Università degli Studi di Pisa
Dipartimento di Statistica e Matematica Applicata all'Economia

Report n. 260

Small Area Estimation: the EBLUP estimator using the CAR model

Nicola Salvati

Pisa, Febbraio 2005

- Stampato in Proprio –

Via Cosimo Ridolfi, 10 – 56124 PISA – Tel. Segr. Amm. 050 2216231 Segr. Stud. 050 2216317 Fax 050 2216375
Cod. Fisc. 80003670504 - P. IVA 00286820501 - Web http://statmat.ec.unipi.it/ - E-mail: dipstat@ec.unipi.it

Small Area Estimation: the EBLUP estimator using the CAR model.

Nicola Salvati *

Abstract

This paper deals with small area estimates based on components-of-variance models and, in particular it shows an EBLUP estimator with spatially correlated random area effects taking into account the information provided by neighboring areas (Spatial EBLUP). Introducing spatial dependence through two unknown parameters is low-premium insurance against the concern (expressed by Freedman and Navidi, 1986) that an important explanatory variable may have been missed , or that the linear functional relationship is in fact more complicated. Moreover the estimator of Mean Squared Error of the Spatial EBLUP is presented.

The properties of proposed estimator are evaluated by applying it both to the results of the sample survey on the Life Conditions in Tuscany (Italy) and to the erosion data collected in the Rathbun Lake Watershed in Iowa.

Key words: small area estimation, spatial correlation, CAR model, Spatial EBLUP, lattice data.

1 Introduction

The demand of reliable statistics for small areas, when only reduced sizes of the samples are available, has promoted the development of statistical methods from both the theoretical and empirical point of view. In this perspective the model based methodologies allow for the construction of efficient estimators and their confidence intervals. These small area estimators have several fields of application: from the production of social data to the production of environmental data.

Small area models make use of explicit linking models based on random area-specific effects that account for between area variation beyond that is explained by auxiliary variables included in the model. Under this class of models,

*University of Pisa, Department of Statistics and Mathematics Applied to Economics, Via Ridolfi, 10 - 50126 Pisa -Italy *e-mail:* `salvati@ec.unipi.it`

when only aggregate specific covariates are available, the Best Linear Unbiased Predictor (BLUP) is obtained under the assumption of uncorrelated random area effects (Fay and Herriot, 1979). Details about this predictor, and its empirical version (EBLUP), for small area parameters (total y_i, mean $\bar{y}_i$) can be found in Ghosh and Rao (1994), Rao (1999), Datta and Lahiri (2000) and Rao (2003). The EBLUP estimator borrows strength from other small areas. From this point of view the potentialities of Geographical Information System (GIS) as a tool for the compilation of statistics, particularly in the field of small area statistics, are large. For example, especially in most of applications on environmental data, the land use, the quote, the slope can be used as auxiliary information to estimate the mean in the small areas. The use of GIS is due to the implicit conviction that the data of neighboring areas are correlated and the correlation decays to zero as distance increases. This suggests that further improvement in the EBLUP estimator can be gained by including eventual spatial interaction between random area effects. Introducing spatial dependence through two unknown parameters is low-premium insurance against the concern (expressed by Freedman and Navidi, 1986) that an important explanatory variable may have been missed, or that the linear functional relationship is in fact more complicated.

The only attempt to generalize the Fay-Herriot model, considering correlated random area effects between the neighboring areas, has been made by Cressie (1991). In the context of U.S. census under-count, it was modeled a state adjustment factor using the basic area level model allowing spatial correlation between the random state effects under the Conditional Autoregressive (CAR) spatial model. In the context of U.S. census undercount, the adjustment factor for each small area was determined as a function of spatial correlation, but it was not associated with any area-specific measure of variability. Until now estimators of Mean Squared Error (MSE) of the EBLUP estimators under a spatial model have not been spelled out.

This papers considers an BLUP estimator with spatially correlated random area effects taking into account the information provided by neighboring areas through the Conditional Autoregressive (CAR) process (Spatial BLUP - Section 2). Its empirical version (EBLUP) is obtained and an estimator of its MSE is proposed . The article also proposes a procedure to estimate the variance components of the model using maximum likelihood or restricted maximum likelihood, combining the Nelder-Mead method with "scoring" algorithm (Section 3). The properties of various estimators are evaluated in Section 4 and 5 by analyzing both data from the survey on the Life Conditions in Tuscany and erosion data collected in the Rathbun Lake Watershed in Iowa. Conclusions can be found in Section 6, where, theoretical and applied advantages of the methodology proposed here are summarized.

2 Spatial BLUP

Let $\boldsymbol{\theta}$ be the $m \times 1$ vector of the parameter of inferential interest (small area total y_i, small area mean $\bar{y}_i$ with $i = 1...m$) and assume that the $m \times 1$ vector of the direct estimator $\hat{\boldsymbol{\theta}}$ is available and design unbiased

$$\hat{\boldsymbol{\theta}} = \boldsymbol{\theta} + \mathbf{e} \tag{1}$$

with $\mathbf{e}$ the vector of independent sampling errors with mean $\mathbf{0}$ and known diagonal variance matrix $\boldsymbol{\psi}$. When all the small areas are sampled, it can be of interest to summarize the data of each small area and to refer the result to a location inside the area. Generally the location is the centroid of the small area itself. When this situation arises the sample data can be considered as lattice data. The spatial relationship among data at different locations is usually based on developing neighborhoods and the autocorrelation of locations within neighborhoods. The spatial dependence among small areas is introduced by specifying a linear mixed model with spatially correlated random effects for the $\boldsymbol{\theta}$ parameter:

$$\boldsymbol{\theta} = \mathbf{X}\boldsymbol{\beta} + \mathbf{Z}\mathbf{v} \tag{2}$$

where $\mathbf{X}$ is the $m \times p$ matrix of the area specific auxiliary covariates $\mathbf{x}_i = (x_{i1}, x_{i2}, ..., x_{ip})$, $\boldsymbol{\beta}$ is the regression parameters vector $p \times 1$, $\mathbf{Z}$ is a $m \times m$ matrix of known positive constants, $\mathbf{v}$ is the $m \times 1$ vector of the second order variation. In general the behavior of spatial phenomena is often the result of a mixture of both first order and second order effects. First order effects relate to the variation in the mean value of the process in the space (a global or large scale trend). Second order effects result from the spatial correlation structure, or the spatial dependence in the process; in other words, the tendency for deviations in value of the process from its mean to follow each other in neighboring sites (local or small scale effects). Basically there are two approaches to describe the spatial second order variation: Simultaneously Autoregressive models (SAR) and Conditional Autoregressive models (CAR). These models produce spatial dependence in the covariance structure as a function of a neighborhood matrix $\mathbf{W}$ and a fixed unknown spatial correlation parameter (Wall, 2004). The CAR model is very different from SAR model: the conditional model assumes that the probability of observing a particular value at a given site is a conditional probability, i.e. it depends on the value of Y in the neighborhood of the site. The SAR model states that the probability is a product of functions which can not be interpreted as conditional probabilities.

In the economy of this paper the deviations from the fixed part of the model $\mathbf{X}\boldsymbol{\beta}$ are the result of a conditional autoregressive process (Besag, 1974):

$$v_i|\{v_j \in N_i\} \sim (\rho \sum_{j \in N_i} w_{ij} v_j, \sigma_u^2) \tag{3}$$

where N_i represents a set of neighboring areas of the i-th area, parameter ρ is the spatial autoregressive coefficient and σ_u^2 is the variance. Combining (1) and (2), with $\mathbf{e}$ independent of $\mathbf{v}$, the model with spatially correlated random area effects is:

$$\hat{\boldsymbol{\theta}} = \mathbf{X}\boldsymbol{\beta} + \mathbf{Z}\mathbf{v} + \mathbf{e}. \tag{4}$$

The error terms $\mathbf{v}$ and $\mathbf{e}$ have respectively $m \times m$ covariance matrices:

$$\mathbf{G} = \sigma_u^2(\mathbf{I} - \rho\mathbf{W})^{-1} \tag{5}$$

that is the Conditional Autoregressive (CAR) dispersion matrix, where $\mathbf{I}$ is the $m \times m$ identity matrix, and

$$\mathbf{R} = \boldsymbol{\psi} = diag(\psi_i). \tag{6}$$

Thus the covariance matrix of the $\hat{\boldsymbol{\theta}}$ is:

$$\mathbf{V} = \mathbf{R} + \mathbf{Z}\mathbf{G}\mathbf{Z}^T = diag(\psi_i) + \mathbf{Z}\sigma_u^2(\mathbf{I} - \rho\mathbf{W})^{-1}\mathbf{Z}^T. \tag{7}$$

The $\mathbf{W}$ matrix describes the neighborhood structure of the small areas whereas ρ defines the strength of the spatial relationship among the random effects associated with neighboring areas. The spatial weight matrix represents the potential interaction between locations. One common way to do this is to define $w_{ij} = 1$ if region i shares a common edge or border with region j or 0 otherwise. So $\mathbf{W}$ is the first-order neighbor proximity matrix and here ρ is called a spatial autoregression parameter. Again, in CAR model $\mathbf{W}$ needs to be symmetrical and $(\mathbf{I} - \rho\mathbf{W})$ needs to be strictly positive definite to ensure the existence and symmetry of $\sigma_u^2(\mathbf{I} - \rho\mathbf{W})^{-1}$ in the conditional scheme (Upton and Fingleton, 1985). This is guaranteed if $\rho \in (\frac{1}{min(\lambda_i)}, \frac{1}{max(\lambda_i)})$ where λ_i's are the eigenvalues of matrix $\mathbf{W}$. There are other ways to define $\mathbf{W}$ as restricting rows of the neighborhood matrix to sum to 1 or creating more elaborate weights as functions of the length of borders (Wall, 2004).

Under the model, the Spatial Best Linear Unbiased Predictor (Spatial BLUP) estimator of θ_i is:

$$\tilde{\theta}_i^S(\sigma_u^2, \rho) = \mathbf{x}_i\hat{\boldsymbol{\beta}} + \mathbf{b}_i^T\{\sigma_u^2(\mathbf{I} - \rho\mathbf{W})^{-1}\}\mathbf{Z}^T \times$$
$$\times\{diag(\psi_i) + \mathbf{Z}\sigma_u^2(\mathbf{I} - \rho\mathbf{W})^{-1}\mathbf{Z}^T\}^{-1}(\hat{\boldsymbol{\theta}} - \mathbf{X}\hat{\boldsymbol{\beta}}) \tag{8}$$

where $\hat{\boldsymbol{\beta}} = (\mathbf{X}^T\mathbf{V}^{-1}\mathbf{X})^{-1}\mathbf{X}^T\mathbf{V}^{-1}\hat{\boldsymbol{\theta}}$ and $\mathbf{b}_i^T$ is $1 \times m$ vector $(0, 0, ...0, 1, 0, ...0)$ with 1 in the i-th position. The proposed predictor is obtained from Henderson 1975 results for general linear mixed models involving fixed and random effects. The Spatial BLUP is equal to the traditional BLUP under the random area specific effects model when $\rho = 0$.

The MSE of Spatial BLUP can be obtained, under the specified model, as indicated in Rao (2003). The $MSE[\tilde{\theta}_i^S(\sigma_u^2, \rho)]$, depending on two variance components (σ_u^2, ρ), can be expressed as:

$$MSE[\tilde{\theta}_i^S(\sigma_u^2, \rho)] = g_{1i}(\sigma_u^2, \rho) + g_{2i}(\sigma_u^2, \rho) \tag{9}$$

where the first term $g_{1i}(\sigma_u^2, \rho)$ is due to the estimation of random effects and is of order $O(1)$ while the second term $g_{2i}(\sigma_u^2, \rho)$ is due to the estimation of $\boldsymbol{\beta}$ and is of order $O(m^{-1})$ for large m (Rao, 2003). The details of the calculation are reported in Appendix A.

3 Spatial EBLUP

The estimator $\tilde{\theta}_i^S(\sigma_u^2, \rho)$ depends on the unknown variance components σ_u^2 and ρ. Replacing the parameters with asymptotically consistent estimators $\hat{\sigma}_u^2$, $\hat{\rho}$, a two stage estimator $\tilde{\theta}_i^S(\hat{\sigma}_u^2, \hat{\rho})$ is obtained and is called Spatial EBLUP:

$$\tilde{\theta}_i^S(\hat{\sigma}_u^2, \hat{\rho}) = \mathbf{x}_i\hat{\boldsymbol{\beta}} + \mathbf{b}_i^T\{\hat{\sigma}_u^2(\mathbf{I} - \hat{\rho}\mathbf{W})^{-1}\}\mathbf{Z}^T \times$$
$$\times\{diag(\psi_i) + \mathbf{Z}\hat{\sigma}_u^2(\mathbf{I} - \hat{\rho}\mathbf{W})^{-1}\mathbf{Z}^T\}^{-1}(\hat{\boldsymbol{\theta}} - \mathbf{X}\hat{\boldsymbol{\beta}}) \tag{10}$$

with $\mathbf{b}_i^T = (0, 0, ...0, 1, 0, ...0)$ with 1 referring to the i-th area. The expected value $E[\tilde{\theta}^S(\hat{\sigma}_u^2, \hat{\rho})]$ is finite, the estimator is unbiased for θ and $\hat{\sigma}_u^2, \hat{\rho}$ are any translation invariant estimators of σ_u^2 and ρ (Kackar and Harville, 1984).

The variance components σ_u^2 and ρ can be estimated either by Maximum Likelihood (ML) or Restricted Maximum Likelihood (REML) methods, assuming normality of the random effects, or by the method of fitting constants. The ML and the REML estimators can be obtained iteratively using the "Nelder-Mead" algorithm (Nelder and Mead, 1965) and the "scoring" algorithm in sequence. The use of these procedures one after the other is necessary because the log-likelihood function has a global maximum and some local maximums.

