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Abstract

The method of maximum likelihood leads to an ill-posed optimization problem in the case of relaxing the exclusion restriction when using the instrumental variables method for estimating causal effects. Estimation is reformulated using simple constraints into an optimization problem having a strongly consistent global solution.

1 Introduction

Estimation of mixed distribution models is a frequent and usually not easy task in evaluating causal effects under mild assumption. For example this is the case of evaluating causal effects for compliers without assuming the exclusion restriction, or in quantifying indirect causal effects in principal stratification models when the monotonicity assumption is relaxed. Indeed the estimation of mixed distributions models implies analytical and computational difficulties principally due to the fact that mixed models are weakly identified, in the sense that corresponding likelihood functions usually have more than one maximum points.

This paper proposes a way for evaluating the local average treatment effect, that is the causal effect for compliers using the randomized experiment with non-compliance theoretical framework, when the exclusion restriction is fully relaxed. Indeed the assumption of exclusion restriction can be often unrealistic in practice. However testing the violation of the assumption is not straightforward, since the assumption is directly related to the identifiability of the instrumental variables model. Given this identifiability problem, previous studies demonstrated the possibility of testing the assumption using weakly identified models in the Bayesian framework (Hirano et al., 2000; Imbens and Rubin, 1997). Without the exclusion restriction, the instrumental variables models are considered as weakly identified, since they show proper posterior distributions, but they do not have unique maximum likelihood estimates. In these models relaxation of the exclusion restriction assumption relies on auxiliary information such as from proper priors.

The current study explores a new option, where we does not introduce extra informations apart assuming a normal distribution for the outcome. We show that relaxing the exclusion restriction introduce in the likelihood two mixtures of distributions producing a likelihood having more than one maximum points. For analysis purposes we propose a maximization constrained to a parametric sub-space having a strongly consistent global solution.

2 Some problems in maximizing the likelihood when the exclusion restriction is fully relaxed

Assume the simplest experimental setting where there is only one outcome measure (Y_i), the treatment assignment (Z_i) is binary ($1 =$treatment, $0 =$control) and the treatment received (D_i) has only two levels ($1 =$received, $0 =$not received). Angrist et al. (1996) defined four behavior types based on treatment receipt status of individuals given treatment assignment status. Let $D_i(1)$ denote the potential treatment receipt status for individual i when assigned to the treatment condition, and $D_i(0)$ denote the potential treatment receipt status for individual i when assigned to the control condition. Compliers are subjects who do what they are assigned to do [$D_i(1) = 1$ and $D_i(0) = 0$]. Never-takers are subjects who do not receive the treatment condition [$D_i(1) = 0$ and $D_i(0) = 0$]. Defiers are subjects who do the opposite of what they are assigned to do [$D_i(1) = 0$ and $D_i(0) = 1$]. Always-takers are subjects who always receive the treatment no matter which condition they are assigned to [$D_i(1) = 1$ and $D_i(0) = 1$]. This classification in groups will be called compliance status in the rest of the paper. It is worth to remember that the template of a randomized experiment with imperfect compliance can be adopted for the identification and estimation of treatment causal effects also in non-experimental situations. Angrist et al. (1996) show under which set of assumptions, a regression analysis supported by the use of instrumental variables identifies causal treatment effects in observational studies. The template is that of a randomized experiment with imperfect compliance in the sense that the particular instrumental variable adopted should have the role of a random assignment for which the treatment does not necessarily comply.

