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Abstract

This paper deals with small area indirect estimators under area level random effect models when only area level data are available and the random effects are correlated.

Under a Simultaneously Autoregressive model (SAR) for the area effects the Empirical Best Linear Unbiased Predictor (EBLUP) for small area means is obtained (Spatial EBLUP). Its Mean Squared Error (MSE) and an estimator of MSE are also proposed. We explore the performance of the estimators with a Monte Carlo simulation study on lattice data and apply them to the results of the sample survey on Life Conditions in Tuscany (Italy). A clear tendency in our empirical findings is that the introduction of spatially correlated random area effects reduce both the variance and the bias of the EBLUP estimator. Despite some residual bias, the coverage rate of our confidence intervals comes close to a nominal 95%.

Key words: small area estimation, spatial correlation, SAR model, Spatial EBLUP, lattice data.

1 Introduction

Small area indirect estimators are often based on area level random effects models. Under this class of models, when only aggregate specific covariates are available, the Best Linear Unbiased Predictor (BLUP) is obtained under the assumption of uncorrelated random area effects (Fay and Herriot, 1979). Details about this predictor, and its empirical version (EBLUP), for small area parameters (total y_i, mean $\bar{y}_i$) can be found in Ghosh and Rao (1994), Rao

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(1999), Datta and Lahiri (2000) and Rao (2003). The EBLUP takes advantage of the between small area-variation. The evidence is that the EBLUP estimator is significantly better than the sample-size dependent estimators, especially when the between small area-variation is not large relative to the within small area-variation (Rao and Choudhry, 1995). This suggests that the location of the small areas may also be relevant in modeling the small area parameters and that further improvement in the EBLUP estimator can be gained by including eventual spatial interaction among random area effects. Spatially correlated effects can also have a pragmatic role (Cressie, 1991). Ideally all relevant variables are chosen in the model, or proxies for them appear in the regression relation. These variables - and the dependent variable - often all vary spatially, so the benefit obtained from including spatial dependence is presumed to be considerable. In addition, it should be noted that small area boundaries are generally defined according to administrative criteria without considering the eventual spatial interaction of the variable of interest. As a result, there is no reason to exclude the assumption that the random effects between the neighboring areas are correlated and that the correlation decays to zero as distance increases.

In a recent paper, Pfeffermann (2002) has examined how much can be gained by accounting for the existing correlation between the area effects. A working model is defined with no covariates and with equal area sample size. However, in the model the correlation between pairs of areas is inserted without considering the spatial weight matrix (contiguity matrix). Instead, the spatial correlation is closely linked with the kind of contiguity matrix. Essential to this concept is the definition of a neighborhood set for each location. Typically, the weights matrix is based on the geographic distribution of the observations, or contiguity. Generally, weights are non-zero when two locations share a common boundary, or are within a given distance of each other. The only attempt to generalize the Fay-Herriot model, considering correlated random area effects between the neighboring areas, has been made by Cressie (1991). In the context of U.S. census undercount, a state adjustment factor was modeled using the basic area level model, allowing spatial correlation between the random state effects under the Conditional Autoregressive (CAR) spatial model. The adjustment factor for each small area was determined as a function of spatial correlation, but it was not associated with any area-specific measure of variability. Until now estimators of Mean Squared Error (MSE) of the EBLUP estimators under a spatial model have not been spelled out.

This paper extends the Fay-Herriot model with spatial correlation between the random small area effects modeled through the Simultaneously Autoregressive (SAR) process. The best linear unbiased predictor under this model is called Spatial BLUP (Section 2). Its empirical version (EBLUP) is obtained and an estimator of its MSE is proposed. The article also proposes a procedure to estimate the variance components of the model using maximum likelihood or restricted maximum likelihood, combining the Nelder-Mead method with "scoring" algorithm (Section 3). Relative performances of the Spatial EBLUP are evaluated through a Monte Carlo experiment. We studied the relative performance of the post-stratified estimator (direct estimator), regression synthetic estimator, composite estimator, EBLUP and Spatial EBLUP estimators. The study is carried out on spatially correlated synthetic populations generated under a nested error regression model (Rao and Choudhry, 1995). The empirical sampling distribution of the estimators is obtained under 7 different patterns of spatial correlation. Comparisons are made according to the customary repeated sampling approach as well as a conditional framework by conditioning on the realized sample sizes in the small areas (Section 4). The properties of various estimators are evaluated in Section 5 by analyzing data from the survey on Life Conditions in Tuscany (Italy). Conclusions can be found in Section 6, where the theoretical and applied advantages of the methodology proposed here are summarized.