The "Nelder-Mead" method for the maximization of a function does not depend on the selected starting point and is computationally compact but not fully efficient: it achieves a point that is close to the global maximum. For this reason it is necessary to use the "scoring" algorithm, selecting as a starting point the maximum that has been obtained by the "Nelder-Mead" method. The log-likelihood function, its partial derivatives and the information matrix are described in Appendix B and C.

The ML and REML estimators are robust, as they produce acceptable results even under non-normal distribution of the random effects (Jiang, 1996).

The MSE of the Spatial EBLUP estimator appears to be insensitive to the choice of the estimators $\hat{\sigma}_u^2$ and $\hat{\rho}$ (Kackar and Harville, 1984). Given normality of random effects, an approximation to the $MSE[\tilde{\theta}_i^S(\hat{\sigma}_u^2, \hat{\rho})]$ is:

$$MSE[\tilde{\theta}_i^S(\hat{\sigma}_u^2, \hat{\rho})] \approx g_{1i}(\sigma_u^2, \rho) + g_{2i}(\sigma_u^2, \rho) + g_{3i}(\sigma_u^2, \rho) \tag{11}$$

where $g_{3i}(\sigma_u^2, \rho)$ is due to the estimation of the variance components, and it is obtained by following the results of Kackar and Harville (1984):

$$g_{3i}(\sigma_u^2, \rho) = tr\left\{ \begin{bmatrix} \mathbf{b}_i^T \left(\mathbf{D}^{-1}\mathbf{Z}^T\mathbf{V}^{-1} + \sigma_u^2\mathbf{D}^{-1}\mathbf{Z}^T(-\mathbf{V}^{-1}\mathbf{Z}\mathbf{D}^{-1}\mathbf{Z}^T\mathbf{V}^{-1})\right) \\ \mathbf{b}_i^T \left(\sigma_u^2\mathbf{D}^{-1}\mathbf{W}\mathbf{D}^{-1}\mathbf{Z}^T\mathbf{V}^{-1} + \sigma_u^2\mathbf{D}^{-1}\mathbf{Z}^T(-\mathbf{V}^{-1}\mathbf{Z}\sigma_u^2\mathbf{D}^{-1}\mathbf{W}\mathbf{D}^{-1}\mathbf{Z}^T\mathbf{V}^{-1})\right) \end{bmatrix} \mathbf{V} \times \right.$$

$$\left. \times \begin{bmatrix} \mathbf{b}_i^T \left(\mathbf{D}^{-1}\mathbf{Z}^T\mathbf{V}^{-1} + \sigma_u^2\mathbf{D}^{-1}\mathbf{Z}^T(-\mathbf{V}^{-1}\mathbf{Z}\mathbf{D}^{-1}\mathbf{Z}^T\mathbf{V}^{-1})\right) \\ \mathbf{b}_i^T \left(\sigma_u^2\mathbf{D}^{-1}\mathbf{W}\mathbf{D}^{-1}\mathbf{Z}^T\mathbf{V}^{-1} + \sigma_u^2\mathbf{D}^{-1}\mathbf{Z}^T(-\mathbf{V}^{-1}\mathbf{Z}\sigma_u^2\mathbf{D}^{-1}\mathbf{W}\mathbf{D}^{-1}\mathbf{Z}^T\mathbf{V}^{-1})\right) \end{bmatrix}^T \bar{\mathbf{V}}(\hat{\sigma}_u^2, \hat{\rho}) \right\}. \tag{12}$$

with with $\mathbf{D} = (\mathbf{I} - \rho\mathbf{W})$ and $\bar{\mathbf{V}}(\hat{\sigma}_u^2, \hat{\rho})$ is the asymptotic covariance matrix of $\hat{\sigma}_u^2$ and $\hat{\rho}$. In practical application the estimator $\tilde{\theta}_i^S(\hat{\sigma}_u^2, \hat{\rho})$ has to be associated with an estimator of $MSE[\tilde{\theta}_i^S(\hat{\sigma}_u^2, \hat{\rho})]$. An approximately unbiased estimator of it is given by:

$$mse[\tilde{\theta}_i^S(\hat{\sigma}_u^2, \hat{\rho})] \approx g_{1i}(\hat{\sigma}_u^2, \hat{\rho}) + g_{2i}(\hat{\sigma}_u^2, \hat{\rho}) + 2g_{3i}(\hat{\sigma}_u^2, \hat{\rho}) \tag{13}$$

if $\hat{\sigma}_u^2$ and $\hat{\rho}$ are REML estimators. Otherwise, if the ML procedure is used, the $mse[\tilde{\theta}_i^S(\hat{\sigma}_u^2, \hat{\rho})]$ is given by

$$mse[\tilde{\theta}_i^S(\hat{\sigma}_u^2, \hat{\rho})] \approx g_{1i}(\hat{\sigma}_u^2, \hat{\rho}) - \mathbf{b}_{ML}^T(\hat{\sigma}_u^2, \hat{\rho}) \bigtriangledown g_{1i}(\hat{\sigma}_u^2, \hat{\rho}) + g_{2i}(\hat{\sigma}_u^2, \hat{\rho}) + 2g_{3i}(\hat{\sigma}_u^2, \hat{\rho}). \tag{14}$$

The term $\mathbf{b}_{ML}^T(\hat{\sigma}_u^2, \hat{\rho}) \bigtriangledown g_{1i}(\hat{\sigma}_u^2, \hat{\rho})$ is an extra term due to the bias of $g_{1i}(\hat{\sigma}_u^2, \hat{\rho})$ and is calculated in Appendix D. If this term is ignored, the use of ML estimators could lead to underestimation of the approximation of MSE.

4 Application 1: survey on Life Conditions in Tuscany

The region of Tuscany is divided into 10 provinces and 287 municipalities. Each province area is obtained by aggregating a varying number of municipalities. In order to analyze the local economic systems of the region, the territory has been officially divided into 43 sub-regions called Local Economy Systems (LESs). These areas are aggregations of municipalities but they are different from provinces. The main town of the region (Florence) is a separate LES. In this application the small area parameter of interest is the annual per-capita mean income for each LES in the year 2001. A map of the LESs of Tuscany and a description of them can be found in Appendix E.

The primary source of data is the survey on Life Conditions (LC) in Tuscany, which provides the survey estimates of per-capita mean income at region level. The survey on Life Conditions was carried out in 2002 in order to measure the life conditions and to collect data relating to household budgets in Tuscany (IRPET, 2004).

We evaluated the performance of our proposed Spatial EBLUP estimator, comparing it with the corresponding EBLUP estimator under the Fay-Herriot model, and the direct estimator under the LC sampling design. In the sample 40 out of the 43 LESs are represented. The distribution of the sample by LES is reported in Appendix E.

Our attempt to find a parsimonious model led to the choice of the following LES-level predictors: ageing index (x_1 - population over 64/ population under 15) and the percentage of employees in industry (x_2). Moreover, the neighborhood structure $\mathbf{W}$ is defined as follows: spatial weight, w_{ij}, is $1/(distance)^2$ if distance between LES i and LES j is less than 30 Km and 0 otherwise. Regarding the sampling variances ψ_i, they are estimated through a Jackknife procedure (Verma, 2004). The estimated variance $\hat{\psi}_i$ is then treated as a proxy to ψ_i. As result the $mse[\tilde{\theta}_i^S(\hat{\sigma}_u^2, \hat{\rho}, \hat{\psi}_i)]$ is greater than $mse[\tilde{\theta}_i^S(\hat{\sigma}_u^2, \hat{\rho}, \psi_i)]$.

Using $\mathbf{x} = \{x_1, x_2\}$, assuming that the random effects are the result of a conditional autoregressive process, ML and REML estimates of $\boldsymbol{\beta}$, σ_u^2 and ρ, the resulting Spatial EBLUP estimates and their estimated MSE were calculated. The estimated spatial autocorrelation coefficient $\hat{\rho}$ is 73.49 ($s.e. = 4.94$) with the ML procedure and 73.18 ($s.e. = 6.07$) with the REML method: this suggests the existence of a strong spatial relationship considering that $\rho \in (-91.71, 76.24)$. For the non-sampled area, the Spatial EBLUP of θ_i is equal to $\mathbf{x}_i\hat{\boldsymbol{\beta}}$. The estimated variance components are applied to the complete neighborhood structure to calculate the mse estimator for each small area.

Software to estimate the parameters (σ_u^2, ρ) and to perform the calculation of the Spatial EBLUP and the EBLUP estimators was written by the authors in R environment.

In order to appreciate the results obtained with the introduction of spatial information, the EBLUP estimates, using the same covariates, are also computed. Figure 1 displays the maps of the Spatial EBLUP and EBLUP estimates of annual per-capita mean income for the LESs in Tuscany. The maps give a visual representation of the different estimates. The territorial distribution of our estimates appears to be more variable than that obtained with the traditional EBLUP. At a larger range (3,905 Euros vs. 3,755) there is a wider diversification of annual per-capita mean income per LES. Given the same explanatory variables, the result is due to the additional spatial information inserted in our estimator. This moderates the smoothing effect resulting from the application of the traditional EBLUP and makes the specific characteristics of the LES evident. This happens without losing precision in the estimates. Our estimator, in fact, is less variable in each small area. The coefficient of variation (CV) per LES is mainly about $6-8\%$ for our estimator, while it is $8-10\%$ for the EBLUP. The results are clear from Table 2, which shows the distribution of the CV of small areas for the estimators. Direct, EBLUP and Spatial EBLUP estimates of the annual per-capita mean income and their estimated standard errors are reported in Appendix E.

Inferences from model-based estimators refer to the distribution implied by the assumed model. Model selection and validation play an important role in model-based estimation, in fact if the assumed models do not perform a good fit to the data, the estimators will be model-biased and can lead to erroneous inferences.

An evaluation of our spatial model is performed by treating the standard residuals $r = \tilde{\boldsymbol{\theta}}^S(\hat{\sigma}_u^2, \hat{\rho}) - \mathbf{X}\boldsymbol{\beta}/(diag(\mathbf{V}))^{1/2}$ as iid $N(0,1)$. In particular, to check the normality of the standardized residuals r and to detect outlier r, a normal q-q plot is examined (Figure 2). It can be noted that there are few outliers r and that they correspond to neighboring medium-income level areas in the zona of Florence. Not other significant departures from the assumed model were observed. The Shapiro-Wilk W statistic gave a value of 0.959 for small area effects, yielding a p-value of 0.108, which suggests no evidence against the hypothesis of normality.

5 Application 2: survey on erosion level in Rathbun Lake Watershed

In 1999, an environmental health study for the Rathbun Lake Watershed was started by a team of researchers from Iowa State University and the Chariton Valley Resource Conservation and Development office. The Rathbun Lake Watershed covers more than 365,000 acres (147,710 ha) in six counties of Iowa (USA), and is divided into 61 sub-watersheds, with an average size of 5,800 acres (2,350 ha) each. One of the major objectives of the project was the estimation of the surface water pollution by sub-watershed. Since erosion on agricultural land is known to be a major component of water pollution as well as the major source for lake sedimentation, a survey was designed to estimate the amount of erosion delivered to streams in the watershed. In this application the small area parameter of interest is the average total erosion for each sub-watershed.

According to the sampling design and data collection methodology described in Opsomer *et al.* (2001), each small area (domain) has been divided in plots (total 2146), each plot has been sequentially labeled and a systematic sampling of plots has been selected. The fractional interval (Särndal *et al.*, 1992, p. 77) has been fixed in order to select four units from each small area (domain). Not all these 4×61 units have been included in the sample. From each domain a simple random sample of 3 units has been drawn. Then within each sub-watershed, three 160-acre (64 ha) plots were selected, as is showed in Figure 3, and a sample of 183 units was obtained. The final sample can be reasonably assimilated to a simple random sample from the domains and the sampling variance ψ_i at the domain level can be estimated by $\left\{(1-\frac{n_i}{N_i})\frac{\hat{\sigma}_i^2}{n_i}\right\}$, where $n_i = 3$ and N_i is the number of plots in the i-th area (Opsomer et al., 2003).

Auxiliary data at the sub-watershed level are required in order to apply an area level random effect model. For this application the land use and the topography data, that are considered major determinants of the erosion, were available. Data related to these factor were available for the study region in the form of digital elevation and land use classification coverages. An initial set of eight variables for a prediction model was constructed combining soil slope class and vegetation types. The variables which was used as predictors were: "0 to 9% Slope-Row Crop" and "9 to 18% Slope-Row Crop".

To implement a EBLUP estimator combined with a CAR model a neighborhood structure $\mathbf{W}$ has to be defined for the 61 small area within the study region. The spatial weight matrix represents the potential interaction between locations. A general spatial weight matrix can be defined by a symmetric binary contiguity matrix, which can be generated from the topological information given by Geographical Information System (GIS) based on adjacency criteria:

the spatial weight, w_{ij} , is set 1 if area i shares an edge with area j and 0 otherwise.

Figure 4 displays the map of the Rathbun Lake Watershed with the Spatial EBLUP estimates for average erosion per acre in 17 small areas, which are the result of sub-watersheds grouping at higher hierarchical hydrological level. The value of the estimated spatial autocorrelation coefficient $\hat{\rho}$ is 0.156 ($s.e. = 0.021$) using the ML procedure and 0.160 ($s.e. = 0.022$) with the REML method, which suggests a strong spatial relationship considering that $\rho \in (-0.349, 0.171)$.

In order to asses the achieved results with the introduction of the spatial information in the small area estimation, the EBLUP estimator and the Direct estimator are also calculated. In Table 2 are reported the average of the estimated standard errors and its variability per acre of the Spatial EBLUP, the EBLUP and the Direct estimators. Table 2 shows also the average of the estimated mse per acre and its breakdown into g_1, due to the random effects, g_2, which accounts for the variability in the estimator $\hat{\beta}$ and g_3 due to estimate ρ and σ_u^2.