Imposing the exclusion restriction is a necessary condition for identifying causal effects using Instrumental Variable Estimator. This assumption states that the assignment to treatment has no direct effect on the outcome. The current study employs a maximum likelihood estimation approach, which is known to be often more efficient than the traditional Instrumental Variables approach in the estimation of Complier Average Causal Effect (Imbens and Rubin, 1997; Little and Yau, 1998). The likelihood function for a randomized experiment with non-compliance without the exclusion restriction, using the classical parameterization, and under this set of assumptions:

Assumption 1 *S.U.T.V.A. (Stable Unit Treatment Value Assumption)* by which for any unit the potential quantities are unrelated to the other units treatments (Angrist et al., 1996),

Assumption 2 *"Random assignment to treatment"* by which the probability to be assigned to the treatment is the same for every unit (Angrist et al., 1996),

Assumption 3 *"Monotonicity"* imposing the absence of defiers (Angrist et al., 1996),

Assumption 4 normal distribution for the outcome,

can be written:

$$L(\boldsymbol{\theta}) = \prod_{i\in(D_i=1,Z_i=0)} (1-\pi)\cdot\omega_a\cdot N(y_i|\mu_{a0},\sigma_{a0}) \times \prod_{i\in(D_i=0,Z_i=1)} \pi\cdot\omega_n\cdot N(y_i|\mu_{n1},\sigma_{n1})$$

$$\times \prod_{i\in(D_i=1,Z_i=1)} \pi\cdot[\omega_a\cdot N(y_i|\mu_{a1},\sigma_{a1}) + \omega_c\cdot N(y_i|\mu_{c1},\sigma_{c1})]$$

$$\times \prod_{i\in(D_i=0,Z_i=0)} (1-\pi)\cdot[\omega_n\cdot N(y_i|\mu_{n0},\sigma_{n0}) + \omega_c\cdot N(y_i|\mu_{c0},\sigma_{c0})], \tag{1}$$

$$\Omega : \left\{ \boldsymbol{\theta} = (\omega_t, \mu_{tz}, \sigma_{tz}, \pi) \in R^{16} | \sum_t \omega_t = 1;\ \omega_t > 0 \vee t;\ \sigma_{tz} > 0 \vee t \vee z;\ 0 < \pi < 1 \right\}$$

where ω_t is the mixing probability, that is the probability of being in the t group, $t = a, n, c$; μ_{tz} is the mean of Y_i for the units in the t group and assigned to z; σ_{tz} is the standard error for the units in the t group and assigned to z, and π is the probability of assignment to treatment $Z_i = 1$.

The maximization of (1) faces analytical and computational difficulties due to the two mixture of normal distributions involved. Indeed the mixtures of normal distributions presents some analytical characteristics causing .... In order to clarify the issue, let consider the density for a mixture of two normal distributions with unequal variances:

$$f(y; \boldsymbol{\theta}) = \sum_{i=1}^{2} \omega_i \cdot N(y; \mu_i, \sigma_i),$$

$$\Omega : \boldsymbol{\theta} = \left\{ (\omega_1, \omega_2, \mu_1, \mu_2, \sigma_1, \sigma_2) \in R^6 | \sum_{i=1}^{2} \omega_i = 1;\ \omega_i > 0,\ \sigma_i > 0 \vee i \right\},$$

$$L(\boldsymbol{\theta}) = \prod_{j=1}^{n} \sum_{i=1}^{2} \omega_i \cdot N(y_j; \mu_i, \sigma_i). \tag{2}$$

The first problem associated with maximum likelihood estimation arises from the unboundedness of (2) on Ω (Day, 1969). A global maximum-likelihood estimate always fails to exist. In addition the unboundedness of (2) causes failures of optimization algorithms of both the EM (Redner and Walker, 1984) and quasi-Newton (Fowlkes, 1979) types.

In spite of the unboundedness of (2), statistical theory (Kiefer, 1978) guarantees that a particular local maximizer of (2) is strongly consistent and asymptotically efficient. Several maximizers can exist for a given sample, and the other major maximum-likelihood difficulty is in determining when the correct one has been found. Day (1969) noted that spurious maximizers, corresponding to parameter points having a component standard deviation, σ_1^2 or σ_2^2, very small, are generated by any small number of sample points grouped sufficiently close together. The spurious maximizers, like the unboundedness of (2), can create difficulties when using the EM or quasi-Newton algorithms.

