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A model for analyzing the effects of information asymmetries of the traders

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Abstract. In this paper it is considered a stylized market with heterogeneously informed traders which is a generalisation of the market described in Schredelseker (1999). Such market model suggests some unusual points of view of more realistic markets and it leads to some interesting considerations about the expected gains both of the traders and of the intermediation system, the necessity of informative levels differentiation, the significance of the volatility ...

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0. Introduction

When a whatever fixing price mechanism is based on bid and ask prices announced by the traders, the evaluation of the advantage of the most informed traders respect to the others, is a subject widely considered in the literature.

In Glosten, Milgrom (1985) and in Khrishna (2002) it is underlined that in case of repeated trading, the most informed traders, while they raise some gains, progressively release their informations to less informed traders, whereas in Cenci, Cerquetti (1991) it is analyzed the effect of imitative behaviours from less informed traders about the possibility of reaching equilibria.

In Schredelseker (1999) it is considered a market model of a perfectly divisible security in which operate a finite number of traders with different levels of correct information, which is supposed cost-free. Furthermore, it is assumed that each trader is a risk neutral expected gain maximizer and that he is willing to buy or to sell at most one unit of the security at a price that will be fixed by intermediation system (or intermediary tout court) before transactions time, while the true value of the security will be common knowledge only after the transactions time.

One of the aims of Schredelseker's paper is to show that, with this kind of traders, in some cases the "medium informed" ones reach worse results not only of those of more informed ones, and it was only to be expected, but also, and this is really unexpected, of those of less informed ones. Author shows that for some traders it is profitable to not exploit correct informations.

In this paper we consider one of the most significative developments of such model, that is the introduction of transaction costs which level is, obviously, supposed known by all traders. In Schredelseker's model the intermediary, on the basis of bid and ask prices announced by each trader (but, let observe that without transaction costs these two prices coincide), fixes the market price with the aim of maximizing trading volumes, while in this paper, in which transaction costs are considered (and in this case, obviously, bid and ask prices of each trader are not coincident), the intermediary acts with the aim of maximizing the expected value of the collected commissions.

Hence, the aim of this paper is not only to give a contribution about the analysis of the effects of different levels of information in terms of the expected gains of the traders, but also to show how information asymmetries of the traders act on the gain of the intermediation system.

Let observe that whatever the price fixed by the intermediary is, on the basis of bid and ask prices announced by each trader, the set of traders is divided in three subsets: one of sellers, one of buyers (and not necessarily these two sets have the same cardinality) and the last one of traders that do not partecipate in trading. As already mentioned, the true value of the traded security is known only after transactions time: buyers gain if intermediary has sufficiently underpriced the security and lose if intermediary has sufficiently overpriced it. Vice versa, obviously, for sellers. "Sufficiently" is referred to the fact that the payment of transactions costs by the traders who partecipate in trading, implies that the algebraic sum of their gains is negative and the opposite figure is just the gain of the intermediary.

Concerning to the possibility of describing real markets, the market model proposed in this paper is a quite good approximation of equities opening market, see for example Caparrelli (1995). Anyway, the main aim of such paper is undoubtedly theoretical and it consists in giving an original point of view of economic relationships between more informed traders (paradoxally from insider traders to institutional investors ...), less informed traders (from small savers that know all the informations given by the media to less and less informed investors that choose their investments randomly) and the intermediary of the market in which they operate.

The paper has the following structure. In the first section the market model is described: in particular, how is defined the price at which transactions happen and how are determined the gains of the traders and of the intermediary. In the second one it is considered a model with only two groups (in terms of informative level) of traders in order to explain some details of the fixing price mechanism and of the determination of traders and intermediary's gains. In the third section we propose some numerical examples in order to describe the sensitivity analysis of the results respect to variations of parameters values. In the last section some conclusions are suggested.

1. The market model

In this section it is described the generalisation, with the introduction of transaction costs, of the market model described in Schredelseker (1999).

Indicating with p the market price for unit of security fixed by the intermediary, the transaction costs, that has to be payed both by sellers and by buyers, are equal to αp, where α, the commission level, is a positive real coefficient (in practice to be considered relatively near to 0): so the intermediary receives $2\alpha p$ for each traded unit. It is obvious that if $\alpha = 0$ the market model without transaction costs is reproduced.

