV \\
&-\psi\left(r^{2}m+Y^{\top}k_{\Psi}k_{\pi_e}^{\top}Y\right)V+\psi rcV,
\end{aligned}
\tag{75}
$$

where $\bar{Q}\equiv\bar{Q}^{1/2}(\bar{Q}^{1/2})^{\top}$. Hence, setting

$$
\begin{aligned}
I(t,Y,m,a,q) \stackrel{\text{def}}{=} &-\frac{1}{2}\mathbf{tr}\left(\left(\bar{Q}^{1/2}+\bar{K}_{\Phi}q\right)^{\top}L\left(\bar{Q}^{1/2}+\bar{K}_{\Phi}q\right)\right)V \\
&+\frac{1}{2}Y^{\top}\left(L+\psi rpk_{\Psi}^{\top}\right)^{\top}\bar{Q}\left(L+\psi rpk_{\Psi}^{\top}\right)YV \\
&-Y^{\top}\left(L+\psi rpk_{\Psi}^{\top}\right)^{\top}\bar{A}YV \\
&-\rho V-\psi\left(r^{2}m+Y^{\top}k_{\Psi}k_{\pi_e}^{\top}Y\right)V,
\end{aligned}
\tag{76}
$$

and

$$
J(t,Y,m,c)\stackrel{\text{def}}{=}\psi rcV-e^{-(\rho t+\psi c)},
\tag{77}
$$

(66) becomes

$$
\max_{a,q}\left\{I(t,Y,m,a,q)\right\}+\max_{c}\left\{J\left(t,Y,m,c\right)\right\}=0.
\tag{78}
$$

Maximizing $I(t,Y,m,a,q)$ with respect to q, the first order conditions yield

$$
\partial_{q_j}\mathbf{tr}\left(\left(\bar{Q}^{1/2}+\bar{K}_{\Phi}q\right)^{\top}L\left(\bar{Q}^{1/2}+\bar{K}_{\Phi}q\right)\right)=0,
\tag{79}
$$

for $j = 1, 2, 3$. On the other hand,

$$\begin{aligned}
&\partial_{q_j} \mathbf{tr}\left(\left(\bar{Q}^{1/2} + \bar{K}_\Phi q\right)^\top L \left(\bar{Q}^{1/2} + \bar{K}_\Phi q\right)\right) \\
&= \mathbf{tr}\left(\partial_{q_j}\left(\bar{Q}^{1/2} + \bar{K}_\Phi q\right)^\top L \left(\bar{Q}^{1/2} + \bar{K}_\Phi q\right)\right) \\
&= \mathbf{tr}\left(e_j^\top \bar{K}_\Phi^\top L \left(\bar{Q}^{1/2} + \bar{K}_\Phi q\right) + \left(\bar{Q}^{1/2} + \bar{K}_\Phi q\right)^\top L \bar{K}_\Phi e_j\right) \\
&= \mathbf{tr}\left(e_j^\top \bar{K}_\Phi^\top L \left(\bar{Q}^{1/2} + \bar{K}_\Phi q\right)\right) + \mathbf{tr}\left(\left(\bar{Q}^{1/2} + \bar{K}_\Phi q\right)^\top L \bar{K}_\Phi e_j\right) \\
&= 2\mathbf{tr}\left(e_j^\top \bar{K}_\Phi^\top L \left(\bar{Q}^{1/2} + \bar{K}_\Phi q\right)\right) \\
&= 2\bar{K}_\Phi^\top L \left(\bar{Q}^{1/2} + \bar{K}_\Phi q\right) e_j.
\end{aligned}$$

Therefore, from (79), we obtain

$$q = -(\bar{K}_\Phi^\top L \bar{K}_\Phi)^{-1} \bar{K}_\Phi^\top L \bar{Q}^{1/2}. \tag{80}$$

and

$$\begin{aligned}
\max_{a,q}\{I(t, Y, m, a, q)\} &= -\frac{1}{2}\mathbf{tr}\left(\Upsilon^\top L \Upsilon\right) \\
&+ \frac{1}{2} Y^\top \left(L + \psi r p k_\Psi^\top\right)^\top \bar{Q} \left(L + \psi r p k_\Psi^\top\right) Y V \\
&- Y^\top \left(L + \psi r p k_\Psi^\top\right)^\top \bar{A} Y V - \rho V - \psi\left(r^2 m + Y^\top k_\Psi k_{\pi_e}^\top Y\right) V.
\end{aligned} \tag{81}$$

Similarly, maximizing $J(t, Y, m, c)$ with respect to c, the first order condition yields

$$e^{-(\rho t + \psi c)} + rV = 0, \tag{82}$$

and this clearly implies

$$c = \frac{\frac{1}{2} Y^\top L Y + \psi r m(t) + \lambda - \ln(r)}{\psi},$$

and

$$\max_c \{J(t, Y, m, c)\} = r(\frac{1}{2} Y^\top L Y + \psi r m + \lambda - \ln(r) + 1) V. \tag{83}$$

Finally, combining (81) with (83), we obtain

$$\begin{aligned}
&\max_{a,q}\{I(t,Y,m,c)\}+\max_{c}\{J(t,Y,m,c)\}\\
&\quad=\left(r\left(\lambda-\ln(r)+1\right)-\rho\right)V-\frac{1}{2}\mathbf{tr}\left(\Upsilon^{\top}L\Upsilon\right)V\\
&\quad+Y^{\top}\Big(\frac{1}{2}rL+\frac{1}{2}\left(L+\psi rpk_{\Psi}^{\top}\right)^{\top}\bar{Q}\left(L+\psi rpk_{\Psi}^{\top}\right)\\
&\quad-\left(L+\psi rpk_{\Psi}^{\top}\right)^{\top}\bar{A}-\psi k_{\Psi}k_{\pi_e}^{\top}\Big)YV.
\end{aligned}$$

The latter clearly implies that $V(t,Y,m)$ is a solution of the Bellman equation (66) provided that (40) and (41) are satisfied.

Now, to show that the trasversality condition (67) holds true we apply the Itô formula to the identity

$$\begin{aligned}
&V(t+\Delta t,\mathring{Y}(t+\Delta t),\mathring{m}(t+\Delta t))-V(t,\mathring{Y}(t),\mathring{m}(t))\\
&\quad=\int_t^{t+\Delta t}dV(s,\mathring{Y}(s),\mathring{m}(s)).
\end{aligned}$$

Therefore, we can write

$$\begin{aligned}
&V(t+\Delta t,\mathring{Y}(t+\Delta t),\mathring{m}(t+\Delta t))-V(t,\mathring{Y}(t),\mathring{m}(t)) \qquad (84)\\
&\quad=\int_t^{t+\Delta t}\left(\partial_s V(s,\mathring{Y}(s),\mathring{m}(s))+\mathcal{L}V(s,\mathring{Y}(s),\mathring{m}(s))\right)ds\\
&\quad+\int_t^{t+\Delta t}\sigma(\mathring{Y}(s),\mathring{m}(s))\nabla_{Y,m}V(s,\mathring{Y}(s),\mathring{m}(s)))\,dw(s),
\end{aligned}$$

where $\sigma(\mathring{Y}(t),\mathring{m}(t))$ denotes the diffusion matrix of the process $(\mathring{Y}(t),\mathring{m}(t))$ and $\nabla_{Y,m}$ denotes the gradient operator in the state space of $(\mathring{Y}(t),\mathring{m}(t))$. On the other hand, since $V(t,Y,m)$ is a solution of the Bellman equation (66) and $(\mathring{Y}(t),\mathring{m}(t))$ corresponds to an optimal control, we have

$$\begin{aligned}
&\int_t^{t+\Delta t}\left(\partial_s V(s,\mathring{Y}(s),\mathring{m}(s))+\mathcal{L}V(s,\mathring{Y}(s),\mathring{m}(s))\right)ds\\
&\quad=\int_t^{t+\Delta t}e^{-(\rho s+\psi\hat{c}(s))}\,ds.
\end{aligned}$$

On account of the latter, by applying the expectation operator on both the sides of (84), we obtain

$$\frac{\mathbf{E}_{t,Y,m}\left[V(t+\Delta t,\mathring{Y}\left(t+\Delta t\right),\mathring{m}\left(t+\Delta t\right))\right]-\mathbf{E}_{t,Y,m}\left[\mathring{V}(t,\mathring{Y}(t),\mathring{m}(t))\right]}{\Delta t}$$
$$=\frac{1}{\Delta t}\mathbf{E}_{t,Y,m}\left[\int_{t}^{t+\Delta t}e^{-(\rho s+\psi\mathring{c}(s))}\,ds\right],$$

and, passing to the limit as $\Delta t \to 0$, it then follows

$$\frac{d\mathbf{E}_{t,Y,m}\left[\mathring{V}(t,\mathring{Y}(t),\mathring{m}(t))\right]}{dt}=\mathbf{E}_{t,Y,m}\left[e^{-(\rho t+\psi\mathring{c}(t))}\right],$$

where, by virtue of (82),

$$e^{-(\rho t+\psi c^*(t))}=-r\mathring{V}(t,\mathring{Y}(t),\mathring{m}(t)).$$

Therefore $\mathbf{E}_{t,Y,m}\left[\mathring{V}(t,\mathring{Y}(t),\mathring{m}(t))\right]$ satisfies the differential equation

$$\frac{d\mathbf{E}_{t,Y,m}\left[\mathring{V}(t,\mathring{Y}(t),\mathring{m}(t))\right]}{dt}=-r\mathbf{E}_{t,Y,m}\left[\mathring{V}(t,\mathring{Y}(t),\mathring{m}(t))\right],$$

and the desired transversality condition clearly follows.