2 Spatial BLUP

Let $\boldsymbol{\theta}$ be the $m \times 1$ vector of the parameter of inferential interest (small area total y_i, small area mean $\bar{y}_i$ with $i = 1...m$) and assume that the $m \times 1$ vector of the direct estimator $\hat{\boldsymbol{\theta}}$ is available and design unbiased

$$\hat{\boldsymbol{\theta}} = \boldsymbol{\theta} + \mathbf{e} \tag{1}$$

with $\mathbf{e}$ the vector of independent sampling errors with mean $\mathbf{0}$ and known diagonal variance matrix $\boldsymbol{\psi}$. When all the small areas are sampled, it can be of interest to summarize the data of each small area and to refer the result to a location inside the area. Generally the location is the centroid of the small area itself. When this situation arises the sample data can be considered as lattice data. The spatial relationship among data at different locations is usually based on developing neighborhoods and the autocorrelation of locations within neighborhoods. The spatial dependence among small areas is introduced by specifying a linear mixed model with spatially correlated random effects for the $\boldsymbol{\theta}$ parameter:

$$\boldsymbol{\theta} = \mathbf{X}\boldsymbol{\beta} + \mathbf{Z}\mathbf{v} \tag{2}$$

where $\mathbf{X}$ is the $m \times p$ matrix of the area specific auxiliary covariates $\mathbf{x}_i = (x_{i1}, x_{i2}, ..., x_{ip})$, $\boldsymbol{\beta}$ is the regression parameters vector $p \times 1$, $\mathbf{Z}$ is a $m \times m$

matrix of known positive constants, $\mathbf{v}$ is the $m \times 1$ vector of the second order variation. Basically there are two approaches to describe the spatial second order variation: Simultaneously Autoregressive models (SAR) and Conditional Autoregressive models (CAR). In the economy of this paper the deviations from the fixed part of the model $\mathbf{X}\boldsymbol{\beta}$ are the result of a simultaneously autoregressive process with parameter ρ (spatial autoregressive coefficient) and $m \times m$ proximity matrix $\mathbf{W}$ (Cressie, 1993; Anselin, 1992):

$$\mathbf{v} = \rho\mathbf{W}\mathbf{v} + \mathbf{u} \Rightarrow \mathbf{v} = (\mathbf{I} - \rho\mathbf{W})^{-1}\mathbf{u} \tag{3}$$

where $\mathbf{u}$ is a $m \times 1$ vector of independent error terms with zero mean and constant variance σ_u^2 and $\mathbf{I}$ is the $m \times m$ identity matrix.

Combining (1) and (2), with $\mathbf{e}$ independent of $\mathbf{v}$, the model with spatially correlated random area effects is:

$$\hat{\boldsymbol{\theta}} = \mathbf{X}\boldsymbol{\beta} + \mathbf{Z}(\mathbf{I} - \rho\mathbf{W})^{-1}\mathbf{u} + \mathbf{e}. \tag{4}$$

The error terms $\mathbf{v}$ and $\mathbf{e}$ have respectively $m \times m$ covariance matrices:

$$\mathbf{G} = \sigma_u^2[(\mathbf{I} - \rho\mathbf{W})(\mathbf{I} - \rho\mathbf{W}^T)]^{-1} \tag{5}$$

that is the Simultaneously Autoregressive (SAR) dispersion matrix and

$$\mathbf{R} = \psi = diag(\psi_i). \tag{6}$$

Thus the covariance matrix of the $\hat{\boldsymbol{\theta}}$ is:

$$\mathbf{V} = \mathbf{R} + \mathbf{Z}\mathbf{G}\mathbf{Z}^T = diag(\psi_i) + \mathbf{Z}\sigma_u^2[(\mathbf{I} - \rho\mathbf{W})(\mathbf{I} - \rho\mathbf{W}^T)]^{-1}\mathbf{Z}^T. \tag{7}$$