The Spatial EBLUP method provides estimates with smaller average estimated standard errors than the Direct and the EBLUP estimators. We note that g_1 is lower for the Spatial EBLUP compared to that in the EBLUP, while g_2 and g_3 are larger than those of the EBLUP estimator. These results are motivated, respectively, by the precise fit of the correlated random effects model to the data (g_1) and by the additional estimation of parameter ρ that is not included in the model underlying the EBLUP estimator (g_2 and g_3). The estimates of the average erosion in each of the 61 small area are reported in Annex F.

An evaluation of our spatial model is performed by treating the standard residuals r. The Shapiro-Wilk W statistic gives a value of 0.976 for small area effects, yielding a p-value of 0.289 that suggests no evidence against the hypothesis of normality.

A simple exploratory device enables another evaluation of the model: the plotting of estimated random effects $\hat{\mathbf{v}}$ against $\mathbf{W}\hat{\mathbf{v}}$, which corresponds to an average of the estimated random effects for nearest neighbors (Figure 5a). It is interesting to confront this plot with an equivalent one (Figure 5b), where $\mathbf{W}\hat{\mathbf{v}}$ is defined as an average of "randomly chosen neighbors" (rather that near neighbors). If there is no spatial correlation in the area effects, then one would expect the slope of a least squares line fitted to the data not to be significantly different from zero. This is the case of the plot in Figure 5b: the comparison between plot (a) and (b) suggests that the random area effects are truly spatially correlated.

6 Final remarks

In this paper we have developed an indirect small area estimator based on a CAR spatial model. The proposed estimator performs very well when applied to the LS data and to the Rathbun Lake Watershed data.

The results of this study suggest that the proposed spatial small area estimator, which takes into account the spatial dimensions of the data modeling the spatial correlation between small area random effects, allows to obtain an appreciable improvement of the small area estimates. The results lead to small area estimates that are better than the usual EBLUP estimates. The point estimators (13) and (14) of MSE of the Spatial EBLUP generated by their three components g_1, g_2 and g_3 are smaller than those of EBLUP estimator. We note that g_1 is lower for the Spatial EBLUP compared to that in the EBLUP. This results is motivated by the precise fit of the correlated random effects model to the data. Thus, considering the cases study, the use of the Spatial EBLUP methodology, which takes into account the CAR model in the small area estimation, reduces the confidence interval. Moreover, the methodology presented allows to make use of all the informative components of the survey data including the geographical ones.

Finally, in practical applications, some of the areas are much smaller than the others and it may occur that these areas of interest are not represented in the sample. Even in this case the Spatial EBLUP can be employed: it can be used the neighborhood structure of those areas which are represented in the sample to estimate the parameters σ_u^2 and ρ. For the area i-th with sample observation the Spatial EBLUP is applied; for the area i-th with non-sample observations, the Spatial EBLUP of θ_i is equal to $\mathbf{x}_i\hat{\boldsymbol{\beta}}$. The estimated parameters are applied to the complete neighborhood structure to calculate the *mse* estimator for each small area.

The developments of the Spatial EBLUP can regard either theoretical work or empirical studies. For example we have considered only the estimation of population totals and means of continuous response variables, but most surveys involve other parameters of interest, such as counts and percentages when the response variable is discrete. Empirical studies are also important to gain further experience with the approach that we propose. It would be useful to verify the performance of the methods using more complex spatial contiguity matrices (Cliff and Ord, 1981; Dacey, 1965; Getis and Aldstadt, 2004).

Acknowledgements: the author is grateful to Jean Opsomer for data availability. I also thanks Monica Pratesi for many suggestions which improved the article.

Estimate	Coefficients of Variation				
	$4-6\%$	$6-8\%$	$8-10\%$	$10-12\%$	**Total**
$\tilde{\theta}^S(\hat{\sigma}^2_{u_{ML}}, \hat{\rho}_{ML})$	2	35	6	0	43
$\tilde{\theta}(\hat{\sigma}^2_{u_{ML}})$	2	16	24	1	43
$\tilde{\theta}^S(\hat{\sigma}^2_{u_R}, \hat{\rho}_R)$	1	18	20	4	43
$\tilde{\theta}(\hat{\sigma}^2_{u_R})$	1	12	26	4	43

Table 1: Class distribution of coefficients of variation of small areas for EBLUP and Spatial EBLUP estimators.

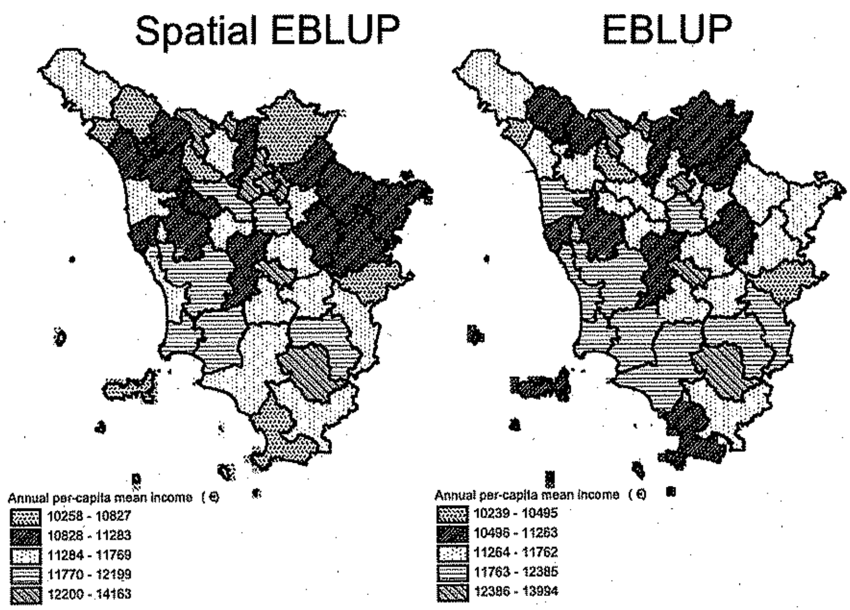

Figure 1: Map of the LESs of the region of Tuscany with Spatial EBLUP and EBLUP estimates for annual per-capita mean income.

Estimator	*A.E.Se.*	*A.E.mse*	*A.E.*(g_1)	*A.E.*(g_2)	*A.E.*(g_3)
$\tilde{\theta}^S(\hat{\sigma}^2_{u_{ML}}, \hat{\rho}_{ML})$	0.520	47.12	42.85	2.07	1.40
$\tilde{\theta}(\hat{\sigma}^2_{u_{ML}})$	0.549	52.38	47.88	2.57	0.93
$\tilde{\theta}^S(\hat{\sigma}^2_{u_R}, \hat{\rho}_R)$	0.531	50.20	44.80	2.10	1.65
$\tilde{\theta}(\hat{\sigma}^2_{u_R})$	0.560	54.79	50.19	2.57	1.01
DIRECT θ	0.886	–	–	–	–

Table 2: Average of the Estimated Standard Errors (A.E.Se.) and average of the Estimated Mean Squared Error (A.E.mse.) of Spatial EBLUP, EBLUP and Direct estimators.

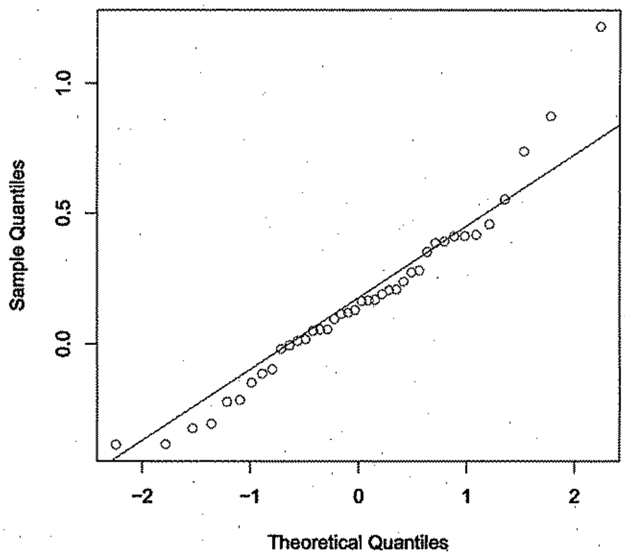

Figure 2: Normal q-q plot to check the normality of the standardized residuals r.

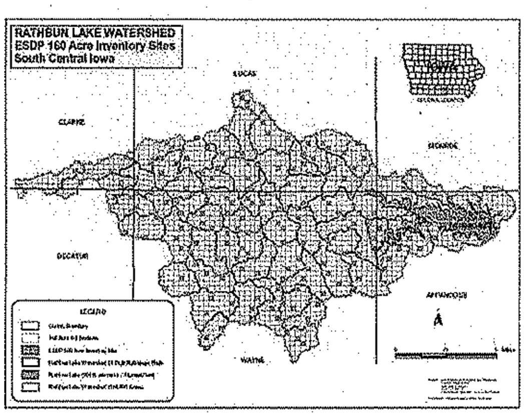

Figure 3: Map of Rathbun Lake watershed. Delineations within the watershed are the 61 sub-watersheds, and the dark squares are erosion assessment sample location.

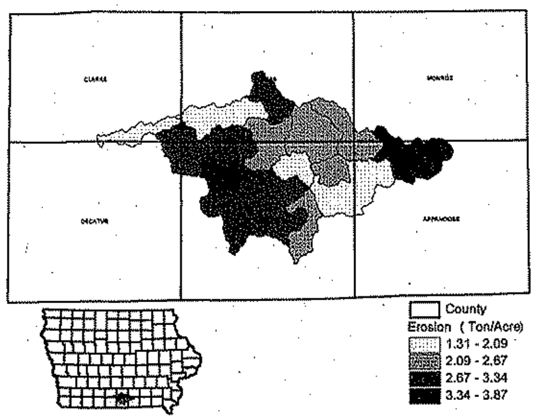

Figure 4: Map of Rathbun Lake watershed with Spatial EBLUP estimates for average erosion. ML procedure.

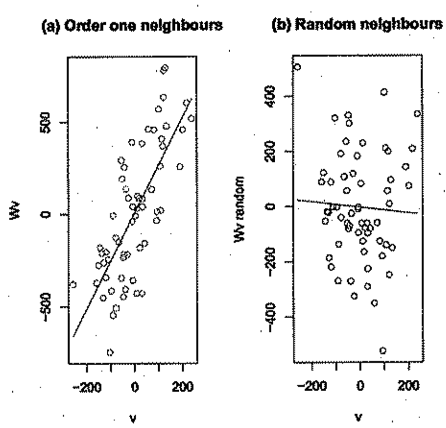

Figure 5: The estimated random effects $\hat{\mathbf{v}}$ against an average of estimated random effects for nearest neighbors $\mathbf{W}\hat{\mathbf{v}}$ (a) and an average of "randomly chosen neighbors" (b).

A Appendix

The $MSE[\tilde{\theta}_i^S(\sigma_u^2,\rho)]$, depending on two parameters (σ_u^2,ρ), can be expressed as:

$$MSE[\tilde{\theta}_i^S(\sigma_u^2,\rho)] = g_{1i}(\sigma_u^2,\rho) + g_{2i}(\sigma_u^2,\rho) \tag{A-1}$$

with with

$$g_{1i}(\sigma_u^2,\rho) = \mathbf{b}_i^T\{\sigma_u^2(\mathbf{I}-\rho\mathbf{W})^{-1} - \sigma_u^2(\mathbf{I}-\rho\mathbf{W})^{-1}\mathbf{Z}^T\times$$
$$\times\{diag(\psi_i) + \mathbf{Z}\sigma_u^2(\mathbf{I}-\rho\mathbf{W})^{-1}\mathbf{Z}^T\}^{-1}\mathbf{Z}\sigma_u^2(\mathbf{I}-\rho\mathbf{W})^{-1}\}\mathbf{b}_i \tag{A-2}$$

and

$$g_{2i}(\sigma_u^2,\rho) = (\mathbf{x}_i - b_i^T\sigma_u^2(\mathbf{I}-\rho\mathbf{W})^{-1}\mathbf{Z}^T\{diag(\psi_i) + \mathbf{Z}\sigma_u^2(\mathbf{I}-\rho\mathbf{W})^{-1}\mathbf{Z}^T\}^{-1}\mathbf{X})\times$$
$$\times(\mathbf{X}^T\{diag(\psi_i) + \mathbf{Z}\sigma_u^2(\mathbf{I}-\rho\mathbf{W})^{-1}\mathbf{Z}^{-1}\}^{-1}\mathbf{X})^{-1}\times$$
$$\times(\mathbf{x}_i - \mathbf{b}_i^T\sigma_u^2(\mathbf{I}-\rho\mathbf{W})^{-1}\mathbf{Z}^T\{diag(\psi_i) + \mathbf{Z}\sigma_u^2(\mathbf{I}-\rho\mathbf{W})^{-1}\mathbf{Z}^T\}^{-1}\mathbf{X})^T. \tag{A-3}$$