Finally, we are only left with the task of proving that $(\mathring{\underline{q}}(t),\mathring{\bar{q}}(t),\mathring{c}(t))$ given by (42)-(44) satisfies (68). To this, let's begin by observing that, on account of (39), for an optimal sample path $Y(t)$ of the state vector of the economy, we must have

$$\bar{K}_{\Phi}^{\top}\left(L+\psi rpk_{\Psi}^{\top}\right)Y(t)=0$$

for every $t\geq 0$. Differentiating the latter and substituting $dY(t)$ with its expression (26) we obtain

$$\bar{K}_{\Phi}^{\top}\left(L+\psi rpk_{\Psi}^{\top}\right)(\bar{A}Y(t)\,dt+\bar{Q}^{1/2}\,dw(t)+\bar{K}_{\Phi}\,d\Phi(t))=0,$$

which implies

$$\begin{aligned}-\bar{K}_{\Phi}^{\top}L\bar{K}_{\Phi}\,d\Phi(t)&=\bar{K}_{\Phi}^{\top}L\bar{A}Y(t)\,dt+\psi r\bar{K}_{\Phi}^{\top}pk_{\Psi}^{\top}(\bar{A}Y(t)\,dt+\bar{Q}^{1/2}\,dw(t)\\&\quad+\bar{K}_{\Phi}\,d\Phi(t))+\bar{K}_{\Phi}^{\top}L\bar{Q}^{1/2}\,dw(t)\\&=\bar{K}_{\Phi}^{\top}L\bar{A}Y(t)\,dt+\psi r\bar{K}_{\Phi}^{\top}pk_{\Psi}^{\top}\,dY(t)+\bar{K}_{\Phi}^{\top}L\bar{Q}^{1/2}\,dw(t).\end{aligned}$$

Hence, taking into account that

$$k_\Psi^\top \, dY(t) = d\Psi(t) = \Phi(t)\, dt = k_\Phi^\top Y(t)\, dt$$

we end up with

$$\begin{aligned} &d\Phi(t) \\ &= -(\bar{K}_\Phi^\top L \bar{K}_\Phi)^{-1} \left(\left(\bar{K}_\Phi^\top L \bar{A} + \psi r \bar{K}_\Phi^\top p k_\Phi^\top \right) Y(t)\, dt + \bar{K}_\Phi^\top L \bar{Q}^{1/2}\, dw(t) \right). \end{aligned}$$

Therefore, from the previous computations, it clearly follows that we have

$$\max_{a(t),q(t),c(t)} \{\mathcal{L}V(t,Y(t),m(t)) - e^{-(\rho t + \psi c(t))}\} = \mathcal{L}V(t,\mathring{Y}(t),\mathring{m}(t)) - e^{-(\rho t + \psi \mathring{c}(t))},$$

where $(\mathring{Y}(t), \mathring{m}(t))$ is the solution of (34) and (35) corresponding to the choice of $(\mathring{a}(t), \mathring{q}(t), \mathring{c}(t))$, which in turn satisfies (42)-(44). □

Proof of Proposition 5

The algebraic computations exploited in the proof are rather heavy. We give here only a sketch, informing the reader that a more detailed version is available from the authors upon request.

We divide our Proof in several steps.

As a first step, setting

$$\Sigma \equiv \begin{pmatrix} \sigma_{1,1} & \sigma_{1,2} & \sigma_{1,3} & \sigma_{1,4} & \sigma_{1,5} \\ \sigma_{1,2} & \sigma_{2,2} & \sigma_{2,3} & \sigma_{2,4} & \sigma_{2,5} \\ \sigma_{1,3} & \sigma_{2,3} & \sigma_{3,3} & \sigma_{3,4} & \sigma_{3,5} \\ \sigma_{1,4} & \sigma_{2,4} & \sigma_{3,4} & \sigma_{4,4} & \sigma_{4,5} \\ \sigma_{1,5} & \sigma_{2,5} & \sigma_{3,5} & \sigma_{4,5} & \sigma_{5,5} \end{pmatrix},$$

we note that, since $\Sigma \hat{M} = 0$ (see Section 3), we must have

$$\sigma_{1,1} = 0, \quad \sigma_{1,2} = 0, \quad \sigma_{1,3} = 0, \quad \sigma_{1,4} = 0, \quad \sigma_{1,5} = 0, \tag{85}$$

and

$$\sigma_{2,4} = -\sigma_{2,3}, \quad \sigma_{3,4} = -\sigma_{3,3}, \quad \sigma_{4,4} = -\sigma_{3,3}, \quad \sigma_{4,5} = -\sigma_{3,5}. \tag{86}$$

In addition, it can be shown that (45) implies

$$\sigma_{2,2} = 0.$$

From the latter, taking into account that Σ is a variance-covariance matrix, it follows

$$\sigma_{2,3} = 0, \quad \sigma_{2,4} = 0, \quad \sigma_{2,5} = 0.$$

Hence, (45) gives rise to the following equations

$$\begin{aligned} &((\alpha_\Theta + a_4 - a_3)\sigma_{3,3} - a_5\sigma_{3,5})^2 \\ &-2\sigma_\Theta(\sigma_\Theta + q_2)((\alpha_\Theta + a_4 - a_3)\sigma_{3,3} - a_5\sigma_{3,5}) \\ &+2\alpha_\Theta(\sigma_\Theta + q_2)^2\sigma_{3,3} = 0, \end{aligned} \tag{87}$$

$$\begin{aligned} &((\alpha_\Theta + a_4 - a_3)\sigma_{3,3} - a_5\sigma_{3,5})\left((\alpha_\Theta + a_4 - a_3)\sigma_{3,5} - a_5\sigma_{5,5}\right) \\ &-\sigma_\Theta(\sigma_\Theta + q_2)\left((\alpha_\Theta + a_4 - a_3)\sigma_{3,5} - a_5\sigma_{5,5}\right) \\ &+(\sigma_\Theta + q_2)^2(\sigma_{3,3} + \alpha_\Theta\sigma_{3,5}) = 0, \end{aligned} \tag{88}$$

$$((\alpha_\Theta + a_4 - a_3)\sigma_{3,5} - a_5\sigma_{5,5})^2 + 2(\sigma_\Theta + q_2)^2\sigma_{3,5} = 0. \tag{89}$$

As a second step, by virtue of (46), we can prove that

$$\bar{K}_\Phi^\top L\bar{Q}^{1/2} = 0.$$

This implies

$$q_1 = q_2 = 0. \tag{90}$$

Moreover, it can be shown that

$$\bar{K}_\Phi^\top p = 0,$$

which gives

$$(a^\top, b^\top) = -(\bar{K}_\Phi^\top L\bar{K}_\Phi)^{-1}\bar{K}_\Phi^\top L\bar{A}.$$

On the other hand, computing $\bar{K}_\Phi^\top L\bar{K}_\Phi$ and $\bar{K}_\Phi^\top L\bar{A}$ on account of (46), we obtain

$$a_1 = a_2 = a_3 = a_4 = a_5 = b_1 = b_2 = b_3 = b_5 = 0, \tag{91}$$

and

$$b_4 = -(\ell_{4,9} - \ell_{8,9} + \ell_{9,9})^{-1}(\ell_{4,9} + \ell_{8,9}). \tag{92}$$

As a third step, we note that, thanks to (90) and (91), Equation (87) yields

$$\sigma_{3,3} = 0$$

and, on account of the nature of Σ,

$$\sigma_{3,4} = 0, \quad \sigma_{3,5} = 0.$$

As a consequence, (88) and (89) are satisfied for any positive value of $\sigma_{5,5}$. Therefore, we obtain (49).

As a fourth step, it remains to prove that $b_4 = 0$ and (50) holds true. To this task, by a straightforward computation, it follows that (47) gives rise to the following system of quadratic equations

$$\begin{aligned} &r(\ell_{1,1} + 2\ell_{1,6} + \ell_{6,6}) + \sigma_D^2(\ell_{1,1} + 2\ell_{1,6} + \ell_{6,6})^2 \\ &+\sigma_\Theta^2(\ell_{1,3} - \ell_{1,4} + \ell_{3,6} - \ell_{4,6})^2 - 4(\ell_{1,2} + \ell_{2,6}) = 0, \end{aligned} \tag{93}$$

$$\begin{aligned} &r(\ell_{1,2} + \ell_{2,6}) + \sigma_D^2(\ell_{1,1} + 2\ell_{1,6} + \ell_{6,6})(\ell_{1,2} + \ell_{2,6}) \\ &+\sigma_\Theta^2(\ell_{1,3} - \ell_{1,4} + \ell_{3,6} - \ell_{4,6})(\ell_{2,3} - \ell_{2,4}) - 2\ell_{2,2} = 0, \end{aligned} \tag{94}$$

$$\begin{aligned} &r(\ell_{1,3} + \ell_{1,9} + \ell_{3,6} + \ell_{6,9}) \\ &+\sigma_D^2(\ell_{1,1} + 2\ell_{1,6} + \ell_{6,6})(\ell_{1,3} + \ell_{1,9} + \ell_{3,6} + \ell_{6,9}) \\ &+\sigma_\Theta^2(\ell_{1,3} - \ell_{1,4} + \ell_{3,6} - \ell_{4,6})(\ell_{3,3} - \ell_{3,4} + \ell_{3,9} - \ell_{4,9}) \\ &-2(\ell_{2,3} + \ell_{2,9}) - (b_4 + \alpha_\Theta)(\ell_{1,8} + \ell_{6,8}) + \alpha_\Theta(\ell_{1,3} + \ell_{3,6}) = 0, \end{aligned} \tag{95}$$

$$\begin{aligned} &r(\ell_{1,4} + \ell_{1,9} + \ell_{4,6} + \ell_{6,9}) \\ &+\sigma_D^2(\ell_{1,1} + 2\ell_{1,6} + \ell_{6,6})(\ell_{1,4} + \ell_{1,9} + \ell_{4,6} + \ell_{6,9}) \\ &+\sigma_\Theta^2(\ell_{1,3} - \ell_{1,4} + \ell_{3,6} - \ell_{4,6})(\ell_{3,4} - \ell_{4,4} + \ell_{3,9} - \ell_{4,9}) \\ &-(b_4 + \alpha_\Theta)(\ell_{1,8} + \ell_{6,8}) - 2(\ell_{2,4} + \ell_{2,9}) = 0, \end{aligned} \tag{96}$$

