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**Estimating Multidimensional Poverty:  
A Comparison of Three Diffused Methods °**

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- Stampato in Proprio -

# Estimating Multidimensional Poverty: a Comparison of Three Diffused Methods<sup>°</sup>

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## Abstract

The aim of this paper is to compare three methods used for poverty analysis in a multidimensional context. The methods considered are those proposed by Cheli and Lemmi in 1995 (Totally Fuzzy and Relative method), Nolan and Whelan in 1996 (ESRI method) and Lolloivier and Verger in 1997 (INSEE method). These three approaches are briefly described with the purpose of understanding their similarities and differences, highlighting especially the choice concerning the indicators of poverty, the selection of the dimensions of poverty and the statistical-mathematical techniques. An empirical illustration is conducted on the first wave (1994) of the European Community Household Panel concerning four Mediterranean countries (France, Italy, Portugal and Spain).

What we present here is just a preliminary analysis that we plan to develop in the future.

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## Introduction

The crucial problem in Sen's capability approach is perhaps the difficulty to shift from the conceptual level to the empirical one. The main obstacle is given by the fact that capabilities are very difficult to observe and measure in practice, particularly in the very common case when the available statistical information is that provided by one of the existing surveys on households' living conditions. Therefore, the researcher who aims to analyse poverty or well-being according to Sen's approach generally has to make a drastic but unavoidable restriction: renouncing to capabilities and focusing exclusively on functionings as indicators of people's living standard.

Nevertheless, even operating in the sole space of functionings requires much more statistical information than the traditional income based approach to poverty analysis. In practice, we need to follow a multidimensional approach and consider a variety of living condition indicators.

At this point other problems must be faced that concern the statistical methodology to adopt in order:

- i) to choose the relevant items (or indicators) among the available ones;
- ii) to reduce the dimensions of the items' space by focusing on a comparatively small number of well-being (or poverty) factors (functionings);
- iii) to perform the well-being (or poverty) analysis and measurement according to the functioning factors previously identified.

In this framework it is important to ascertain to what extent the results obtained by the implementation of Sen's conceptual model are influenced by the choice of the specific technique chosen for the statistical analysis.

Several Authors (among them, Chiappero Martinetti, 1994, 2000; Lelli, 2001; Clark and Qizilbash, 2002) indicated the so called Totally Fuzzy and Relative (TFR) approach (Cheli and Lemmi, 1995) as a particularly effective technique to analyse well-being and poverty in the framework of Sen's functionings and consequently applied it to different realities. Moreover, Lelli (2001) made an empirical comparison between the fuzzy sets approach (including the TFR) and factor analysis which is preferred by certain Authors and found that the results obtained by the empirical application of Sen's conceptual model seem not to be appreciably influenced by which of these two techniques we use.

The same paper also contributes to the research on the fuzzy sets approach by assessing its robustness in relation to several different specifications of the membership function. The empirical evidence clearly suggests that the degree of such robustness is high. This finding is corroborated by another recent study (Filippone, Cheli, D'Agostino, 2001) that shows that robustness is high also respect to the choice of the weight function.

In this paper we aim to compare three methods for poverty analysis that follow a multidimensional approach, hence are virtually suitable to operationalize Sen's theory. They are: the TFR method – in the shape adopted by EUROSTAT (Betti and Verma, 2002), the INSEE method (Lolloivier and Verger, 1997) and the ESRI method (Nolan and Whelan, 1996).

This choice is motivated by the fact that they have been widely applied to European countries at the official level.

The comparison will be carried out at both the methodological and empirical level by considering the quantitative and qualitative results that we have obtained for four Mediterranean countries: France, Italy, Portugal and Spain.

## The TFR method for the analysis of multidimensional deprivation

In this section we describe the TFR method in the shape recently proposed by Betti and Verma (1998) and applied by EUROSTAT (Betti and Verma, 2002).

The conventional approach to the measurement of poverty involves a rather simplistic division of the population into the 'poor' and the 'non-poor' in terms of the person's position in relation to some arbitrarily chosen point in the income distribution. Such measures have a number of serious limitations. The TFR approach uses measures of *degree of poverty*, thus avoiding the need to choose an arbitrary poverty line and a weighting procedure which allows to incorporate different aspects of *deprivation* into multidimensional indices.

The degree of poverty of each household that replaces the simple poor/non-poor dichotomy by a measure of the degree of (or propensity to) poverty as a function of the household's position in the indicators distribution.