B Appendix

The log-likelihood function is:

$$l(\boldsymbol{\beta},\sigma_u^2,\rho) = -\frac{1}{2}m\log 2\pi - \frac{1}{2}\log|\mathbf{V}| - \frac{1}{2}(\hat{\boldsymbol{\theta}} - \mathbf{X}\boldsymbol{\beta})^T\mathbf{V}^{-1}(\hat{\boldsymbol{\theta}} - \mathbf{X}\boldsymbol{\beta}) \tag{B-1}$$

with $\mathbf{V}$ as represented in (7) and the partial derivatives of $l(\boldsymbol{\beta},\sigma_u^2,\rho)$ with respect to σ_u^2 and ρ given by

$$\begin{array}{rl} s_{\sigma_u^2}(\boldsymbol{\beta},\sigma_u^2,\rho) = \frac{\partial l}{\partial\sigma_u^2} = & -\frac{1}{2}tr\{\mathbf{V}^{-1}\mathbf{Z}\mathbf{D}^{-1}\mathbf{Z}^T\} + \frac{1}{2}(\hat{\boldsymbol{\theta}} - \mathbf{X}\boldsymbol{\beta})^T(\mathbf{V}^{-1}\mathbf{Z}\mathbf{D}^{-1}\mathbf{Z}^T\mathbf{V}^{-1})(\hat{\boldsymbol{\theta}} - \mathbf{X}\boldsymbol{\beta}) \\ s_{\rho}(\boldsymbol{\beta},\sigma_u^2,\rho) = \frac{\partial l}{\partial\rho} = & -\frac{1}{2}tr\{\mathbf{V}^{-1}\mathbf{Z}\sigma_u^2[\mathbf{D}^{-1}\mathbf{W}\mathbf{D}^{-1}]\mathbf{Z}^T\} + \\ & +\frac{1}{2}(\hat{\boldsymbol{\theta}} - \mathbf{X}\boldsymbol{\beta})^T(\mathbf{V}^{-1}\mathbf{Z}\sigma_u^2[\mathbf{D}^{-1}\mathbf{W}\mathbf{D}^{-1}]\mathbf{Z}^T\mathbf{V}^{-1})(\hat{\boldsymbol{\theta}} - \mathbf{X}\boldsymbol{\beta}) \end{array} \tag{B-2}$$

with $\mathbf{D} = (\mathbf{I} - \rho\mathbf{W})$. The matrix of expected second derivatives of $-l(\boldsymbol{\beta},\sigma_u^2,\rho)$ with respect to σ_u^2 and ρ is given by

$$\mathcal{I}(\sigma_u^2,\rho) = \begin{bmatrix} \mathcal{I}_{11} & \mathcal{I}_{12} \\ \mathcal{I}_{21} & \mathcal{I}_{22} \end{bmatrix} \tag{B-3}$$

where

$$\mathcal{I}_{11} = \frac{1}{2}tr\{\mathbf{V}^{-1}\mathbf{Z}\mathbf{D}^{-1}\mathbf{Z}^T\mathbf{V}^{-1}\mathbf{Z}\mathbf{D}^{-1}\mathbf{Z}^T\}$$

$$\mathcal{I}_{12} = \frac{1}{2}tr\{\mathbf{V}^{-1}\mathbf{Z}\mathbf{C}^{-1}\mathbf{Z}^T\mathbf{V}^{-1}\mathbf{Z}\sigma_u^2\mathbf{D}^{-1}\mathbf{W}\mathbf{D}^{-1}\mathbf{Z}^T\}$$

$$\mathcal{I}_{21} = \frac{1}{2} tr\{\mathbf{V}^{-1}\mathbf{Z}\sigma_u^2\mathbf{D}^{-1}\mathbf{W}\mathbf{D}^{-1}\mathbf{Z}^T\mathbf{V}^{-1}\mathbf{Z}\mathbf{D}^{-1}\mathbf{Z}^T\}$$

$$\mathcal{I}_{22} = \frac{1}{2} tr\{\mathbf{V}^{-1}\mathbf{Z}\sigma_u^2\mathbf{D}^{-1}\mathbf{W}\mathbf{D}^{-1}\mathbf{Z}^T\mathbf{V}^{-1}\mathbf{Z}\sigma_u^2\mathbf{D}^{-1}\mathbf{W}\mathbf{D}^{-1}\mathbf{Z}^T\}.$$

The ML estimators $\hat{\sigma}^2_{u_{ML}}$ and $\hat{\rho}_{ML}$ can be obtained iteratively using the "scoring" algorithm:

$$\begin{bmatrix} \sigma_u^2 \\ \rho \end{bmatrix}^{(n+1)} = \begin{bmatrix} \sigma_u^2 \\ \rho \end{bmatrix}^{(n)} + [\mathcal{I}(\sigma_u^{2^{(n)}}, \rho^{(n)})]^{-1} \cdot s\left[\hat{\boldsymbol{\beta}}(\sigma_u^{2^{(n)}}, \rho^{(n)}), \sigma_u^{2^{(n)}}, \rho^{(n)}\right] \quad \text{(B-4)}$$

where n indicates the number of iteration.

C Appendix

The partial derivatives of the restricted log-likelihood function $l_R(\sigma_u^2, \rho)$ with respect to variance components are:

$$\begin{aligned} s_{R_{\sigma_u^2}}(\sigma_u^2, \rho) = \frac{\partial l_R}{\partial \sigma_u^2} = & \ -\tfrac{1}{2} tr\{\mathbf{P}\mathbf{Z}\mathbf{D}^{-1}\mathbf{Z}^T\} + \tfrac{1}{2}\hat{\boldsymbol{\theta}}^T\mathbf{P}\mathbf{Z}\mathbf{D}^{-1}\mathbf{Z}^T\mathbf{P}\hat{\boldsymbol{\theta}} \\ s_{R_\rho}(\sigma_u^2, \rho) = \frac{\partial l_R}{\partial \rho} = & \ -\tfrac{1}{2} tr\{\mathbf{P}\mathbf{Z}\sigma_u^2[\mathbf{D}^{-1}\mathbf{W}\mathbf{D}^{-1}]\mathbf{Z}^T\} + \\ & +\tfrac{1}{2}\hat{\boldsymbol{\theta}}^T\mathbf{P}\mathbf{Z}\sigma_u^2[\mathbf{D}^{-1}\mathbf{W}\mathbf{C}^{-1}]\mathbf{Z}^T\mathbf{P}\hat{\boldsymbol{\theta}} \end{aligned} \quad \text{(C-1)}$$

with $\mathbf{P} = \mathbf{V}^{-1} - \mathbf{V}^{-1}\mathbf{X}(\mathbf{X}^T\mathbf{V}^{-1}\mathbf{X})^{-1}\mathbf{X}^T\mathbf{V}^{-1}$ and with $\mathbf{D} = (\mathbf{I} - \rho\mathbf{W})$.

The $\mathcal{I}_R(\sigma_u^2, \rho)$ matrix assumes the form:

$$\mathcal{I}_R(\sigma_u^2, \rho) = \begin{bmatrix} \mathcal{I}_{11} & \mathcal{I}_{12} \\ \mathcal{I}_{21} & \mathcal{I}_{22} \end{bmatrix} \quad \text{(C-2)}$$

where

$$\mathcal{I}_{11} = \frac{1}{2} tr\{\mathbf{P}\mathbf{Z}\mathbf{D}^{-1}\mathbf{Z}^T\mathbf{P}\mathbf{Z}\mathbf{D}^{-1}\mathbf{Z}^T\}$$

$$\mathcal{I}_{12} = \frac{1}{2} tr\{\mathbf{P}\mathbf{Z}\mathbf{D}^{-1}\mathbf{Z}^T\mathbf{P}\mathbf{Z}\sigma_u^2\mathbf{D}^{-1}\mathbf{W}\mathbf{D}^{-1}\mathbf{Z}^T\}$$

$$\mathcal{I}_{21} = \frac{1}{2} tr\{\mathbf{P}\mathbf{Z}\sigma_u^2\mathbf{D}^{-1}\mathbf{W}\mathbf{D}^{-1}\mathbf{Z}^T\mathbf{P}\mathbf{Z}\mathbf{D}^{-1}\mathbf{Z}^T\}$$

$$\mathcal{I}_{22} = \frac{1}{2} tr\{\mathbf{P}\mathbf{Z}\sigma_u^2\mathbf{D}^{-1}\mathbf{W}\mathbf{D}^{-1}\mathbf{Z}^T\mathbf{P}\mathbf{Z}\sigma_u^2\mathbf{D}^{-1}\mathbf{W}\mathbf{D}^{-1}\mathbf{Z}^T\}.$$

Then the "Nelder-Mead" method and the "scoring" algorithm (B-4) are used and at convergence the REML estimators are obtained. The asymptotic covariance matrix of $\hat{\boldsymbol{\beta}}_R$, $\hat{\sigma}^2_{u_R}$ and $\hat{\rho}_R$ has a diagonal structure $diag\left[\bar{\mathbf{V}}(\hat{\boldsymbol{\beta}}_R), \bar{\mathbf{V}}(\hat{\sigma}^2_{u_R}, \hat{\rho}_R)\right] \approx diag\left[\bar{\mathbf{V}}(\hat{\boldsymbol{\beta}}_{ML}), \bar{\mathbf{V}}(\hat{\sigma}^2_{u_{ML}}, \hat{\rho}_{ML})\right]$ with

$$\bar{\mathbf{V}}(\hat{\boldsymbol{\beta}}_R) \approx \bar{\mathbf{V}}(\hat{\boldsymbol{\beta}}_{ML}) = (\mathbf{X}^T\mathbf{V}^{-1}\mathbf{X})^{-1}$$

$$\bar{\mathbf{V}}(\hat{\sigma}^2_{u_R}, \hat{\rho}_R) \approx \bar{\mathbf{V}}(\hat{\sigma}^2_{u_{ML}}, \hat{\rho}_{ML}) = \mathcal{I}^{-1}(\sigma^2_u, \rho). \tag{C-3}$$

D Appendix

If the ML procedure is used to estimate the variance components the term $\mathbf{b}^T_{ML}(\hat{\sigma}^2_u, \hat{\rho}) \bigtriangledown g_{1i}(\hat{\sigma}^2_u, \hat{\rho})$ of the MSE estimator of Spatial EBLUP is given by:

$$\mathbf{b}^T_i \left\{ \begin{array}{c} (\mathbf{D}^{-1} - [\mathbf{D}^{-1}\mathbf{Z}^T\mathbf{V}^{-1}\mathbf{Z}\sigma^2_u\mathbf{D}^{-1} + \sigma^2_u\mathbf{D}^{-1}\mathbf{Z}^T(-\mathbf{V}^{-1}\mathbf{Z}\mathbf{D}^{-1}\mathbf{Z}^T\mathbf{V}^{-1})\mathbf{Z}\sigma^2_u\mathbf{D}^{-1} + \\ (\sigma^2_u\mathbf{D}^{-1}\mathbf{W}\mathbf{D}^{-1} - [\sigma^2_u\mathbf{D}^{-1}\mathbf{W}\mathbf{D}^{-1}\mathbf{Z}^T\mathbf{V}^{-1}\mathbf{Z}\sigma^2_u\mathbf{D}^{-1} + \\ +\sigma^2_u\mathbf{D}^{-1}\mathbf{Z}^T V^{-1}\mathbf{Z}\mathbf{D}^{-1}]) \\ \sigma^2_u\mathbf{D}^{-1}\mathbf{Z}^T(-\mathbf{V}^{-1}\mathbf{Z}\sigma^2_u\mathbf{D}^{-1}\mathbf{W}\mathbf{D}^{-1}\mathbf{Z}^T\mathbf{V}^{-1})\mathbf{Z}\sigma^2_u\mathbf{D}^{-1} + \sigma^2_u\mathbf{D}^{-1}\mathbf{Z}^T\mathbf{V}^{-1}\mathbf{Z}\sigma^2_u\mathbf{D}^{-1}\mathbf{W}\mathbf{D}^{-1}]) \end{array} \right\} \mathbf{b}_i \tag{D-1}$$

and

$$\mathbf{b}_{ML}(\sigma^2_u, \rho) = \frac{1}{2m} \left\{ \mathcal{I}^{-1}(\sigma^2_u, \rho) \left[\begin{array}{c} tr[(\mathbf{X}^T\mathbf{V}^{-1}\mathbf{X})^{-1}\mathbf{X}^T(-\mathbf{V}^{-1}\mathbf{Z}\mathbf{D}^{-1}\mathbf{Z}^T\mathbf{V}^{-1})\mathbf{X}] \\ tr[(\mathbf{X}^T\mathbf{V}^{-1}\mathbf{X})^{-1}\mathbf{X}^T(-\mathbf{V}^{-1}\mathbf{Z}\sigma^2_u\mathbf{D}^{-1}\mathbf{W}\mathbf{D}^{-1}\mathbf{Z}^T\mathbf{V}^{-1})\mathbf{X}] \end{array} \right] \right\} \tag{D-2}$$

where $\mathcal{I}(\sigma^2_u, \rho)$ is given by (B-3), $\mathbf{D} = (\mathbf{I} - \rho\mathbf{W})$.

E Appendix

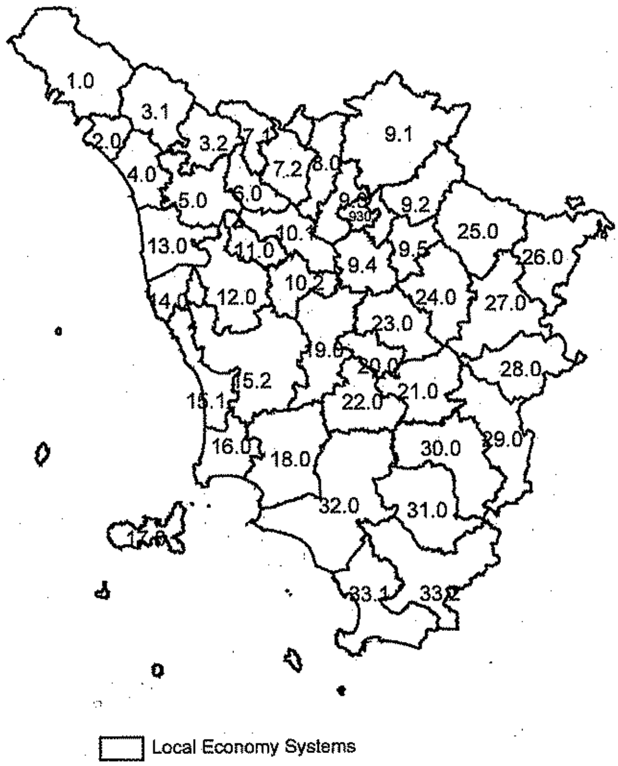

Figure E-1: Map of the LES of Tuscany.