$$\begin{aligned} &r(\ell_{1,5} + \ell_{5,6}) + \sigma_D^2(\ell_{1,1} + 2\ell_{1,6} + \ell_{6,6})(\psi + \ell_{1,5} + \ell_{5,6}) \\ &+\sigma_\Theta^2(\ell_{1,3} - \ell_{1,4} + \ell_{3,6} - \ell_{4,6})(\ell_{3,5} - \ell_{4,5}) \\ &-(\ell_{1,4} + \ell_{4,6}) - 2\ell_{2,5} = 0, \end{aligned} \tag{97}$$

$$\begin{aligned} &r(\ell_{1,7} + \ell_{6,7}) + \sigma_D^2(\ell_{1,1} + 2\ell_{1,6} + \ell_{6,6})(\ell_{1,7} + \ell_{6,7}) \\ &+\sigma_\Theta^2(\ell_{1,3} - \ell_{1,4} + \ell_{3,6} - \ell_{4,6})(\ell_{3,7} - \ell_{4,7}) - 2\ell_{2,7} = 0, \end{aligned} \tag{98}$$

$$\begin{aligned} &r(\ell_{1,8} - \ell_{1,9} + \ell_{6,8} - \ell_{6,9}) \\ &+\sigma_D^2(\ell_{1,1} + 2\ell_{1,6} + \ell_{6,6})(\ell_{1,8} - \ell_{1,9} + \ell_{6,8} - \ell_{6,9}) \\ &+\sigma_\Theta^2(\ell_{1,3} - \ell_{1,4} + \ell_{3,6} - \ell_{4,6})(\ell_{3,8} - \ell_{3,9} - \ell_{4,8} + \ell_{4,9}) \\ &+2(b_4 + \alpha_\Theta)(\ell_{1,8} + \ell_{6,8}) + \alpha_\Theta(\ell_{1,3} + \ell_{3,6}) - 2(\ell_{2,8} - \ell_{2,9}) = 0, \end{aligned} \tag{99}$$

$$r(\ell_{1,10}+\ell_{6,10})+\sigma_D^2(\ell_{1,1}+2\ell_{1,6}+\ell_{6,6})(\ell_{1,10}+\ell_{6,10}) \\ +\sigma_\Theta^2(\ell_{1,3}-\ell_{1,4}+\ell_{3,6}-\ell_{4,6})(\ell_{3,10}-\ell_{4,10}) \\ -(\ell_{1,9}+\ell_{6,9})-2\ell_{2,10}=0, \tag{100}$$

$$r\ell_{2,2}+\sigma_D^2(\ell_{1,2}+\ell_{2,6})^2+\sigma_\Theta^2(\ell_{2,3}-\ell_{2,4})^2=0, \tag{101}$$

$$r(\ell_{2,3}+\ell_{2,9})+\sigma_D^2(\ell_{1,2}+\ell_{2,6})(\ell_{1,3}+\ell_{3,6}+\ell_{1,9}+\ell_{6,9}) \\ +\sigma_\Theta^2(\ell_{2,3}-\ell_{2,4})(\ell_{3,3}-\ell_{3,4}+\ell_{3,9}-\ell_{4,9}) \\ -(b_4+\alpha_\Theta)\ell_{2,8}+\alpha_\Theta\ell_{2,3}=0, \tag{102}$$

$$r(\ell_{2,4}+\ell_{2,9})+\sigma_D^2(\ell_{1,2}+\ell_{2,6})(\ell_{1,4}+\ell_{1,9}+\ell_{4,6}+\ell_{6,9}) \\ +\sigma_\Theta^2(\ell_{2,3}-\ell_{2,4})(\ell_{3,4}+\ell_{3,9}-\ell_{4,4}-\ell_{4,9})-(b_4+\alpha_\Theta)\ell_{2,8}=0, \tag{103}$$

$$r\ell_{2,5}+\sigma_D^2(\ell_{1,2}+\ell_{2,6})(\psi+\ell_{1,5}+\ell_{5,6}) \\ +\sigma_\Theta^2(\ell_{2,3}-\ell_{2,4})(\ell_{3,5}-\ell_{4,5})-\ell_{2,4}=0, \tag{104}$$

$$r\ell_{2,7}+\sigma_D^2(\ell_{1,2}+\ell_{2,6})(\ell_{1,7}+\ell_{6,7})+\sigma_\Theta^2(\ell_{2,3}-\ell_{2,4})(\ell_{3,7}-\ell_{4,7})=0, \tag{105}$$

$$r(\ell_{2,8}-\ell_{2,9})+\sigma_D^2(\ell_{1,2}+\ell_{2,6})(\ell_{1,8}-\ell_{1,9}+\ell_{6,8}-\ell_{6,9}) \\ +\sigma_\Theta^2(\ell_{2,3}-\ell_{2,4})(\ell_{3,8}-\ell_{3,9}-\ell_{4,8}+\ell_{4,9}) \\ +\alpha_\Theta\ell_{2,3}+2(b_4+\alpha_\Theta)\ell_{2,8}=0, \tag{106}$$

$$r\ell_{2,10}+\sigma_D^2(\ell_{1,2}+\ell_{2,6})(\ell_{1,10}+\ell_{6,10}) \\ +\sigma_\Theta^2(\ell_{2,3}-\ell_{2,4})(\ell_{3,10}-\ell_{4,10})-\ell_{2,9}=0, \tag{107}$$

$$r(\ell_{3,3}+2\ell_{3,9}+\ell_{9,9})+\sigma_D^2(\ell_{1,3}+\ell_{3,6}+\ell_{1,9}+\ell_{6,9})^2 \\ +\sigma_\Theta^2(\ell_{3,3}-\ell_{3,4}+\ell_{3,9}-\ell_{4,9})^2 \\ -2\,(b_4+\alpha_\Theta)\,(\ell_{3,8}+\ell_{8,9})+2\alpha_\Theta\,(\ell_{3,3}+\ell_{3,9})=0, \tag{108}$$

$$r(\ell_{3,4}+\ell_{3,9}+\ell_{4,9}+\ell_{9,9}) \\ +\sigma_D^2(\ell_{1,3}+\ell_{1,9}+\ell_{3,6}+\ell_{6,9})(\ell_{1,4}+\ell_{1,9}+\ell_{4,6}+\ell_{6,9}) \\ +\sigma_\Theta^2(\ell_{3,3}-\ell_{3,4}+\ell_{3,9}-\ell_{4,9})(\ell_{3,4}+\ell_{3,9}-\ell_{4,4}-\ell_{4,9}) \\ +\alpha_\Theta\,(\ell_{3,4}+\ell_{3,9})-(b_4+\alpha_\Theta)(\ell_{3,8}+2\ell_{8,9}+\ell_{4,8})=0, \tag{109}$$

$$r(\ell_{3,5}+\ell_{5,9})+\sigma_D^2(\ell_{1,3}+\ell_{1,9}+\ell_{3,6}+\ell_{6,9})(\psi+\ell_{1,5}+\ell_{5,6}) \\ +\sigma_\Theta^2(\ell_{3,3}-\ell_{3,4}+\ell_{3,9}-\ell_{4,9})(\ell_{3,5}-\ell_{4,5}) \\ +\alpha_\Theta\ell_{3,5}-(\ell_{3,4}+\ell_{4,9})-(b_4+\alpha_\Theta)\,\ell_{5,8}=0, \tag{110}$$

$$r(\ell_{3,7}+\ell_{7,9})+\sigma_D^2(\ell_{1,3}+\ell_{1,9}+\ell_{3,6}+\ell_{6,9})(\ell_{1,7}+\ell_{6,7}) \\ +\sigma_\Theta^2(\ell_{3,3}-\ell_{3,4}+\ell_{3,9}-\ell_{4,9})(\ell_{3,7}-\ell_{4,7}) \\ +\alpha_\Theta\ell_{3,7}-(b_4+\alpha_\Theta)\,\ell_{7,8}=0, \tag{111}$$

$$r(\ell_{3,8}-\ell_{3,9}+\ell_{8,9}-\ell_{9,9}) \\ +\sigma_D^2(\ell_{1,3}+\ell_{1,9}+\ell_{3,6}+\ell_{6,9})(\ell_{1,8}-\ell_{1,9}+\ell_{6,8}-\ell_{6,9}) \\ +\sigma_\Theta^2(\ell_{3,3}-\ell_{3,4}+\ell_{3,9}-\ell_{4,9})(\ell_{3,8}-\ell_{3,9}-\ell_{4,8}+\ell_{4,9}) \\ +\alpha_\Theta\,(\ell_{3,3}+\ell_{3,8})+(b_4+\alpha_\Theta)(2\ell_{3,8}+3\ell_{8,9}-\ell_{8,8})=0, \tag{112}$$

$$r(\ell_{3,10}+\ell_{9,10})+\sigma_D^2(\ell_{1,3}+\ell_{1,9}+\ell_{3,6}+\ell_{6,9})(\ell_{1,10}+\ell_{6,10}) \\ +\sigma_\Theta^2(\ell_{3,10}-\ell_{4,10})(\ell_{3,3}-\ell_{3,4}+\ell_{3,9}-\ell_{4,9}) \\ -(\ell_{3,9}+\ell_{9,9})+\alpha_\Theta\ell_{3,10}-(b_4+\alpha_\Theta)\,\ell_{8,10}=0, \tag{113}$$

$$r(\ell_{4,4}+2\ell_{4,9}+\ell_{9,9})+\sigma_D^2(\ell_{1,4}+\ell_{1,9}+\ell_{4,6}+\ell_{6,9})^2 \\ +\sigma_\Theta^2(\ell_{3,4}+\ell_{3,9}-\ell_{4,4}-\ell_{4,9})^2 \\ -2(b_4+\alpha_\Theta)(\ell_{4,8}+\ell_{8,9})=0, \tag{114}$$