Suppose that for each unit (household) we observe a vector  $\mathbf{X}$  of  $k$  characteristics  $X_1, \dots, X_k$ . The  $i$ -th unit's membership function (m.f.) in the fuzzy subset of the poor can be defined as follows:

$$f(\mathbf{x}_i) = \frac{\sum_{j=1}^k \mu(x_{ij}) w_j}{\sum_{j=1}^k w_j} \quad (i=1, \dots, n) \quad (1)$$

where  $w_1, \dots, w_k$  represent a generic system of weights.  $f(\mathbf{x}_i)$  is an individual index of deprivation referred to the considered set of items, whereas  $\mu(x_{ij})$  measures the specific deprivation for item  $j$ .

$\mu(\ )$  represents the specific m.f. for indicator  $X_j$  and is defined in terms of the its distribution function. By such a specification of  $\mu(x_{ij})$ , we give a deprivation score to each household that depends only on how many have less and how many have more than it does. In the particular case in which  $X_j$  is dichotomous  $\mu(x_{ij})$  takes value 1 if the lack or presence of the item denotes a symptom of poverty and 0 in the other case.

From a relative point of view, the importance of an indicator for the measurement of poverty depends on how representative it is of the community's lifestyle. This justifies the choice of defining the weights  $w_j$  in formula (1) as a decreasing function of the proportion of the deprived. Here we adopt the specification proposed by Betti and Verma (1998), according to which the weight of the  $j$ -th item is proportional to the product of two components as follows:

$$w_j \propto w_j^{(a)} \cdot w_j^{(b)} \quad (2)$$

where  $w_j^{(a)}$  depends on the distribution of  $X_j$  only and is a decreasing function of the proportion of deprived in this item; on the other side,  $w_j^{(b)}$  considers the correlation between  $X_j$  and the other indicators in order to reduce the redundancy produced by highly correlated indicators<sup>1</sup>.

The overall index of poverty for the entire population is given by the arithmetic mean of the poverty indices of each statistical unit as follows:

$$P = \frac{1}{n} \sum_{i=1}^n f(\mathbf{x}_i) \quad (3)$$

<sup>1</sup> For a complete description of this weighting system see Betti and Verma (1998).

The index  $P$  can be evaluated using either all the available poverty indicators or particular groups of them, referring for instance to housing conditions or to the ownership of durable goods. In the first case we obtain a collective measure of *global* poverty whereas in the second a collective measure of a *specific* type of deprivation.

### The ESRI Method

Nolan and Whelan take deprivation to mean "an inability to obtain the goods, facilities and opportunities to participate identified as appropriate in the community in question" (Nolan and Whelan, 2001). In order to be consistent with such definition the Authors prefer to use indicators whose absence will most often be attributable to limited resources rather than other constraints such as poor health or differences in taste. This helps to select the indicators of deprivation by considering those that are likely to be directly affected by access to financial resources.

Once the items have been identified, the first stage is to examine their range so as to see whether they cluster into distinct groups. Exploratory factor analysis allows to identify the number of dimensions that it is necessary to take into account. Subsequently, confirmatory factor analysis is made in order to compare solutions constrained to be equal across countries with those allowing cross-national variation<sup>2</sup>. The choice of the model is carried out by comparing a variety of indices which assess absolute, comparative and parsimonious fit<sup>3</sup>. After identifying the dimensions of deprivation, this approach proceeds to the construction of a set of deprivation indices and provides an assessment of the extent to which they can be considered to constitute reliable measures<sup>4</sup>. In combining the items into scales of deprivation this approach allows to use a number of options: i) use of factor scores; ii) weighting each item by the extent to which this deprivation is experienced (i.e. the proportion of households possessing that item in each country); iii) an additive procedure that gives us the number of items respect to which the household is deprived. However, Nolan and Whelan show a slight preference for the last option because of the benefits of simplicity and transparency in cross-national comparisons. For this reason, we refer to the last option in the application that we present here.

### The INSEE Method

The INSEE approach (Lolloivier and Verger, 1997; Dell and Legendre, 2003) considers a condition of insufficient money for affording a decent life, a condition of deprivation of ordinary goods and basic consumption and a condition of difficulty to obtain a judged minimum level of existence.

Starting from these three aspects of deprivation, the INSEE approach considers an *a priori* classification of the indicators in three categories: the first one refers to income, the second one to living condition and the third one reflects subjective appraisals on one's material situation. The considered units of analysis (the households) are classified as "poor" according to these three dimensions.