It is assumed that the true value of the security, V, is unknown before transactions time. V is given by the number of heads, multiplied by a real positive coefficient c obtained with n independent tosses of a coin that provides the result "head" with probability π. In Schredelseker's model it is $c = 1$ and $\pi = 0.5$.

Hence, the ratio V/c has a binomial distribution with parameters n and π. It holds

$$E\left[V\right] = cn\pi \tag{1}$$

and

$$\sigma^2\left[V\right] = c^2 n\pi\left(1-\pi\right) \tag{2}$$

Let observe that to obtain a generic couple $E\left[V\right]$, $\sigma^2\left[V\right]$ one of three parameters c, π and n, can be arbitrarily fixed.

About the traders it is assumed that

- the set of traders has cardinality f
- such set is divided at most in $n+1$ groups
- the generic group h, $h = 0, 1, ..., n$, has cardinality f_h, with $f_h \geq 0$ and $f_0 + f_1 + ... + f_n = f$
- each generic trader of the group h, $h = 0, 1, ..., n$, knows the result of the first h tosses of the coin (notice that the information held by each trader is "correct")
- each trader is willing to buy or to sell at most one unit of the security that is assumed perfectly divisible
- each trader is a risk neutral expected gain maximizer

In Schredelseker's market model it is supposed that each trader can use three kinds of strategies: the active, the passive and the contrarian.

The so-called passive strategy provides that the trader does not exploit his correct information and, whatever the price may be, he decides to buy or to sell by chance, advising the intermediary of his decision before of price fixing. The so-called contrarian strategy provides decisions that are the opposite of those implied by active strategy.

In this paper it is assumed that each trader applies the active strategy. Such strategy provides that if the trader of the generic group h has observed i heads on the first h tosses then he will propose as his ask price

$$a_{hi} = \frac{c\,(i + \pi\,(n-h))}{1+\alpha} \tag{3}$$

and as his bid price

$$b_{hi} = \frac{c\,(i + \pi\,(n-h))}{1-\alpha} \tag{4}$$

since he estimates as expected value of the security

$$v_{hi} = c\,(i + \pi(n-h))$$

and he has to pay αp as transaction cost. With α positive, for each trader ask price is lower than bid price, so for $h = 0, 1, ..., n$ and $i = 0, 1, ..., h$, it is $a_{hi} < b_{hi}$.

Since it is assumed that each trader is risk neutral, if the generic trader of the group h has observed i heads on the first h tosses, $i = 0, 1, ..., h$, if p is the market price fixed by the intermediary

- he buys if and only if $p < a_{hi}$
- he sells if and only if $p > b_{hi}$
- he does not partecipate in trading if $a_{hi} < p < b_{hi}$

For traders with $p = a_{hi}$ or $p = b_{hi}$ (indifference situations) it is supposed that anyway they prefer to partecipate to trading and that they respectively buy or sell.

Summarizing, whatever the market price is, no trader is willing both to buy and to sell and then the set of traders is divided in three subsets: one of sellers, one of buyers (and not necessarily these two sets have the same cardinality) and the last one of traders that do not partecipate in trading.

Paradoxally, if all traders have the same informative level and, hence, they announce the same ask and bid prices then there is no trading: so, some information asymmetries are necessary for the existence of the market!

From the previous assumptions it follows that each market scenario is described by a sequence of heads and tails obtainable with n tosses of a coin, and the admissible scenarios are obviously 2^n. Given the commission level α and one of the market scenarios, there is a set of intervals of price such that to each of them can be associated a constant number of traded units. While in Schredelseker's model the price is fixed in the middle point of the interval for which the number of traded units is maximum, in this paper the price will be fixed in order to maximize the value of the collected commissions and it always happens in an upper extreme of one of such intervals.