$$r(\ell_{4,5}+\ell_{5,9})+\sigma_D^2(\ell_{1,4}+\ell_{1,9}+\ell_{4,6}+\ell_{6,9})(\psi+\ell_{1,5}+\ell_{5,6}) \\ +\sigma_\Theta^2(\ell_{3,4}+\ell_{3,9}-\ell_{4,4}-\ell_{4,9})(\ell_{3,5}-\ell_{4,5}) \\ -(\ell_{4,4}+\ell_{4,9})-(b_4+\alpha_\Theta)\,\ell_{5,8}=0, \tag{115}$$

$$r(\ell_{4,7}+\ell_{7,9})+\sigma_D^2(\ell_{1,4}+\ell_{1,9}+\ell_{4,6}+\ell_{6,9})(\ell_{1,7}+\ell_{6,7}) \\ +\sigma_\Theta^2(\ell_{3,4}+\ell_{3,9}-\ell_{4,4}-\ell_{4,9})(\ell_{3,7}-\ell_{4,7}) \\ -(b_4+\alpha_\Theta)\,\ell_{7,8}=0, \tag{116}$$

$$r(\ell_{4,8}-\ell_{4,9}+\ell_{8,9}-\ell_{9,9}) \\ +\sigma_D^2(\ell_{1,4}+\ell_{1,9}+\ell_{4,6}+\ell_{6,9})(\ell_{1,8}-\ell_{1,9}+\ell_{6,8}-\ell_{6,9}) \\ +\sigma_\Theta^2(\ell_{3,4}+\ell_{3,9}-\ell_{4,4}-\ell_{4,9})(\ell_{3,8}-\ell_{3,9}-\ell_{4,8}+\ell_{4,9}) \\ +\alpha_\Theta(\ell_{3,4}+\ell_{3,9})+(b_4+\alpha_\Theta)(2\ell_{4,8}+3\ell_{8,9}-\ell_{8,8})=0, \tag{117}$$

$$r(\ell_{4,10}+\ell_{9,10})+\sigma_D^2(\ell_{1,4}+\ell_{1,9}+\ell_{4,6}+\ell_{6,9})(\ell_{1,10}+\ell_{6,10})$$
$$+\sigma_\Theta^2(\ell_{3,4}+\ell_{3,9}-\ell_{4,4}-\ell_{4,9})(\ell_{3,10}-\ell_{4,10})$$
$$-(\ell_{4,9}+\ell_{9,9})-(b_4+\alpha_\Theta)\,\ell_{8,10}=0, \tag{118}$$

$$r\ell_{5,5}+\sigma_D^2(\psi+\ell_{1,5}+\ell_{5,6})^2+\sigma_\Theta^2(\ell_{3,5}-\ell_{4,5})^2-2\ell_{4,5}=0, \tag{119}$$

$$r\ell_{5,7}+\sigma_D^2(\psi+\ell_{1,5}+\ell_{5,6})(\ell_{1,7}+\ell_{6,7})$$
$$+\sigma_\Theta^2(\ell_{3,5}-\ell_{4,5})(\ell_{3,7}-\ell_{4,7})-\ell_{4,7}-\psi=0, \tag{120}$$

$$r(\ell_{5,8}-\ell_{5,9})+\sigma_D^2(\psi+\ell_{1,5}+\ell_{5,6})(\ell_{1,8}-\ell_{1,9}+\ell_{6,8}-\ell_{6,9})$$
$$+\sigma_\Theta^2(\ell_{3,5}-\ell_{4,5})(\ell_{3,8}-\ell_{3,9}-\ell_{4,8}+\ell_{4,9})$$
$$+\alpha_\Theta\ell_{3,5}-(\ell_{4,8}-\ell_{4,9})+2(b_4+\alpha_\Theta)\ell_{5,8}=0, \tag{121}$$

$$r\ell_{5,10}+\sigma_D^2(\psi+\ell_{1,5}+\ell_{5,6})(\ell_{1,10}+\ell_{6,10})$$
$$+\sigma_\Theta^2(\ell_{3,5}-\ell_{4,5})(\ell_{3,10}-\ell_{4,10})-(\ell_{5,9}+\ell_{4,10})=0, \tag{122}$$

$$r\ell_{7,7}+\sigma_D^2(\ell_{1,7}+\ell_{6,7})^2+\sigma_\Theta^2(\ell_{3,7}-\ell_{4,7})^2=0, \tag{123}$$

$$r(\ell_{7,8}-\ell_{7,9})+\sigma_D^2(\ell_{1,7}+\ell_{6,7})(\ell_{1,8}-\ell_{1,9}+\ell_{6,8}-\ell_{6,9})$$
$$+\sigma_\Theta^2(\ell_{3,7}-\ell_{4,7})(\ell_{3,8}-\ell_{3,9}-\ell_{4,8}+\ell_{4,9})$$
$$+\alpha_\Theta\ell_{3,7}+2(b_4+\alpha_\Theta)\ell_{7,8}=0, \tag{124}$$

$$r\ell_{7,10}+\sigma_D^2(\ell_{1,7}+\ell_{6,7})(\ell_{1,10}+\ell_{6,10})$$
$$+\sigma_\Theta^2(\ell_{3,7}-\ell_{4,7})(\ell_{3,10}-\ell_{4,10})-\ell_{7,9}=0, \tag{125}$$

$$r(\ell_{8,8}-2\ell_{8,9}+\ell_{9,9})+\sigma_D^2(\ell_{1,8}-\ell_{1,9}+\ell_{6,8}-\ell_{6,9})^2$$
$$+\sigma_\Theta^2(\ell_{3,8}-\ell_{3,9}-\ell_{4,8}+\ell_{4,9})^2$$
$$+2\alpha_\Theta\,(\ell_{3,8}-\ell_{3,9})+4\,(b_4+\alpha_\Theta)\,(\ell_{8,8}-\ell_{8,9})=0, \tag{126}$$

$$r(\ell_{8,10}-\ell_{9,10})+\sigma_D^2(\ell_{1,8}-\ell_{1,9}+\ell_{6,8}-\ell_{6,9})(\ell_{1,10}+\ell_{6,10})$$
$$+\sigma_\Theta^2(\ell_{3,8}-\ell_{3,9}-\ell_{4,8}+\ell_{4,9})(\ell_{3,10}-\ell_{4,10})$$
$$+\alpha_\Theta\ell_{3,10}-(\ell_{8,9}-\ell_{9,9})+2\,(b_4+\alpha_\Theta)\,\ell_{8,10}=0, \tag{127}$$

$$r\ell_{10,10} + \sigma_D^2(\ell_{1,10} + \ell_{6,10})^2 + \sigma_\Theta^2(\ell_{3,10} - \ell_{4,10})^2 - 2\ell_{9,10} = 0. \tag{128}$$

In addition, (46) yields the linear system

$$\ell_{1,4} + \ell_{1,8} + \ell_{4,6} + \ell_{6,8} = 0, \tag{129}$$

$$\ell_{2,4} + \ell_{2,8} = 0, \tag{130}$$

$$\ell_{3,4} + \ell_{3,8} + \ell_{4,9} + \ell_{8,9} = 0, \tag{131}$$

$$\ell_{4,4} + \ell_{4,8} + \ell_{4,9} + \ell_{8,9} = 0, \tag{132}$$

$$\ell_{4,5} + \ell_{5,8} = 0, \tag{133}$$

$$\ell_{4,7} + \ell_{7,8} = 0, \tag{134}$$

$$\ell_{4,8} - \ell_{4,9} - \ell_{8,9} + \ell_{8,8} = 0, \tag{135}$$

$$\ell_{4,10} + \ell_{8,10} = 0, \tag{136}$$

and, from (92), it follows that we must have

$$\ell_{4,9} - \ell_{8,9} + \ell_{9,9} \neq 0 \tag{137}$$

and

$$(1 + b_4)\ell_{4,9} + (1 - b_4)\ell_{8,9} + b_4\ell_{9,9} = 0. \tag{138}$$

Hence, combining (129)-(136) with (138), we obtain

$$\ell_{3,4} = -\ell_{3,8} - \frac{b_4}{1 + b_4}(2\ell_{8,9} - \ell_{9,9}), \tag{139}$$

$$\ell_{4,4} = \ell_{8,8} - \frac{2b_4}{1 + b_4}(2\ell_{8,9} - \ell_{9,9}), \tag{140}$$

$$\ell_{4,8} = -\ell_{8,8} + \frac{b_4}{1 + b_4}(2\ell_{8,9} - \ell_{9,9}), \tag{141}$$

$$\ell_{4,9} = -\ell_{8,9} + \frac{b_4}{1 + b_4}(2\ell_{8,9} - \ell_{9,9}), \tag{142}$$

$$\ell_{2,8} = -\ell_{2,4}, \tag{143}$$

$$\ell_{5,8} = -\ell_{4,5}, \tag{144}$$

$$\ell_{7,8} = -\ell_{4,7}, \tag{145}$$

$$\ell_{8,10} = -\ell_{4,10}, \tag{146}$$

$$\ell_{1,8} = -(\ell_{1,4} + \ell_{4,6} + \ell_{6,8}), \tag{147}$$

and

$$\ell_{4,9} - \ell_{8,9} + \ell_{9,9} = -\frac{1}{1 + b_4}(2\ell_{8,9} - \ell_{9,9}). \tag{148}$$