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<sup>2</sup> The acceptance of the constrained model means the validation of the hypothesis that the latent dimensions are the same (are interpretable in the same way) for all countries.

<sup>3</sup> The indices used for such aim are: Root Mean Squared Error of Approximation (RMSEA), Adjusted Goodness of Fit Index (AGFI), Normal Fit Index (NFI), Comparative Fit Index (CFI), Parsimonious Goodness of Fit Index (PGFI). For the meaning and the interpretation of these indices, see Kelloway (1998).

<sup>4</sup> The reliability is measured by the Cronbach's coefficient alpha which is a measure of internal consistency from one set of measurements to another.  $\alpha$  takes value between 0 and 1, that represent respectively the minimum and maximum degree of reliability.

The poverty measure for each dimension is based on the lack of the correspondent items. Starting from a set of indicators, that are considered as essential elements of a standard material well-being - largely spread in the population -, the condition of poverty is identified by the lack of several items and not by the lack of each single item. The degree of deprivation is obtained by calculating a deprivation score for each dimension which is based on the households distribution respect to the number of deprivation items. The same proportion of the most deprived households (the closest to the one obtained by the income approach) is approximately isolated for each distribution, constituted by the households with the highest number of deprivation items.

By this way, three sub-populations are identified which do not necessarily coincide. Generally they overlap partially, showing that material well-being is not dependent only the level of current income but also on the past conditions and on the accessibility to goods and services not always available by money.

### **The choice of indicators**

All the methods analysed start from an *a priori* choice of indicators based on several and different considerations supplied by the first wave of the European Community Household Panel (ECHP).

On the basis of the above considerations, we consider the following 24 non-monetary items available in the ECHP and included in the analysis as indicators of lifestyle deprivation.

*Enforced lack of widely desired possessions.* The following six items represent enforced lack of widely desired consumer durable items. Respondents were asked about these items in the following format: for each household it was established whether the item was possessed/availed of, and if not, a follow-up question was asked to establish whether this was due to an inability to afford the item concerned or not. A household is considered to be deprived only if the absence (of a car or van, a colour TV, a video recorder, a micro wave, a dishwasher, a telephone) is stated to be due to lack of resources.

*Absence of basic housing facilities.* Three items relate to absence of housing facilities. They are so basic that one can presume all households would wish to have them: a bath or shower, an indoor flushing toilet, hot running water.

*Problems with accommodation and the environment.* A further set of items relating to problems with accommodation and the environment contained the implicit assumption that households wish to avoid such difficulties. These include the following eight items: shortage of space, noise from neighbours or outside, too dark/not enough light, leaky roof, damp walls, floors, foundation etc., rot in window frames or floors, pollution, grime or other environmental problems caused by traffic or industry, vandalism or crime in the area.

*Lack of ability to afford most basic requirements.* For some items the absence and affordability elements were incorporated into a single question, as follows: 'There are some things many people cannot afford even if they would like them. Can I just check whether your household can afford these if you want them'. The following six items were administered in this fashion in ECHP: keeping the home (household's principal accommodation) adequately warm, paying for a week's annual holiday away from home, replacing any worn-out furniture, buying new, rather than second hand clothes, eating meat chicken or fish every second day, if the household wanted to, having friends or family for a drink or meal at least once a month.

*Inability to meet payment schedules.* The final item (based on three separate variables in the ECHP Users' Data Base.) relates to arrears on payment of scheduled mortgage payments, utility bills or hire purchase instalments. A household is considered as experiencing deprivation in terms

of this item if it was unable at any time during the past twelve months to pay scheduled mortgage payments, utility bills or hire purchase instalments.

Official application of both of TFR and ESRI methods considered all these 24 indicators whereas the INSEE analyses excluded the micro-wave and the dish-washer, and included the presence of separate kitchen in the house, centralised heating facilities and garden or terrace.

Moreover as already underlined, the INSEE also considers few subjective indicators of financial difficulties (ability to make ends meet, income situation as judged by the household, income level equal or lower than the minimum considered as necessary) together with the indicators of the inability to meet payment schedules also considered in the other two approaches. The results obtained by applying the original proposal of the authors is reported in Appendix 1

### **Methodologies applied**

Both the TFR and ESRI applications carry out a preliminary factor analysis, in order to obtain the underlying dimensions from the 24 indicators<sup>5</sup>. Once the variables have been chosen and codified, the deprivation structure is drawn by a confirmatory factor analysis, which allows the comparison and the choice of the best model among several alternatives models with different dimensions of deprivation. Moreover it is possible to test the constrained model (the parameters do not change across countries) and non constrained one (the parameters may vary across countries). In this way the underlying dimensions of deprivation can be identified by studying the association between indicators and factors.