The $\mathbf{W}$ matrix describes the neighborhood structure of the small areas whereas ρ defines the strength of the spatial relationship among the random effects associated with neighboring areas. The spatial weight matrix represents the potential interaction between locations. A general spatial weight matrix can be defined by a symmetric binary contiguity matrix, which can be generated from topological information provided by the Geographical Information System (GIS) based on adjacency criteria: the element of the spatial weight matrix $\{w_{ij}\}$ is one if location i is adjacent to location j, and zero otherwise. So $\mathbf{W}$ is the first-order neighbor proximity matrix and here ρ is called a spatial autoregression parameter. Again, $\mathbf{I} - \rho\mathbf{W}$ will be nonsingular if $\rho \in (\frac{1}{min(\lambda_i)}, \frac{1}{max(\lambda_i)})$ where λ_i's are the eigenvalues of matrix $\mathbf{W}$. Generally, for ease of interpretation, the general spatial weight matrix is defined in row standardized form, in which the row elements sum to one. In this case $\mathbf{W}$ is not symmetric but is row stochastic, and ρ is called a spatial autocorrelation parameter. With $\mathbf{W}$ row stochastic the eigenvalues of $\mathbf{W}$ are all less than or equal to 1, then $\mathbf{I}-\rho\mathbf{W}$ will be nonsingular if $\rho \in (-1,1)$, justifying referring to ρ as an autocorrelation parameter (Banerjee *et al.*, 2004, p. 85). Some more complex spatial weight matrices, for more precise spatial linkages, are proposed by Cliff and Ord (1981), Dacey (1965) and Getis and Aldstadt (2004).

Under the model, the Spatial Best Linear Unbiased Predictor (Spatial BLUP) estimator of θ_i is:

$$\tilde{\theta}_i^S(\sigma_u^2,\rho) = \mathbf{x}_i\hat{\boldsymbol{\beta}} + \mathbf{b}_i^T\{\sigma_u^2[(\mathbf{I}-\rho\mathbf{W})(\mathbf{I}-\rho\mathbf{W}^T)]^{-1}\}\mathbf{Z}^T \times$$

$$\times\{diag(\psi_i) + \mathbf{Z}\sigma_u^2[(\mathbf{I}-\rho\mathbf{W})(\mathbf{I}-\rho\mathbf{W}^T)]^{-1}\mathbf{Z}^T\}^{-1}(\hat{\boldsymbol{\theta}} - \mathbf{X}\hat{\boldsymbol{\beta}}) \qquad (8)$$

where $\hat{\boldsymbol{\beta}} = (\mathbf{X}^T\mathbf{V}^{-1}\mathbf{X})^{-1}\mathbf{X}^T\mathbf{V}^{-1}\hat{\boldsymbol{\theta}}$ and $\mathbf{b}_i^T$ is $1\times m$ vector $(0,0,...0,1,0,...0)$ with 1 in the i-th position. The proposed predictor is obtained from Henderson's 1975 results for general linear mixed models involving fixed and random effects. The Spatial BLUP is equal to the traditional BLUP under the random area specific effects model when $\rho = 0$.

The MSE of Spatial BLUP can be obtained, under the specified model, as indicated in Rao (2003). The $MSE[\tilde{\theta}_i^S(\sigma_u^2,\rho)]$, depending on two variance components (σ_u^2,ρ), can be expressed as:

$$MSE[\tilde{\theta}_i^S(\sigma_u^2,\rho)] = g_{1i}(\sigma_u^2,\rho) + g_{2i}(\sigma_u^2,\rho) \qquad (9)$$

where the first term $g_{1i}(\sigma_u^2,\rho)$ is due to the estimation of random effects and is of order $O(1)$ while the second term $g_{2i}(\sigma_u^2,\rho)$ is due to the estimation of $\boldsymbol{\beta}$ and is of order $O(m^{-1})$ for large m (Rao, 2003). The details of the calculation are reported in Appendix A.

3 Spatial EBLUP

The estimator $\tilde{\theta}_i^S(\sigma_u^2,\rho)$ depends on the unknown variance components σ_u^2 and ρ. Replacing the parameters with asymptotically consistent estimators $\hat{\sigma}_u^2$, $\hat{\rho}$, a two stage estimator $\tilde{\theta}_i^S(\hat{\sigma}_u^2,\hat{\rho})$ is obtained and is called Spatial EBLUP:

$$\tilde{\theta}_i^S(\hat{\sigma}_u^2,\hat{\rho}) = \mathbf{x}_i\hat{\boldsymbol{\beta}} + \mathbf{b}_i^T\{\hat{\sigma}_u^2[(\mathbf{I}-\hat{\rho}\mathbf{W})(\mathbf{I}-\hat{\rho}\mathbf{W}^T)]^{-1}\}\mathbf{Z}^T \times$$

$$\times\{diag(\psi_i) + \mathbf{Z}\hat{\sigma}_u^2[(\mathbf{I}-\hat{\rho}\mathbf{W})(\mathbf{I}-\hat{\rho}\mathbf{W}^T)]^{-1}\mathbf{Z}^T\}^{-1}(\hat{\boldsymbol{\theta}} - \mathbf{X}\hat{\boldsymbol{\beta}}) \qquad (10)$$

with $\mathbf{b}_i^T = (0,0,...0,1,0,...0)$ with 1 referring to the i-th area. The expected value $E[\tilde{\theta}^S(\hat{\sigma}_u^2,\hat{\rho})]$ is finite, the estimator is unbiased for θ and $\hat{\sigma}_u^2,\hat{\rho}$ are any translation invariant estimators of σ_u^2 and ρ (Kackar and Harville, 1984).