Given a possible scenario, let respectively $\varphi_b\,(p)$ and $\varphi_s\,(p)$ the cardinalities of the set of buyers and of the set of sellers if the market price is p. It is $\varphi_b\,(p) + \varphi_s\,(p) \leq f$ and the number of traded units is just

$$\varphi(p) = \min(\varphi_b(p), \varphi_s(p)) \leq f/2$$

If $\varphi(p) = \varphi_b(p)$ then each buyer will buy one unit and each seller will sell $\frac{\varphi_b(p)}{\varphi_s(p)}$ units, while if $\varphi(p) = \varphi_s(p)$ then each seller will sell one unit and each buyer will buy $\frac{\varphi_s(p)}{\varphi_b(p)}$ units. Naturally, in order to maximize his own gain, the intermediary will fix the market price p^* that satisfies

$$p^* = \arg\max_p [2\alpha p\varphi(p)]$$

After the transactions have happened at price p, the true value of the security v (v is one of the possible realizations of the random variable V) becomes common knowledge. The gains of the traders that partecipate in trading and of the intermediary are

- if $v \geq p(1+\alpha)$ (the intermediary has underpriced the security)

  each buyer gains $\frac{\varphi(p)}{\varphi_b(p)}(v - p(1+\alpha))$

  each seller loses $\frac{\varphi(p)}{\varphi_s(p)}(v - p(1-\alpha))$

  the intermediary gains $2\alpha p\varphi(p)$

- if $v \leq p(1-\alpha)$ (the intermediary has overpriced the security)

  each seller gains $\frac{\varphi(p)}{\varphi_s(p)}(p(1-\alpha) - v)$

  each buyer loses $\frac{\varphi(p)}{\varphi_b(p)}(p(1+\alpha) - v)$

  the intermediary gains $2\alpha p\varphi(p)$

- if $v \in (p(1-\alpha), p(1+\alpha))$ (the intermediary has fixed a price "near" to security value)

  each buyer loses $\frac{\varphi(p)}{\varphi_s(p)}(v - p(1-\alpha))$,

  each seller loses $\frac{\varphi(p)}{\varphi_b(p)}(p(1+\alpha) - v)$,

  the intermediary gains $2\alpha p\varphi(p)$

Let indicate with

- v_r the true value of the security

- $q_r = \pi^{\frac{v_r}{c}} (1-\pi)^{n-\frac{v_r}{c}}$ the probability
- p_r the market price
- φ_r the number of traded units

relative to the generic r-th market scenario, with $r = 1, 2, ..., 2^n$.

Indicating with G_h the random gain of the trader of group h, $h = 0, 1, ..., n$, and with g_{hr} the realization of such random variable in the r-th market scenario (it is $g_{hr} = 0$ if the traders of group h do not partecipate in trading), for $r = 1, ..., 2^n$, the expected gain of such trader, $E[G_h]$, is given by

$$\gamma_h \triangleq E[G_h] = \sum_{r=1}^{2^n} q_r \, g_{hr} \tag{5}$$

Notice that to obtain the aggregate gain of a group of traders with the same informative level, h, is sufficient to multiply such individual gain, g_{hr}, by the cardinality of the group, f_h.

Indicating with G_I the random gain of the intermediary, and with g_{Ir} the realization of such random variable in the r-th market scenario, $r = 1, ..., 2^n$, it holds

$$g_{Ir} = 2\alpha \, p_r \, \varphi_r$$

from which the intermediary's expected gain is given by

$$\gamma_I \triangleq E[G_I] = \sum_{r=1}^{2^n} q_r \, g_{Ir} = 2\alpha \sum_{r=1}^{2^n} q_r \, p_r \, \varphi_r \tag{6}$$

The aggregate gain of all the groups of traders and of the intermediary has to be null in each scenario, that is

$$g_{Ir} + \sum_{h=0}^{n} g_{hr} \, f_h = 0, \text{ for each } r = 1, 2, ..., 2^n$$

from which it follows for the expected gains

$$\gamma_I + \sum_{h=0}^{n} \gamma_h \, f_h = 0$$

In order to measure the distorsive effect generated by the fixing price mechanism previously described, we consider the same index used in Schredelseker (1999) that is the variance of the market mispricing. Such index is here given by

$$d = \frac{\sum_{r=1}^{2^n} q_r \ \varphi_r \ (p_r - v_r)^2}{\sum_{r=1}^{2^n} q_r \ \varphi_r} \tag{7}$$

The bigger is the value taken by such index, the bigger is the distorsive effect generated by the fixing price mechanism, and we could say the less efficient is the market. The denominator of the above ratio is just the average number of traded units, hereafter indicated with $\overline{\varphi}$.