LES code	*LES description*	**Sample size**
1	Lunigiana	44
2	Area di Massa-Carrara	144
3.1	Valle del Serchio - Quadrante Garfagnana	24
3.2	Valle del Serchio - Quadrante Media Valle	23
4	Versilia	99
5	Area Lucchese	113
6	Val di Nievole	109
7.1	Area Pistoiese - Quadrante Montano	–
7.2	Area Pistoiese - Quadrante Metropolitano	46
8	Area Pratese	117
9.1	Area Fiorentina - Quadrante Mugello	13
9.2	Area Fiorentina - Quadrante Val di sieve	10
9.3	Area Fiorentina - Quadrante Centrale	173
9.4	Area Fiorentina - Quadrante Chianti	76
9.5	Area Fiorentina - Quadrante Valdarno Superiore Nord	34
10.1	Circondario di Empoli - Quadrante empolese	81
10.2	Circondario di Empoli - Quadrante Valdelsano	26
11	Valdarno Inferiore	57
12	Val d'Era	61
13	Area Pisana	104
14	Area Livornese	69
15.1	Val di Cecina - Quadrante Costiero	35
15.2	Val di Cecina - Quadrante Interno	51
16	Val di Cornia	75
17	Arcipelago	–
18	Colline Metallifere	29
19	Alta Val d'Elsa	25
20	Area Urbana	47
21	Crete Senesi - Val d'Arbia	19
22	Val di Merse	16
23	Chianti	10
24	Valdarno Superiore Sud	43
25	Casentino	10
26	Alta Val Tiberina	56
27	Area Aretina	115
28	Val di Chiana Aretina	53
29	Val di Chiana Senese	53
30	Amiata - Val d'Orcia	50
31	Amiata Grossetano	–
32	Area Grossetana	108
33.1	Albegna-Fiora - Quadrante Costa d'Argento	39
33.2	Albegna-Fiora - Quadrante Colline Interne	45
930	Firenze	310

Table E-1: LES code and sample size, Tuscany.

LES code	$\hat{\theta}_{ML}$	*s.e.*	$\hat{\theta}^S_{ML}$	*s.e.*	$\hat{\theta}^S_{REML}$	*s.e.*	$\hat{\theta}_{REML}$	*s.e.*	**Direct**	*s.e.*
1	11768.79	837.46	11624.83	951.11	11626.38	938.50	11512.37	992.45	10167.49	1371.97
2	10257.92	649.92	10360.55	663.65	10206.44	678.05	10316.08	673.45	9938.32	741.38
3.1	10827.02	779.35	10859.42	929.00	10667.83	908.97	10730.42	987.45	8580.21	1596.80
3.2	10969.85	861.07	11109.03	989.74	10804.14	1009.70	10976.56	1050.30	8677.20	1763.12
4	10972.81	766.86	11423.12	870.03	11034.59	863.06	11461.78	907.06	11774.57	1201.40
5	11228.15	730.87	11540.03	814.09	11258.32	821.85	11554.48	846.34	11691.50	1084.32
6	10519.94	702.27	10238.74	715.54	10387.11	751.05	10164.98	732.36	9531.66	844.43
7.1	12797.61	1174.51	12938.85	1257.32	12733.47	1317.63	12888.73	1332.04	–	–
7.2	11487.80	874.28	11543.28	935.99	11464.00	999.21	11562.91	994.21	11899.10	1604.70
8	11111.78	739.60	11093.38	793.09	11092.00	816.30	11093.31	812.50	11074.33	952.47
9.1	10781.07	849.53	11187.55	1019.39	10804.10	997.89	11205.44	1104.18	11532.61	2904.06
9.2	11157.86	855.31	11262.88	1016.98	11155.44	1019.99	11251.72	1102.63	10600.48	3012.66
9.3	12648.86	704.12	11710.43	699.56	12639.37	758.57	11756.74	713.77	12063.35	808.79
9.4	12082.45	801.53	12088.28	911.23	12220.06	921.22	12192.04	959.91	13646.16	1406.75
9.5	11458.63	889.22	11435.75	1001.85	11540.81	1047.11	11487.48	1070.11	12587.11	1980.34
10.1	11857.39	796.28	11720.64	892.74	11886.05	892.93	11757.56	930.15	12240.36	1240.74
10.2	11759.01	899.31	11753.27	1075.98	11778.42	1020.43	11754.07	1147.15	12223.22	2284.77
11	11250.30	884.39	11363.73	977.87	11269.92	970.80	11374.65	1015.69	11579.70	1359.23
12	11198.00	814.10	11207.06	906.21	11184.70	910.07	11179.13	947.96	10859.71	1309.80
13	11688.44	805.81	12118.07	904.71	11847.04	908.24	12233.52	946.08	13550.35	1297.51
14	10932.57	755.27	10785.86	829.39	10877.84	834.21	10729.10	859.10	10067.99	1079.06
15.1	11726.21	809.98	12191.20	996.42	11859.57	957.52	12297.88	1075.86	15723.00	2433.36
15.2	12198.81	840.29	12384.70	979.65	12241.69	960.66	12400.34	1038.19	13007.86	1701.97
16	12102.65	816.06	12211.55	904.72	12081.15	902.62	12181.30	940.20	12022.66	1239.00
17	10621.41	963.58	10933.47	1133.52	10667.75	1084.41	10970.25	1200.62	–	–
18	11891.30	820.79	12046.34	1007.89	11911.26	943.96	12050.71	1079.42	12330.66	2119.80
19	11130.72	891.61	11093.33	969.04	11162.10	1069.75	11059.27	1033.54	10343.28	1805.38
20	13121.92	1043.36	13068.80	1063.26	13335.24	1433.50	13209.51	1135.22	17720.15	2288.80
21	11609.55	851.64	11762.20	993.50	11658.54	1026.40	11764.17	1072.37	12075.41	2410.07
22	11586.01	827.29	11573.28	986.87	11614.22	1016.61	11550.47	1067.04	10856.30	2450.98
23	11624.25	976.78	11336.27	1070.15	11811.39	1369.18	11402.75	1164.23	18190.08	5459.31
24	11184.55	846.58	11057.87	920.01	11171.50	962.03	11009.07	969.71	10323.60	1436.21
25	11259.04	866.89	11533.57	1024.72	11248.46	1006.31	11517.60	1110.73	11039.84	3131.57
26	11283.02	777.07	11469.42	881.99	11262.28	881.29	11435.54	928.51	11047.62	1334.14
27	11212.73	742.66	11689.11	800.54	11319.35	812.39	11744.98	829.17	12227.06	1034.10
28	10470.40	764.64	10495.40	844.29	10392.67	853.95	10414.44	881.32	9434.48	1168.40
29	11586.42	782.87	11856.36	912.75	11585.91	894.04	11849.33	964.83	11829.12	1462.18
30	12036.42	844.51	12139.25	958.32	11997.36	946.76	12097.72	1005.91	11747.44	1471.12
31	13058.41	1078.89	13044.94	1266.53	13003.45	1172.91	12999.81	1336.35	–	–
32	11530.45	762.63	11898.49	841.80	11649.68	849.18	11965.32	871.14	12559.21	1090.79
33.1	10612.06	866.16	10762.34	983.71	10579.28	968.83	10728.35	1027.67	9951.94	1438.63
33.2	11706.09	852.64	11463.72	953.11	11537.58	942.07	11335.25	990.95	9802.54	1329.82
930	14163.32	687.59	13994.07	693.81	14280.94	735.72	14121.33	704.92	15144.66	784.65

Table E-2: Direct, EBLUP and Spatial EBLUP estimates for annual per-capita mean income for each LES of the Tuscany region.

F Appendix

Area Code	$\tilde{\theta}^S(\hat{\sigma}^2_{u_R}, \hat{\rho}_R)$	$\widehat{\text{s.q.m.}}$	$\tilde{\theta}(\hat{\sigma}^2_{u_R})$	$\widehat{\text{s.q.m.}}$	$\hat{\theta}$	$\widehat{\text{s.q.m.}}$
10280201040010	2.85	0.22	2.83	0.22	2.80	0.23
10280201040020	4.24	0.25	4.25	0.25	4.36	0.26
10280201040030	3.46	0.40	3.43	0.40	3.31	0.42
10280201040040	4.31	0.53	4.28	0.56	5.08	0.70
10280201040050	3.44	0.64	3.14	0.67	3.57	0.99
10280201040060	4.44	0.19	4.43	0.19	4.51	0.19
10280201040070	3.54	0.74	2.83	0.81	3.40	2.52
10280201040080	2.56	0.53	2.36	0.56	1.94	0.71
10280201040090	3.47	0.49	3.54	0.50	3.88	0.61
10280201040100	1.98	0.39	1.97	0.39	1.79	0.43
10280201040110	2.73	0.56	2.58	0.58	2.15	0.76
10280201040120	2.20	0.57	2.17	0.60	1.75	0.81
10280201040130	3.37	0.55	3.22	0.56	3.68	0.71
10280201040140	3.70	0.71	3.11	0.75	3.77	1.54
10280201040150	3.57	0.40	3.47	0.40	3.70	0.44
10280201040160	1.99	0.47	2.01	0.48	1.66	0.57
10280201040170	2.91	0.71	3.06	0.78	4.45	1.57
10280201040180	4.22	0.55	4.09	0.55	4.95	0.70
10280201040190	3.32	0.45	3.25	0.46	3.54	0.53
10280201040200	2.41	0.39	2.55	0.40	2.41	0.44
10280201040210	2.15	0.58	2.26	0.60	2.15	0.78
10280201040220	1.27	0.30	1.27	0.31	1.02	0.33
10280201040230	1.99	0.61	2.11	0.63	1.49	0.84
10280201040240	2.70	0.67	2.58	0.68	2.67	1.04
10280201040250	1.90	0.54	2.18	0.56	2.01	0.71
10280201040260	1.80	0.46	1.78	0.47	1.44	0.55
10280201040270	1.99	0.60	2.03	0.60	1.16	0.81
10280201040280	3.09	0.31	3.18	0.31	3.22	0.33
10280201040290	2.28	0.57	2.50	0.59	2.31	0.79
10280201050010	2.63	0.66	2.79	0.72	2.77	1.27
10280201050020	2.60	0.62	2.82	0.65	3.18	0.96

Table F-1: Estimate of the total watershed in each small area and estimated Standard Errors (E.Se.) of Spatial EBLUP, EBLUP and Direct estimators. REML procedure.

(Continued)

Area Code	$\tilde{\theta}^S(\hat{\sigma}^2_{u_R}, \hat{\rho}_R)$	$\widehat{\text{s.q.m.}}$	$\tilde{\theta}(\hat{\sigma}^2_{u_R})$	$\widehat{\text{s.q.m.}}$	$\hat{\theta}$	$\widehat{\text{s.q.m.}}$
10280201060010	3.91	0.79	3.73	0.86	4.37	1.96
10280201060020	3.02	0.55	2.96	0.56	2.72	0.67
10280201060030	3.52	0.34	3.47	0.34	3.61	0.37
10280201060040	2.94	0.52	2.75	0.53	2.80	0.64
10280201060050	3.70	0.56	3.83	0.59	4.50	0.79
10280201060060	3.36	0.69	3.42	0.75	3.46	1.26
10280201060070	3.59	0.73	3.10	0.78	4.50	1.87
10280201060080	3.03	0.44	2.88	0.44	2.99	0.50
10280201060090	3.18	0.73	3.10	0.76	4.78	1.61
10280201060100	2.86	0.65	2.78	0.69	2.32	1.09
10280201060110	2.63	0.55	2.56	0.56	2.56	0.73
10280201060120	3.00	0.75	3.00	0.75	3.45	1.36
10280201060130	2.12	0.26	2.10	0.26	2.04	0.27
10280201060140	2.54	0.72	2.90	0.70	2.94	1.13
10280201060150	2.72	0.62	2.66	0.65	2.82	0.95
10280201060160	2.84	0.73	2.75	0.80	4.09	1.90
10280201060170	3.17	0.70	3.26	0.76	5.93	1.52
10280201060180	2.25	0.44	2.25	0.45	2.12	0.52
10280201060190	1.72	0.41	1.69	0.41	1.42	0.46
10280201060200	2.50	0.60	2.58	0.63	2.42	0.91
10280201060210	2.74	0.78	3.20	0.83	6.02	3.13
10280201060220	2.81	0.25	2.86	0.25	2.87	0.26
10280201060230	1.93	0.61	2.05	0.64	1.63	0.91
10280201060240	1.97	0.45	1.99	0.46	1.73	0.53
10280201060250	1.72	0.27	1.74	0.28	1.62	0.29
10280201070010	2.34	0.50	2.41	0.50	2.08	0.57
10280201070020	3.19	0.59	3.37	0.62	3.84	0.88
10280201070030	2.62	0.70	2.56	0.74	2.78	1.38
10280201070040	2.81	0.63	2.88	0.66	2.80	0.94
10280201070050	2.51	0.65	2.65	0.69	2.39	1.13

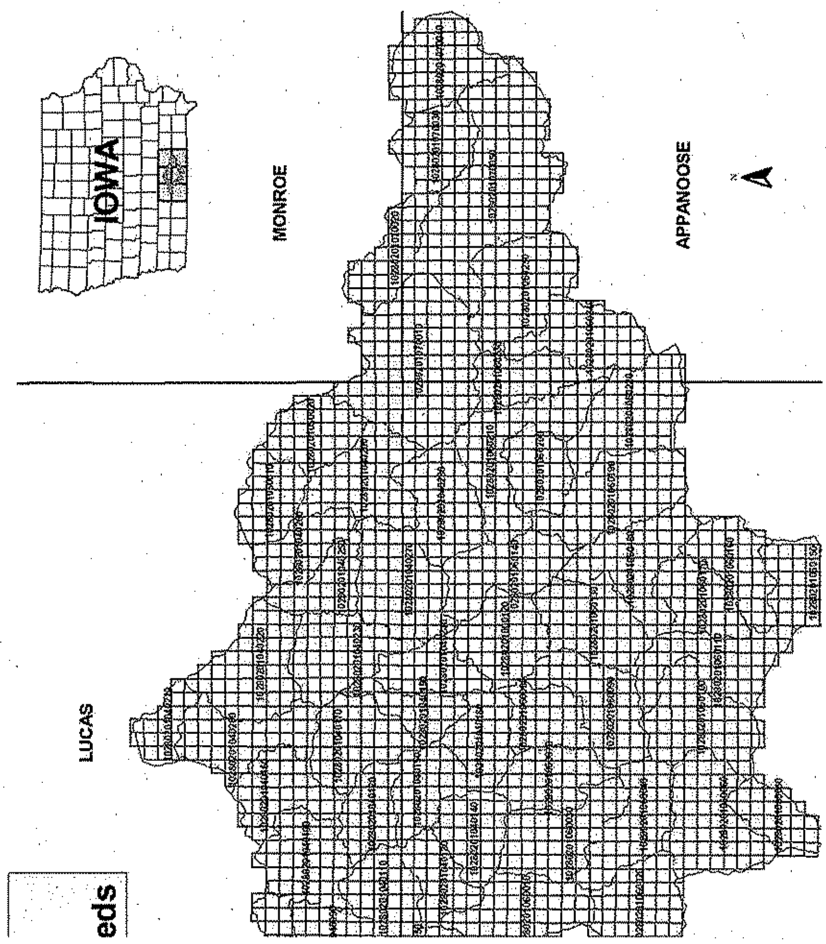

Figure F-1: Map of the area code in the Rathbun Lake Wathershed

References

ANSELIN, L. (1992): *Spatial Econometrics: Method and Models.* Kluwer Academic Publishers, Boston.