*TFR approach.* After having identified the underlying dimensions, all the indicators can be combined into a single summary index of the degree of non-monetary or lifestyle deprivation. This would need to consider weights for each of the 24 indicators.

Such a single overall index can provide a powerful tool for analysis of lifestyle deprivation within and across countries, and greatly enrich the picture emerging from the study of income poverty risk alone. However, it should be kept in view that putting together all the information into a single index and simply ignoring the different underlying dimensions of deprivation can lead to misleading conclusions regarding the determinants of deprivation. For this reason, besides reporting on the analysis using a single index of overall lifestyle also on analyses that relate to the underlining dimensions separately is considered.

*ESRI approach.* The main scores, as indices of deprivation, can be calculated for each country and for each dimension considered, in order to try a comparison of situation within and across countries, even if it is not the real purpose of the proposed approach.

The analysis in fact is mainly devoted to a measure of poverty named "Current Life-Style Deprivation" (CLSD), that includes only two dimensions (Basic and Secondary, outlined below) that display the highest correlation with income. Moreover the relationship between the subjective difficulty to make ends meet, the income levels and the CLSD index of deprivation is analysed.

*INSEE approach.* The INSEE approach, as already mentioned, considers an *a priori* classification of the indicators in three categories: the first referring to income, the second to the living condition (that can be considered mainly related to the dimension explored by the first two approaches considered) and a third one based on the so called subjective approach.

An index of deprivation is calculated for each dimension and it is obtained by selecting all the households having more than a given number of deprivation items. The number of items for each

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<sup>5</sup> Although the majority of TFR applications do not make use of factor analysis, we follow this procedure in view of the fact that it has been used in the EUROSTAT application (Betti and Verma, 2002).

dimension is chosen so that the percentage of households classified as deprived respect to each specific dimension is similar of that obtained applying the poverty line approach.

The relationship between the three dimensions of poverty is investigated, especially in terms of income and poverty line.

### Main results from different approaches

In order to compare the different approaches in empirical terms, we simply applied the methodologies on the first wave data of the ECHP.

Both the TFR and the ESRI approaches start with the factor analysis. An exploratory factor analysis suggests that a five-factor solution is optimal. On the basis of this analysis we compare goodness of fit for three to six-factor solutions. In Table 1 a series of indices is reported, obtained by a LISREL application, that allow to conclude that a five factors solution is the best one<sup>6</sup>.

**Table 1. Four Country Constrained Confirmatory Factor Analysis of Five Alternative Oblique Factor Solutions for the Deprivation Items**

Oblique Model	X <sup>2</sup>	Df	RMSEA	AGFI	NFI	PGFI	CFI
3 Factors	34889.54	249	0.073	0.85	0.78	0.73	0.79
4 Factors	23976.25	246	0.061	0.89	0.85	0.75	0.85
5 Factors	21126.25	242	0.058	0.90	0.87	0.74	0.87
6 Factors	17255.96	237	0.053	0.92	0.89	0.74	0.89

Once the five factors solution is chosen, it is possible to test the constrained model against the non constrained one. Given the evidence that the difference between them is not so relevant, the constrained one has been chosen:

**Table 2. Unconstrained and Constrained Oblique and Orthogonal Five-Factor Deprivation Solutions**

Model	X <sup>2</sup>	df	RMSEA	AGFI	NFI	PGFI	CFI
<i>Orthogonal</i>							
Unconstrained	29910.99	1008	0.033	0.94	0.77	0.66	0.78
Constrained	37725.42	252	0.076	0.84	0.77	0.72	0.77
<i>Oblique</i>							
Unconstrained	17135.60	968	0.025	0.96	0.87	0.71	0.87
Constrained	21126.25	242	0.058	0.90	0.87	0.74	0.87

Given the association between indicators and the five factors reported in Table 3 below

<sup>6</sup> Although all the indices show the six-factor solution to be slightly superior to the five, the latter still provides an excellent fit. Given this, and the fact that the six-factor solution involves specifying a factor with only two items, we opt in what follows for the five-factor solution.