2. Two groups of traders

For illustrative purposes, but also as application for special markets, here it is considered a market with only two groups of traders. This example allows to explain some details of the fixing price mechanism and of the determination of traders and intermediary's gains.

Let h and k the two informative levels with $h, k \in \{0, 1, .., n\}$ and $0 \leq h < k \leq n$, so the traders of group k is more informed than the traders of group h. Let f_h and f_k the cardinalities of such two groups, with $f_h + f_k = f$.

Let assume that the traders of the group h and those of the group k have observed respectively i and j heads, with obviously $i \leq h$ and $i \leq j \leq k$. Ask and bid prices announced by the traders of the two groups are respectively obtainable according to (3) and (4).

Notice that the only two possibilities for the number of traded units are 0 or $\min(f_h, f_k)$. To have transactions if the buyers are the traders of group h and the sellers ones of group k then it has to result

$$a_{hi} = \frac{c\,(i + \pi\,(n-h))}{1+\alpha} \geq \frac{c\,(j + \pi\,(n-k))}{1-\alpha} = b_{kj}$$

that is

$$(1-\alpha)\,(i + \pi\,(n-h)) \geq (1+\alpha)\,(j + \pi\,(n-k)) \tag{8}$$

while, if the buyers are the traders of group k and the sellers ones of group h then it has to result

$$a_{kj} = \frac{c\,(j + \pi\,(n-k))}{1+\alpha} \geq \frac{c\,(i + \pi\,(n-h))}{1-\alpha} = b_{hi}$$

that is

$$(1-\alpha)\,(j + \pi\,(n-k)) \geq (1+\alpha)\,(i + \pi\,(n-h)) \tag{9}$$

Notice that for positive α neither (8) nor (9) are satisfied if $j - \pi k = i - \pi h$. Given h and k, for each couple (i, j), with $j - \pi k \neq i - \pi h$, (at each market scenario is associable univocally the couple (i, j), but it does not true the vice versa) it is possible to determine the highest commission level such that the transactions happen. Specifically for such highest level it has to result from (8)

$$\alpha \leq \frac{c\,(i-j+(k-h)\,\pi)}{i+j+(2n-h-k)\,\pi}$$

or from (9)

$$\alpha \leq \frac{c\,(j-i+(h-k)\,\pi)}{i+j+(2n-h-k)\,\pi}$$

Let observe that given h and k, the number of the different possible couples (i,j) is $(h+1)\,(k-h+1)$ since i can assume $h+1$ different values and j can assume $k-h+1$ different values. The probability of the couple (i,j), $P((i,j))$ is given by

$$P((i,j)) = \binom{h}{i}\binom{k-h}{j-i}\pi^{i+j}(1-\pi)^{k-(i+j)}$$

Such probability could be seen as the sum of the probabilities of all the scenarios that imply the couple (i,j) and, hence, the same ask and bid prices.

For illustrative purposes let consider the case $h=1$ and $k=3$, with $f_1=3$ and $f_3=2$ (and hence $f=5$). Furthermore, let assume $n=4$, $\pi=0.8$, $c=0.3125$ and so it is $E[V]=1$ and $\sigma^2[V]=0.0625$. Ask and bid prices depend on the couple (i,j) of observed heads respectively after the first and after the third toss and they are represented in the following table

(i,j)	a_{1i}	b_{1i}	a_{3j}	b_{3j}
$(0,0)$	$\frac{0.75}{1+\alpha}$	$\frac{0.75}{1-\alpha}$	$\frac{0.25}{1+\alpha}$	$\frac{0.25}{1-\alpha}$
$(0,1)$	$\frac{0.75}{1+\alpha}$	$\frac{0.75}{1-\alpha}$	$\frac{0.5625}{1+\alpha}$	$\frac{0.5625}{1-\alpha}$
$(0,2)$	$\frac{0.75}{1+\alpha}$	$\frac{0.75}{1-\alpha}$	$\frac{0.875}{1+\alpha}$	$\frac{0.875}{1-\alpha}$
$(1,1)$	$\frac{1.0625}{1+\alpha}$	$\frac{1.0625}{1-\alpha}$	$\frac{0.5625}{1+\alpha}$	$\frac{0.5625}{1-\alpha}$
$(1,2)$	$\frac{1.0625}{1+\alpha}$	$\frac{1.0625}{1-\alpha}$	$\frac{0.875}{1+\alpha}$	$\frac{0.875}{1-\alpha}$
$(1,3)$	$\frac{1.0625}{1+\alpha}$	$\frac{1.0625}{1-\alpha}$	$\frac{1.1875}{1+\alpha}$	$\frac{1.1875}{1+\alpha}$

