



Università degli Studi di Pisa  
Dipartimento di Statistica e Matematica  
Applicata all'Economia

---

**Report n. 252**

**A Treshold Model for Prevaccination Measles  
Data: Some Empirical Results for  
England and Italy**

**Eugene M. Cleur**

**Pisa, Maggio 2004**

**- Stampato in Proprio -**

# A TRESHOLD MODEL FOR PREVACCINATION MEASLES DATA : SOME EMPIRICAL RESULTS FOR ENGLAND AND ITALY

Eugene M. Cleur

Dipartimento di Statistica e Matematica Applicata all'Economia

Università di Pisa.

cleur@ec.unipi.it

**Abstract.** This paper considers the application of threshold autoregressive models to pre-vaccination measles time series in an attempt to explain differences in the epidemic dynamics as a function of the level reached by the series in a recent past. Data for the English cities of, London, Manchester and Cambridge, and for the provinces of Milano, Roma and Pisa in Italy are used in the illustration. The parametric, or autoregressive, spectrum, calculated for each piece-wise model clearly evidences such differences.

*Key words: threshold autoregressive models, autoregressive (gain) spectrum.*

## 1. INTRODUCTION

The dynamics of the spread of measles, in the absence of vaccination, is generally thought to follow a non-linear pattern in time and has been largely modelled in the past by the use of causal compartmental mathematical models (see Anderson and May (1991), Finkenstadt and Grenfell (2000), Bjornstad et al. (2002), Grenfell et al. (2002)). These models have incorporated the interactions between the number of reported cases and variables such as births, age structures, residential population and population densities. Pure time series analysis has generally been discarded as being inadequate for describing the phenomena; the time series of reported cases was itself plagued by under-reporting which is thought to have been approximately 50 percent in the UK (see Manfredi et al. (2002) for an evaluation of the Italian data). However, if the under-reporting varied little over time, time series methods could still be applied in shedding light on interesting aspects of the dynamics of the phenomena and such information might ultimately be useful to the model builder himself.

This paper was initially motivated by an interest in Italian data for the pre-vaccination period 1949-76 collected on a regional basis in which a well defined minimum, in August, is rapidly reached, but the maximum is often spread over two or three months during the period January – March. Cleur (2004) investigated the presence of specific types of non-linearities in this data and estimated time series models with autoregressive conditional heteroschedastic (ARCH) errors or bilinear models when these were identified as possible generators of the data. The present paper advances the idea that there does not exist one model for the whole series, but that it could change with the level (number of reported cases) reached. This idea, which is sustained by results obtained from the estimation of Self Exciting Threshold Autoregressive (SETAR) models for the Italian data, is further confirmed when applied to similar data for the UK. The empirical results for many series, when put together, become repetitive and therefore it was decided to present the analysis carried out on bi-weekly data

covering the period 1944 - 1965 for the cities of, London, Manchester and Cambridge and for monthly data covering the period 1949 - 1976 for the Provinces of Milano, Roma and Pisa; data for the single towns and cities in Italy were not available, but a comparison with the UK data can, in any case, be made on the basis of the populations of the territorial units considered.

## 2. METHODS

The time series for London, Manchester and Cambridge were obtained from the web site [www.zoo.cam.ac.uk/zoostaff/grenfell/measles.htm](http://www.zoo.cam.ac.uk/zoostaff/grenfell/measles.htm) and are part of the data set reporting pre-vaccination measles time series for 60 towns and cities in England and Wales for the period 1944 - 1965. London and Manchester were chosen for their population size as well as geographical location. Cambridge instead is a town with approximately 100.000 inhabitants and a low number of reported cases.

The data for the provinces of Milano, Roma and Pisa are part of the data published by the Istituto Nazionale di Statistica (ISTAT), Italy's Central Statistical Office, on measles time series for all the provinces in Italy. Roma and Milano were selected for the same reasons as London and Manchester. Pisa is a province with approximately 100.000 inhabitants and comparable to Cambridge for the number of reported cases.

Each series was log transformed to stabilise the variance (a value of 1 was added to all the data before the transformation in order to avoid the problem of log(zero) for those months without reported cases), and then centred for the mean. This means that negative values of a series correspond to low values of reported cases, whereas high values, and in particular very high positive values, correspond to epidemic outbursts. One should be particularly interested in adequately modelling the positive values.

After experimenting various forms of the SETAR model it was decided to estimate the following for each log transformed mean-centered time series (see Priestley (1988) and Tong et al. (1980) for an introduction to SETAR models):

$$X_t = c_1 + \sum_{j=1}^6 \alpha_j X_{t-j} + \varepsilon_{1t} \quad \text{if } 0 \leq X_{t-d} < T \quad (1a)$$

$$X_t = c_2 + \sum_{j=1}^6 \beta_j X_{t-j} + \varepsilon_{2t} \quad \text{if } X_{t-d} \geq T \quad (1b)$$

$$X_t = c_3 + \sum_{j=1}^6 \gamma_j X_{t-j} + \varepsilon_{3t} \quad \text{if } X_{t-d} < 0 \quad (1c)$$

In other words, whereas two "regimes" (or subsets or states) are defined for the positive data, only one model is fitted to the negative values. The fitted model for each "regime" is a "best subset" autoregressive model with maximum lag 6, i. e. only significant lags up to lag 6 are retained. The optimum values of the threshold coefficient  $T$  and the lag  $d$  are obtained using AIC statistics as in Tong and Lim (1980); the only difference with their method is that we estimate the "best subset" and not the full model of order 6 and the values of  $T$  and  $d$  are found simultaneously by minimising the AIC statistic, using a grid search method, over a two dimensional grid of values;  $d$  varied in the interval from

1 to 4 bi-weeks for the UK data and from 1 to 2 months for the Italian data. These values of  $d$  were forced on the grounds of their practical importance. The results are reported in Tables (1a) – (1f).

Differences in the SETAR model coefficients are useful for shedding light on the different characteristics of each series when it varies at different levels. However, in order to make such differences more evident and interesting, the autoregressive (or parametric or gain) spectrum (see Hamilton (1994) and Priestley (1981)) for each estimated autoregressive model is calculated. The results are reported in Figure 2 .

In Figure 1 fitted values from each estimated model are compared with the observed counterpart and will be fundamental in evaluating the opportunity of applying SETAR models.

### 3. RESULTS

Figure 1 is useful in our bid to find a model which adapts well to the data. As can be seen, the threshold models identified provide an excellent fit for London and Manchester; these two cities are characterised by a large number of reported cases. Overall, for the remaining four series, the threshold model gives a very good representation of the positive values of the mean centred data, but the local minimums are always over estimated. This is particularly true of Cambridge and the province of Pisa where the local minimums correspond to a large number of fade outs, i.e. zero reported cases; obviously, (1c) in the threshold model is unable to adequately represent this phenomena. In fact the presence of many fade outs in these two series reduces  $R^2$  quite sensibly. However, since it is the outburst of an epidemic which is of prime concern and not that of having few reported cases, the use of a SETAR model to predict epidemics can be considered satisfactory. These results may also be interpreted as suggesting a phenomena which could be amplitude dependent.

There are some differences between the threshold values,  $T$ , but the lag value  $d$  found by the selection procedure for each SETAR model defines a 2 months span.

A much more interesting result that emerges from the estimated SETAR models are the differences in their frequency domain properties. The autoregressive spectrum was calculated for each of the three autoregressive models defined in equations (1a) – (1c). In Figure 2 the spectrum is reproduced in the band (0-0.8) since it tails off and rapidly converges to zero beyond frequency  $\lambda = 0.8$ ; (the spectrum is generally estimated in the frequency interval  $(0, \pi)$  ).