BANERJEE, S., CARLIN, B.P., GELFAND, A.E. (2004): *Hierarchical Modeling and Analysis for Spatial Data.* Chapman & Hall, New York.

BAILEY, T.C., GATRELL, A.C. (1995): *Interactive Spatial Data Analysis.* Longman, London.

CLIFF, A., ORD, J.K. (1981): *Spatial Processes.* Pion, London.

CRESSIE, N. (1991): Small-Area Prediction of Undercount Using the General Linear Model. *Proceedings of the Statistic Symposium 90: Measurement and Improvement of Data Quality*, Ottawa: Statistics Canada, 93–105.

CRESSIE, N. (1992): REML Estimation in Empirical Bayes Smoothing of Census Undercount. *Survey Methodology, 18, 1*, 75–94.

CRESSIE, N. (1993): *Statistics for Spatial Data.* John Wiley & Sons, New York.

DACEY, M.F. (1965): A review on measures of contiguity for two and k-color maps. *Technical Report, 2, Spatial Diffusion Study*, Department of Geography, Evanston: Northwestern University.

DATTA, G.S., LAHIRI, P. (2000): A unified measure of uncertainty of estimated best linear unbiased predictors in small area estimation problems. *Statistica Sinica, 10*, 613–627.

FAY, R.E., HERRIOT, R.A. (1979): Estimates of income for small places: an application of James-Stein procedures to census data. *Journal of the American Statistical Association, 74*, 269–277.

GETIS, A., ALDSTADT, J. (2004): Constructing the Spatial Weights Matrix Using a Local Statistic. *Geographical Analysis, 32, 2*, 90–104.

GHOSH, M., RAO, J.N.K. (1994): Small Area Estimation: An Appraisal (with discussion). *Statistical Science, 9, 1*, 55–93.

HENDERSON, C.R. (1975): Best linear unbiased estimation and prediction under a selection model. *Biometrics, 31*, 423–447.

IRPET. (2004): Indagine sulle condizioni di vita delle famiglie toscane. *Technical Report*, Istituto Regionale Programmazione Economica Toscana, Florence.

JIANG, J. (1996): REML Estimation: Asymptotic Behavior and Related Topics. *Annals of Statistics, 24*, 255–286.

KACKAR, R.N., HARVILLE, D.A. (1984): Approximations for standard errors of estimators for fixed and random effects in mixed models. *Journal of the American Statistical Association, 79*, 853–862.

NELDER, J.A., MEAD, R. (1965): A simplex method for function minimization. *Computer Journal, 7*, 308–313.

OPSOMER, J.D., BOTTS, C., KIM, J.Y. (2003): Small Area Estimation in Watershed Erosion Assessment Survey. *Journal of Agricultural, Biological, and Environmental Statistics, 8, 2*, 139–152.

ORD, K., GETIS, A. (1995) Local spatial autocorrelation statistics: distributional issues and application. *Geographical Analysis, 27, 3*, 286–296.

PFEFFERMANN, D. (2002): Small Area Estimation - New Developments and Directions. *International Statistical Review, 70, 1*, 125–143.

PRASAD, N., RAO, J.N.K. (1990): The Estimation of the Mean Squared Error of Small-Area Estimators. *Journal of the American Statistical Association, 85, 409*, 163–171.

RAO, J.N.K., CHOUDHRY, G.H. (1995): Small Area Estimation: Overview and Empirical Study in *Business Survey Method*, Edited by Cox, Binder, Chinnappa, Christianson, Colledge, Kott, John Wiley & Sons, 38, 527–540.

RAO, J.N.K. (1998): Small Area Estimation. *Encyclopedia of Statistical Sciences, 2*, 621–628.

RAO, J.N.K. (1999): Some Recent Advances in Model-Based Small Area Estimation. *Survey Methodology, 25, 2*, 175–186.

RAO, J.N.K. (2003): *Small Area Estimation*. Wiley, London.

ROBINSON, G.K. (1991): That BLUP is a Good Thing: The Estimation of Random Effects. *Statistical Science, 6, 1*, 15–51.

SAEI, A., CHAMBERS, R. (2003): Small Area Estimation: A Review of Methods based on the Application of Mixed Models. *Southampton Statistical Sciences Research Institute, WP M03/16*, Southampton.

SALVATI, N. (2004): *La correlazione spaziale nella stima per piccole aree: metodi proposti e casi di studio*. Doctoral thesis in Applied Statistics, "G. Parenti" Department of Statistics, University of Florence.

UPTON, G.J.G., FINGLETON, B. (1985): *Spatial Data Analysis by Example.* John Wiley & Sons, New York.

VERMA, V. (2004): Sampling errors and design effects for poverty measures and other complex statistics. *Proceedings of the VII International Meeting QUANTITATIVE METHODS FOR APPLIED SCIENCES: Sampling designs for environmental, economic and social surveys: theoretical and practical perspectives,* Siena.

WALL, M.M. (2004). A close look at the spatial structure implied by the CAR and SAR models. *Journal of Statistical Planning and Inference,* Article in press (accepted November 2002).

Anno: 1987

- ☐ n. 1 Alberto Cambini - Laura Martein, Some Optimality Conditions in Vector Optimization
- ☐ n. 2 Alberto Cambini - Laura Martein - S.Schaibel, On Maximizing a Sum of Ratios
- ☐ n. 3 Giuliano Gasparotto, On the Charnes-Cooper Transformation in linear Fractional Programming.
- ☐ n. 4 Alberto Cambini, Non-linear separation Theorems, Duality and Optimality
- ☐ n. 5 Giovanni Boletto, Indicizzazione parziale: aspetti metodologici e riflessi economici
- ☐ n. 6 Alberto Cambini - Claudio Sodini, On Parametric Linear Fractional Programming
- ☐ n. 7 Alberto Bonaguidi, Alcuni aspetti meno noti delle migrazioni in Italia
- ☐ n. 8 Laura Martein - S. Schaible, On Solving a Linear Program with one Quadratic Constraint

Anno: 1988

- ☐ n. 9 Ester Lari, Alcune osservazioni sull'equazione funzionale Ø(x,y,z)=Ø(Ø(x,y,t),t,z)
- ☐ n. 10 F. Bartiaux, Une étude par ménage des migrations des personnes âgées: comparaison des résultats pour l'Italie et les Etats-Unis
- ☐ n. 11 Giovanni Boletto, Metodi di scomposizione del tasso di inflazione
- ☐ n. 12 Claudio Sodini, A New Algorithm for the Strictly Convex Quadratic Programming Problem
- ☐ n. 13 Laura Martein, On Generating the Set of all Efficient Points of a Bicriteria Fractional Problem
- ☐ n. 14 Laura Martein, Applicazioni della programmazione frazionaria nel campo economico-finanziario
- ☐ n. 15 Laura Martein, On the Bicriteria Maximization Problem
- ☐ n. 16 Paolo Manca, Un prototipo di sistema esperto per la consulenza finanziaria rivolta ai piccoli risparmiatori
- ☐ n. 17 Paolo Manca, Operazioni Finanziarie di Soper e Operazioni di puro Investimento secondo Teichroew-Robichek-Montalbano
- ☐ n. 18 Paolo Carraresi - Claudio Sodini, A k - Shortest Path Approach to the Minimum Cost Matching Problem.
- ☐ n. 19 Odo Barsotti - Marco Bottai, Sistemi gravitazionali e fasi di transazione della crescita Demografica
- ☐ n. 20 Giovanni Boletto, Metodi di scomposizione dell'inflazione aggregata : recenti sviluppi.
- ☐ n. 21 Marc Termote - Alberto Bonaguidi, Multiregional Stable Populatioin as a Tool for Short-term Demographic Analysis
- ☐ n. 22 Marco Bottai, Storie familiari e storie migratorie: un'indagine in Italia
- ☐ n. 23 Maria Francesca Romano - Marco Marchi, Problemi connessi con la disomogeneità dei gruppi sottoposti a sorveglianza statistico-epidemiologica.
- ☐ n. 24 Franca Orsi, Un approcio logico ai problemi di scelta finanziaria.

Anno: 1989

- ☐ n. 25 Vincenzo Bruno, Attrazione ed entropia.
- ☐ n. 26 Giorgio Giorgi - S. Mititelu, Invexity in nonsmooth Programming.
- ☐ n. 28 Alberto Cambini - Laura Martein, Equivalence in linear fractional programming.

Anno: 1990

- ☐ n. 27 Vincenzo Bruno, Lineamenti econometrici dell'evoluzione del reddito nazionale in relazione ad altri fenomeni economici
- ☐ n. 29 Odo Barsotti - Marco Bottai - Marco Costa, Centralità e potenziale demografico per l'analisi dei comportamenti demografici: il caso della Toscana
- ☐ n. 30 Anna Marchi, A sequential method for a bicriteria problem arising in portfolio selectiontheory.
- ☐ n. 31 Marco Bottai, Mobilità locale e pianificazione territoriale.
- ☐ n. 32 Anna Marchi, Solving a quadratic fractional program by means of a complementarity approach
- ☐ n. 33 Anna Marchi, Sulla relazione tra un problema bicriteria e un problema frazionario.

Anno: 1991

- ☐ n. 34 Enrico Gori, Variabili latenti e "self-selection" nella valutazione dei processi formativi.
- ☐ n. 35 Piero Manfredi - E. Salinelli, About an interactive model for sexual Populations.
- ☐ n. 36 Giorgio Giorgi, Alcuni aspetti matematici del modello di sraffa a produzione semplice
- ☐ n. 37 ALberto Cambini - S.Schaibl - Claudio Sodini, Parametric linear fractional programming for an unbounded feasible Region.
- ☐ n. 38 I.Emke - Poulopoulos - V.Gozàlves Pèrez - Odo Barsotti - Laura Lecchini, International migration to northern Mediterranean countries the cases of Greece, Spain and Italy.
- ☐ n. 39 Giuliano Gasparotto, A LP code implementation
- ☐ n. 40 Riccardo Cambini, Un problema di programmazione quadratica nella costituzione di capitale.
- ☐ n. 41 Gilberto Ghilardi, Stime ed errori campionari nell'indagine ISTAT sulle forze di lavoro.
- ☐ n. 42 Vincenzo Bruno, Alcuni valori medi, variabilità paretiana ed entropia.
- ☐ n. 43 Giovanni Boletto, Gli effetti del trascinamento dei prezzi sulle misure dell'inflazione: aspetti metodologici
- ☐ n. 44 P. Paolicchi, Gli abbandoni nell'università: modelli interpretativi.
- ☐ n. 45 Maria Francesca Romano, Da un archivio amministrativo a un archivio statistico: una proposta metodologica per i dati degli studenti universitari.
- ☐ n. 46 Maria Francesca Romano, Criteri di scelta delle variabili nei modelli MDS: un'applicazione sulla popolazione studentesca di Pisa.
- ☐ n. 47 Odo Barsotti - Laura Lecchini, Les parcours migratoires en fonction de la nationalité. Le cas de l'Italie.
- ☐ n. 48 Vincenzo Bruno, Indicatori statistici ed evoluzione demografica, economica e sociale delle province toscane.
- ☐ n. 49 Alberto Cambini - Laura Martein, Tangent cones in optimization.
- ☐ n. 50 Alberto Cambini - Laura Martein, Optimality conditions in vector and scalar optimization: a unified approach.

Anno: 1992

- ☐ n. 51 Gilberto Ghilardi, Elementi di uno schema di campionamento areale per alcune rilevazioni ufficiali in Italia.
- ☐ n. 52 Paolo Manca, Investimenti e finanziamenti generalizzati.
- ☐ n. 53 Laura Lecchini - Odo Barsotti, Le rôle des immigrés extra- communautaires dans le marché du travail

- ☐ **n. 54** Riccardo Cambini, Alcune condizioni di ottimalità relative ad un insieme stellato.
- ☐ **n. 55** Gilberto Ghilardi, Uno schema di campionamento areale per le rilevazionninsulle famiglie in Italia.
- ☐ **n. 56** Riccardo Cambini, Studio di una classe di problemi non lineari: un metodo sequenziale.
- ☐ **n. 57** Riccardo Cambini, Una nota sulle possibili estensioni a funzioni vettoriali di significative classi di funzioni concavo-generalizzate.
- ☐ **n. 58** Alberto Bonaguidi - Valerio Terra Abrami, Metropolitan aging transition and metropolitan redistribution of the elderly in Italy.
- ☐ **n. 59** Odo Barsotti - Laura Lecchini, A comparison of male and female migration strategies: the cases of African and Filipino Migrants to Italy.
- ☐ **n. 60** Gilberto Ghilardi, Un modello logit per lo studio del fenomeno delle nuove imprese.
- ☐ **n. 61** S. Schaible, Generalized monotonicity.
- ☐ **n. 62** Vincenzo Bruno, Dell'elasticità in economia e dell'incertezza statistica.
- ☐ **n. 63** Laura Martein, Alcune classi di funzioni concave generalizzate nell'ottimizzazione vettoriale
- ☐ **n. 64** Anna Marchi, On the relationships between bicriteria problems and non-linear programming problems.
- ☐ **n. 65** Giovanni Boletto, Considerazioni metodologiche sul concetto di elasticità prefissata.
- ☐ **n. 66** Laura Martein, Soluzione efficienti e condizioni di ottimalità nell'otti-mizzazione vettoriale.