Some alternative methods were proposed in the literature to obtain a maximum likelihood estimation of an univariate normal mixture distribution model. For example a general approach, that is a sequence of unrestricted maximizations and a subsequent analysis of the local maximum points in order to detect the spurious maximum points (McLachlan e Peel, 2000). This method is straightforward, even if a sufficiently exhaustive detection of maximum points could be time consuming. Alternatively, other methods were proposed but at the cost of introducing extra informations respect to the general approach. For example, Furman (1994) proposed a likelihood maximization restricted to appropriate parameter subspaces identified by exploiting a priori informations about the mixing probabilities and the variance components order. For what concern the (2) this approach suggests a maximization restricted to the parameter subspace satisfying:

$$\omega_i > \varepsilon, \quad \sigma_i \geq c\,\sigma_{i+1}, \quad i = 1, 2;\ \varepsilon > 0;\ c > 0.$$

Another method concerns the introduction of a penalized term in the (2) (Ridolfi and Idier, 2002): $L^P(\boldsymbol{\theta}) \propto L(\boldsymbol{\theta})\ \ p(\sigma_1,\, \sigma_2)$, in order to obtain a bounded likelihood. The authors show that if the term $p(\sigma_1,\, \sigma_2)$ is the product of two inverse Gamma distributions then the $L^P(\boldsymbol{\theta})$ is bounded.

The analysis of (1) is harder respect to (2) because of the label switching problem, that occurs when some of the labels of the mixture components permute. It is well known (McLachlan and Peel, 2000) that in a finite mixture of distributions in the same class, $f(\mathbf{x}; \boldsymbol{\theta}) = \sum_{i=1}^{g} \omega_i\, f_i(\mathbf{x}; \boldsymbol{\theta}_i)$, the parameter vector $\boldsymbol{\theta}$ is not identified. Because of $f(\mathbf{x}; \boldsymbol{\theta})$ is invariant under the $g!$ permutations of the component label in $\boldsymbol{\theta}$, then only a class of distributions $f(\mathbf{x}; \boldsymbol{\theta})$ is identified. The presence of two component densities $N(y|\boldsymbol{\theta}_1)$ and $N(y|\boldsymbol{\theta}_2)$, with $\boldsymbol{\theta}_i = (\mu_i, \sigma_i)$ $i = 1, 2$, in (2) implies that $f(y; \boldsymbol{\theta}) = f(y; \boldsymbol{\theta}^*)$ if the component labels 1 and 2 are interchanged in $\boldsymbol{\theta}$. This means that only the set of parameter vectors invariant respect to the order of labelling the components is identified. Consequently the likelihood functions for mixtures having all the g components in the same class are invariant respect to the $g!$ permutations in the labels. Despite the label switching is not a relevant problem in the maximum likelihood estimation of a same class components mixture model for cluster analysis purposes, the estimation of a randomized experiment with imperfect compliance without exclusion restriction can suffer from this inconvenience. An alternative choice of the parameter vector, more natural in this mixture based approach, can be now introduced. The sub-vector $\boldsymbol{\omega}_t = (\omega_a, \omega_n, \omega_c)$ can be indeed substituted with $\boldsymbol{\omega}_{tz} = (\omega_{a0}, \omega_{a1}, \omega_{n0}, \omega_{n1}, \omega_{c0}, \omega_{c1})$, where ω_{tz} is probability of being in the $v(t,z)$ group of the units in the compliance status t and assigned to z. The proposed decomposition is possible if taking into account that $\omega_{tz} = \omega_t \, I(z=1)\, \pi + \omega_t \, I(z=0)\,(1-\pi)$, and it produces a likelihood function equivalent to the (1):

$$L(\boldsymbol{\theta}) = \prod_{i \in (D_i=1, Z_i=0)} \omega_{a0} \cdot N(y_i | \mu_{a0}, \sigma_{a0}) \times \prod_{i \in (D_i=0, Z_i=1)} \omega_{n1} \cdot N(y_i | \mu_{n1}, \sigma_{n1})$$