Assuming normality of the random effects, σ_u^2 and ρ can be estimated both by Maximum Likelihood (ML) and Restricted Maximum Likelihood (REML) procedures. The ML estimators, $\hat{\sigma}^2_{u_{ML}}$ and $\hat{\rho}_{ML}$, can be obtained iteratively using the "Nelder-Mead" algorithm (Nelder and Mead, 1965) and the "scoring" algorithm in sequence. The use of these procedures one after the other is necessary because the log-likelihood function has a global maximum and some local maximums.

The ML estimator obtained with the "scoring" algorithm depends on the selected starting point, while the "Nelder-Mead" method for the maximization of a function of q variables depends on the comparison of function values at the $(q+1)$ vertices of a general simplex; it adapts itself to the local landscape, and contracts on the final maximum. It does not depend on the selected starting point and is computationally compact but not fully efficient: it achieves a point that is close to the global maximum. For this reason it is necessary to use the "scoring" algorithm, selecting as a starting point the maximum that has been obtained by the "Nelder-Mead" method. The log-likelihood function, its partial derivatives and the information matrix are described in Appendix B.

The ML procedure to estimate σ_u^2 and ρ does not consider the loss in degrees of freedom due to estimating $\boldsymbol{\beta}$. This drawback motivates the use of the REML method. The loss in degrees of freedom is taken into account in the REML method by using the transformed data $\boldsymbol{\theta}^* = \mathbf{F}^T\hat{\boldsymbol{\theta}}$, where $\mathbf{F}$ is any $m \times (m-p)$ matrix of full rank orthogonal to the $m \times p$ matrix $\mathbf{X}$ (see Appendix C).

The ML and REML estimators are robust, as they produce acceptable results even under non-normal distribution of the random effects (Jiang, 1996).

The MSE of the Spatial EBLUP estimator appears to be insensitive to the choice of the estimators $\hat{\sigma}_u^2$ and $\hat{\rho}$ (Kackar and Harville, 1984). Given normality of random effects, an approximation to the $MSE[\tilde{\theta}_i^S(\hat{\sigma}_u^2, \hat{\rho})]$ is:

$$MSE[\tilde{\theta}_i^S(\hat{\sigma}_u^2, \hat{\rho})] \approx g_{1i}(\sigma_u^2, \rho) + g_{2i}(\sigma_u^2, \rho) + g_{3i}(\sigma_u^2, \rho) \tag{11}$$

where $g_{3i}(\sigma_u^2, \rho)$ is due to the estimation of the variance components, and it is obtained by following the results of Kackar and Harville (1984):

$$g_{3i}(\sigma_u^2, \rho) = tr\left\{\begin{bmatrix} \mathbf{b}_i^T\left(\mathbf{C}^{-1}\mathbf{Z}^T\mathbf{V}^{-1} + \sigma_u^2\mathbf{C}^{-1}\mathbf{Z}^T(-\mathbf{V}^{-1}\mathbf{Z}\mathbf{C}^{-1}\mathbf{Z}^T\mathbf{V}^{-1})\right) \\ \mathbf{b}_i^T\left(\mathbf{A}\mathbf{Z}^T\mathbf{V}^{-1} + \sigma_u^2\mathbf{C}^{-1}\mathbf{Z}^T(-\mathbf{V}^{-1}\mathbf{Z}\mathbf{A}\mathbf{Z}^T\mathbf{V}^{-1})\right) \end{bmatrix} \mathbf{V} \times \right.$$

$$\left. \times \begin{bmatrix} \mathbf{b}_i^T\left(\mathbf{C}^{-1}\mathbf{Z}^T\mathbf{V}^{-1} + \sigma_u^2\mathbf{C}^{-1}\mathbf{Z}^T(-\mathbf{V}^{-1}\mathbf{Z}\mathbf{C}^{-1}\mathbf{Z}^T\mathbf{V}^{-1})\right) \\ \mathbf{b}_i^T\left(\mathbf{A}\mathbf{Z}^T\mathbf{V}^{-1} + \sigma_u^2\mathbf{C}^{-1}\mathbf{Z}^T(-\mathbf{V}^{-1}\mathbf{Z}\mathbf{A}\mathbf{Z}^T\mathbf{V}^{-1})\right) \end{bmatrix}^T \bar{\mathbf{V}}(\hat{\sigma}_u^2, \hat{\rho})\right\} \tag{12}$$