Anno: **1993**

- ☐ **n. 67** Maria Francesca Romano, Le rilevazioni ufficiali ISTAT della popolazione universitaria: problemi e definizioni alternative.
- ☐ **n. 68** Marco Bottai - Odo Barsotti, La ricerca "Spazio Utilizzato" Obiettivi e primi risultati.
- ☐ **n. 69** Marco Bottai - F.Bartiaux, Composizione familiare e mobilità delle persone anziane. Una analisi regionale.
- ☐ **n. 70** Anna Marchi - Claudio Sodini, An algorithm for a non-differentiable non -linear fractional programming problem.
- ☐ **n. 71** Claudio Sodini - S.Schaible, An finite algorithm for generalized linear multiplicative programming.
- ☐ **n. 72** Alberto Cambini - Laura Martein, An approach to optimality conditions in vector and scalar optimization.
- ☐ **n. 73** Alberto Cambini - Laura Martein, Generalized concavity and optimality conditions in vector and scalar optimization.
- ☐ **n. 74** Riccardo Cambini, Alcune nuove classi di funzioni concavo-generalizzate.

Anno: **1994**

- ☐ **n. 75** Alberto Cambini - Anna Marchi - Laura Martein, On nonlinear scalarization in vector optimization.
- ☐ **n. 76** Maria Francesca Romano - Giovanna Nencioni, Analisi delle carriere degli studenti immatricolati dal 1980 al 1982.
- ☐ **n. 77** Gilberto Ghilardi, Indici statistici della congiuntura.
- ☐ **n. 78** Riccardo Cambini, Condizioni di efficienza locale nella ottimizzazione vettoriale.
- ☐ **n. 79** Odo Barsotti - Marco Bottai, Funzioni di utilizzazione dello spazio.
- ☐ **n. 80** Vincenzo Bruno, Alcuni aspetti dinamici della popolazione dei comuni della Toscana, distinti per ampiezza demografica e per classi di urbanità e di ruralità.
- ☐ **n. 81** Giovanni Boletto, I numeri indici del potere d'acquisto della moneta.
- ☐ **n. 82** Alberto Cambini - Laura Martein - Riccardo Cambini, Some optimality conditions in multiobjective programming.
- ☐ **n. 83** S. Schaible, Fractional programming with sume of ratios.
- ☐ **n. 84** Stefan Tigan - I.M.Stancu-Minasian, The minimun-risk approach for continuous time linear-fractional programming.
- ☐ **n. 85** Vasile Preda - I.M.Stancu-Minasian, On duality for multiobjective mathematical programming of n-set.
- ☐ **n. 86** Vasile Preda - I.M.Stancu-Minasian - Anton Batatorescu, Optimality and duality in nonlinear programming involving semilocally preinvex and related functions.

Anno: **1995**

- ☐ **n. 87** Elena Melis, Una nota storica sulla programmazione lineare: un problema di Kantorovich rivisto alla luce del problema degli zeri.
- ☐ **n. 88** Vincenzo Bruno, Mobilità territoriale dell'Italia e di tre Regioni tipiche: Lombardia, Toscana, Sicilia.
- ☐ **n. 89** Antonio Cortese, Bibliografia sulla presenza straniera in Italia
- ☐ **n. 90** Riccardo Cambini, Funzioni scalari affini generalizzate.
- ☐ **n. 91** Piero Manfredi - Fabio Tarini, Modelli epidemiologici: teoria e simulazione. (I)
- ☐ **n. 92** Marco Bottai - Maria Caputo - Laura Lecchini, The "OLIVAR" survey.Methodology and quality.
- ☐ **n. 93** Laura Lecchini - Donatella Marsiglia - Marco Bottai, Old people and social network.
- ☐ **n. 94** Gilberto Ghilardi, Uno studio empirico sul confronto tra alcuni indici statistici della congiuntura.
- ☐ **n. 95** Vincenzo Bruno, Il traffico nei porti italiani negli anni recenti.
- ☐ **n. 96** Alberto Cambini - Anna Marchi - Laura Martein - S. Schaible, An analysis of the falk-palocsay algorithm.
- ☐ **n. 97** Alberto Cambini - Laura Carosi, Sulla esistenza di elementi massimali.

Anno: **1996**

- ☐ **n. 98** Riccardo Cambini - S. Komlòsi, Generalized concavity and generalized monotonicity concepts for vector valued.
- ☐ **n. 99** Riccardo Cambini, Second order optimality conditions in the image space.
- ☐ **n. 100** Vincenzo Bruno, La stagionalità delle correnti di navigazione marittima.
- ☐ **n. 101** Eugene Maurice Cleur, A comparison of alternative discrete approximations of the Cox -I ngersoll - ross model.
- ☐ **n. 102** Gilberto Ghilardi, Sul calcolo del rapporto di concentrazione del Gini.
- ☐ **n. 103** Alberto Cambini - Laura Martein - Riccardo Cambini, A new approach to second order optimality conditions in vector optimization.
- ☐ **n. 104** Fausto Gozzi, Alcune osservazioni sull'immunizzazione semideterministica.
- ☐ **n. 105** Emilio Barucci - Fausto Gozzi, Innovation and capital accumulation in a vintage capital model an infinite dimensional control approach.
- ☐ **n. 106** Alberto Cambini - Laura Martein - I.M.Stancu-Minasian., A survey of bicriteria fractional problems.
- ☐ **n. 107** Luciano Fanti - Piero Manfredi, Viscosità dei salari, offerta di lavoro endogena e ciclo.
- ☐ **n. 108** Piero Manfredi - Luciano Fanti, Ciclo di vita di nuovi prodotti: modellistica non lineare.
- ☐ **n. 109** Piero Manfredi, Crescita con ciclo, gestazione dei piani di investimento ed effetti.
- ☐ **n. 110** Luciano Fanti - Piero Manfredi, Un modello "classico" di ciclo con crescita ed offerta di lavoro endogena.
- ☐ **n. 111** Anna Marchi, On the connecttedness of the efficient frontier : sets without local maxima.

- n. 112 Riccardo Cambini, Generalized concavity for bicriteria functions.
- n. 113 Vincenzo Bruno, Variazioni dinamiche (1971-1981-1991)dei fenomeni demografici dei comuni (urbani e rurali) della Lombardia, in relazione ad alcune caratteristiche di mobilità territoriale.

Anno: 1997

- n. 114 Piero Manfredi - Fabio Tarini - J.R. Williams - A. Carducci - B. Casini, Infectious diseases: epidemiology, mathematical models, and immunization policies.
- n. 115 Eugene Maurice Cleur - Piero Manfredi, One dimensional SDE models, low order numerical methods and simulation based estimation: a comparison of alternative estimators.
- n. 116 Luciano Fanti - Piero Manfredi, Point stability versus orbital stability (or instability): remarks on policy implications in classical growth cycle model.
- n. 117 Piero Manfredi - Francesco Billari, transition into adulthood, marriage, and timing of life in a stable popolation framework.
- n. 118 Laura Carosi, Una nota sul concetto di estremo superiore di insiemi ordinati da coni convessi.
- n. 119 Laura Lecchini - Donatella Marsiglia, Reti sociali degli anziani: selezione e qualità delle relazioni.
- n. 120 Piero Manfredi - Luciano Fanti, Gestation lags and efficiency wage mechanisms in a goodwin type growth model.
- n. 121 G.Rivellini, La metodologia statistica multilevel come possibile strumento per lo studio delle interazioni tra il comportamento procreativo individuale e il contesto
- n. 122 Laura Carosi, Una nota sugli insiemi C-limitati e L-limitati.
- n. 123 Laura Carosi, Sull'estremo superiore di una funzione lineare fratta ristretta ad un insieme chiuso e illimitato.
- n. 124 Piero Manfredi, A demographic framework for the evaluation of the impact of imported infectious diseases.
- n. 125 Alessandro Valentini, Calo della fecondità ed immigrazione: scenari e considerazioni sul caso italiano.
- n. 126 Alberto Cambini - Laura Martein, Second order optimality conditions.

Anno: 1998

- n. 127 Piero Manfredi and Alessandro Valentini, Populations with below replacementfertility: theoretical considerations and scenarioes from the italian laboratory.
- n. 128 Alberto Cambini - Laura Martein - E. Moretti, Programmazione frazionaria e problemi bicriteria.
- n. 129 Emilio Barucci - Fausto Gozzi - Andrej Swiech, Incentive compatibility constraints and dynamic programming in continuous time.

Anno: 1999

- n. 130 Alessandro Valentini, Impatto delle immigrazioni sulla popolazione italiana: confronto tra scenari alternativi.
- n. 131 K. Iglicka - Odo Barsotti - Laura Lecchini, Recent developement of migrations from Poland to Europe with a special emphsis on Italy K.Iglika - Le Migrazioni est-ovest: le unioni miste in Italia
- n. 132 Alessandro Valentini, Proiezioni demografiche multiregionali a due sessi, con immigrazioni internazionali e vincoli di consistenza.
- n. 133 Fabio Antonelli - Emilio Barucci - Maria Elvira Mancino, Backward-forward stochastic differential utility: existence, consumption and equilibrium analysis.
- n. 134 Emilio Barucci - Maria Elvira Mancino, Asset pricing with endogenous aspirations.
- n. 135 Eugene Maurice Cleur, Estimating a class of diffusion models: an evaluation of the effects of sampled discrete observations.
- n. 136 Luciano Fanti - Piero Manfredi, Labour supply, time delays, and demoeconomic oscillations in a solow-typegrowth model.
- n. 137 Emilio Barucci - Sergio Polidoro - Vincenzo Vespri, Some results on partial differential equations and Asian options.
- n. 138 Emilio Barucci - Maria Elvira Mancino, Hedging european contingent claims in a Markovian incomplete market.
- n. 139 Alessandro Valentini, L'applicazione del modello multiregionale-multistato alla popolazione in Italia mediante l'utilizzo del Lipro: procedura di adattamento dei dati e particolarità tecniche del programma.
- n. 140 I.M.Stancu-Minasian, optimality conditions and duality in fractional programming-involving semilocally preinvex and related functions.
- n. 141 Alessandro Valentini, Proiezioni demografiche con algoritmi di consistenza per la popolazione in Italia nel periodo 1997-2142: presentazione dei risultati e confronto con metodologie di stima alternative.
- n. 142 Laura Carosi, Competitive equilibria with money and restricted partecipation.
- n. 143 Laura Carosi, Monetary policy and Pareto improvability in a financial economy with restricted partecipation
- n. 144 Bruno Cheli, Misurare il benessere e lo sviluppo dai paradossi del Pil a misure di benessere economico sostenibile, con uno sguardo allo sviluppo umano
- n. 145 Bruno Cheli - Laura Lecchini - Lucio Masserini, The old people's perception of well-being: the role of material and non material resources
- n. 146 Eugene Maurice Cleur, Maximum likelihood estimation of one-dimensional stochastic differential equation models from discrete data: some computational resultas
- n. 147 Alessandro Valentini - Francesco Billari - Piero Manfredi, Utilizzi empirici di modelli multistato continui con durate multiple
- n. 148 Francesco Billari - Piero Manfredi - Alberto Bonaguidi - Alessandro Valentini, Transition into adulthoold: its macro-demographic consequences in a multistatew stable population framework
- n. 149 Francesco Billari - Piero Manfredi - Alessandro Valentini, Becoming Adult and its Macro-Demographic Impact: Multistate Stable Population Theory and an Application to Italy
- n. 150 Alessandro Valentini, Le previsioni demografiche in presenza di immigrazioni: confronto tra modelli alternativi e loro utilizzo empirico ai fini della valutazione dell'equilibrio nel sistema pensionistico
- n. 151 Emilio Barucci - Roberto Monte, Diffusion processes for asset prices under bounded rationality
- n. 152 Emilio Barucci - P. Cianchi - L. Landi - A. Lombardi, Reti neurali e analisi delle serie storiche: un modello per la previsione del BTP future
- n. 153 Alberto Cambini - Laura Carosi - Laura Martein, On the supremum in fractional programming
- n. 154 Riccardo Cambini - Laura Martein, First and second order characterizations of a class of pseudoconcave vector functions
- n. 155 Piero Manfredi and Luciano Fanti, Embedding population dynamics in macro-economic models. The case of the goodwin's growth cycle
- n. 156 Laura Lecchini e Odo Barsotti, Migrazioni dei preti dalla Polonia in Italia
- n. 157 Vincenzo Bruno, Analisi dei prezzi, in Italia dal 1975 in poi
- n. 158 Vincenzo Bruno, Analisi del commercio al minuto in Italia
- n. 159 Vincenzo Bruno, Aspetti ciclici della liquidità bancaria, dal 1971 in poi
- n. 160 Anna Marchi, A separation theorem in alternative theorems and vector optimization