Now, substituting (139)-(147) in (114), (117), and (126), we obtain

$$\begin{aligned} &r(\ell_{8,8} - 2\ell_{8,9} + \ell_{9,9}) \\ &+\sigma_D^2(\ell_{1,4} + \ell_{1,9} + \ell_{4,6} + \ell_{6,9})^2 + \sigma_\Theta^2(\ell_{3,8} - \ell_{3,9} + \ell_{8,9} - \ell_{9,9})^2 \\ &-2\,(b_4 + \alpha_\Theta)\,(\ell_{8,9} - \ell_{8,8} + \frac{b_4}{1+b_4}(2\ell_{8,9} - \ell_{9,9})) = 0, \end{aligned} \tag{149}$$

$$\begin{aligned} &-r(\ell_{8,8} - 2\ell_{8,9} + \ell_{9,9}) \\ &-\sigma_D^2(\ell_{1,4} + \ell_{1,9} + \ell_{4,6} + \ell_{6,9})^2 - \sigma_\Theta^2(\ell_{3,8} - \ell_{3,9} + \ell_{8,8} - \ell_{8,9})^2 \\ &+\frac{(2b_4 + \alpha_\Theta)b_4}{1+b_4}(2\ell_{8,9} - \ell_{9,9}) \\ &+3\,(b_4 + \alpha_\Theta)\,(\ell_{8,9} - \ell_{8,8}) - \alpha_\Theta(\ell_{3,8} - \ell_{3,9}) = 0, \end{aligned} \tag{150}$$

$$\begin{aligned} &r(\ell_{8,8} - 2\ell_{8,9} + \ell_{9,9}) - \sigma_D^2(\ell_{1,4} + \ell_{1,9} + \ell_{4,6} + \ell_{6,9})^2 \\ &+\sigma_\Theta^2(\ell_{3,8} - \ell_{3,9} + \ell_{8,8} - \ell_{8,9})^2 \\ &+2\alpha_\Theta\,(\ell_{3,8} - \ell_{3,9}) + 4\,(b_4 + \alpha_\Theta)\,(\ell_{8,8} - \ell_{8,9}) = 0, \end{aligned} \tag{151}$$

Hence, summing term by term (149) and (150), we have

$$\begin{aligned} &(b_4 + \alpha_\Theta)\,(\ell_{8,9} - \ell_{8,8}) - \alpha_\Theta(\ell_{3,8} - \ell_{3,9}) \\ &-\frac{\alpha_\Theta b_4}{1+b_4}(2\ell_{8,9} - \ell_{9,9}) = 0. \end{aligned} \tag{152}$$

and subtracting term by term (149) and (151), it follows

$$-2\,(b_4 + \alpha_\Theta)\,(\ell_{8,8} - \ell_{8,9}) - 2\alpha_\Theta\,(\ell_{3,8} - \ell_{3,9}) \tag{153}$$

$$-2\,(b_4 + \alpha_\Theta)\,\frac{b_4}{1+b_4}(2\ell_{8,9} - \ell_{9,9}) = 0. \tag{154}$$

Therefore, combining (152) and (153), we obtain

$$\frac{2b_4^2}{1+b_4}(2\ell_{8,9} - \ell_{9,9}) = 0.$$

This, by virtue of (137) and (148), implies

$$b_4 = 0. \tag{155}$$

Finally, on account of what shown above, it is possible to obtain (50). □

References

[1] B.D.O. Anderson, J.B. Moore: *Linear Optimal Control*, Prentice-Hall Inc., New York, 1971.

[2] K. Back: Insider Trading in Continuous Time, *The Review of Financial Studies*, **5**, 3, 387-409 (1992).

[3] S. Bittanti, A.J. Laub, J.C. Willems: *The Riccati Equation*, Springer-Verlag, Berlin, 1991.

[4] J.Y. Campbell, A.S. Kyle: Smart Money, Noise Trading and Stock Price Behaviour, *Review of Economic Studies*, **60**, 1-34 (1993).

[5] W.H. Fleming, R.W. Rishel: *Deterministic and Stochastic Optimal Control* Springer Verlag, New York, 1975.

[6] W.H. Fleming, H.M. Soner: *Controlled Markov Processes and Viscosity Solutions* Springer Verlag, New York, 1993.

[7] G. Gennotte: Optimal Portfolio Choice under Incomplete Information, *The Journal of Finance*, **XLI**, 3, 732-746 (1986).

[8] G. Gennotte, A.S. Kyle: *Intertemporal Information Aggregation*, Working Paper, University of California, Berkley (1991).

[9] Grossman, S. (1989) *The Informational Role of Prices.* MIT press.

[10] Holden, C. and Subrahmanyan, A. (1992) Long-lived private information and imperfect competition. *Journal of Finance*, 47:247–270.

[11] I. Karatzas, S.E. Shreve: *Brownian Motion and Stochastic Calculus.* Springer-Verlag, New York (1988).

[12] A.S. Kyle: Continuous Auctions and Insider Trading, *Econometrica*, **53**, 1315-1335 (1985).

[13] A.S. Kyle: Informed Speculation with Imperfect Competition, *The Review of Economic Studies*, **56**, 317-335 (1989).

[14] R.S. Lipster, A.N. Shiryaev: *Statistic of Random Processes I*, Springer Verlag, New York, 2001.

[15] P. Milgrom and N. Stokey Information, trade and common knowledge. *Journal of Economic Theory*, 1982, 26:17–27.

[16] P. Veronesi: Stock Market Overreaction to Bad News in Good Times: A Rational Expectation Equilibrium Model *The Review of Financial Studies*, **12**, 5, 975-1007 (1999).

[17] J. Wang: A Model of Intertemporal Asset Prices Under Asymmetric Information, *The Review of Economic Studies*, **60**, 249-282 (1993)

[18] J. Wang: Differential Information and Dynamic Behavior of Stock Trading Volume, *The Review of Financial Studies*, **8**, 919-972 (1995)

[19] W.M. Wonham: On a Matrix Riccati Equation of Stochastic Control, *SIAM J. Control*, **6**, 4, 681-697 (1968).

[20] J. Yong, X.Y. Zhou: *Stochastic Controls: Hamiltonian Systems and HJB Equations* Springer Verlag, Berlin, 1999.

Elenco dei report pubblicati

Anno: **1987**

- ☐ **n. 1** Alberto Cambini - Laura Martein, Some Optimality Conditions in Vector Optimization
- ☐ **n. 2** Alberto Cambini - Laura Martein - S.Schaibel, On Maximizing a Sum of Ratios
- ☐ **n. 3** Giuliano Gasparotto, On the Charnes-Cooper Transformation in linear Fractional Programming.
- ☐ **n. 4** Alberto Cambini, Non-linear separation Theorems, Duality and Optimality
- ☐ **n. 5** Giovanni Boletto, Indicizzazione parziale: aspetti metodologici e riflessi economici
- ☐ **n. 6** Alberto Cambini - Claudio Sodini, On Parametric Linear Fractional Programming
- ☐ **n. 7** Alberto Bonaguidi, Alcuni aspetti meno noti delle migrazioni in Italia
- ☐ **n. 8** Laura Martein - S. Schaible, On Solving a Linear Program with one Quadratic Constraint

Anno: **1988**

- ☐ **n. 9** Ester Lari, Alcune osservazioni sull'equazione funzionale Ø(x,y,z)=Ø(Ø(x,y,t),t,z)
- ☐ **n. 10** F. Bartiaux, Une étude par ménage des migrations des personnes âgées: comparaison des résulttats pour l'Italie et les Etats-Unis
- ☐ **n. 11** Giovanni Boletto, Metodi di scomposizione del tasso di inflazione
- ☐ **n. 12** Claudio Sodini, A New Algorithm for the Strictly Convex Quadratic Programming Problem
- ☐ **n. 13** Laura Martein, On Generating the Set of all Efficient Points of a Bicriteria Fractional Problem
- ☐ **n. 14** Laura Martein, Applicazioni della programmazione frazionaria nel campo economico-finanziario
- ☐ **n. 15** Laura Martein, On the Bicriteria Maximization Problem
- ☐ **n. 16** Paolo Manca, Un prototipo di sistema esperto per la consulenza finanziaria rivolta ai piccoli risparmiatori
- ☐ **n. 17** Paolo Manca, Operazioni Finanziarie di Soper e Operazioni di puro Investimento secondo Teichroew-Robichek-Montalbano
- ☐ **n. 18** Paolo Carraresi - Claudio Sodini, A k - Shortest Path Approach to the Minimum Cost Matching Problem.
- ☐ **n. 19** Odo Barsotti - Marco Bottai, Sistemi gravitazionali e fasi di transazione della crescita Demografica
- ☐ **n. 20** Giovanni Boletto, Metodi di scomposizione dell'inflazione aggregata : recenti sviluppi.
- ☐ **n. 21** Marc Termote - Alberto Bonaguidi, Multiregional Stable Populatioin as a Tool for Short-term Demographic Analysis
- ☐ **n. 22** Marco Bottai, Storie familiari e storie migratorie: un'indagine in Italia
- ☐ **n. 23** Maria Francesca Romano - Marco Marchi, Problemi connessi con la disomogeneità dei gruppi sottoposti a sorveglianza statistico-epidemiologica.
- ☐ **n. 24** Franca Orsi, Un approcio logico ai problemi di scelta finanziaria.

Anno: **1989**

- ☐ **n. 25** Vincenzo Bruno, Attrazione ed entropia.
- ☐ **n. 26** Giorgio Giorgi - S. Mititelu, Invexity in nonsmooth Programming.
- ☐ **n. 28** Alberto Cambini - Laura Martein, Equivalence in linear fractional programming.

Anno: **1990**

- ☐ **n. 27** Vincenzo Bruno, Lineamenti econometrici dell'evoluzione del reddito nazionale in relazione ad altri fenomeni economici
- ☐ **n. 29** Odo Barsotti - Marco Bottai - Marco Costa, Centralità e potenziale demografico per l'analisi dei comportamenti demografici: il caso della Toscana
- ☐ **n. 30** Anna Marchi, A sequential method for a bicriteria problem arising in portfolio selectiontheory.
- ☐ **n. 31** Marco Bottai, Mobilità locale e pianificazione territoriale.
- ☐ **n. 32** Anna Marchi, Solving a quadratic fractional program by means of a complementarity approach
- ☐ **n. 33** Anna Marchi, Sulla relazione tra un problema bicriteria e un problema frazionario.