$$\times \prod_{i \in (D_i=1, Z_i=1)} \left[\omega_{a1} \cdot N(y_i | \mu_{a1}, \sigma_{a1}) + \omega_{c1} \cdot N(y_i | \mu_{c1}, \sigma_{c1})\right]$$

$$\times \prod_{i \in (D_i=0, Z_i=0)} \left[\omega_{n0} \cdot N(y_i | \mu_{n0}, \sigma_{n0}) + \omega_{c0} \cdot N(y_i | \mu_{c0}, \sigma_{c0})\right], \qquad (3)$$

$$\Omega : \left\{ \boldsymbol{\theta} = (\omega_{tz}, \mu_{tz}, \sigma_{tz}) \in R^{21} | \sum_t \sum_z \omega_{tz} = 1;\, \omega_{tz} > 0,\, \sigma_{tz}, > 0 \vee t \vee z \right\}.$$

The parameter π, that in a mixtures based analysis of (1) can be considered as a bisturbance, has been eliminated by this new definition for $\boldsymbol{\theta}$. Moreover, the new parameterization allows a direct introduction of the six counterfactual groups, $v(t,z)$, in which the population can be subdivided. Like every maximum likelihood analysis of a finite mixtures model for cluster analysis purposes, the wrong labelling of the components for at least one mixture in the (3) does not imply difficulties in the identification of the model. But this is not the our case; the causal effects from a counterfactual point of view are defined by the three differences $\Delta_t = (\mu_{t1} - \mu_{t0})$, where $t = a, n, c$, and consequently their identification implies a right labelling of all the components. For example, let consider an hypotetical local maximum point for the likelihood function, $\hat{\boldsymbol{\theta}}$, for which the component labels of the mixture formed by assigned always-takers and assigned compliers permute. The corresponding mixture density function is:

$$f(y; \omega_{a1}, \omega_{c1}, \mu_{a1}, \mu_{c1}, \sigma_{a1}, \sigma_{c1}) = \omega_{a1} \cdot N(y | \mu_{a1}, \sigma_{a1}) + \omega_{c1} \cdot N(y | \mu_{c1}, \sigma_{c1}). \quad (4)$$

In the (4) the causal effects of the assignment to treatment for always-takers and compliers are not identified because of the permutation of labels components in $\hat{\boldsymbol{\theta}}$. Indeed, the causal effect for compliers Δ_c in $\hat{\boldsymbol{\theta}}$ is wrongly identified by $(\mu_{a1} - \mu_{c0})$ instead of $(\mu_{c1} - \mu_{c0})$, and the causal effect for always-takers Δ_a is wrongly identified by $(\mu_{c1} - \mu_{a0})$ instead of $(\mu_{a1} - \mu_{a0})$.

3 A restricted maximization procedure

In the recent literature some methods for relaxing the exclusion restriction were proposed on the bases of exploiting extra informations respect to the assumptions 1-4 presented in the previous Section. For example, Hirano et al. (2000) worked in a Bayesian context and they adopted a relatively diffuse but proper prior distribution, and more recently Jo (2002) studied alternative model specifications allowing the identification of causal effects in the presence of observed pre-treatment informations. Moreover, the general approach for analyzing a mixture model is not feasible in a maximum likelihood analysis of the (1) because of the label switching problem. An alternative approach can be proposed if considering that the estimation of the mixing proportions $(\omega_a, \omega_n, \omega_c)$ can be a straightforward task even out of a maximum likelihood context and without introducing extra assumptions respect to the 1-4 of the previous Section. The estimated mixing proportions can be exploited in a maximum likelihood estimation of $\boldsymbol{\theta}$, by constraining the analysis to appropriate parametric subspaces. Then, in this Section a constrained maximization facilitating the identification of the consistent maximum point is proposed. We show also how the EM algorithm can help in making easier the detection.