Anno: 2000

- ☐ **n. 161** Piero Manfredi and Luciano Fanti, Labour suppley, population dynamicics and persistent oscillations in a Goodwin-type growth cycle model
- ☐ **n. 162** Luciano Fanti and Piero Manfredi, Neo-classical labour market dynamics and chaos (and the Phillips curve revisited)
- ☐ **n. 163** Piero Manfredi - and Luciano Fanti, Detection of Hopf bifurcations in continuos-time macro- economic models, with an application to reducible delay-systems.
- ☐ **n. 164** Fabio Antonelli - Emilio Barucci, The Dynamics of pareto allocations with stochastic differential utility
- ☐ **n. 165** Eugene M. Cleur, Computing maximum likelihood estimates of a class of One-Dimensional stochastic differential equation models from discrete Date*
- ☐ **n. 166** Eugene M. Cleur, Estimatng the drift parameter in diffusion processes more efficiently at discrete times:a role of indirect estimation
- ☐ **n. 167** Emilio Barucci - Vincenzo Valori, Forecasting the forecasts of others e la Politica di Inflation targeting
- ☐ **n. 168** A.Cambini - L. Martein, First and second order optimality conditions in vector optimization
- ☐ **n. 169** A. Marchi, Theorems of the Alternative by way of Separation Theorems
- ☐ **n. 170** Emilio Barucci - Maria Elvira Mancino, Asset Pricing and Diversification with Partally Exchangeable random Variables
- ☐ **n. 171** Piero Manfredi - Luciano Fanti, Long Term Effects of the Efficiency Wage Hypothesis in Goodwin-Type Economies.
- ☐ **n. 172** Piero Manfredi - Luciano Fanti, Long Term Effects of the Efficiency wage Hypothesis in Goodwin-type Economies: a reply.
- ☐ **n. 173** Luciano Fanti, Innovazione Finanziaria e Domanda di Moneta in un Modello dinamico IS-LM con Accumulazione.
- ☐ **n. 174** P.Manfredi, A.Bonaccorsi, A.Secchi, Social Heterogeneities in Classical New Product Diffusion Models. I: "External" and "Internal" Models.
- ☐ **n. 175** Piero Manfredi - Ernesto Salinelli, Modelli per formazione di coppie e modelli di Dinamica familiare.
- ☐ **n. 176** P.Manfredi, E. Salinelli, A.Melegaro, A.Secchi, Long term Interference Between Demography and Epidemiology: the case of tuberculosis
- ☐ **n. 177** Piero Manfredi - Ernesto Salinelli, Toward the Development of an Age Structure Teory for Family Dynamics I: General Frame.
- ☐ **n. 178** Piero Manfredi - Luciano Fanti, Population heterogeneities, nonlinear oscillations and chaos in some Goodwin-type demo-economic models Paper to be presented at the: Second workshop on "nonlinear demography" Max Planck Institute for demographic Research Rostock, Germany, May 31-June 2, 2
- ☐ **n. 179** E. Barucci - M.E. Mancini - Roberto Renò, Volatility Estimation via Fourier Analysis
- ☐ **n. 180** Riccardo Cambini, Minimum Principle Type Optimality Conditions
- ☐ **n. 181** E. Barucci, M. Giuli, R. Monte, Asset Prices under Bounded Rationality and Noise Trading
- ☐ **n. 182** A. Cambini, D.T.Luc, L.Martein, Order Preserving Transformations and application.
- ☐ **n. 183** Vincenzo Bruno, Variazioni dinamiche (1971-1981-1991) dei fenomeni demografici dei comuni urbani e rurali della Sicilia, in relazione ad alcune caratteristiche di mobilità territoriale.
- ☐ **n. 184** F.Antonelli, E.Barucci, M.E.Mancino, Asset Pricing with a Backward-Forward Stochastic Differential Utility
- ☐ **n. 185** Riccardo Cambini - Laura Carosi, Coercivity Concepts and Recession Functions in Constrained Problems
- ☐ **n. 186** John R. Williams, Piero Manfredi, The pre-vaccination dynamics of measles in Italy: estimating levels of under-reporting of measles cases
- ☐ **n. 187** Piero Manfredi, John R. Williams, To what extent can inter-regional migration perturbe local endemic patterns? Estimating numbers of measles cases in the Italian regions
- ☐ **n. 188** Laura Carosi, Johannes Jahn, Laura Martein, On The Connections between Semidefinite Optimization and Vector Optimization
- ☐ **n. 189** Alberto Cambini, Jean-Pierre Crouzeix, Laura Martein, On the Pseudoconvexity of a Quadratic Fractional Function
- ☐ **n. 190** Riccardo Cambini - Claudio Sodini, A finite Algorithm for a Particular d.c. Quadratic Programming Problem.
- ☐ **n. 191** Riccardo Cambini - Laura Carosi, Pseudoconvexity of a class of Quadratic Fractional Functions.
- ☐ **n. 192** Laura Carosi, A note on endogenous restricted partecipation on financial markets: an existence result.
- ☐ **n. 193** Emilio Barucci - Roberto Monte - Roberto Renò, Asset Price Anomalies under Bounded Rationality.
- ☐ **n. 194** Emilio Barucci - Roberto Renò, A Note on volatility estimate-forecast with GARCH models.
- ☐ **n. 195** Bruno Cheli, Sulla misura del benessere economico: i paradossi del PIL e le possibili correzioni in chiave etica e sostenibile, con uno spunto per l'analisi della povertà
- ☐ **n. 196** M.Bottai, M.Bottai, N. Salvati, M.Toigo, Le proiezioni demografiche con il programma Nostradamus. (Applicazione all'area pisana)
- ☐ **n. 197** A. Lemmi - B. Cheli - B. Mazzolli, La misura della povertà multidimensionale: aspetti metodologici e analisi della realtà italiana alla metà degli anni '90
- ☐ **n. 198** C.R. Bector - Riccardo Cambini, Generalized B-invex vector valued functions
- ☐ **n. 199** Luciano Fanti - Piero Manfredi, The workers' resistance to wage cuts is not necessarily detrimental for the economy: the case of a Goodwin's growth model with endogenous population.
- ☐ **n. 200** Emilio Barucci - Roberto Renò, On Measuring volatility of diffusion processes with high frequency data
- ☐ **n. 201** Piero Manfredi - Luciano Fanti, Demographic transition and balanced growth

Anno: 2001

- ☐ **n. 202** E.Barucci - M. E. Mancini - E. Vannucci, Asset Pricing, Diversification and Risk Ordering with Partially Exchangeable random Variables
- ☐ **n. 203** E. Barucci - R. Renò - E. Vannucci, Executive Stock Options Evaluation.
- ☐ **n. 204** Odo Barsotti - Moreno Toigo, Dimensioni delle rimesse e variabili esplicative: un'indagine sulla collettività marocchina immigrata nella Toscana Occidentale
- ☐ **n. 205** Vincenzo Bruno, I Consumi voluttuari, nell'ultimo trentennio, in Italia
- ☐ **n. 206** Michele Longo, The monopolist choice of innovation adoption: A regular-singular stochastic control problem
- ☐ **n. 207** Michele Longo, The competitive choice of innovation adoption: A finite-fuel singular stochastic control problem.
- ☐ **n. 208** Riccardo Cambini - Laura Carosi, On the pseudoaffinity of a class of quadratic fractional functions
- ☐ **n. 209** Riccardo Cambini - Claudio Sodini, A Finite Algorithm for a Class of Non Linear Multiplicative Programs.
- ☐ **n. 210** Alberto Cambini - Dinh The Luc - Laura Martein, A method for calculating subdifferential Convex vector functions
- ☐ **n. 211** Alberto Cambini - Laura Martein, Pseudolinearity in scalar and vector optimization.
- ☐ **n. 212** Riccardo Cambini, Necessary Optimality Conditions in Vector Optimization.
- ☐ **n. 213** Riccardo Cambini - Laura Carosi, On generalized convexity of quadratic fractional functions.
- ☐ **n. 214** Riccardo Cambini - Claudio Sodini, A note on a particular quadratic programming problem.
- ☐ **n. 215** Michele Longo - Vincenzo Valori, Existence end stability of equilibria in OLG models under adaptive expectations.

- ☐ **n. 216** Luciano Fanti - Piero Manfredi, Population, unemployment and economic growth cycles: a further explanatory perspective
- ☐ **n. 217** J.R.Williams,P.Manfredi,S.Salmaso,M.Ciofi, Heterogeneity in regional notification patterns and its impact on aggregate national case notification data: the example of measles in Italy.
- ☐ **n. 218** Anna Marchi, On the connectedness of the efficient frontier: sets without local efficient maxima
- ☐ **n. 219** Laura Lecchini - Odo Barsotti, Les disparités territoriales au Maroc au travers d'une optique de genre.

Anno: **2002**

- ☐ **n. 220** Gilberto Ghilardi - Nicola Orsini, Sull'uso dei modelli statistici lineari nella valutazione dei sistemi formativi.
- ☐ **n. 221** Andrea Mercatanti, Un'analisi descrittiva dei laureati dell'Università di Pisa
- ☐ **n. 222** E. Barucci - C. Impenna - R. Renò, The Italian Overnight Market: microstructure effects, the martingale hypothesis and the payment system.
- ☐ **n. 223** E. Barucci, P.Malliavin, M.E.Mancino, R.Renò, A.Thalmaier, The Price-volatility feedback rate: an implementable mathematical indicator of market stability.
- ☐ **n. 224** Andrea Mercatanti, Missing at random in randomized experiments with imperfect compliance
- ☐ **n. 225** Andrea Mercatanti, Effetto dell'uso di carte Bancomat e carte di Credito sulla liquidità familiare: una valutazione empirica
- ☐ **n. 226** Piero Manfredi - John R. Williams, Population decline and population waves: their impact upon epidemic patterns and morbidity rates for childhood infectious diseases. Measles in Italy as an example.
- ☐ **n. 227** Piero Manfredi - Marta Ciofi degli Atti, La geografia pre-vaccinale del morbillo in Italia. I. Comportamenti di contatto e sforzi necessari all'eliminazione: predizioni dal modello base delle malattie prevenibili da vaccino.
- ☐ **n. 228** I.M.Stancu-Minasian, Optimality Conditions and Duality in Fractional Programming Involving Semilocally Preinvex and Related
- ☐ **n. 229** Nicola Salvati, Un software applicativo per un'analisi di dati sui marchi genetici (Genetic Markers)
- ☐ **n. 230** Piero Manfredi, J. R. Williams, E. M. Cleur, S. Salmaso, M. Ciofi, The pre-vaccination regional landscape of measles in Italy: contact patterns and related amount of needed eradication efforts (and the "EURO" conjecture)
- ☐ **n. 231** Andrea Mercatanti, I tempi di laurea presso l'Università di Pisa: un'applicazione dei modelli di durata in tempo discreto
- ☐ **n. 232** Andrea Mercatanti, The weak version of the exclusion restriction in causal effects estimation: a simulation study
- ☐ **n. 233** Riccardo Cambini and Laura Carosi, Duality in multiobjective optimization problems with set constraints
- ☐ **n. 234** Riccardo Cambini and Claudio Sodini, Decomposition methods for nonconvex quadratic programs
- ☐ **n. 235** R.Cambini and L. Carosi and S.Schaible, Duality in fractional optimization problems with set constraints
- ☐ **n. 236** Anna Marchi, On the mix-efficient points

Anno: **2003**

- ☐ **n. 237** Emanuele Vannucci, The valuation of unit linked policies with minimal return guarantees under symmetric and asymmetric information hypoteses
- ☐ **n. 238** John R Williams - Piero Manfredi, Ageing populations and childhood infections: the potential impact on epidemic patterns and morbidity
- ☐ **n. 239** Bruno Cheli, Errata Corrige del Manuale delle Impronte Ecologiche (2002) ed alcuni utili chiarimenti
- ☐ **n. 240** Alessandra Petrucci-Nicola Salvati-Monica Pratesi, Stimatore Combinato r Correlazione Spaziale nella Stima per Piccole Aree
- ☐ **n. 241** Riccardo Cambini - Laura Carosi, Mixed Type Duality for Multiobjective Optimization Problems with set constraints
- ☐ **n. 242** O.Barsotti, L.Lecchini, F.Benassi, Foreigners from central and eastern European countries in Italy: current and future perspectives of eu enlargement
- ☐ **n. 243** A. Cambini - L. Martein - S. Schaible, Pseudoconvexity under the Charnes-Cooper transformation
- ☐ **n. 244** Eugene M. Cleur, Piero Manfredi, and John R. William, The pre-and post-Vaccination regional dynamics of measles in Italy: Insights from time series analysis

Anno: **2004**

- ☐ **n. 245** Emilio Barucci - Jury Falini, Determinants of Corporate Governance in Italy: Path dependence or convergence?
- ☐ **n. 246** R. Cambini - A. Marchi, A note on the connectedness of the efficient frontier
- ☐ **n. 247** Laura Carosi - Laura Martein, On the pseudoconvexity and pseudolinearity of some classes of fractional functions
- ☐ **n. 248** E. Barucci - R. Monte - B. Trivellato, Bayesian nash equilibrium for insider trading in continuous time
- ☐ **n. 249** Eugene M. Cleur, A Time Series Analysis of the Inter-Epidemic Period for Measles in Italy
- ☐ **n. 250** Andrea Mercatanti, Causal inference methods without exclusion restrictions: an economic application.
- ☐ **n. 251** Eugene M. Cleur, Non-Linearities in Monthly Measles data for Italy
- ☐ **n. 252** Eugene M. Cleur, A Treshold Model for Prevaccination Measles Data: Some Empirical Results for England and Italy
- ☐ **n. 253** Andrea Mercatanti, La gestione dei dati mancanti nei modelli di inferenza causale: il caso degli esperimenti naturali.
- ☐ **n. 254** Andrea Mercatanti, Rilevanza delle analisi di misture di distribuzioni nelle valutazioni di efficacia
- ☐ **n. 255** Andrea Mercatanti, Local estimation of mixtures in instrumental variables models
- ☐ **n. 256** Monica Pratesi - Nicola Salvati, Spatial EBLUP in agricultural surveys: an application based on italian census data.
- ☐ **n. 257** Emanuele Vannucci, A model analyzing the effects of information asymmetries of the traders
- ☐ **n. 258** Monica Pratesi-Emilia Rocco, Two-Step centre sampling for estimating elusive population size
- ☐ **n. 259** A.Lemmi, N.Pannuzi, P.Valentini, B.Cheli, G.Berti, Estimating Multidimensional Poverty: A Comparison of Three Diffused Methods°

Anno: **2005**

- ☐ **n. 260** Nicola Salvati, Small Area estimation: the EBLUP estimator using the CAR model