with $\mathbf{C} = [(\mathbf{I} - \rho\mathbf{W})(\mathbf{I} - \rho\mathbf{W}^T)]$, $\mathbf{A} = \sigma_u^2[-\mathbf{C}^{-1}(2\rho\mathbf{W}\mathbf{W}^T - 2\mathbf{W})\mathbf{C}^{-1}]$ and $\bar{\mathbf{V}}(\hat{\sigma}_u^2, \hat{\rho})$ is the asymptotic covariance matrix of $\hat{\sigma}_u^2$ and $\hat{\rho}$. In practical application the estimator $\tilde{\theta}_i^S(\hat{\sigma}_u^2, \hat{\rho})$ has to be associated with an estimator of $MSE[\tilde{\theta}_i^S(\hat{\sigma}_u^2, \hat{\rho})]$. An approximately unbiased estimator of it is given by:

$$mse[\tilde{\theta}_i^S(\hat{\sigma}_u^2, \hat{\rho})] \approx g_{1i}(\hat{\sigma}_u^2, \hat{\rho}) + g_{2i}(\hat{\sigma}_u^2, \hat{\rho}) + 2g_{3i}(\hat{\sigma}_u^2, \hat{\rho}) \quad (13)$$

if $\hat{\sigma}_u^2$ and $\hat{\rho}$ are REML estimators. Otherwise, if the ML procedure is used, the $mse[\tilde{\theta}_i^S(\hat{\sigma}_u^2, \hat{\rho})]$ is given by

$$mse[\tilde{\theta}_i^S(\hat{\sigma}_u^2, \hat{\rho})] \approx g_{1i}(\hat{\sigma}_u^2, \hat{\rho}) - \mathbf{b}_{ML}^T(\hat{\sigma}_u^2, \hat{\rho}) \bigtriangledown g_{1i}(\hat{\sigma}_u^2, \hat{\rho}) + g_{2i}(\hat{\sigma}_u^2, \hat{\rho}) + 2g_{3i}(\hat{\sigma}_u^2, \hat{\rho}). \quad (14)$$

The term $\mathbf{b}_{ML}^T(\hat{\sigma}_u^2, \hat{\rho}) \bigtriangledown g_{1i}(\hat{\sigma}_u^2, \hat{\rho})$ is an extra term due to the bias of $g_{1i}(\hat{\sigma}_u^2, \hat{\rho})$ and is calculated in Appendix D. If this term is ignored, the use of ML estimators could lead to underestimation of the approximation of MSE.

4 Simulation study

In order to assess the use of the developed methodology, simulated experiments were carried out. It is reasonable to assume that the Spatial EBLUP estimator performs better when the spatially correlated random effects model provides a good fit than in the case of an imprecise fit, so we generated a synthetic population of y-values, using the spatial nested error regression model with random area effects of neighboring areas correlated according to the SAR dispersion matrix with established spatial autoregressive coefficient. It is a model for the values y_{ij} of the study variable for the unit j in the small area i:

$$y_{ij} = x_{ij}\boldsymbol{\beta} + v_i + e_{ij}x_{ij}^{1/2} \quad i = 1 \dots m, \quad j = 1 \dots N_i \quad (15)$$

where x_{ij} is the value of auxiliary variable x, v_i is the random area specific effect and e_{ij} is the individual error. In order to compare the results for the Spatial EBLUP with the ones obtained for the EBLUP, the experiment was designed following the example of Rao and Choudry (1995, Section 27.2.3). Letting $\beta = 0.21$, $\sigma_u^2 = 100$, and $\sigma^2 = 1.34$, we generated independent random variables $\mathbf{v} = [v_1, v_2, \dots, v_m]^T$ and $\mathbf{e} = [e_{11}, e_{12}, ..., e_{ij} ..., e_{mN_m}]^T$, respectively, from a $MVN(0, \sigma_u^2[(\mathbf{I} - \rho\mathbf{W})(\mathbf{I} - \rho\mathbf{W}^T)]^{-1})$ and $N(0, \sigma^2)$; x_{ij} values were generated from a uniform distribution between 0 and 10, then we obtained y_{ij} for each x_{ij} from $y_{ij} = 0.21x_{ij} + v_i + e_{ij}x_{ij}^{1/2}$. The SAR dispersion matrix was generated with ρ equal to 0, ± 0.25, ± 0.5, ± 0.75 and neighborhood structure ($\mathbf{W}$) obtained randomly assigning neighbors for each area as follows: fixed $m = 42$, the number of small areas, the value 1 is assigned to the spatial weight w_{ij} if the value drawn from a uniform distribution [0,1] is greater than 0.5, 0 otherwise. The maximum number of neighbors for each area was 5, and the **W** matrix was standardized by row, that is, the row elements sum to one, thus we can refer to ρ as an autocorrelation parameter. The **W** matrix was kept fixed for all simulations.