Under the parameterization (1), the three probabilities sub-vector $\boldsymbol{\omega}_t = (\omega_a, \omega_n, \omega_c)$ can be estimated respectively by (Imbens and Rubin, 1997):

- the proportion of treated units in the group of not assigned units: $\hat{\phi}_a = \#(D=1,\, Z=0)/\#(Z=0)$;
- the proportion of untreated units in the group of assigned units: $\hat{\phi}_n = \#(D=0,\, Z=1)/\#(Z=1)$;
- the difference: $\hat{\phi}_c = 1 - \hat{\phi}_a - \hat{\phi}_n$.

In the (3), the estimated vector of $\boldsymbol{\omega}_{tz}$, $\hat{\boldsymbol{\phi}}_{tz} = \left(\hat{\phi}_{a0}, \hat{\phi}_{a1}, \hat{\phi}_{n0}, \hat{\phi}_{n1}, \hat{\phi}_{c0}, \hat{\phi}_{c1}\right)$ can be obtained by a transformation of $\hat{\boldsymbol{\phi}}_t = (\hat{\phi}_a, \hat{\phi}_n, \hat{\phi}_c)$:

$$\hat{\phi}_{a0} = \frac{\#(D=1,\, Z=0)}{N},\ \hat{\phi}_{a1} = \hat{\phi}_a - \hat{\phi}_{a0},$$

$$\hat{\phi}_{n0} = \hat{\phi}_n - \hat{\phi}_{n1},\ \hat{\phi}_{n1} = \frac{\#(D=0,\, Z=1)}{N},$$

$$\hat{\phi}_{c0} = \frac{\#(D=0,\, Z=0)}{N} - \hat{\phi}_{n0},\ \hat{\phi}_{c1} = \frac{\#(D=1,\, Z=1)}{N} - \hat{\phi}_{a1},$$

where N is the sample size. Given this set of informations a constrained maximization of (3) to a non-spherical neighborhood of $\hat{\boldsymbol{\phi}}_{tz}$ can be proposed. This procedure would identify the local maximum $\hat{\boldsymbol{\theta}}^{ML}$ satisfying the constrains:

$$|\hat{\phi}_{a1} - \hat{\omega}_{a1}^{ML}| \leq c_{a1},\ |\hat{\phi}_{n0} - \hat{\omega}_{n0}^{ML}| \leq c_{n0},$$

$$|\hat{\phi}_{c1} - \hat{\omega}_{c1}^{ML}| \leq c_{c1},\ |\hat{\phi}_{c0} - \hat{\omega}_{c0}^{ML}| \leq c_{c0}. \tag{5}$$

A difficulty in running the proposed restricted procedure emerges if considering that the constrains c_{a1}, c_{n0}, c_{c1}, and c_{c0} have to be calibrated taking into account the values $\hat{\phi}_{a1}$, $\hat{\phi}_{n0}$, $\hat{\phi}_{c1}$, and $\hat{\phi}_{c0}$. The relative weight for a certain value of the generic constraint c_{tz} is clearly proportional to the corresponding value of $\hat{\phi}_{tz}$. A more direct control about the two mixtures in the (1) or in the (3) could be achieved by imposing some constraints on the conditional mixing probabilities: $\omega_{t|dz} = P(C_i = t|D_i = d, Z_i = z)$. This requires reformulating the (3) in order to make the likelihood as a function of the probabilities $\omega_{t|dz}$; the task is not difficult if taking into account the relationship:

$$\omega_{tz} = \omega_{0z}\,\omega_{t|0z} + \omega_{1z}\,\omega_{t|1\,z},$$

where ω_{dz} is the probability to take the treatment d, if assigned to z. The result is:

$$L(\boldsymbol{\theta}) = \prod_{i\in(D_i=1,Z_i=0)} \omega_{10} \cdot N\left(y_i|\mu_{a0}, \sigma_{a0}\right) \times \prod_{i\in(D_i=0,Z_i=1)} \omega_{01} \cdot N\left(y_i|\mu_{n1}, \sigma_{n1}\right)$$

$$\times \prod_{i \in (D_i=1, Z_i=1)} \omega_{11} \left[\omega_{a|11} \cdot N\left(y_i|\mu_{a1}, \sigma_{a1}\right) + \omega_{c|11} \cdot N\left(y_i|\mu_{c1}, \sigma_{c1}\right)\right]$$

$$\times \prod_{i \in (D_i=0, Z_i=0)} \omega_{00} \left[\omega_{n|00} \cdot N\left(y_i|\mu_{n0}, \sigma_{n0}\right) + \omega_{c|00} \cdot N\left(y_i|\mu_{c0}, \sigma_{c0}\right)\right]. \quad (6)$$