The three spectra for any one series are different, at times in form (some are peaked whilst others are not), but more often in terms of the peak frequency thereby confirming the conjecture that the dynamic properties vary with the level reached. Sharp peaks are present in the spectra generated by models (1a) and (1b) for London, Manchester, and the provinces of Milano and Roma. Given the nature of these peaks as well as the very high values of  $R^2$ , they might indicate true cyclical behaviour; thus, for London, for example, the peaks at frequencies 0.27 and 0.19 correspond, respectively, to cycles with periodicity of approximately 12 and 16 months. For Cambridge and the

province of Pisa, the peaks present are not as marked as in the other cases, but nonetheless indicate differences in the dynamics of the series at different levels.

#### 4. DISCUSSION

The results presented above have been obtained from data where negative values correspond to a small number of reported cases during a two week, for bi-weekly data, or monthly interval. Positive values, and particularly large positive values, are a result of outbursts of epidemics and it is for this reason that we concentrate our attention mainly on models (1a) and (1b). The peak in the spectrum from model (1b) for London, for instance, means that the response 2 months after a positive value crosses the threshold level of 1.1 (converted to the original frequency of reported cases by taking the exponential of the sum of 1.1 and the mean value of 6.1144 for London means nearly 500 reported cases in a bi-week) is cyclical with periodicity of approximately 16 months; similar interpretations may be given to a peak in the spectrum for the other two models.

The threshold model is seen to give very good results for territorial units with a large number of reported cases. The presence of fade-outs and a relatively low level of reported cases is not always adequately represented by the autoregressive structure imbedded in the threshold model.

The results reported correspond to the models selected automatically using the AIC statistic in the procedure described above. Different spectral properties are obtained from other manually selected models which further confirms the belief that the phenomena of measles is amplitude dependent

It emerges from the published literature that the dynamics of the spread of measles are best understood when considered along with demographic variables such as birth rates, age structures and community size in causal models such as the SEIR and TSIR models (see Finkenstadt et al. (2000) and Grenfell et al.(2002)). However, classical time series methods could also be useful for understanding the phenomena. This paper has shown that non-linearities in measles time series in a pre-vaccination period are amplitude dependent and can be profitably explained in piece-wise linear models like the threshold autoregressive model. This approach, when transformed to the frequency domain, has enabled us to identify different dynamics at different levels reached by a series which further confirms the amplitude dependent hypothesis. The estimated threshold model, like most time series models, given the excellent fit to the empirical data, should be useful for purposes of forecasting within a pre-vaccination era. On the other hand, the results from the frequency domain analysis should be useful for identifying the information that has to be incorporated into a causal mathematical model.

## BIBLIOGRAPHY

- Anderson R.M., B.T. Grenfell and R.M. May. 1984. Oscillatory fluctuations in the incidence of infectious diseases and the impact of vaccination: time series analysis, *Journ. of Hygiene Cambridge*, 93, 587-698
- Anderson R.M. and R.M. May. 1991. *Infectious diseases of humans: dynamics and control*, Oxford University Press.
- Bjornstad. O. N., B. F. Grenfell, and B. F. Finkenstadt. 2002. Dynamics of Measles Epidemics: Estimation of Scaling of Transmission Rates using a Time Series SIR Model. *Ecological Monographs*, 72, 169-184.
- Cleur, E. M. 2004. Nonlinearities in monthly measles data for Italy, Working Paper 251, Dipartimento di Statistica e Matematica Applicata all'Economia, Università di Pisa.
- Finkenstadt, B. F. Ad B. T. Grenfell. 2000. Time series modelling of childhood diseases: a dynamical systems approach., *Journal of the Royal Statistical Society, Series C*, 49, 187-205.
- Grenfell, B. F., O. N. Bjornstad and B. F. Finkenstadt. 2002. Dynamics of Measles Epidemics: Scaling Noise, Determinism, and Predictability with the TSIR Model. *Ecological Monographs*, 72, 185-202.
- Hamilton, J. D. 1994. *Time Series Analysis*, Princeton University Press, Princeton, New Jersey.
- Manfredi, P., J. W. Williams., E. M. Cleur, S. Salmaso. and M. Ciofi. 2002. The pre-vaccination regional landscape of measles in Italy: contact patterns and related amount of needed eradication efforts (and the "EURO" conjecture), Working Paper 230 , Dipartimento di Statistica e Matematica Applicata all'Economia, Università di Pisa.
- Priestley, M. B. (1981), *Spectral Analysis and Time Series Volume 1*, Academic Press Inc., London.
- Shumway, R. H. and D. S. Stoffer. 2000. *Time Series Analysis and its Applications*, Springer-Verlag, New York.
- Tong, T. and K. S. Lim. 1980. Threshold autoregression, limit cycles and cyclical data, *Journal of the Royal Statistica Society, Serie B*, 42, 245-292.

**TABLE 1.** Estimates of SETAR model (1a)-(1c)(1a)  $0 < X_{t-d} < T$  (1b)  $X_{t-d} > T$  (1c)  $X_{t-d} < 0$ .

N = number of data points used in estimation of model in column 1

1a. London :  $T = 1.1, d = 4$ 

	N	$\mu$	$\phi_1$	$\phi_2$	$\phi_3$	$\phi_4$	$\phi_5$	$\phi_6$	$R^2$
(1a)	158	0.05	1.47	-0.37	-0.13	-	-	-0.13	0.95
(1b)	147	0.03	1.23	-0.27	-	0.24	-0.22	-0.09	0.92
(1c)	289	0.09	1.13	-	-0.11	-	-	-0.03	0.92

1b. Manchester :  $T = 1.1, d = 4$ 

	N	$\mu$	$\phi_1$	$\phi_2$	$\phi_3$	$\phi_4$	$\phi_5$	$\phi_6$	$R^2$
(1a)	164	0.06	1.20	-	-0.20	-	-0.14	-	0.89
(1b)	139	0.15	1.08	-	-0.11	-	-	-0.14	0.89
(1c)	291	0.04	0.90	0.12	-	-0.08	-	-	0.82

1c. Cambridge :  $T = 1.1, d = 4$ 

	N	$\mu$	$\phi_1$	$\phi_2$	$\phi_3$	$\phi_4$	$\phi_5$	$\phi_6$	$R^2$
(1a)	96	0.04	0.89	0.05	-	-	-	-0.12	0.72
(1b)	164	0.41	1.03	-	-0.14	-0.17	-	-	0.78
(1c)	334	-0.12	0.83	-	-	-	-0.07	-	0.57

1d. Milano :  $T = 0.8, d = 2$ 

	N	$\mu$	$\phi_1$	$\phi_2$	$\phi_3$	$\phi_4$	$\phi_5$	$\phi_6$	$R^2$
(1a)	92	-0.09	1.14	-0.30	-	-0.15	-	-0.23	0.82
(1b)	85	-0.26	1.27	-	-0.28	-0.06	-	-0.17	0.81
(1c)	133	0.06	1.00	-0.33	-	-	0.22	-0.15	0.60