From these indications it is easy to determine for each couple (i,j) the range of values for α, indicated with $[0,\overline{\alpha}]$, that allows the transactions between the two groups of traders

(i,j)	$[0,\overline{\alpha}]$
$(0,0)$	$[0, 0.5000]$
$(0,1)$	$[0, 0.1429]$
$(0,2)$	$[0, 0.0769]$
$(1,1)$	$[0, 0.3077]$
$(1,2)$	$[0, 0.0968]$
$(1,3)$	$[0, 0.0556]$

Since in this market the trading is always possible because $j - 3\pi = i - \pi$ is never verified, if the intermediary will fix α in the interval $[0, 0.0556]$ the average number of traded units, $\overline{\varphi}$, reaches its upper limit $2 = \min(f_1, f_3) = \min(3, 2)$.

For illustrative purposes let assume $\alpha = 0.0556$. With such choice, the intermediary, in order to maximize his own gain, will determine case for case, on the basis of such commission level and of the bid and the ask prices announced by the traders, the highest price p that allows the transactions. The details are given in the following table

(i,j)	Buying group	p	$p(1+\alpha)$	$p(1-\alpha)$
$(0,0)$	1	0.7105	0.7500	0.6710
$(0,1)$	1	0.7105	0.7500	0.6710
$(0,2)$	3	0.8289	0.8750	0.7829
$(1,1)$	1	1.0065	1.0625	0.9506
$(1,2)$	1	1.0065	1.0625	0.9506
$(1,3)$	3	1.1250	1.1875	1.0625

Let indicate with

- β_1 the gain of each trader of the group 1, for which it is
  $\beta_1 = \frac{2}{3}(p(1-\alpha) - v)$ if traders of group 1 sell
  $\beta_1 = \frac{2}{3}(v - p(1+\alpha))$ if traders of group 1 buy
- β_3 the gain of each trader of the group 3, for which it is
  $\beta_3 = (p(1-\alpha) - v)$ if traders of group 3 sell
  $\beta_3 = (v - p(1+\alpha))$ if traders of group 3 buy
- β_I the gain of the intermediary, for which it is
  $\beta_I = 4p\alpha$ in each case

Such gains are respectively reported in the last three columns of the following table in which the 3-tuple $\left(i, j, \frac{v}{c}\right)$ represents both the informations known by the two groups of traders and the true value of the security and $P\left(\left(i, j, \frac{v}{c}\right)\right)$ is its probability.

$(i,j,\frac{v}{c})$	$P\left((i,j,\frac{v}{c})\right)$	β_1	β_3	β_I
$(0,0,0)$	0.0016	−0.5000	0.6710	0.1579
$(0,0,1)$	0.0064	−0.2916	0.3585	0.1579
$(0,1,1)$	0.0128	−0.2916	0.3585	0.1579
$(0,1,2)$	0.0512	−0.0833	0.0460	0.1579
$(0,2,2)$	0.0256	0.1052	−0.2500	0.1842
$(0,2,3)$	0.1024	−0.1030	0.0624	0.1842
$(1,1,1)$	0.0064	−0.5000	0.6381	0.2237
$(1,1,2)$	0.0256	−0.2916	0.3256	0.2237
$(1,2,2)$	0.0512	−0.2916	0.3256	0.2237
$(1,2,3)$	0.2048	−0.0833	0.0131	0.2237
$(1,3,3)$	0.1024	0.0833	−0.2500	0.2500
$(1,3,4)$	0.4096	−0.1250	0.0625	0.2500

In conclusion with $\alpha = 0.0556$, we have $\overline{\varphi} = 2$ (obviously), $d = 0.0317$ and for the individual expected gains

$$\gamma_1 = -0.1039 \qquad \gamma_3 = 0.0421 \qquad \gamma_I = 0.2274$$

Another choice for the commission level could be $\alpha = 0.0769$, that is the highest value that allows the transactions for all couples (i,j), except that for $(i,j) = (1,3)$. With such choice we have $\overline{\varphi} = 0.9760$ (let notice that it is $P\left((1,3)\right) = 0.5120$), $d = 0.040$ and for the individual expected gains

$$\gamma_1 = -0.0640 \qquad \gamma_3 = 0.0285 \qquad \gamma_I = 0.1348$$

Let observe that generally an increase of the commission level reduces the average number of traded units and, in this example, it advantages the traders with negative expected gains to the damage of the traders with positive expected gains and of the intermediary.