**Table 3. Confirmatory Factor Analysis Oblique Five-factor solution – factor loadings**

Indicators	1 factor	2 factor	3 factor	4 factor	5 factor
can the household afford replacing any worn-out furniture?	0.70				
can the household afford paying for a week's annual holiday?	0.74				
can the household afford buying new, rather than second-hand clothes?	0.64				
can the household afford having friends or family for drink?	0.60				
can the household afford keeping its home adequately warm?	0.66				
can the household afford eating meat, chicken or fish every second day?	0.40				
has the household been unable to pay scheduled rent for the past 12 months?	0.21				
possession of a micro wave		0.77			
possession of a dishwasher		0.73			
possession of a video recorder		0.66			
possession of a car or van (for private use)		0.52			
possession of a telephone		0.47			
possession of colour TV		0.35			
does the dwelling have bath or shower ?			0.88		
does the dwelling have indoor flushing toilet ?			0.78		
does the dwelling have hot running water ?			0.77		
does the accommodation have damp walls, floors, foundations etc.?				0.65	
does the accommodation have rot in window frames or floors ?				0.63	
does the accommodation have leaky roof ?				0.60	
does the accommodation have noise from neighbours or outside?					0.57
is there any pollution, grime or other environmental problem?					0.52
does the accommodation have shortage of space ?					0.25
is the accommodation too dark/not enough light ?					0.26
is there crime or vandalism in the area ?					0.39

they can be so described and summarised:

- *Basic life-style deprivation* - comprising items such as food and clothing, a holiday at least once a year, replacing worn-out furniture and the experience of arrears for scheduled payments.
- *Secondary life-style deprivation* - comprising items that are less likely to be considered essential such as a car, a phone, a colour television, a video recorder, a microwave and a dishwasher.
- *Housing facilities* - housing services such the availability of a bath or shower, an indoor flushing toilet and running water.
- *Housing deterioration* - the existence of problems such as a leaking roof, dampness and rotting in window frames and floors.

- *Environmental problems* - problems relating to noise, pollution, vandalism and inadequate space and light.<sup>7</sup>

Summing up the single variables for each dimension, it is possible to calculate the factor scores and to judge the reliability of each index by the Cronbach's index:

**Table 4. Cronbach's coefficient index**

Variable	Cronbach's index
Basic life-style	0.76
Secondary life-style	0.76
Housing facilities	0.85
Housing deterioration	0.66
Environmental problems	0.48

Table 4 shows that the alpha level is quite high for basic, secondary, and housing facilities and deterioration dimensions. The lowest value of 0.48 is observed for the environmental dimension suggesting that this area would benefit from further attempts at scale development.

In the correlation matrix, the basic life-style dimension and the secondary life-style show a high correlation, whereas both of them seem to be almost independent from the environmental dimension.

**Table 5. Pearson Correlation Coefficients**

	Basic	Secondary	House fac.	House det.	Environm.
Basic	1.00	0.54	0.32	0.32	0.14
Secondary	0.54	1.00	0.32	0.29	0.06
House fac.	0.32	0.32	1.00	0.33	0.04
House det.	0.32	0.29	0.33	1.00	0.21
Environm.	0.14	0.06	0.04	0.21	1.00

From this point the two approaches (TFR and ESRI) follow a different procedure.

### TFR approach

The calculation of the weights for each country and for each indicator is presented in Table 6. What we observe is that for each country the "less important" items are those referred to the affordability by the household of a video recorder or a dish washer. On the other hand different is the situation if we consider the items which present the highest weight. For France the affordability of a week's annual holiday seem to be one of the main indicators of deprivation, followed by the bad conditions of the house. For Italy and Spain bad housing conditions represent a important symptom of poverty, whereas for Portugal eating fish or meat every day or having friend or family

<sup>7</sup> It is worth noting that the models reported in Table 1 have been built as follows: 1) the three factor solution distinguishing basic, secondary and housing/environment dimensions; 2) the four factor option distinguishing basic, secondary, housing facilities /environment dimensions; 3) the six-factor solution separating out the space and light items from those relating to noise, pollution and vandalism.

for a drink at least once a month present the highest score. By the way different culture and different life style, other than different degree of deprivation, seem to be behind those very simple results.