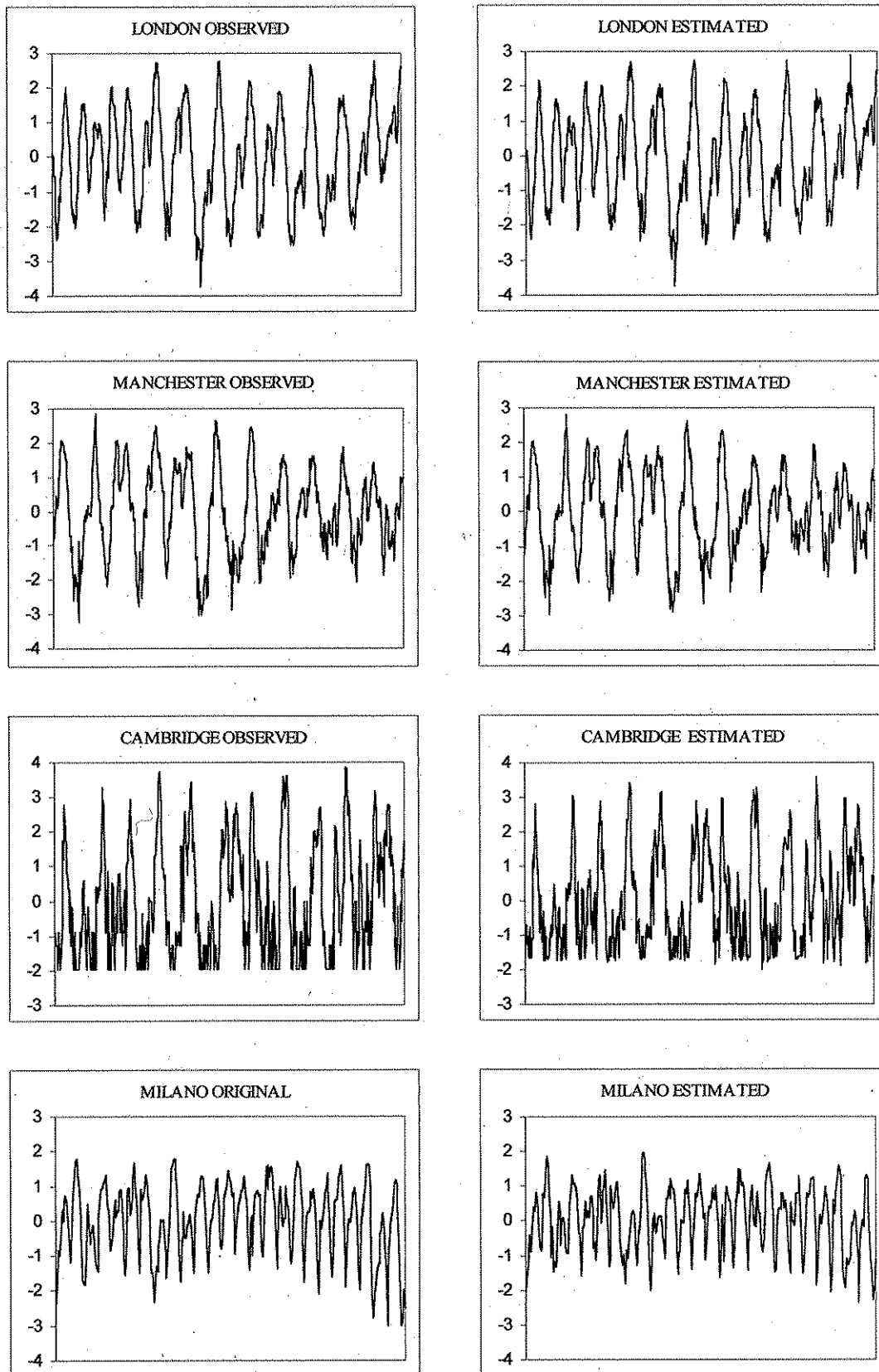
1e. Roma :  $T = 1.3, d = 3$ 

	N	$\mu$	$\phi_1$	$\phi_2$	$\phi_3$	$\phi_4$	$\phi_5$	$\phi_6$	$R^2$
(1a)	86	-0.07	1.22	-0.43	-	-	-0.33	0.12	0.88
(1b)	85	-0.24	1.56	-0.67	-	-	-	-0.13	0.92
(1c)	141	-0.02	0.95	-	-	-0.40	-	0.26	0.74

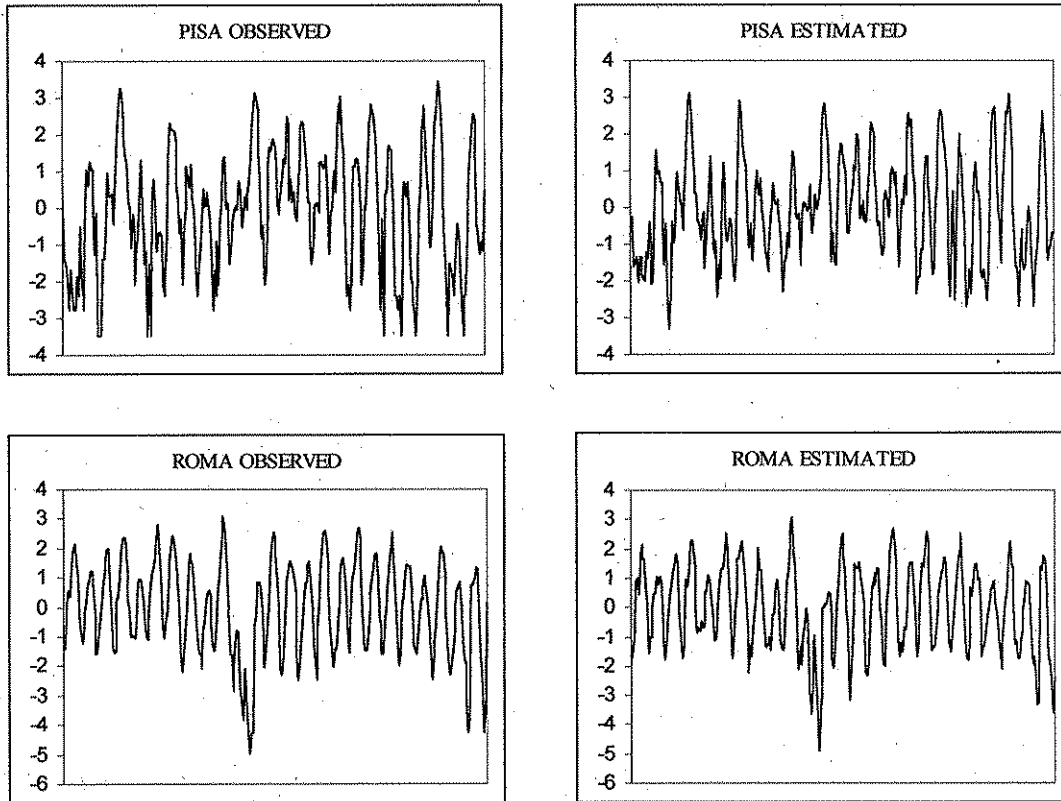
1f. Pisa :  $T = 1.3, d = 23$ 

	N	$\mu$	$\phi_1$	$\phi_2$	$\phi_3$	$\phi_4$	$\phi_5$	$\phi_6$	$R^2$
(1a)	90	-0.10	0.96	-	-0.09	-0.15	-	-	0.63
(1b)	72	-0.11	1.07	-	-	-0.34	-	-	0.81
(1c)	148	-0.01	0.76	0.21	-0.28	-	-	-	0.58