The three likelihood functions (1), (3) and (6) are equivalent for maximum likelihood purposes. Again, the estimated vector $\hat{\phi}_{t|dz}$ of the conditional probabilities $\omega_{t|dz}$, are easily obtainable out of a maximum likelihood context given the conditions:

$$\sum_t \hat{\phi}_{t|dz} = 1, \omega_{n|11} = \omega_{a|00} = \omega_{a|01} = \omega_{n|10} = \omega_{c|01} = \omega_{c|10} = 0. \quad (7)$$

The results are:

$$\hat{\phi}_{a|11} = \frac{\hat{\phi}_{a1}}{\hat{\phi}_{a1} + \hat{\phi}_{c1}}, \hat{\phi}_{c|11} = \frac{\hat{\phi}_{c1}}{\hat{\phi}_{a1} + \hat{\phi}_{c1}}, \hat{\phi}_{n|00} = \frac{\hat{\phi}_{n0}}{\hat{\phi}_{n0} + \hat{\phi}_{c0}}, \hat{\phi}_{c|00} = \frac{\hat{\phi}_{c0}}{\hat{\phi}_{n0} + \hat{\phi}_{c0}},$$

$$\hat{\phi}_{a|10} = 1, \hat{\phi}_{n|01} = 1.$$

The new formulation (6) for the likelihood allows a restriction of the analysis to a spherical neighborhood of $\hat{\boldsymbol{\phi}}_{t|dz} = (\hat{\phi}_{a|11}, \hat{\phi}_{c|11}, \hat{\phi}_{n|00}, \hat{\phi}_{c|00})$. This procedure would identify the local maximum point $\hat{\boldsymbol{\theta}}^{ML}$ satisfying:

$$|\hat{\phi}_{a|11} - \hat{\omega}^{ML}_{a|11}| \leq c, |\hat{\phi}_{c|11} - \hat{\omega}^{ML}_{c|11}| \leq c,$$

$$|\hat{\phi}_{n|00} - \hat{\omega}^{ML}_{n|00}| \leq c, |\hat{\phi}_{c|00} - \hat{\omega}^{ML}_{c|00}| \leq c. \quad (8)$$

Respect to the previous set of constraints (5) is now possible to perform a maximization restricted to a spherical neighborhood, then bypassing the problem related to the relative weights of the constraints. The probabilities comparing in (8) are indeed expressed conditionally on the mixture belongness and for this reason the value of $\hat{\phi}_{a|11}$ does not matter in the choice of c.

The proposed restrict procedure is then equivalent to a likelihood maximization over the set:

$$\Omega_c^{\hat{\phi}}: \Big\{\boldsymbol{\theta} = (\omega_{t|dz}, \omega_{dz}, \mu_{tz}, \sigma_{tz}) \in R^{28} | \omega_{n|11} = \omega_{a|00} = \omega_{a|01} = \omega_{n|10} = \omega_{c|01} = \omega_{c|10} = 0;$$

$$|\hat{\phi}_{t|dz} - \omega_{t|dz}| \leq c; \sum_t \omega_{t|dz} = \sum_d \sum_z \omega_{dz} = 1; \ \omega_{t|dz} > 0, \ \omega_{dz} > 0, \sigma_{tz}, > 0 \ \forall t \ \forall z \Big\}.$$