To make unconditional comparison between estimators under customary repeated sampling, ($T = 500$) samples of average size $\bar{n} = 510$ ($s.e. = 64.1$) were selected from each of the 7 synthetic populations by simple random sampling. For each sample drawn, the mean of each small area has been estimated by:

1. *Post-stratified* estimator:

$$\hat{Y}_i(pst) = \sum_{j \in s_i} y_{ij} \quad if \quad n_i > 1 \tag{16}$$

   where s_i is the set of n_i sample units falling in the ith small area.

2. *Composite* estimator:

$$\hat{Y}_i(comp) = \alpha_i \hat{Y}_i(pst) + (1 - \alpha_i)\hat{Y}_i(reg) \tag{17}$$

   where $\hat{Y}_i(reg) = \bar{X}_i\hat{\beta}$ with $\hat{\beta} = \sum_{j \in s} x_j y_j (\sum_{j \in s} x_j x_j^T)^{-1}$ and $\alpha_i = 1$ if $n_i/n \geq \delta(N_i/N)$ and $\alpha_i = (1/\delta)\frac{n_i/n}{N_i/N}$ if $n_i/n < \delta(N_i/N)$. In this study δ is equal to 1.

3. *EBLUP* estimator.

4. *Spatial EBLUP* estimator.

For each estimator we computed the Average Absolute Relative Bias ($\overline{ARB}$), the Average Relative Efficiency ($\overline{EFF}$), the Average Absolute Relative Error ($\overline{ARE}$) and the Average Relative Root MSE ($\overline{RRMSE}$), defined as follows:

$$\overline{ARB} = \frac{1}{m}\sum_{i}^{m}\left|\frac{1}{T}\sum_{t=1}^{T}(\hat{Y}_{it}/Y_i - 1)\right| \tag{18}$$

$$\overline{EFF} = \left\{\frac{\overline{MSE}(\hat{Y}(pst))}{\overline{MSE}(\hat{Y})}\right\}^{1/2} \tag{19}$$

$$\overline{ARE} = \frac{1}{m}\sum_{i}^{m}\frac{1}{T}\sum_{t=1}^{T}(|\hat{Y}_{it}/Y_i - 1|) \tag{20}$$

$$\overline{RRMSE} = \frac{1}{m}\sum_{i}^{m}\frac{[\overline{MSE}(\hat{Y}_i)^{1/2}]}{Y_i} \tag{21}$$

where

$$\overline{MSE} = \frac{1}{m}\sum_{i}^{m}\frac{1}{T}\sum_{t=1}^{T}(\hat{Y}_{it} - Y_i)^2. \tag{22}$$

Comparisons are also made under a conditional framework by conditioning on the realized sample size in the small areas. Conditioning on the sample size, we also studied the effect of sample size n_i on the values of $\overline{ARB}$, $\overline{EFF}$, $\overline{ARE}$ and $\overline{RRMSE}$.

For the whole study a new programme, running under the R environment, was implemented to estimate the parameters (σ_u^2, ρ) and to calculate the Spatial EBLUP and the EBLUP estimators.

4.1 Unconditional comparisons

We selected 500 simple random samples from each synthetic population and then computed $\overline{ARB}$, $\overline{EFF}$, $\overline{ARE}$ and $\overline{RRMSE}$. Table 1 reports these values. Table 2 reports the estimation of the MSE and its breakdown into the three parts g_1, g_2 and g_3.

Table 1 shows that the Spatial EBLUP estimator performs significantly better than post-stratified and composite estimators in terms of $\overline{EFF}$, $\overline{ARE}$ and $\overline{RRMSE}$. Moreover, the Spatial EBLUP performs generally better than the traditional EBLUP in the case of both in case of positive and of negative spatial correlation. The results are the same with both the ML and the REML estimation methods of σ_u^2, ρ and $\boldsymbol{\beta}$. In the following we comment on the ML results and indicate in brackets those obtained with REML; in the tables, ML stands for maximum likelihood procedure, while R refers to restricted maximum likelihood procedure.