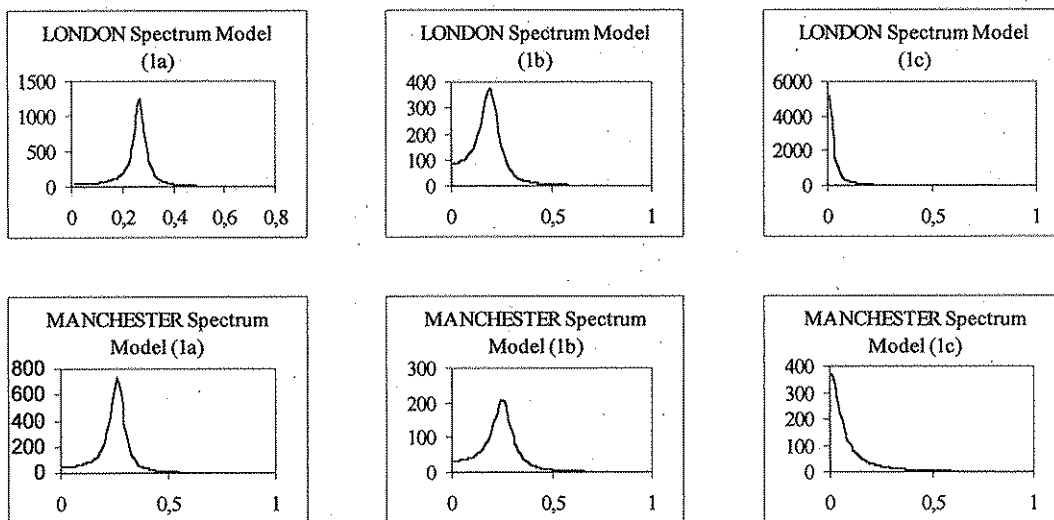
**FIGURE 1.** Observed data and estimated values using threshold model (1a) – (1c)

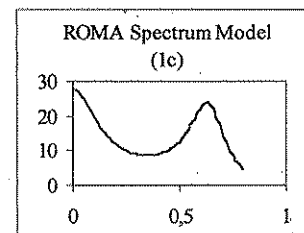
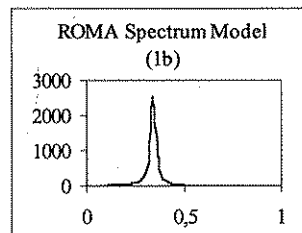
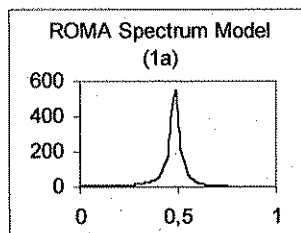
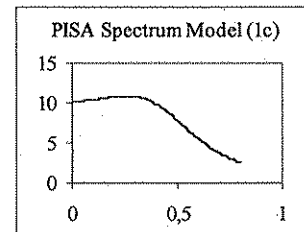
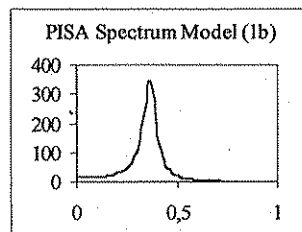
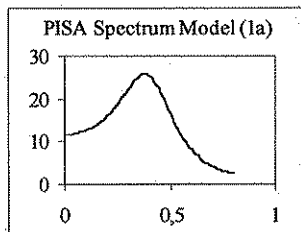
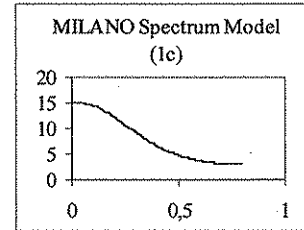
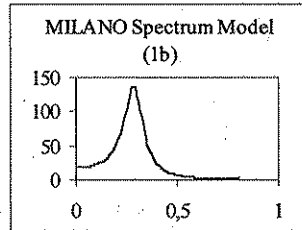
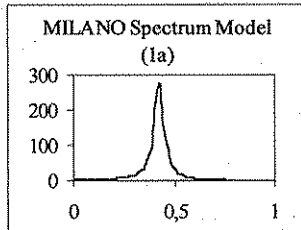
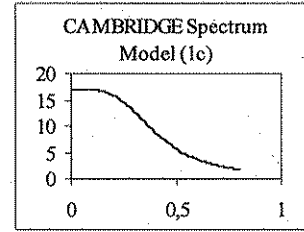
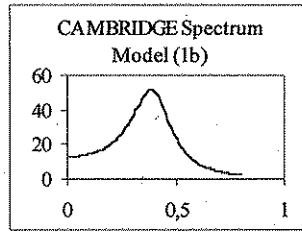
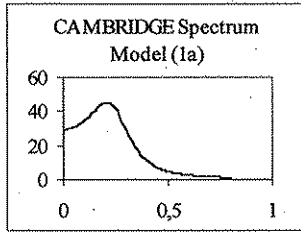






**FIGURE 2.** Autoregressive (gain) spectra from SETAR Model (1a) – (1c)





## Elenco dei report pubblicati

---

### Anno: 1987

---

- n. 1 Alberto Cambini - Laura Martein, Some Optimality Conditions in Vector Optimization
- n. 2 Alberto Cambini - Laura Martein - S.Schaibel, On Maximizing a Sum of Ratios
- n. 3 Giuliano Gasparotto, On the Charnes-Cooper Transformation in linear Fractional Programming.
- n. 4 Alberto Cambini, Non-linear separation Theorems, Duality and Optimality
- n. 5 Giovanni Boletto, Indicizzazione parziale: aspetti metodologici e riflessi economici
- n. 6 Alberto Cambini - Claudio Sodini, On Parametric Linear Fractional Programming
- n. 7 Alberto Bonaguidi, Alcuni aspetti meno noti delle migrazioni in Italia
- n. 8 Laura Martein - S. Schaible, On Solving a Linear Program with one Quadratic Constraint

### Anno: 1988

---

- n. 9 Ester Lari, Alcune osservazioni sull'equazione funzionale  $\emptyset(x,y,z)=\emptyset(\emptyset(x,y,t),t,z)$
- n. 10 F. Barfiaux, Une étude par ménage des migrations des personnes âgées: comparaison des résultats pour l'Italie et les Etats-Unis
- n. 11 Giovanni Boletto, Metodi di scomposizione del tasso di inflazione
- n. 12 Claudio Sodini, A New Algorithm for the Strictly Convex Quadratic Programming Problem
- n. 13 Laura Martein, On Generating the Set of all Efficient Points of a Bicriteria Fractional Problem
- n. 14 Laura Martein, Applicazioni della programmazione frazionaria nel campo economico-finanziario
- n. 15 Laura Martein, On the Bicriteria Maximization Problem
- n. 16 Paolo Manca, Un prototipo di sistema esperto per la consulenza finanziaria rivolta ai piccoli risparmiatori
- n. 17 Paolo Manca, Operazioni Finanziarie di Soper e Operazioni di puro Investimento secondo Teichroew-Robichek-Montalbano
- n. 18 Paolo Carraresi - Claudio Sodini, A k - Shortest Path Approach to the Minimum Cost Matching Problem.
- n. 19 Odo Barsotti - Marco Bottai, Sistemi gravitazionali e fasi di transazione della crescita Demografica
- n. 20 Giovanni Boletto, Metodi di scomposizione dell'inflazione aggregata: recenti sviluppi.
- n. 21 Marc Termote - Alberto Bonaguidi, Multiregional Stable Population as a Tool for Short-term Demographic Analysis
- n. 22 Marco Bottai, Storie familiari e storie migratorie: un'indagine in Italia
- n. 23 Maria Francesca Romano - Marco Marchi, Problemi connessi con la disomogeneità dei gruppi sottoposti a sorveglianza statistico-epidemiologica.
- n. 24 Franca Orsi, Un approccio logico ai problemi di scelta finanziaria.