**Table 6. TFR weights of indicators by country**

Country	ind.1	ind.2	ind.3	ind.4	ind.5	ind.6
France	0.17	0.10	0.09	0.14	0.16	0.13
Italy	0.10	0.08	0.06	0.13	0.14	0.10
Spain	0.07	0.07	0.06	0.15	0.20	0.14
Portugal	0.04	0.05	0.04	0.07	0.20	0.12
	ind.7	ind.8	ind.9	ind.10	ind.11	ind.12
France	0.24	0.28	0.19	0.19	0.20	0.25
Italy	0.32	0.29	0.21	0.22	0.22	0.30
Spain	0.21	0.31	0.17	0.13	0.12	0.21
Portugal	0.15	0.23	0.12	0.11	0.10	0.18
	ind. 13	ind. 14	ind. 15	ind. 16	ind. 17	ind. 18
France	0.25	0.27	0.27	0.19	0.24	0.27
Italy	0.27	0.30	0.30	0.16	0.20	0.23
Spain	0.30	0.33	0.33	0.18	0.24	0.27
Portugal	0.23	0.24	0.22	0.37	0.29	0.41
	ind. 19	ind. 20	ind. 21	ind. 22	ind. 23	ind. 24
France	0.29	0.23	0.26	0.23	0.21	0.16
Italy	0.26	0.26	0.27	0.18	0.22	0.18
Spain	0.29	0.24	0.31	0.23	0.22	0.20
Portugal	0.26	0.21	0.24	0.40	0.39	0.34

The global index and the specific indexes for each dimension (see Table 7) show the different situation of deprivation for each country considered.

**Table 7. TFR indices of poverty by country**

Country	global	basic	secondary	house facilities	house deterioration	environmental problems
France	0.110	0.148	0.063	0.034	0.117	0.184
Italy	0.110	0.171	0.082	0.021	0.076	0.208
Spain	0.156	0.205	0.143	0.027	0.149	0.242
Portugal	0.229	0.232	0.266	0.181	0.270	0.213

If we look at the global index we observe that the degree of deprivation is lower for Italy and France, whereas Portugal is the country with the highest value. If the specific indices are taken into account it is possible to observe that the different dimensions of poverty affect the four countries in

different ways. The highest degree of deprivation for Italy, France and Spain is observed for environmental problems followed by basic life-style. Very low is the deprivation that refers to house facilities (availability of a bath or shower, an indoor flushing toilet and running water). In Portugal the highest deprivation is observed for housing deterioration (the existence of problems such as a leaking roof, dampness and rotting in window frames and floors), followed by the secondary life-style (non affordability of a car, a phone, a colour TV, a video recorder, a microwave oven and a dishwasher). Again the house facilities dimension displays the lowest level of deprivation.

ESRI approach

According to the ESRI analysis Portugal is again the most deprived country compared to the others, except for environmental problems. The situation seems to be particularly worrying for the secondary life-style dimension and for the housing condition. France and Italy appear as the less deprived countries in all dimensions (except environmental problems).

**Table 8. Mean Scores on Five-Factor Solution by Country**

Country	Basic	Secondary	Housing Facilities	Housing Deterioration	Environment
France	1.22	0.41	0.10	0.37	0.93
Italy	1.49	0.54	0.06	0.23	1.05
Spain	2.17	1.08	0.09	0.48	1.19
Portugal	3.27	1.93	0.64	0.77	0.86

The factor scores of each household are defined as the number of items respect to which the household is deprived. Hence, the mean scores reported in Table 8 simply represent the average number of items denoting deprivation in the correspondent dimension.

In this case the comparison between different dimensions is problematic because it depends on the characteristics and number of the indicators considered. On the other hand the comparison among values in the same dimension between countries gives results that are influenced by the fact that the indicator weights are the same for each country.

Then the approach introduces a new variable, called Current Life Style Deprivation (CLSD), that is obtained by summing up the first two dimensions and investigates its correlation with income.

Table 9 shows that CLSD decreases as the equivalent income level increases.

The level of CLSD is lowest for France and Italy, the countries showing even the strong relationship between income and CLSD index of deprivation, for Spain and Portugal symptoms of deprivation are present even in the top deciles of the equivalent income distribution.