Anno: **1991**

- ☐ **n. 34** Enrico Gori, Variabili latenti e "self-selection" nella valutazione dei processi formativi.
- ☐ **n. 35** Piero Manfredi - E. Salinelli, About an interactive model for sexual Populations.
- ☐ **n. 36** Giorgio Giorgi, Alcuni aspetti matematici del modello di sraffa a produzione semplice
- ☐ **n. 37** ALberto Cambini - S.Schaibl - Claudio Sodini, Parametric linear fractional programming for an unbounded feasible Region.
- ☐ **n. 38** I.Emke - Poulopoulos - V.Gozàlves Pèrez - Odo Barsotti - Laura Lecchini, International migration to northern Mediterranean countries the cases of Greece, Spain and Italy.
- ☐ **n. 39** Giuliano Gasparotto, A LP code implementation
- ☐ **n. 40** Riccardo Cambini, Un problema di programmazione quadratica nella costituzione di capitale.
- ☐ **n. 41** Gilberto Ghilardi, Stime ed errori campionari nell'indagine ISTAT sulle forze di lavoro.
- ☐ **n. 42** Vincenzo Bruno, Alcuni valori medi, variabilità paretiana ed entropia.
- ☐ **n. 43** Giovanni Boletto, Gli effetti del trascinamento dei prezzi sulle misure dell'inflazione: aspetti metodologici
- ☐ **n. 44** P. Paolicchi, Gli abbandoni nell'università: modelli interpretativi.
- ☐ **n. 45** Maria Francesca Romano, Da un archivio amministrativo a un archivio statistico: una proposta metodologica per i dati degli studenti universitari.
- ☐ **n. 46** Maria Francesca Romano, Criteri di scelta delle variabili nei modelli MDS: un'applicazione sulla popolazione studentesca di Pisa.
- ☐ **n. 47** Odo Barsotti - Laura Lecchini, Les parcours migratoires en fonction de la nationalité. Le cas de l'Italie.
- ☐ **n. 48** Vincenzo Bruno, Indicatori statistici ed evoluzione demografica, economica e sociale delle province toscane.
- ☐ **n. 49** Alberto Cambini - Laura Martein, Tangent cones in optimization.
- ☐ **n. 50** Alberto Cambini - Laura Martein, Optimality conditions in vector and scalar optimization: a unified approach.

Anno: **1992**

- ☐ **n. 51** Gilberto Ghilardi, Elementi di uno schema di campionamento areale per alcune rilevazioni ufficiali in Italia.
- ☐ **n. 52** Paolo Manca, Investimenti e finanziamenti generalizzati.
- ☐ **n. 53** Laura Lecchini - Odo Barsotti, Le rôle des immigrés extra- communautaires dans le marché du travail

- ☐ **n. 54** Riccardo Cambini, Alcune condizioni di ottimalità relative ad un insieme stellato.
- ☐ **n. 55** Gilberto Ghilardi, Uno schema di campionamento areale per le rilevazionninsulle famiglie in Italia.
- ☐ **n. 56** Riccardo Cambini, Studio di una classe di problemi non lineari: un metodo sequenziale.
- ☐ **n. 57** Riccardo Cambini, Una nota sulle possibili estensioni a funzioni vettoriali di significative classi di funzioni concavo-generalizzate.
- ☐ **n. 58** Alberto Bonaguidi - Valerio Terra Abrami, Metropolitan aging transition and metropolitan redistribution of the elderly in Italy.
- ☐ **n. 59** Odo Barsotti - Laura Lecchini, A comparison of male and female migration strategies: the cases of African and Filipino Migrants to Italy.
- ☐ **n. 60** Gilberto Ghilardi, Un modello logit per lo studio del fenomeno delle nuove imprese.
- ☐ **n. 61** S. Schaible, Generalized monotonicity.
- ☐ **n. 62** Vincenzo Bruno, Dell'elasticità in economia e dell'incertezza statistica.
- ☐ **n. 63** Laura Martein, Alcune classi di funzioni concave generalizzate nell'ottimizzazione vettoriale
- ☐ **n. 64** Anna Marchi, On the relationships between bicriteria problems and non-linear programming problems.
- ☐ **n. 65** Giovanni Boletto, Considerazioni metodologiche sul concetto di elasticità prefissata.
- ☐ **n. 66** Laura Martein, Soluzione efficienti e condizioni di ottimalità nell'otti-mizzazione vettoriale.

Anno: **1993**

- ☐ **n. 67** Maria Francesca Romano, Le rilevazioni ufficiali ISTAT della popolazione universitaria: problemi e definizioni alternative.
- ☐ **n. 68** Marco Bottai - Odo Barsotti, La ricerca "Spazio Utilizzato" Obiettivi e primi risultati.
- ☐ **n. 69** Marco Bottai - F.Bartiaux, Composizione familiare e mobilità delle persone anziane. Una analisi regionale.
- ☐ **n. 70** Anna Marchi - Claudio Sodini, An algorithm for a non-differentiable non -linear fractional programming problem.
- ☐ **n. 71** Claudio Sodini - S.Schaible, An finite algorithm for generalized linear multiplicative programming.
- ☐ **n. 72** Alberto Cambini - Laura Martein, An approach to optimality conditions in vector and scalar optimization.
- ☐ **n. 73** Alberto Cambini - Laura Martein, Generalized concavity and optimality conditions in vector and scalar optimization.
- ☐ **n. 74** Riccardo Cambini, Alcune nuove classi di funzioni concavo-generalizzate.

Anno: **1994**

- ☐ **n. 75** Alberto Cambini - Anna Marchi - Laura Martein, On nonlinear scalarization in vector optimization.
- ☐ **n. 76** Maria Francesca Romano - Giovanna Nencioni, Analisi delle carriere degli studenti immatricolati dal 1980 al 1982.
- ☐ **n. 77** Gilberto Ghilardi, Indici statistici della congiuntura.
- ☐ **n. 78** Riccardo Cambini, Condizioni di efficienza locale nella ottimizzazione vettoriale.
- ☐ **n. 79** Odo Barsotti - Marco Bottai, Funzioni di utilizzazione dello spazio.
- ☐ **n. 80** Vincenzo Bruno, Alcuni aspetti dinamici della popolazione dei comuni della Toscana, distinti per ampiezza demografica e per classi di urbanità e di ruralità.
- ☐ **n. 81** Giovanni Boletto, I numeri indici del potere d'acquisto della moneta.
- ☐ **n. 82** Alberto Cambini - Laura Martein - Riccardo Cambini, Some optimality conditions in multiobjective programming.
- ☐ **n. 83** S. Schaible, Fractional programming with sume of ratios.
- ☐ **n. 84** Stefan Tigan - I.M.Stancu-Minasian, The minimun-risk approach for continuous time linear-fractional programming.
- ☐ **n. 85** Vasile Preda - I.M.Stancu-Minasian, On duality for multiobjective mathematical programming of n-set.
- ☐ **n. 86** Vasile Preda - I.M.Stancu-Minasian - Anton Batatorescu, Optimality and duality in nonlinear programming involving semilocally preinvex and related functions.

Anno: **1995**

- ☐ **n. 87** Elena Melis, Una nota storica sulla programmazione lineare: un problema di Kantorovich rivisto alla luce del problema degli zeri.
- ☐ **n. 88** Vincenzo Bruno, Mobilità territoriale dell'Italia e di tre Regioni tipiche: Lombardia, Toscana, Sicilia.
- ☐ **n. 89** Antonio Cortese, Bibliografia sulla presenza straniera in Italia
- ☐ **n. 90** Riccardo Cambini, Funzioni scalari affini generalizzate.
- ☐ **n. 91** Piero Manfredi - Fabio Tarini, Modelli epidemiologici: teoria e simulazione. (I)
- ☐ **n. 92** Marco Bottai - Maria Caputo - Laura Lecchini, The "OLIVAR" survey.Methodology and quality.
- ☐ **n. 93** Laura Lecchini - Donatella Marsiglia - Marco Bottai, Old people and social network.
- ☐ **n. 94** Gilberto Ghilardi, Uno studio empirico sul confronto tra alcuni indici statistici della congiuntura.
- ☐ **n. 95** Vincenzo Bruno, Il traffico nei porti italiani negli anni recenti.
- ☐ **n. 96** Alberto Cambini - Anna Marchi - Laura Martein - S. Schaible, An analysis of the falk-palocsay algorithm.
- ☐ **n. 97** Alberto Cambini - Laura Carosi, Sulla esistenza di elementi massimali.

Anno: **1996**

- ☐ **n. 98** Riccardo Cambini - S. Komlòsi, Generalized concavity and generalized monotonicity concepts for vector valued.
- ☐ **n. 99** Riccardo Cambini, Second order optimality conditions in the image space.
- ☐ **n. 100** Vincenzo Bruno, La stagionalità delle correnti di navigazione marittima.
- ☐ **n. 101** Eugene Maurice Cleur, A comparison of alternative discrete approximations of the Cox -I ngersoll - ross model.
- ☐ **n. 102** Gilberto Ghilardi, Sul calcolo del rapporto di concentrazione del Gini.
- ☐ **n. 103** Alberto Cambini - Laura Martein - Riccardo Cambini, A new approach to second order optimality conditions in vector optimization.
- ☐ **n. 104** Fausto Gozzi, Alcune osservazioni sull'immunizzazione semideterministica.
- ☐ **n. 105** Emilio Barucci - Fausto Gozzi, Innovation and capital accumulation in a vintage capital model an infinite dimensional control approach.
- ☐ **n. 106** Alberto Cambini - Laura Martein - I.M.Stancu-Minasian., A survey of bicriteria fractional problems.
- ☐ **n. 107** Luciano Fanti - Piero Manfredi, Viscosità dei salari, offerta di lavoro endogena e ciclo.
- ☐ **n. 108** Piero Manfredi - Luciano Fanti, Ciclo di vita di nuovi prodotti: modellistica non lineare.
- ☐ **n. 109** Piero Manfredi, Crescita con ciclo, gestazione dei piani di investimento ed effetti.
- ☐ **n. 110** Luciano Fanti - Piero Manfredi, Un modello "classico" di ciclo con crescita ed offerta di lavoro endogena.
- ☐ **n. 111** Anna Marchi, On the connecttedness of the efficient frontier : sets without local maxima.

- ☐ **n. 112** Riccardo Cambini, Generalized concavity for bicriteria functions.
- ☐ **n. 113** Vincenzo Bruno, Variazioni dinamiche (1971-1981-1991)dei fenomeni demografici dei comuni (urbani e rurali) della Lombardia, in relazione ad alcune caratteristiche di mobilità territoriale.