### Anno: 1989

---

- n. 25 Vincenzo Bruno, Attrazione ed entropia.
- n. 26 Giorgio Giorgi - S. Mititelu, Invexity in nonsmooth Programming.
- n. 28 Alberto Cambini - Laura Martein, Equivalence in linear fractional programming.

### Anno: 1990

---

- n. 27 Vincenzo Bruno, Lineamenti econometrici dell'evoluzione del reddito nazionale in relazione ad altri fenomeni economici
- n. 29 Odo Barsotti - Marco Bottai - Marco Costa, Centralità e potenziale demografico per l'analisi dei comportamenti demografici: il caso della Toscana
- n. 30 Anna Marchi, A sequential method for a bicriteria problem arising in portfolio selection theory.
- n. 31 Marco Bottai, Mobilità locale e pianificazione territoriale.
- n. 32 Anna Marchi, Solving a quadratic fractional program by means of a complementarity approach
- n. 33 Anna Marchi, Sulla relazione tra un problema bicriteria e un problema frazionario.

### Anno: 1991

---

- n. 34 Enrico Gori, Variabili latenti e "self-selection" nella valutazione dei processi formativi.
- n. 35 Piero Manfredi - E. Salinelli, About an interactive model for sexual Populations.
- n. 36 Giorgio Giorgi, Alcuni aspetti matematici del modello di sraffa a produzione semplice
- n. 37 Alberto Cambini - S.Schaibl - Claudio Sodini, Parametric linear fractional programming for an unbounded feasible Region.
- n. 38 I.Emke - Pouloupoulos - V.Gozálves Pérez - Odo Barsotti - Laura Lecchini, International migration to northern Mediterranean countries the cases of Greece, Spain and Italy.
- n. 39 Giuliano Gasparotto, A LP code implementation
- n. 40 Riccardo Cambini, Un problema di programmazione quadratica nella costituzione di capitale.
- n. 41 Gilberto Ghilardi, Stime ed errori campionari nell'indagine ISTAT sulle forze di lavoro.
- n. 42 Vincenzo Bruno, Alcuni valori medi, variabilità paretiana ed entropia.
- n. 43 Giovanni Boletto, Gli effetti del trascinarsi dei prezzi sulle misure dell'inflazione: aspetti metodologici
- n. 44 P. Paolicchi, Gli abbandoni nell'università: modelli interpretativi.
- n. 45 Maria Francesca Romano, Da un archivio amministrativo a un archivio statistico: una proposta metodologica per i dati degli studenti universitari.
- n. 46 Maria Francesca Romano, Criteri di scelta delle variabili nei modelli MDS: un'applicazione sulla popolazione studentesca di Pisa.
- n. 47 Odo Barsotti - Laura Lecchini, Les parcours migratoires en fonction de la nationalité. Le cas de l'Italie.
- n. 48 Vincenzo Bruno, Indicatori statistici ed evoluzione demografica, economica e sociale delle province toscane.
- n. 49 Alberto Cambini - Laura Martein, Tangent cones in optimization.
- n. 50 Alberto Cambini - Laura Martein, Optimality conditions in vector and scalar optimization: a unified approach.

### Anno: 1992

---

- n. 51 Gilberto Ghilardi, Elementi di uno schema di campionamento areale per alcune rilevazioni ufficiali in Italia.
- n. 52 Paolo Manca, Investimenti e finanziamenti generalizzati.
- n. 53 Laura Lecchini - Odo Barsotti, Le rôle des immigrés extra- communautaires dans le marché du travail

## ***Elenco dei report pubblicati***

---

- n. 54 Riccardo Cambini, Alcune condizioni di ottimalità relative ad un insieme stellato.
- n. 55 Gilberto Ghilardi, Uno schema di campionamento areale per le rilevazioni sulle famiglie in Italia.
- n. 56 Riccardo Cambini, Studio di una classe di problemi non lineari: un metodo sequenziale.
- n. 57 Riccardo Cambini, Una nota sulle possibili estensioni a funzioni vettoriali di significative classi di funzioni concavo-generalizzate.
- n. 58 Alberto Bonaguidi - Valerio Terra Abrami, Metropolitan aging transition and metropolitan redistribution of the elderly in Italy.
- n. 59 Odo Barsotti - Laura Lecchini, A comparison of male and female migration strategies: the cases of African and Filipino Migrants to Italy.
- n. 60 Gilberto Ghilardi, Un modello logit per lo studio del fenomeno delle nuove imprese.
- n. 61 S. Schaible, Generalized monotonicity.
- n. 62 Vincenzo Bruno, Dell'elasticità in economia e dell'incertezza statistica.
- n. 63 Laura Martein, Alcune classi di funzioni concave generalizzate nell'ottimizzazione vettoriale
- n. 64 Anna Marchi, On the relationships between bicriteria problems and non-linear programming problems.
- n. 65 Giovanni Boletto, Considerazioni metodologiche sul concetto di elasticità prefissata.
- n. 66 Laura Martein, Soluzione efficienti e condizioni di ottimalità nell'ottimizzazione vettoriale.

### **Anno: 1993**

---

- n. 67 Maria Francesca Romano, Le rilevazioni ufficiali ISTAT della popolazione universitaria: problemi e definizioni alternative.
- n. 68 Marco Bottai - Odo Barsotti, La ricerca "Spazio Utilizzato" Obiettivi e primi risultati.
- n. 69 Marco Bottai - F. Bartiaux, Composizione familiare e mobilità delle persone anziane. Una analisi regionale.
- n. 70 Anna Marchi - Claudio Sodini, An algorithm for a non-differentiable non-linear fractional programming problem.
- n. 71 Claudio Sodini - S. Schaible, An finite algorithm for generalized linear multiplicative programming.
- n. 72 Alberto Cambini - Laura Martein, An approach to optimality conditions in vector and scalar optimization.
- n. 73 Alberto Cambini - Laura Martein, Generalized concavity and optimality conditions in vector and scalar optimization.
- n. 74 Riccardo Cambini, Alcune nuove classi di funzioni concavo-generalizzate.

### **Anno: 1994**

---

- n. 75 Alberto Cambini - Anna Marchi - Laura Martein, On nonlinear scalarization in vector optimization.
- n. 76 Maria Francesca Romano - Giovanna Nencioni, Analisi delle carriere degli studenti immatricolati dal 1980 al 1982.
- n. 77 Gilberto Ghilardi, Indici statistici della congiuntura.
- n. 78 Riccardo Cambini, Condizioni di efficienza locale nella ottimizzazione vettoriale.
- n. 79 Odo Barsotti - Marco Bottai, Funzioni di utilizzazione dello spazio.
- n. 80 Vincenzo Bruno, Alcuni aspetti dinamici della popolazione dei comuni della Toscana, distinti per ampiezza demografica e per classi di urbanità e di ruralità.
- n. 81 Giovanni Boletto, I numeri indici del potere d'acquisto della moneta.
- n. 82 Alberto Cambini - Laura Martein - Riccardo Cambini, Some optimality conditions in multiobjective programming.
- n. 83 S. Schaible, Fractional programming with sum of ratios.
- n. 84 Stefan Tigan - I.M. Stancu-Minasian, The minimum-risk approach for continuous time linear-fractional programming.
- n. 85 Vasile Preda - I.M. Stancu-Minasian, On duality for multiobjective mathematical programming of n-set.
- n. 86 Vasile Preda - I.M. Stancu-Minasian - Anton Batatorescu, Optimality and duality in nonlinear programming involving semilocally preinvex and related functions.