**Table 9. Mean Scores on CLSD by Equivalent Income Decile**

Decile	France	Italy	Spain	Portugal
Bottom	3.29	3.88	5.30	6.75
2	2.86	3.38	5.00	7.18
3	2.56	2.73	4.49	6.97
4	2.05	2.42	4.25	6.42
5	1.63	2.12	3.48	5.68
6	1.28	1.73	3.08	5.30
7	1.00	1.49	2.56	4.82
8	0.78	1.14	2.12	4.26
9	0.52	0.83	1.44	3.14
Top	0.36	0.60	0.78	1.48
Total	1.63	2.03	3.25	5.20

Table 10 shows that the CLSD mean scores decrease as the poverty line increases, confirming the association between two indicators.

**Table 10. Mean Scores on CLSD by Income Poverty Lines**

Country	Mean score 40% Mean Income	Between 40-50% of Mean Income	Between 50-60% of Mean Income
France	3.28	2.95	2.62
Italy	3.86	3.37	2.81
Spain	5.28	5.02	4.51
Portugal	6.98	6.94	6.54

If we consider the relationship between the “Experiencing Difficulty or Great Difficulty in Making Ends Meet” and the CLSD index, it is possible to observe that 1/3 of the Spanish and Portuguese households present difficulties in making ends meet, whereas only less than 1/5 of the Italian and French households experienced this kind of deprivation.

**Table 11. Percentage of Households Experiencing Difficulty or Great Difficulty in Making Ends Meet by CLSD by Country**

	CLSD							
	0	1	2	3	4	5	6+	All
France	2.66	13.14	21.44	33.54	42.50	58.95	69.72	18.92
Italy	0.60	9.64	15.76	27.49	41.11	49.2	67.72	19.56
Spain	4.95	14.73	28.59	35.94	47.76	57.28	70.08	36.23
Portugal	3.91	6.73	10.00	18.81	31.50	37.15	55.84	36.48

### INSEE approach

Although the INSEE approach selects the items in a different way (as we mentioned above), in order to better compare this approach with the other two, we refer to the same set of indicators, considering the five factors solution obtained and used by the other two approaches.

**Table 12. Percentage of deprived households in the INSEE approach by country**

Country	global	basic	secondary	house facilities	house deterioration	environmental problems
France	10.69	16.22	18.61	6.17	8.61	10.45
Italy	16.65	15.02	22.33	4.12	15.17	13.47
Spain	15.46	17.81	24.80	5.14	12.41	16.48
Portugal	25.96	24.56	22.29	25.11	27.23	12.67

In this case the comparison between dimensions is still problematic because the level of deprivation is more or less established *a priori* by considering the percentage of households below the poverty line – income approach – (the percentages of income poor population in each country are reported in Appendix).

Observing Table 12, Portugal still remains the country with the highest value for all the indicators, except environmental problems. The house deterioration conditions seem to be particularly worrying, followed by the secondary life-style dimension. Again, France appears to be the less deprived country.

**Table 13. Percentage of households experiencing several dimension of deprivation by country**

	France	Italy	Spain	Portugal
No dimension	59.46	55.37	49.55	45.38
1 dimension	26.07	27.49	31.85	23.09
2 dimensions	10.41	11.10	12.80	13.96
3 dimensions	3.18	3.92	4.23	10.41
4 dimensions	0.77	1.65	1.34	6.18
5 dimensions	0.10	0.47	0.22	0.98

Even considering the overlapping of dimensions (see Table 13), Portugal is confirmed to be the most deprived country, since the percentage of households without poverty symptoms is the lowest and the percentage of households with all the symptoms is the highest (almost 1%).

All the correlation coefficients between monetary deprivation and the five types of deprivation in living conditions result to be rather low (see Table 14). However, among them the highest values are those ones corresponding to basic and secondary life-style deprivation and (in decreasing order) to house facilities for France and Portugal. Very low is the correlation between monetary poverty and environmental problems deprivation.

**Table 14. Pearson correlation between monetary poverty and the five dimensions by country**

Correlation	France	Italy	Spain	Portugal
Monetary poverty-Basic	0.20	0.23	0.17	0.31
Monetary poverty-Secondary	0.20	0.13	0.09	0.38
Monetary poverty-House facilities	0.16	0.10	0.07	0.31
Monetary poverty-House deterioration	0.07	0.09	0.09	0.15
Monetary poverty-Environmental problems	0.01	0.07	0.01	0.01

### Conclusion

The three approaches examined here use non-monetary poverty indicators that are available from existing sources. However none of them makes a real proposal of what should be analysed, proposal that could be used on designing new surveys.