Anno: **1997**

- ☐ **n. 114** Piero Manfredi - Fabio Tarini - J.R. Williams - A. Carducci - B. Casini, Infectious diseases: epidemiology, mathematical models, and immunization policies.
- ☐ **n. 115** Eugene Maurice Cleur - Piero Manfredi, One dimensional SDE models, low order numerical methods and simulation based estimation: a comparison of alternative estimators.
- ☐ **n. 116** Luciano Fanti - Piero Manfredi, Point stability versus orbital stability (or instability): remarks on policy implications in classical growth cycle model.
- ☐ **n. 117** Piero Manfredi - Francesco Billari, transition into adulthood, marriage, and timing of life in a stable popolation framework.
- ☐ **n. 118** Laura Carosi, Una nota sul concetto di estremo superiore di insiemi ordinati da coni convessi.
- ☐ **n. 119** Laura Lecchini - Donatella Marsiglia, Reti sociali degli anziani: selezione e qualità delle relazioni.
- ☐ **n. 120** Piero Manfredi - Luciano Fanti, Gestation lags and efficiency wage mechanisms in a goodwin type growth model.
- ☐ **n. 121** G.Rivellini, La metodologia statistica multilevel come possibile strumento per lo studio delle interazioni tra il comportamento procreativo individuale e il contesto
- ☐ **n. 122** Laura Carosi, Una nota sugli insiemi C-limitati e L-limitati.
- ☐ **n. 123** Laura Carosi, Sull'estremo superiore di una funzione lineare fratta ristretta ad un insieme chiuso e illimitato.
- ☐ **n. 124** Piero Manfredi, A demographic framework for the evaluation of the impact of imported infectious diseases.
- ☐ **n. 125** Alessandro Valentini, Calo della fecondità ed immigrazione: scenari e considerazioni sul caso italiano.
- ☐ **n. 126** Alberto Cambini - Laura Martein, Second order optimality conditions.

Anno: **1998**

- ☐ **n. 127** Piero Manfredi and Alessandro Valentini, Populations with below replacementfertility: theoretical considerations and scenarioes from the italian laboratory.
- ☐ **n. 128** Alberto Cambini - Laura Martein - E. Moretti, Programmazione frazionaria e problemi bicriteria.
- ☐ **n. 129** Emilio Barucci - Fausto Gozzi - Andrej Swiech, Incentive compatibility constraints and dynamic programming in continuous time.

Anno: **1999**

- ☐ **n. 130** Alessandro Valentini, Impatto delle immigrazioni sulla popolazione italiana: confronto tra scenari alternativi.
- ☐ **n. 131** K. Iglicka - Odo Barsotti - Laura Lecchini, Recent developement of migrations from Poland to Europe with a special emphsis on Italy K.Iglika - Le Migrazioni est-ovest: le unioni miste in Italia
- ☐ **n. 132** Alessandro Valentini, Proiezioni demografiche multiregionali a due sessi, con immigrazioni internazionali e vincoli di consistenza.
- ☐ **n. 133** Fabio Antonelli - Emilio Barucci - Maria Elvira Mancino, Backward-forward stochastic differential utility: existence, consumption and equilibrium analysis.
- ☐ **n. 134** Emilio Barucci - Maria Elvira Mancino, Asset pricing with endogenous aspirations.
- ☐ **n. 135** Eugene Maurice Cleur, Estimating a class of diffusion models: an evaluation of the effects of sampled discrete observations.
- ☐ **n. 136** Luciano Fanti - Piero Manfredi, Labour supply, time delays, and demoeconomic oscillations in a solow-typegrowth model.
- ☐ **n. 137** Emilio Barucci - Sergio Polidoro - Vincenzo Vespri, Some results on partial differential equations and Asian options.
- ☐ **n. 138** Emilio Barucci - Maria Elvira Mancino, Hedging european contingent claims in a Markovian incomplete market.
- ☐ **n. 139** Alessandro Valentini, L'applicazione del modello multiregionale-multistato alla popolazione in Italia mediante l'utilizzo del Lipro: procedura di adattamento dei dati e particolarità tecniche del programma.
- ☐ **n. 140** I.M.Stancu-Minasian, optimality conditions and duality in fractional programming-involving semilocally preinvex and related functions.
- ☐ **n. 141** Alessandro Valentini, Proiezioni demografiche con algoritmi di consistenza per la popolazione in Italia nel periodo 1997-2142: presentazione dei risultati e confronto con metodologie di stima alternative.
- ☐ **n. 142** Laura Carosi, Competitive equilibria with money and restricted partecipation.
- ☐ **n. 143** Laura Carosi, Monetary policy and Pareto improvability in a financial economy with restricted partecipation
- ☐ **n. 144** Bruno Cheli, Misurare il benessere e lo sviluppo dai paradossi del Pil a misure di benessere economico sostenibile, con uno sguardo allo sviluppo umano
- ☐ **n. 145** Bruno Cheli - Laura Lecchini - Lucio Masserini, The old people's perception of well-being: the role of material and non material resources
- ☐ **n. 146** Eugene Maurice Cleur, Maximum likelihood estimation of one-dimensional stochastic differential equation models from discrete data: some computational resultas
- ☐ **n. 147** Alessandro Valentini - Francesco Billari - Piero Manfredi, Utilizzi empirici di modelli multistato continui con durate multiple
- ☐ **n. 148** Francesco Billari - Piero Manfredi - Alberto Bonaguidi - Alessandro Valentini, Transition into adulthoold: its macro-demographic consequences in a multistatew stable population framework
- ☐ **n. 149** Francesco Billari - Piero Manfredi - Alessandro Valentini, Becoming Adult and its Macro-Demographic Impact: Multistate Stable Population Theory and an Application to Italy
- ☐ **n. 150** Alessandro Valentini, Le previsioni demografiche in presenza di immigrazioni: confronto tra modelli alternativi e loro utilizzo empirico ai fini della valutazione dell'equilibrio nel sistema pensionistico
- ☐ **n. 151** Emilio Barucci - Roberto Monte, Diffusion processes for asset prices under bounded rationality
- ☐ **n. 152** Emilio Barucci - P. Cianchi - L. Landi - A. Lombardi, Reti neurali e analisi delle serie storiche: un modello per la previsione del BTP future
- ☐ **n. 153** Alberto Cambini - Laura Carosi - Laura Martein, On the supremum in fractional programming
- ☐ **n. 154** Riccardo Cambini - Laura Martein, First and second order characterizations of a class of pseudoconcave vector functions
- ☐ **n. 155** Piero Manfredi and Luciano Fanti, Embedding population dynamics in macro-economic models. The case of the goodwin's growth cycle
- ☐ **n. 156** Laura Lecchini e Odo Barsotti, Migrazioni dei preti dalla Polonia in Italia
- ☐ **n. 157** Vincenzo Bruno, Analisi dei prezzi, in Italia dal 1975 in poi
- ☐ **n. 158** Vincenzo Bruno, Analisi del commercio al minuto in Italia
- ☐ **n. 159** Vincenzo Bruno, Aspetti ciclici della liquidità bancaria, dal 1971 in poi
- ☐ **n. 160** Anna Marchi, A separation theorem in alternative theorems and vector optimization