Altough it could be an interesting subject, for the lack of brevity, we do not consider the problem of the intermediary's expected gain maximization in the hypothesis in which he can act on some subsets of parameters: for example h, k, f_h and f_k or n, c and π or ...

3. More groups of traders

The description of a market with more groups of traders with different levels of information implies to use a considerable number of parameters: n that describes the level of full information, the $(n+1)$-tuple $(f_0, f_1, ..., f_n)$ of the cardinalities of the groups of traders with different levels of information (from ones that know the results of 0 tosses, that is null information, to ones that know the results of n tosses and, hence, that know the true value of the security before the transactions time), c and π that characterize the distribution of the random value of the unit of traded security, α the commission level.

The expected value, $E[V]$, and the variance, $\sigma^2[V]$, of the random value of the unit of traded security are obtained from n, c and π according to (1) and (2).

In the following examples, for an easier interpretation of the results, we prefer to fix $E[V] = 1$ and to report the value of $\sigma^2[V]$ instead of those of c and π.

In the following subsections we propose some numerical examples to sketch a sensitivity analysis respect to variations of parameters values starting from such reference setting

$$n = 6,\ E[V] = 1,\ \sigma^2[V] = 0.1,\ \alpha = 0.005,\ (f_0, f_1, ..., f_6) = (7, 6, 5, 4, 3, 2, 1)$$

that will be reported in bold type.

In the following subsections we report the expected gain of the generic trader of the group i, γ_i, with $i = 0, 1, ..., n$, skipping the groups of null cardinality, the expected gain of the intermediary, γ_I, the average number of the transactions, $\overline{\varphi}$, and the variance of the market mispricing, d, defined in (7).

3.1 Different sharings of the traders in the groups.

In this subsection we analyze the effects of different sharings of the traders in the various groups. In order to directly compare the results relative to the average number of the traded units and to the expected gain of the intermediary, the total number of traders remains 28 in each case.

In the following table, fixed $n = 6$, $E[V] = 1$, $\sigma^2[V] = 0.1$, $\alpha = 0.005$, we consider the following nine 7-tuples, F_z, $z = 1, 2, ..., 9$, of the cardinalities of the groups of traders $(f_0, f_1, ..., f_6)$

$$F_1 = (7,6,5,4,3,2,1),\ F_2 = (1,2,3,4,5,6,7),\ F_3 = (14,0,0,0,0,0,14),$$
$$F_4 = (0,14,0,0,0,14,0),\ F_5 = (0,0,14,0,14,0,0),\ F_6 = (14,0,0,7,0,0,7),$$
$$F_7 = (16,0,0,8,0,0,4),\ F_8 = (4,0,0,8,0,0,16),\ F_9 = (4,4,4,4,4,4,4)$$

	$\overline{\varphi}$	d	γ_I	γ_0	γ_1	γ_2	γ_3	γ_4	γ_5	γ_6
$\mathbf{F_1}$	12.676	.060	.129	−.090	−.101	.027	.052	.104	.142	.169
F_2	12.644	.016	.130	−.035	−.047	−.061	−.062	−.059	.023	.084
F_3	14.000	.052	.157	−.136						.124
F_4	14.000	.051	.154		−.116				.105	
F_5	14.000	.050	.150			−.083		.072		
F_6	12.486	.073	.134	−.143			.104			.164
F_7	11.133	.073	.119	−.099			.099			.168
F_8	11.133	.022	.120	−.065			−.087			.052
F_9	12.000	.036	.126	−.071	−.084	−.072	−.035	.037	.077	.116

A significative remark is that, as in Schredelseker (1999), the expected gain of the traders with medium-law informative levels (e.g. those of groups 1, 2 and

3) are, in some cases, worse than those obtained by less informed ones (even of those obtained by traders of groups 0 that have null information). On the contrary, the expected gains of the most informed traders (e.g. those of groups 5 and 6) are always the highest ones and they are stable in the positive side. For traders of group 6 such result is obvious since they know the true value of the security before the transaction time.