### **Anno: 1995**

---

- n. 87 Elena Melis, Una nota storica sulla programmazione lineare: un problema di Kantorovich rivisto alla luce del problema degli zeri.
- n. 88 Vincenzo Bruno, Mobilità territoriale dell'Italia e di tre Regioni tipiche: Lombardia, Toscana, Sicilia.
- n. 89 Antonio Cortese, Bibliografia sulla presenza straniera in Italia
- n. 90 Riccardo Cambini, Funzioni scalari affini generalizzate.
- n. 91 Piero Manfredi - Fabio Tarini, Modelli epidemiologici: teoria e simulazione. (I)
- n. 92 Marco Bottai - Maria Caputo - Laura Lecchini, The "OLIVAR" survey. Methodology and quality.
- n. 93 Laura Lecchini - Donatella Marsiglia - Marco Bottai, Old people and social network.
- n. 94 Gilberto Ghilardi, Uno studio empirico sul confronto tra alcuni indici statistici della congiuntura.
- n. 95 Vincenzo Bruno, Il traffico nei porti italiani negli anni recenti.
- n. 96 Alberto Cambini - Anna Marchi - Laura Martein - S. Schaible, An analysis of the falk-palocsay algorithm.
- n. 97 Alberto Cambini - Laura Carosi, Sulla esistenza di elementi massimali.

### **Anno: 1996**

---

- n. 98 Riccardo Cambini - S. Komlósi, Generalized concavity and generalized monotonicity concepts for vector valued.
- n. 99 Riccardo Cambini, Second order optimality conditions in the image space.
- n. 100 Vincenzo Bruno, La stagionalità delle correnti di navigazione marittima.
- n. 101 Eugene Maurice Cleur, A comparison of alternative discrete approximations of the Cox - Ingersoll - Ross model.
- n. 102 Gilberto Ghilardi, Sul calcolo del rapporto di concentrazione del Gini.
- n. 103 Alberto Cambini - Laura Martein - Riccardo Cambini, A new approach to second order optimality conditions in vector optimization.
- n. 104 Fausto Gozzi, Alcune osservazioni sull'immunizzazione semideterministica.
- n. 105 Emilio Barucci - Fausto Gozzi, Innovation and capital accumulation in a vintage capital model an infinite dimensional control approach.
- n. 106 Alberto Cambini - Laura Martein - I.M. Stancu-Minasian, A survey of bicriteria fractional problems.
- n. 107 Luciano Fanti - Piero Manfredi, Viscosità dei salari, offerta di lavoro endogena e ciclo.
- n. 108 Piero Manfredi - Luciano Fanti, Ciclo di vita di nuovi prodotti: modellistica non lineare.
- n. 109 Piero Manfredi, Crescita con ciclo, gestazione dei piani di investimento ed effetti.
- n. 110 Luciano Fanti - Piero Manfredi, Un modello "classico" di ciclo con crescita ed offerta di lavoro endogena.
- n. 111 Anna Marchi, On the connectedness of the efficient frontier : sets without local maxima.

## Elenco dei report pubblicati

- n. 112 Riccardo Cambini, Generalized concavity for bicriteria functions.
- n. 113 Vincenzo Bruno, Variazioni dinamiche (1971-1981-1991) dei fenomeni demografici dei comuni (urbani e rurali) della Lombardia, in relazione ad alcune caratteristiche di mobilità territoriale.

### Anno: 1997

- n. 114 Piero Manfredi - Fabio Tarini - J.R. Williams - A. Carducci - B. Casini, Infectious diseases: epidemiology, mathematical models, and immunization policies.
- n. 115 Eugene Maurice Cleur - Piero Manfredi, One dimensional SDE models, low order numerical methods and simulation based estimation: a comparison of alternative estimators.
- n. 116 Luciano Fanti - Piero Manfredi, Point stability versus orbital stability (or instability): remarks on policy implications in classical growth cycle model.
- n. 117 Piero Manfredi - Francesco Billari, transition into adulthood, marriage, and timing of life in a stable population framework.
- n. 118 Laura Carosi, Una nota sul concetto di estremo superiore di insiemi ordinati da coni convessi.
- n. 119 Laura Lecchini - Donatella Marsiglia, Reti sociali degli anziani: selezione e qualità delle relazioni.
- n. 120 Piero Manfredi - Luciano Fanti, Gestation lags and efficiency wage mechanisms in a goodwin type growth model.
- n. 121 G. Rivellini, La metodologia statistica multilevel come possibile strumento per lo studio delle interazioni tra il comportamento procreativo individuale e il contesto
- n. 122 Laura Carosi, Una nota sugli insiemi C-limitati e L-limitati.
- n. 123 Laura Carosi, Sull'estremo superiore di una funzione lineare fratta ristretta ad un insieme chiuso e illimitato.
- n. 124 Piero Manfredi, A demographic framework for the evaluation of the impact of imported infectious diseases.
- n. 125 Alessandro Valentini, Calo della fecondità ed immigrazione: scenari e considerazioni sul caso italiano.
- n. 126 Alberto Cambini - Laura Martein, Second order optimality conditions.

### Anno: 1998

- n. 127 Piero Manfredi and Alessandro Valentini, Populations with below replacement fertility: theoretical considerations and scenarios from the Italian laboratory.
- n. 128 Alberto Cambini - Laura Martein - E. Moretti, Programmazione frazionaria e problemi bicriteria.
- n. 129 Emilio Barucci - Fausto Gozzi - Andrej Swiech, Incentive compatibility constraints and dynamic programming in continuous time.