Two things stand out in Table 1: the first is that for high and medium spatial correlation among small areas ($\rho = \pm 0.75$, ± 0.50), the introduction of Spatial EBLUP leads to a reduction in the bias of the traditional EBLUP. Secondly, in terms of variability, the Spatial EBLUP reaches lower levels of MSE in comparison to the remaining estimators.

Regarding the bias of the Spatial EBLUP, for example, for $\rho = 0.75$ the $\overline{ARB}$ value for the Spatial EBLUP estimator is 3.23% (3.13%), compared to 3.54% (3.39%) for the EBLUP estimator; the $\overline{ARE}$ value is 5.89% (5.81%) compared to 7.20% (7.18%). In the case of moderate spatial correlation ($\rho = \pm 0.25$) the introduction of correlated random area effects does not always reduce the bias of the Spatial EBLUP estimator. $\overline{ARB}$ and $\overline{ARE}$ values are slightly higher for the Spatial EBLUP estimator. When $\rho = 0.25$ the $\overline{ARB}$ value is 6.04% (5.85%) compared to 5.87% (5.60%) for the EBLUP estimator. The $\overline{ARE}$ is 10.68% (10.65%) versus 10.50% (10.45%). The $\overline{ARB}$ value is lower for the Spatial EBLUP when $\rho = -0.25$: we obtain $\overline{ARB} = 6.33\%$ (6.09%) for the Spatial EBLUP versus the value of 6.45% (6.19%) for the EBLUP.

Looking at the efficiency of the Spatial EBLUP, the $\overline{EFF}$ value is always greater than the traditional post-stratified and composite estimators and greater than the traditional EBLUP estimator. For example, for $\rho = 0.75$ the $\overline{EFF}$ value for the Spatial EBLUP estimator is 155.68% (155.77%) compared to 122.08% (122.18%) for the EBLUP estimator. The results are similar to those obtained for the simulated synthetic populations where the spatial correlation is medium $\rho = 0.50$. In the presence of moderate spatial correlation the results are confirmed only for $\rho = -0.25$. In the case of $\rho = 0.25$ the EBLUP estimator is slightly more efficient than the Spatial EBLUP. Obviously, when $\rho = 0$ the Spatial EBLUP results are similar to those obtained by the application of the EBLUP. Possible differences are likely to be due to the simulation experiment.

In Table 2 we show how the unconditional MSE estimators (13) and (14) performed in the simulation study with respect to coverage of 95% confidence intervals. Our results are encouraging, even if a close inspection of Table 2 indicates that they are very sensitive to the spatial correlation value. The coverage is almost perfect for medium and high correlation. In the case of moderate correlation ($\rho = \pm 0.25$) the coverage rate is about 2% lower than the nominal value. However it does not lead to intervals with coverage less than the lower value achieved by the post-stratified estimator (93%). The cases where the coverage rate drops below the nominal 95% are, for obvious reasons, those where the bias of the Spatial EBLUP estimator remains relatively high, as Table 1 shows. We would expect the variance of the Spatial EBLUP estimator to be lower than the variance of the EBLUP with increasing $|\rho|$. The data in Table 2 confirms this. Not unexpectedly the reduction is much more evident for $\rho > 0$ than for $\rho < 0$. Spatial EBLUP improves the performance of EBLUP because, taking into account of correlation pins out the situations where the between-areas homogeneity is high.

In addition, we would also expect the difference between EBLUP and Spatial EBLUP variance to vanish when the spatial correlation is moderate, and this is confirmed by Table 2 when $\rho = \pm 0.25$. The most extreme illustration of this occurs when $\rho = 0$ and the two estimators are practically identical.

In the simulations, we also studied the point estimators (13) and (14) of the MSE of the Spatial EBLUP generated by their three components g_1, g_2 and g_3, respectively, due to the estimation of the random effects (g_1), due to the estimation of $\boldsymbol{\beta}$ (g_2) and to the estimation of the variance components (g_3). The results are displayed in Table 2.

We found that the three estimated components of MSE are ranked in the same order as those of the EBLUP: g_1 is the largest one.

We note that g_1 is lower for the Spatial EBLUP compared to that in the EBLUP, while g_2 and g_3 are larger than those of the EBLUP estimator. These results are motivated, respectively, by the precise fit of the correlated random effects model to the data (g_1) and by the additional estimation of parameter ρ that is not included in the model underlying the EBLUP estimator (g_2 and g_3).