It is worth to note that under the conditions (7) imposing the four constraints (8) are equivalent, for maximum likelihood purposes, to impose only two constraints, that is a single constraint for any mixtures; for example:

$$|\hat{\phi}_{c11} - \omega_{c11}| < c, \ |\hat{\phi}_{c00} - \omega_{c00}| < c.$$

From a computational point of view, the EM algorithm can be efficiently adopted in this context. This is attractive in making maximum likelihood inference in our context because if the compliance status C_i was known for all units, the likelihood would not involve mixtures. Then the compliance status of the units in one of the two mixtures can be considered as a missing information whose imputation produce the so-called augmented likelihood. Moreover, in our context the augmented log-likelihood function is linear in the missing informations, so the EM algorithm corresponds to fill-in missing data and then upddating parameter estimates. The imputation of the unobserved compliance status is handled by the E-step; it requires the calculation of the conditional expectation of C_i given the observed data and the current fit for $\boldsymbol{\theta}$. The compliance status C_i can be represented by a three components indicator $t = c$ *(complier)*, n *(never-taker)*, a *(always taker)*; and at the k-iteration conditional probabilities of unit i being type t given that the unit is in the $(D_i = d, \ Z_i = z)$ group are obtainable by:

$$\tau_{it|dz}^{(k)}(\hat{\boldsymbol{\theta}}^{(k-1)}) = \frac{\hat{\omega}_{t|dz}^{(k-1)} \cdot N(y_i | \hat{\mu}_{tz}^{(k-1)}, \hat{\sigma}_{tz}^{(k-1)})}{\sum_t \hat{\omega}_{t|dz}^{(k-1)} \cdot N(y_i | \hat{\mu}_{tz}^{(k-1)}, \hat{\sigma}_{tz}^{(k-1)})},$$

where $\hat{\omega}_{t|dz}^{(k-1)}$, $\hat{\mu}_{tz}^{(k-1)}$, and $\hat{\sigma}_{tz}^{(k-1)}$ are the estimates of $\omega_{t|dz}$, μ_{tz}, σ_{tz} calculated during the $(k-1)$ iteration.

The subsequent M-step maximizes the augmented log-likelihood by updating the estimated parameter vector; in particular the updated conditional probability $\hat{\omega}_{t|dz}^{(k)}$ results:

$$\hat{\omega}_{t|dz}^{(k)} = \frac{\sum_{i\in(D_i=d,\, Z_i=z)} \tau_{it|dz}^{(k)}(\hat{\boldsymbol{\theta}}^{(k-1)})}{\#\,(D_i = d,\ Z_i = z)}.$$

In order to satisfy the spherical constraints imposed in the (8), the results from the M-step can be easily checked. This means to introduce another step, immediately after the M-step, for testing if at the k iteration: $\left|\hat{\phi}_{t|dz} - \hat{\omega}_{t|dz}^{(k)}\right| < c$. Eventually the values $\hat{\omega}_{t|dz}^{(k)}$ for which $\hat{\omega}_{t|dz}^{(k)} > \hat{\phi}_{t|dz} + c$ have to be posed $\hat{\omega}_{t|dz}^{(k)} = \hat{\phi}_{t|dz} + c$, and the values for which $\hat{\omega}_{t|dz}^{(k)} < \hat{\phi}_{t|dz} - c$ have to be posed $\hat{\omega}_{t|dz}^{(k)} = \hat{\phi}_{t|dz} - c$.

4 Conclusions

The paper concerns the problem of relaxing the exclusion restriction under the assumptions usually adopted for the identification and estimation of causal effects with instrumental variables. The main difficulties in this task is due to the presence of mixtures of distributions implying weakly identified models.

The imposition of simple constraints of the form (8) yields a maximum-likelihood problem which is well posed optimizationally. The constrained formulation is statistically well posed in that the global solutions are strongly consistent (Kiefer, 1978). Problems associated with singularities do not exist, and those associated with spurious maximizers should be at least lessened. Moreover, for computational purposes and exploiting the particular incomplete structure of the likelihood a constrained EM algorithm can be easily developed.

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