### Anno: 1999

- n. 130 Alessandro Valentini, Impatto delle immigrazioni sulla popolazione italiana: confronto tra scenari alternativi.
- n. 131 K. Iglicka - Odo Barsotti - Laura Lecchini, Recent development of migrations from Poland to Europe with a special emphasis on Italy K. Iglicka - Le Migrazioni est-ovest: le unioni miste in Italia
- n. 132 Alessandro Valentini, Proiezioni demografiche multiregionali a due sessi, con immigrazioni internazionali e vincoli di consistenza.
- n. 133 Fabio Antonelli - Emilio Barucci - Maria Elvira Mancino, Backward-forward stochastic differential utility: existence, consumption and equilibrium analysis.
- n. 134 Emilio Barucci - Maria Elvira Mancino, Asset pricing with endogenous aspirations.
- n. 135 Eugene Maurice Cleur, Estimating a class of diffusion models: an evaluation of the effects of sampled discrete observations.
- n. 136 Luciano Fanti - Piero Manfredi, Labour supply, time delays, and demoeconomic oscillations in a solow-type growth model.
- n. 137 Emilio Barucci - Sergio Polidoro - Vincenzo Vespri, Some results on partial differential equations and Asian options.
- n. 138 Emilio Barucci - Maria Elvira Mancino, Hedging european contingent claims in a Markovian incomplete market.
- n. 139 Alessandro Valentini, L'applicazione del modello multiregionale-multistato alla popolazione in Italia mediante l'utilizzo del Lipro: procedura di adattamento dei dati e particolarità tecniche del programma.
- n. 140 I.M. Stancu-Minasian, optimality conditions and duality in fractional programming-involving semilocally preinvex and related functions.
- n. 141 Alessandro Valentini, Proiezioni demografiche con algoritmi di consistenza per la popolazione in Italia nel periodo 1997-2142: presentazione dei risultati e confronto con metodologie di stima alternative.
- n. 142 Laura Carosi, Competitive equilibria with money and restricted participation.
- n. 143 Laura Carosi, Monetary policy and Pareto improvability in a financial economy with restricted participation
- n. 144 Bruno Cheil, Misurare il benessere e lo sviluppo dai paradossi del Pil a misure di benessere economico sostenibile, con uno sguardo allo sviluppo umano
- n. 145 Bruno Cheil - Laura Lecchini - Lucio Masserini, The old people's perception of well-being: the role of material and non material resources
- n. 146 Eugene Maurice Cleur, Maximum likelihood estimation of one-dimensional stochastic differential equation models from discrete data: some computational results
- n. 147 Alessandro Valentini - Francesco Billari - Piero Manfredi, Utilizzi empirici di modelli multistato continui con durate multiple
- n. 148 Francesco Billari - Piero Manfredi - Alberto Bonaguidi - Alessandro Valentini, Transition into adulthood: its macro-demographic consequences in a multistate stable population framework
- n. 149 Francesco Billari - Piero Manfredi - Alessandro Valentini, Becoming Adult and its Macro-Demographic Impact: Multistate Stable Population Theory and an Application to Italy
- n. 150 Alessandro Valentini, Le previsioni demografiche in presenza di immigrazioni: confronto tra modelli alternativi e loro utilizzo empirico ai fini della valutazione dell'equilibrio nel sistema pensionistico
- n. 151 Emilio Barucci - Roberto Monte, Diffusion processes for asset prices under bounded rationality
- n. 152 Emilio Barucci - P. Cianchi - L. Landi - A. Lombardi, Reti neurali e analisi delle serie storiche: un modello per la previsione del BTP future
- n. 153 Alberto Cambini - Laura Carosi - Laura Martein, On the supremum in fractional programming
- n. 154 Riccardo Cambini - Laura Martein, First and second order characterizations of a class of pseudoconcave vector functions
- n. 155 Piero Manfredi and Luciano Fanti, Embedding population dynamics in macro-economic models. The case of the goodwin's growth cycle
- n. 156 Laura Lecchini e Odo Barsotti, Migrazioni dei preti dalla Polonia in Italia
- n. 157 Vincenzo Bruno, Analisi dei prezzi, in Italia dal 1975 in poi
- n. 158 Vincenzo Bruno, Analisi del commercio al minuto in Italia
- n. 159 Vincenzo Bruno, Aspetti ciclici della liquidità bancaria, dal 1971 in poi
- n. 160 Anna Marchi, A separation theorem in alternative theorems and vector optimization

## *Elenco dei report pubblicati*

---

### Anno: 2000

- n. 161 Piero Manfredi and Luciano Fanti, Labour supply, population dynamics and persistent oscillations in a Goodwin-type growth cycle model
- n. 162 Luciano Fanti and Piero Manfredi, Neo-classical labour market dynamics and chaos (and the Phillips curve revisited)
- n. 163 Piero Manfredi - and Luciano Fanti, Detection of Hopf bifurcations in continuous-time macro- economic models, with an application to reducible delay-systems.
- n. 164 Fabio Antonelli - Emilio Barucci, The Dynamics of pareto allocations with stochastic differential utility
- n. 165 Eugene M. Cleur, Computing maximum likelihood estimates of a class of One-Dimensional stochastic differential equation models from discrete Date\*
- n. 166 Eugene M. Cleur, Estimating the drift parameter in diffusion processes more efficiently at discrete times: a role of indirect estimation
- n. 167 Emilio Barucci - Vincenzo Valori, Forecasting the forecasts of others e la Politica di Inflation targeting
- n. 168 A.Cambini - L. Martein, First and second order optimality conditions in vector optimization
- n. 169 A. Marchi, Theorems of the Alternative by way of Separation Theorems
- n. 170 Emilio Barucci - Maria Elvira Mancino, Asset Pricing and Diversification with Partially Exchangeable random Variables
- n. 171 Piero Manfredi - Luciano Fanti, Long Term Effects of the Efficiency Wage Hypothesis in Goodwin-Type Economies.
- n. 172 Piero Manfredi - Luciano Fanti, Long Term Effects of the Efficiency wage Hypothesis in Goodwin-type Economies: a reply.
- n. 173 Luciano Fanti, Innovazione Finanziaria e Domanda di Moneta in un Modello dinamico IS-LM con Accumulazione.
- n. 174 P.Manfredi, A.Bonaccorsi, A.Secchi, Social Heterogeneities in Classical New Product Diffusion Models. I: "External" and "Internal" Models.
- n. 175 Piero Manfredi - Ernesto Salinelli, Modelli per formazione di coppie e modelli di Dinamica familiare.
- n. 176 P.Manfredi, E. Salinelli, A.Melegaro, A.Secchi, Long term Interference Between Demography and Epidemiology: the case of tuberculosis
- n. 177 Piero Manfredi - Ernesto Salinelli, Toward the Development of an Age Structure Teory for Family Dynamics I: General Frame.
- n. 178 Piero Manfredi - Luciano Fanti, Population heterogeneities, nonlinear oscillations and chaos in some Goodwin-type demo-economic models  
Paper to be presented at the: Second workshop on "nonlinear demography" Max Planck Institute for demographic Research Rostock, Germany, May 31-June 2, 2
- n. 179 E. Barucci - M.E. Mancini - Roberto Renò, Volatility Estimation via Fourier Analysis
- n. 180 Riccardo Cambini, Minimum Principle Type Optimality Conditions
- n. 181 E. Barucci, M. Giuli, R. Monte, Asset Prices under Bounded Rationality and Noise Trading
- n. 182 A. Cambini, D.T.Luc, L.Martein, Order Preserving Transformations and application.
- n. 183 Vincenzo Bruno, Variazioni dinamiche (1971-1981-1991) dei fenomeni demografici dei comuni urbani e rurali della Sicilia, in relazione ad alcune caratteristiche di mobilità territoriale.
- n. 184 F.Antonelli, E.Barucci, M.E.Mancino, Asset Pricing with a Backward-Forward Stochastic Differential Utility
- n. 185 Riccardo Cambini - Laura Carosi, Coercivity Concepts and Recession Functions in Constrained Problems
- n. 186 John R. Williams, Piero Manfredi, The pre-vaccination dynamics of measles in Italy: estimating levels of under-reporting of measles cases
- n. 187 Piero Manfredi, John R. Williams, To what extent can inter-regional migration perturb local endemic patterns? Estimating numbers of measles cases in the Italian regions
- n. 188 Laura Carosi, Johannes Jahn, Laura Martein, On The Connections between Semidefinite Optimization and Vector Optimization
- n. 189 Alberto Cambini, Jean-Pierre Crouzeix, Laura Martein, On the Pseudoconvexity of a Quadratic Fractional Function
- n. 190 Riccardo Cambini - Claudio Sodini, A finite Algorithm for a Particular d.c. Quadratic Programming Problem.
- n. 191 Riccardo Cambini - Laura Carosi, Pseudoconvexity of a class of Quadratic Fractional Functions.
- n. 192 Laura Carosi, A note on endogenous restricted participation on financial markets: an existence result.
- n. 193 Emilio Barucci - Roberto Monte - Roberto Renò, Asset Price Anomalies under Bounded Rationality.
- n. 194 Emilio Barucci - Roberto Renò, A Note on volatility estimate-forecast with GARCH models.
- n. 195 Bruno Cheli, Sulla misura del benessere economico: i paradossi del PIL e le possibili correzioni in chiave etica e sostenibile, con uno spunto per l'analisi della povertà
- n. 196 M.Bottai, M.Bottai, N. Saivati, M.Toigo, Le proiezioni demografiche con il programma Nostradamus. (Applicazione all'area pisana)
- n. 197 A. Lemmi - B. Cheli - B. Mazzolli, La misura della povertà multidimensionale: aspetti metodologici e analisi della realtà italiana alla metà degli anni '90
- n. 198 C.R. Bector - Riccardo Cambini, Generalized B-invex vector valued functions
- n. 199 Luciano Fanti - Piero Manfredi, The workers' resistance to wage cuts is not necessarily detrimental for the economy: the case of a Goodwin's growth model with endogenous population.
- n. 200 Emilio Barucci - Roberto Renò, On Measuring volatility of diffusion processes with high frequency data
- n. 201 Piero Manfredi - Luciano Fanti, Demographic transition and balanced growth