There are two main reasons to investigate non-monetary deprivation: i) income is often unreliable, being affected by errors and survey problems; ii) income is not the only dimension of poverty. In view of point i), we should use the non-monetary indicators that are more related to income in order to represent a sort of *proxy* variables of income itself, so the correlation and the relationship with it should be investigated and analysed. In view of point ii), something substantially different from income/financial dimension, it is important to define it in theoretical terms and define appropriate measures. The analysed approaches present several problematic aspects if they have to be used as measures of a deprivation that is substantially independent on income. The EUROSTAT-TFR approach in fact is the only one that can be used in this term, which considers the correlation among indicators and is the only one which tries to give a real measure of the level of deprivation in quantitative terms. The ESRI approach in fact produces a non standardised measure that cannot be considered a real index of deprivation, since it depends on the number of items and it can be higher or lower independently on the real living condition of the population. The ESRI approach is more devoted to the investigation of the relationship between income, non-monetary deprivation and economic strain subjectively evaluated. By the way it is not really clear if the low association among different dimensions is due to errors in income survey or to a different nature of the deprivation captured by different indicators.

Finally the problematic aspect of the INSEE approach is again that it does not give a real measure of deprivation. It isolates, in each of the considered dimensions, the same percentage of the most deprived households, so that the index obtained for each dimension is, as close as possible, the same. It cannot be considered a real measure but only a way to isolate the low tail of the distribution.

Referring to the comparison between dimensions and countries, the TFR seems to be the only one able to allow a real comparison, for any specific dimension and for each country, given the fact that it takes into account the different situation of each country and reality.

The ESRI approach can allow a comparison among countries, given the fact that the same number of indicators are considered, but it does not take into account the differences in term of importance and diffusion of symptoms in each reality. Moreover the comparison between dimensions is rather difficult, since it depends on the number of indicators included. So the direct comparison is not meaningful.

The comparison with the INSEE approach is problematic both in terms of dimensions and countries. The comparison among different dimensions is meaningless given the fact that the indices values are more or less the same and depend on the extension of monetary income. The comparison among countries is again very problematic, because the results in quantitative terms depend on the income analysis and quantitative comparison are limited to this dimension.

In conclusion, the comparison among the three approaches can only be made at the qualitative level, trying to draw a picture of the dimensions and the country with the most severe profile of deprivation. Portugal appears to be the most deprived country among the four analysed by all the methods considered. In this country, housing conditions reveal the highest degree of deprivation, compared to the situation of the other countries, whereas environmental problems seem to have less weight than elsewhere.

Italy and France generally appear to be the less deprived countries in both monetary and non-monetary terms. France always displays the lowest values of deprivation except for housing conditions which appear to be better for Italy. Spain reveals the highest value, compared to the other countries, referring to the environmental problems, whereas the household facilities seem to be the minor problem. The correlation with income is quite strong especially referring to the first two dimension (basic and secondary life style indicators). In terms of subjective dimensions, by the ESRI approach and the INSEE approach, we discover a quite high correlation between monetary and subjective poverty, considering that the last one is always a perception of the own financial situation.

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## Appendix

### INSEE method application to the original indicators:

Country	Monetary	Living Condition	Subjective
Francia	10,96	9,58	15,68
Italia	13,81	13,23	13,66
Spagna	12,5	12,93	24,17
Portogallo	21,92	21,94	23,87

	France	Italy	Spain	Portugal
<b>No dimensions</b>	<b>72,4</b>	<b>71,53</b>	<b>63,25</b>	<b>54,13</b>
<b>One dimension</b>	<b>20,16</b>	<b>18,87</b>	<b>25,97</b>	<b>27,83</b>
Monetary	6,24	7,23	5,54	8,7
Living condition	3,87	5,81	5,67	7,18
Subjective	10,05	5,83	14,76	11,95
<b>Two dimensions</b>	<b>6,25</b>	<b>6,98</b>	<b>8,72</b>	<b>14,21</b>
Monetary-Living Condition	1,81	1,77	1,37	6,12
Monetary-Subjective	1,72	2,18	3,52	3,27
Subjective- Living Condition	2,72	3,03	3,83	4,82
<b>Three dimensions</b>	<b>1,19</b>	<b>2,63</b>	<b>2,07</b>	<b>3,82</b>

Correlation	France	Italy	Spain	Portugal
Monetary-Living Condition	0,21	0,22	0,17	0,31
Monetary-Subjective	0,12	0,24	0,19	0,15
Subjective- Living Condition	0,23	0,34	0,22	0,23

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