Anno: 2000

- ☐ **n. 161** Piero Manfredi and Luciano Fanti, Labour suppley, population dynamicics and persistent oscillations in a Goodwin-type growth cycle model
- ☐ **n. 162** Luciano Fanti and Piero Manfredi, Neo-classical labour market dynamics and chaos (and the Phillips curve revisited)
- ☐ **n. 163** Piero Manfredi - and Luciano Fanti, Detection of Hopf bifurcations in continuos-time macro- economic models, with an application to reducible delay-systems.
- ☐ **n. 164** Fabio Antonelli - Emilio Barucci, The Dynamics of pareto allocations with stochastic differential utility
- ☐ **n. 165** Eugene M. Cleur, Computing maximum likelihood estimates of a class of One-Dimensional stochastic differential equation models from discrete Date*
- ☐ **n. 166** Eugene M. Cleur, Estimatng the drift parameter in diffusion processes more efficiently at discrete times:a role of indirect estimation
- ☐ **n. 167** Emilio Barucci - Vincenzo Valori, Forecasting the forecasts of others e la Politica di Inflation targeting
- ☐ **n. 168** A.Cambini - L. Martein, First and second order optimality conditions in vector optimization
- ☐ **n. 169** A. Marchi, Theorems of the Alternative by way of Separation Theorems
- ☐ **n. 170** Emilio Barucci - Maria Elvira Mancino, Asset Pricing and Diversification with Partally Exchangeable random Variables
- ☐ **n. 171** Piero Manfredi - Luciano Fanti, Long Term Effects of the Efficiency Wage Hypothesis in Goodwin-Type Economies.
- ☐ **n. 172** Piero Manfredi - Luciano Fanti, Long Term Effects of the Efficiency wage Hypothesis in Goodwin-type Economies: a reply.
- ☐ **n. 173** Luciano Fanti, Innovazione Finanziaria e Domanda di Moneta in un Modello dinamico IS-LM con Accumulazione.
- ☐ **n. 174** P.Manfredi, A.Bonaccorsi, A.Secchi, Social Heterogeneities in Classical New Product Diffusion Models. I: "External" and "Internal" Models.
- ☐ **n. 175** Piero Manfredi - Ernesto Salinelli, Modelli per formazione di coppie e modelli di Dinamica familiare.
- ☐ **n. 176** P.Manfredi, E. Salinelli, A.Melegaro, A.Secchi, Long term Interference Between Demography and Epidemiology: the case of tuberculosis
- ☐ **n. 177** Piero Manfredi - Ernesto Salinelli, Toward the Development of an Age Structure Teory for Family Dynamics I: General Frame.
- ☐ **n. 178** Piero Manfredi - Luciano Fanti, Population heterogeneities, nonlinear oscillations and chaos in some Goodwin-type demo-economic models Paper to be presented at the: Second workshop on "nonlinear demography" Max Planck Institute for demographic Research Rostock, Germany, May 31-June 2, 2
- ☐ **n. 179** E. Barucci - M.E. Mancini - Roberto Renò, Volatility Estimation via Fourier Analysis
- ☐ **n. 180** Riccardo Cambini, Minimum Principle Type Optimality Conditions
- ☐ **n. 181** E. Barucci, M. Giuli, R. Monte, Asset Prices under Bounded Rationality and Noise Trading
- ☐ **n. 182** A. Cambini, D.T.Luc, L.Martein, Order Preserving Transformations and application.
- ☐ **n. 183** Vincenzo Bruno, Variazioni dinamiche (1971-1981-1991) dei fenomeni demografici dei comuni urbani e rurali della Sicilia, in relazione ad alcune caratteristiche di mobilità territoriale.
- ☐ **n. 184** F.Antonelli, E.Barucci, M.E.Mancino, Asset Pricing with a Backward-Forward Stochastic Differential Utility
- ☐ **n. 185** Riccardo Cambini - Laura Carosi, Coercivity Concepts and Recession Functions in Constrained Problems
- ☐ **n. 186** John R. Williams, Piero Manfredi, The pre-vaccination dynamics of measles in Italy: estimating levels of under-reporting of measles cases
- ☐ **n. 187** Piero Manfredi, John R. Williams, To what extent can inter-regional migration perturbe local endemic patterns? Estimating numbers of measles cases in the Italian regions
- ☐ **n. 188** Laura Carosi, Johannes Jahn, Laura Martein, On The Connections between Semidefinite Optimization and Vector Optimization
- ☐ **n. 189** Alberto Cambini, Jean-Pierre Crouzeix, Laura Martein, On the Pseudoconvexity of a Quadratic Fractional Function
- ☐ **n. 190** Riccardo Cambini - Claudio Sodini, A finite Algorithm for a Particular d.c. Quadratic Programming Problem.
- ☐ **n. 191** Riccardo Cambini - Laura Carosi, Pseudoconvexity of a class of Quadratic Fractional Functions.
- ☐ **n. 192** Laura Carosi, A note on endogenous restricted partecipation on financial markets: an existence result.
- ☐ **n. 193** Emilio Barucci - Roberto Monte - Roberto Renò, Asset Price Anomalies under Bounded Rationality.
- ☐ **n. 194** Emilio Barucci - Roberto Renò, A Note on volatility estimate-forecast with GARCH models.
- ☐ **n. 195** Bruno Cheli, Sulla misura del benessere economico: i paradossi del PIL e le possibili correzioni in chiave etica e sostenibile, con uno spunto per l'analisi della povertà
- ☐ **n. 196** M.Bottai, M.Bottai, N. Salvati, M.Toigo, Le proiezioni demografiche con il programma Nostradamus. (Applicazione all'area pisana)
- ☐ **n. 197** A. Lemmi - B. Cheli - B. Mazzolli, La misura della povertà multidimensionale: aspetti metodologici e analisi della realtà italiana alla metà degli anni '90
- ☐ **n. 198** C.R. Bector - Riccardo Cambini, Generalized B-invex vector valued functions
- ☐ **n. 199** Luciano Fanti - Piero Manfredi, The workers' resistance to wage cuts is not necessarily detrimental for the economy: the case of a Goodwin's growth model with endogenous population.
- ☐ **n. 200** Emilio Barucci - Roberto Renò, On Measuring volatility of diffusion processes with high frequency data
- ☐ **n. 201** Piero Manfredi - Luciano Fanti, Demographic transition and balanced growth

Anno: 2001

- ☐ **n. 202** E.Barucci - M. E. Mancini - E. Vannucci, Asset Pricing, Diversification and Risk Ordering with Partially Exchangeable random Variables
- ☐ **n. 203** E. Barucci - R. Renò - E. Vannucci, Executive Stock Options Evaluation.
- ☐ **n. 204** Odo Barsotti - Moreno Toigo, Dimensioni delle rimesse e variabili esplicative: un'indagine sulla collettività marocchina immigrata nella Toscana Occidentale
- ☐ **n. 205** Vincenzo Bruno, I Consumi voluttuari, nell'ultimo trentennio, in Italia
- ☐ **n. 206** Michele Longo, The monopolist choice of innovation adoption: A regular-singular stochastic control problem
- ☐ **n. 207** Michele Longo, The competitive choice of innovation adoption: A finite-fuel singular stochastic control problem.
- ☐ **n. 208** Riccardo Cambini - Laura Carosi, On the pseudoaffinity of a class of quadratic fractional functions
- ☐ **n. 209** Riccardo Cambini - Claudio Sodini, A Finite Algorithm for a Class of Non Linear Multiplicative Programs.
- ☐ **n. 210** Alberto Cambini - Dinh The Luc - Laura Martein, A method for calculating subdifferential Convex vector functions
- ☐ **n. 211** Alberto Cambini - Laura Martein, Pseudolinearity in scalar and vector optimization.
- ☐ **n. 212** Riccardo Cambini, Necessary Optimality Conditions in Vector Optimization.
- ☐ **n. 213** Riccardo Cambini - Laura Carosi, On generalized convexity of quadratic fractional functions.
- ☐ **n. 214** Riccardo Cambini - Claudio Sodini, A note on a particular quadratic programming problem.
- ☐ **n. 215** Michele Longo - Vincenzo Valori, Existence end stability of equilibria in OLG models under adaptive expectations.

- n. 216 Luciano Fanti - Piero Manfredi, Population, unemployment and economic growth cycles: a further explanatory perspective
- n. 217 J.R.Williams,P.Manfredi,S.Salmaso,M.Ciofi, Heterogeneity in regional notification patterns and its impact on aggregate national case notification data: the example of measles in Italy.
- n. 218 Anna Marchi, On the connectedness of the efficient frontier: sets without local efficient maxima
- n. 219 Laura Lecchini - Odo Barsotti, Les disparités territoriales au Maroc au travers d'une optique de genre.

Anno: 2002

- n. 220 Gilberto Ghilardi - Nicola Orsini, Sull'uso dei modelli statistici lineari nella valutazione dei sistemi formativi.
- n. 221 Andrea Mercatanti, Un'analisi descrittiva dei laureati dell'Università di Pisa
- n. 222 E. Barucci - C. Impenna - R. Renò, The Italian Overnight Market: microstructure effects, the martingale hypothesis and the payment system.
- n. 223 E. Barucci, P.Malliavin, M.E.Mancino, R.Renò, A.Thalmaier, The Price-volatility feedback rate: an implementable mathematical indicator of market stability.
- n. 224 Andrea Mercatanti, Missing at random in randomized experiments with imperfect compliance
- n. 225 Andrea Mercatanti, Effetto dell'uso di carte Bancomat e carte di Credito sulla liquidità familiare: una valutazione empirica
- n. 226 Piero Manfredi - John R. Williams, Population decline and population waves: their impact upon epidemic patterns and morbidity rates for childhood infectious diseases. Measles in Italy as an example.
- n. 227 Piero Manfredi - Marta Ciofi degli Atti, La geografia pre-vaccinale del morbillo in Italia. I. Comportamenti di contatto e sforzi necessari all'eliminazione: predizioni dal modello base delle malattie prevenibili da vaccino.
- n. 228 I.M.Stancu-Minasian, Optimality Conditions and Duality in Fractional Programming Involving Semilocally Preinvex and Related
- n. 229 Nicola Salvati, Un software applicativo per un'analisi di dati sui marchi genetici (Genetic Markers)
- n. 230 Piero Manfredi, J. R. Williams, E. M. Cleur, S. Salmaso, M. Ciofi, The pre-vaccination regional landscape of measles in Italy: contact patterns and related amount of needed eradication efforts (and the "EURO" conjecture)
- n. 231 Andrea Mercatanti, I tempi di laurea presso l'Università di Pisa: un'applicazione dei modelli di durata in tempo discreto
- n. 232 Andrea Mercatanti, The weak version of the exclusion restriction in causal effects estimation: a simulation study
- n. 233 Riccardo Cambini and Laura Carosi, Duality in multiobjective optimization problems with set constraints
- n. 234 Riccardo Cambini and Claudio Sodini, Decomposition methods for nonconvex quadratic programs
- n. 235 R.Cambini and L. Carosi and S.Schaible, Duality in fractional optimization problems with set constraints
- n. 236 Anna Marchi, On the mix-efficient points

Anno: 2003

- n. 237 Emanuele Vannucci, The valuation of unit linked policies with minimal return guarantees under symmetric and asymmetric information hypoteses
- n. 238 John R Williams - Piero Manfredi, Ageing populations and childhood infections: the potential impact on epidemic patterns and morbidity
- n. 239 Bruno Cheli, Errata Corrige del Manuale delle Impronte Ecologiche (2002) ed alcuni utili chiarimenti
- n. 240 Alessandra Petrucci-Nicola Salvati-Monica Pratesi, Stimatore Combinato r Correlazione Spaziale nella Stima per Piccole Aree
- n. 241 Riccardo Cambini - Laura Carosi, Mixed Type Duality for Multiobjective Optimization Problems with set constraints
- n. 242 O.Barsotti, L.Lecchini, F.Benassi, Foreigners from central and eastern European countries in Italy: current and future perspectives of eu enlargement
- n. 243 A. Cambini - L. Martein - S. Schaible, Pseudoconvexity under the Charnes-Cooper transformation
- n. 244 Eugene M. Cleur, Piero Manfredi, and John R. William, The pre-and post-Vaccination regional dynamics of measles in Italy: Insights from time series analysis

Anno: 2004

- n. 245 Emilio Barucci - Jury Falini, Determinants of Corporate Governance in Italy: Path dependence or convergence?
- n. 246 R. Cambini - A. Marchi, A note on the connectedness of the efficient frontier
- n. 247 Laura Carosi - Laura Martein, On the pseudoconvexity and pseudolinearity of some classes of fractional functions
- n. 248 E. Barucci - R. Monte - B. Trivellato, Bayesian nash equilibrium for insider trading in continuous time