### Anno: 2001

- n. 202 E.Barucci - M. E. Mancini - E. Vannucci, Asset Pricing, Diversification and Risk Ordering with Partially Exchangeable random Variables
- n. 203 E. Barucci - R. Renò - E. Vannucci, Executive Stock Options Evaluation.
- n. 204 Odo Barsotti - Moreno Toigo, Dimensioni delle rimesse e variabili esplicative: un'indagine sulla collettività marocchina immigrata nella Toscana Occidentale
- n. 205 Vincenzo Bruno, I Consumi voluttuari, nell'ultimo trentennio, in Italia
- n. 206 Michele Longo, The monopolist choice of innovation adoption: A regular-singular stochastic control problem
- n. 207 Michele Longo, The competitive choice of innovation adoption: A finite-fuel singular stochastic control problem.
- n. 208 Riccardo Cambini - Laura Carosi, On the pseudoaffinity of a class of quadratic fractional functions
- n. 209 Riccardo Cambini - Claudio Sodini, A Finite Algorithm for a Class of Non Linear Multiplicative Programs.
- n. 210 Alberto Cambini - Dinh The Luc - Laura Martein, A method for calculating subdifferential Convex vector functions
- n. 211 Alberto Cambini - Laura Martein, Pseudolinearity in scalar and vector optimization.
- n. 212 Riccardo Cambini, Necessary Optimality Conditions in Vector Optimization.
- n. 213 Riccardo Cambini - Laura Carosi, On generalized convexity of quadratic fractional functions.
- n. 214 Riccardo Cambini - Claudio Sodini, A note on a particular quadratic programming problem.
- n. 215 Michele Longo - Vincenzo Valori, Existence and stability of equilibria in OLG models under adaptive expectations.

## Elenco dei report pubblicati

---

- n. 216 Luciano Fanti - Piero Manfredi, Population, unemployment and economic growth cycles: a further explanatory perspective
- n. 217 J.R.Williams, P.Manfredi, S.Salmaso, M.Ciofi, Heterogeneity in regional notification patterns and its impact on aggregate national case notification data: the example of measles in Italy.
- n. 218 Anna Marchi, On the connectedness of the efficient frontier: sets without local efficient maxima
- n. 219 Laura Lecchini - Odo Barsotti, Les disparités territoriales au Maroc au travers d'une optique de genre.

### Anno: 2002

---

- n. 220 Gilberto Ghilardi - Nicola Orsini, Sull'uso dei modelli statistici lineari nella valutazione dei sistemi formativi.
- n. 221 Andrea Mercatanti, Un'analisi descrittiva dei laureati dell'Università di Pisa
- n. 222 E. Barucci - C. Impenna - R. Renò, The Italian Overnight Market: microstructure effects, the martingale hypothesis and the payment system.
- n. 223 E. Barucci, P.Malliavin, M.E.Mancino, R.Renò, A.Thalmaier, The Price-volatility feedback rate: an implementable mathematical indicator of market stability.
- n. 224 Andrea Mercatanti, Missing at random in randomized experiments with imperfect compliance
- n. 225 Andrea Mercatanti, Effetto dell'uso di carte Bancomat e carte di Credito sulla liquidità familiare: una valutazione empirica
- n. 226 Piero Manfredi - John R. Williams, Population decline and population waves: their impact upon epidemic patterns and morbidity rates for childhood infectious diseases. Measles in Italy as an example.
- n. 227 Piero Manfredi - Marta Ciofi degli Atti, La geografia pre-vaccinale del morbillo in Italia. I. Comportamenti di contatto e sforzi necessari all'eliminazione: predizioni dal modello base delle malattie prevenibili da vaccino.
- n. 228 I.M.Stancu-Minasian, Optimality Conditions and Duality in Fractional Programming Involving Semilocally Preinvex and Related
- n. 229 Nicola Salvati, Un software applicativo per un'analisi di dati sui marchi genetici (Genetic Markers)
- n. 230 Piero Manfredi, J. R. Williams, E. M. Cleur, S. Salmaso, M. Ciofi, The pre-vaccination regional landscape of measles in Italy: contact patterns and related amount of needed eradication efforts (and the "EURO" conjecture)
- n. 231 Andrea Mercatanti, I tempi di laurea presso l'Università di Pisa: un'applicazione dei modelli di durata in tempo discreto
- n. 232 Andrea Mercatanti, The weak version of the exclusion restriction in causal effects estimation: a simulation study
- n. 233 Riccardo Cambini and Laura Carosi, Duality in multiobjective optimization problems with set constraints
- n. 234 Riccardo Cambini and Claudio Sodini, Decomposition methods for nonconvex quadratic programs
- n. 235 R.Cambini and L. Carosi and S.Schaible, Duality in fractional optimization problems with set constraints
- n. 236 Anna Marchi, On the mix-efficient points

### Anno: 2003

---

- n. 237 Emanuele Vannucci, The valuation of unit linked policies with minimal return guarantees under symmetric and asymmetric information hypotheses
- n. 238 John R Williams - Piero Manfredi, Ageing populations and childhood infections: the potential impact on epidemic patterns and morbidity
- n. 239 Bruno Cheli, Errata Corrigé del Manuale delle Impronte Ecologiche (2002) ed alcuni utili chiarimenti
- n. 240 Alessandra Petrucci-Nicola Salvati-Monica Pratesi, Stimatore Combinato r Correlazione Spaziale nella Stima per Piccole Aree
- n. 241 Riccardo Cambini - Laura Carosi, Mixed Type Duality for Multiobjective Optimization Problems with set constraints
- n. 242 O.Barsotti, L.Lecchini, F.Benassi, Foreigners from central and eastern European countries in Italy: current and future perspectives of eu enlargement
- n. 243 A. Cambini - L. Martein - S. Schaible, Pseudoconvexity under the Charnes-Cooper transformation
- n. 244 Eugene M. Cleur, Piero Manfredi, and John R. William, The pre-and post-Vaccination regional dynamics of measles in Italy: Insights from time series analysis

### Anno: 2004

---

- n. 245 Emilio Barucci - Jury Falini, Determinants of Corporate Governance in Italy: Path dependence or convergence?
- n. 246 R. Cambini - A. Marchi, A note on the connectedness of the efficient frontier
- n. 247 Laura Carosi - Laura Martein, On the pseudoconvexity and pseudolinearity of some classes of fractional functions
- n. 248 E. Barucci - R. Monte - B. Trivellato, Bayesian nash equilibrium for insider trading in continuous time
- n. 249 Eugene M. Cleur, A Time Series Analysis of the Inter-Epidemic Period for Measles in Italy
- n. 250 Andrea Mercatanti, Causal inference methods without exclusion restrictions: an economic application.
- n. 251 Eugene M. Cleur, Non-Linearities in Monthly Measles data for Italy
- n. 252 Eugene M. Cleur, A Threshold Model for Prevaccination Measles Data: Some Empirical Results for England and Italy