About the expected gain of the intermediary, let observe that the most profitable configurations seem to be those in which the traders are divided in two groups (in particular when the gap of informative levels between the two groups increases): in such case one of the two will be the group of sellers and the other one will be the group of buyers and the average number of traded units reaches the maximum admissible value, namely $f/2 = 14$.

Let observe that the lower is d, that is the more efficient is the maket, the higher are the gains of the less informed traders and the lower are the gains of the most informed ones.

3.2 Reduction of all relative informative levels

Fixed the 7-tuple $(f_0, f_1, ..., f_6)$ let assume increasing values of n without adding any traders: in such case the relative informative level of each group decreases and this reduction is relatively bigger for the groups of the most informed traders respect to the groups of the less informed traders.

In the following table, fixed $E[V] = 1$, $\sigma^2[V] = 0.1$, $\alpha = 0.005$, $(f_0, f_1, ..., f_6) = (7, 6, 5, 4, 3, 2, 1)$, we consider some integer values of $n > 6$, that are reported in the first column, assuming $f_h = 0$ for each $h = 7, 8, ...n$.

n	$\overline{\varphi}$	d	γ_I	γ_0	γ_1	γ_2	γ_3	γ_4	γ_5	γ_6
6	12.676	.060	.129	−.090	−.101	.027	.052	.104	.142	.169
9	12.642	.074	.129	−.070	−.079	.009	.044	.081	.114	.134
12	12.651	.080	.128	−.067	−.063	.006	.045	.070	.090	.114
15	12.506	.085	.127	−.062	−.061	.010	.040	.065	.083	.103

As it was expected, the obtained results point out that the reduction of all relative informative levels (that is when n increases) advantages the traders with low informative levels (those with negative expected gain) to the damage of, especially, the most informed ones (those of groups 4, 5 and 6).

It comes out quite clearly that a reduction of the relative informative level makes the market less efficient, at least in terms of the index d, while the average number of traded units is very little affected by variations of such parameter.

3.3 Different variances of the random value of the security

In this section we study the effects of different values of the variance of the security random value.

In the following table, fixed $n = 6$, $E[V] = 1$, $\alpha = 0.005$, $(f_0, f_1, ..., f_6) = (7, 6, 5, 4, 3, 2, 1)$, we consider some values of $\sigma^2[V]$ that are reported in the first column.

$\sigma^2[V]$	$\overline{\varphi}$	d	γ_I	γ_0	γ_1	γ_2	γ_3	γ_4	γ_5	γ_6
.001	0.236	.002	.002	−.002	−.001	.000	.001	.002	.003	.004
.010	10.234	.007	.103	−.016	−.033	−.014	.015	.029	.042	.052
.050	12.840	.032	.130	−.065	−.086	.021	.049	.076	.095	.117
.100	12.676	.060	.129	−.090	−.101	.027	.052	.104	.142	.169
.200	12.599	.123	.131	−.131	−.127	.021	.097	.149	.187	.237
.300	12.621	.183	.134	−.161	−.173	.048	.118	.186	.233	.292

The most interesting remark concerns the fact that the higher is the variance, the higher are the expected gains of the more informed traders (ones of groups 2, 3, ..., 6) and of the intermediary and the lower are the expected gains of the less informed traders (ones of groups 0 and 1). Such remark suggests that there could be a common interest of the more informed traders and of the intermediary in making the volatility as higher as possible, to the damage of the less informed traders. Furthermore, if the variance tends to 0 (let observe for example the case in which it is $\sigma^2[V] = 0.001$) that is if the value of the security tends to be deterministic, then the trading volumes tends to be null with obvious consequences on the gains of the traders and of the intermediary. Paradoxally, a null variance implies the cancellation of all information asymmetries and, hence, of the conditions to have trading: so, the presence of some uncertainty is necessary for the existence of the market.

Notice that the higher is the variance, the higher is the difference between bid and ask prices of each trader and hence, the larger is the spreading of the set of prices. From such remarks, it seems quite reasonable that an increase of the variance tends to enlarge the mispricing between the market price and the true value of the security and hence, to reduce the efficience of the market.

3.4 Different values of the commission level

In this section we analyze the effects relative to variations of the commission level, which could be considered the parameter somehow managed by the intermediary in order to maximize his own expected gain. Such expected gain depends on the trade-off between the gain relative to a single traded unit and the average number of traded units when the commission level increases.

In the following table, fixed $n = 6$, $E[V] = 1$, $\sigma^2[V] = 0.1$, $(f_0, f_1, ..., f_6) = (7, 6, 5, 4, 3, 2, 1)$ we consider some values of α that are reported in the first column.

α	$\overline{\varphi}$	d	γ_I	γ_0	γ_1	γ_2	γ_3	γ_4	γ_5	γ_6
.0010	12.676	.060	.026	−.085	−.097	.030	.055	.106	.145	.172
.0050	12.676	.060	.129	−.090	−.101	.027	.052	.104	.142	.169
.0100	12.676	.060	.258	−.095	−.107	.023	.048	.100	.137	.164
.0900	7.085	.062	1.267	−.139	−.074	−.053	.011	.045	.073	.096
.0909	7.085	.062	1.279	−.140	−.074	−.053	.010	.044	.072	.095
.0910	6.277	.066	1.133	−.125	−.073	−.040	.003	.045	.071	.096
.0967	6.277	.066	1.198	−.128	−.074	−.043	.000	.042	.069	.093
.0968	6.051	.066	1.156	−.131	−.080	−.020	.000	.042	.064	.091

As it was expected we observe that respect to the commission level, the intermediary has opposite interests of those of the traders when increases of such parameter do not imply a reduction of the expected number of traded units. On the contrary, when an increase of such parameter implies a reduction of the average number of traded units, the expected gains of the most informed traders (e.g. ones of groups 4, 5 and 6) decrease quite univocally, while some traders with negative expected gains reduce their losses.

The trade-off between the gain relative to the single traded unit and the average number of traded units is profitable for the intermediary until that such level reaches a certain value (0.0909 in this case) at which intermediary's expected gain is maximum. It has to be underlined that the aims of maximizing the average number of traded units and of maximizing the intermediary's expected gain do not agree.

Let observe that the efficience of the market seems to be very little affected by variations of the considered parameter.

4 Conclusions

Altough we have considered a quite stylized market model, this paper can suggest some unusual points of view of more realistic market models in which operate heterogeneously informed traders, that could lead to some interesting considerations about financial markets analysis.

Indeed, as in Schredelseker (1999) it comes out that the expected gain of the traders with intermediate informative levels are, in some cases, worse than those obtained by less informed ones. For example, such intermediate levels could be associated to small savers that try to collect all the informations available on the media that, unexpectedly, could not lead to better results than those of less meticulous small savers. On the other hand, as already mentioned, some information asymmetries are necessary for the existence of the market.

Another remarkable suggestion of this paper concerns the significance of the volatility. The obtained results show both that a too much low volatility reduces the trading volumes dramatically, and that the more informed traders and the intermediary could have the common interest of increasing the volatility, since it increases their expected gains.

However, they should consider that from the point of view of the interpretation of the model in terms of Capital Asset Pricing Model theory, the negative expected gains obtained by less informed traders have to be interpreted as partial reductions of extra gains allowed by this "risky" market respect to those allowed by the free-risk return. Indeed, it suggests that they should "control" that the volatility does not exceed the maximum level such that the traders with the worst expected gain prefer to operate in this "risky" market rather than investing in the free-risk return since, in the opposite case the aggregate positive gain of the most informed traders and of the intermediary would be, consequently, smaller.

Considering that the most informed traders could be identified with the institutional investors (which, usually, manage the greater share of the trading volumes), it is not difficult to imagine that the volatility could be somehow "controlled" by such kind of traders in the light of the above considerations.

In this paper we have analyzed one of the most significative development of the model described in Schredelseker (1999), that is the introduction of transaction costs. Some other developments that could be interesting to analyze are the consideration of risk adverse traders, the introduction of the cost of the correct information and the hypothesis that different kinds of traders can exchange different volumes of the security.

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