There are three remarkable aspects in the unconditional comparisons results. First, the gain in relative efficiency of the Spatial EBLUP is relevant especially where the spatial correlation is medium or high. At the same time the estimated MSE is lower and consequently the width of the confidence interval is smaller, given the nominal level of coverage (95%). This happens maintaining or improving the level of empirical coverage of the confidence interval obtained with the EBLUP estimator.

4.2 Conditional comparisons

A more realistic approach is the conditional comparisons of the estimator by conditioning on the realized sample sizes in the small areas, because the domain sample sizes, n_i, are random with known distribution. We made conditional comparisons from the synthetic populations, similar to those in Section 4.1, under repeated sampling: a simple random sample of size $n = 492$ was selected to determine the sample sizes, n_i, in the small areas. Then, considering the n_i's as fixed, $T = 500$ stratified random samples were then selected, treating the small areas as strata. The conditional values of $\overline{ARB}$, $\overline{ARE}$, $\overline{EFF}$, $\overline{RRMSE}$, as well as the estimation of the MSE and its breakdown into the three parts g_1, g_2 and g_3, were computed from the simulated stratified samples. These are reported in Tables 3 and 4. The conditional performances are similar to the unconditional performances. In particular, Table 3 shows that the EBLUP and Spatial EBLUP estimators perform significantly better than post-stratified and composite estimators in terms of $\overline{EFF}$, $\overline{ARE}$, $\overline{RRMSE}$. $\hat{Y}(pst)$ and $\hat{Y}(comp)$ have smaller $\overline{ARB}$. Overall, the Spatial EBLUP is better than the EBLUP for high and medium spatial correlation between small areas ($\rho = \pm 0.75$, ± 0.50): the introduction of the Spatial EBLUP leads to a reduction in the bias of traditional the EBLUP and an improvement in terms of efficiency. As well as in the unconditional comparisons, in the presence of moderate spatial correlation ($\rho = \pm 0.25$) and $\rho = 0$, the results are confirmed only for $\rho = -0.25$. In the case of $\rho = 0$ and $\rho = 0.25$, the EBLUP estimator has lower levels in the bias and higher efficiency than the Spatial EBLUP estimator.

Table 4 displays the point estimators (13) and (14) of the MSE of the Spatial EBLUP. The results in terms of coverage of 95% confidence intervals as well as in terms of the three components g_1, g_2 and g_3 confirm those obtained in the unconditional comparisons. Using the Spatial EBLUP the coverage is almost perfect for medium and high spatial correlation, while the coverage rate is about 2% lower than the nominal value when the correlation is moderate. Regarding the components of the MSE estimator, we note that g_1 is the largest component and that it is lower for the Spatial EBLUP than for the EBLUP.

Finally, we studied the effect of sample size n_i on the values of $\overline{ARB}$, $\overline{ARE}$, $\overline{EFF}$, $\overline{RRMSE}$. We calculated two separate values for each quality measure by averaging first over areas with $n_i < 6$ only and then over areas with $n_i \geq 6$. Tables 5 and 6 report these values. In general, they show that the Spatial EBLUP estimator is better than the EBLUP estimator for high and medium spatial correlation between small areas both for $n_i < 6$ and $n_i \geq 6$ in terms of $\overline{ARB}$, $\overline{ARE}$,$\overline{EFF}$, and $\overline{RRMSE}$. This also occurs when $\rho = -0.25$. In the presence of low positive correlation ($\rho = 0.25$), the Spatial EBLUP estimator performs better then the EBLUP estimator only when $n_i \geq 6$. In the case of no spatial correlation ($\rho = 0$), $\overline{ARB}$, $\overline{ARE}$ and $\overline{RRMSE}$ values are slightly lower using the EBLUP estimator.

Turning to the effect of sample size n_i on the values of quality measures, we note that the $\overline{ARB}$ for $n_i \geq 6$ decreased for all the estimators compared to the case of $n_i < 6$, as expected. The $\overline{EFF}$ value for the Spatial EBLUP estimator is much larger than the values for the EBLUP, post-stratified and composite estimators when the domain sample sizes are small ($n_i < 6$): for example, for $\rho = -0.75$, the $\overline{EFF} = 170.70\%$ (171.57%) for the Spatial EBLUP estimator compared to $\overline{EFF} = 115.45\%$ (115.85%) for the EBLUP estimator and 90.77 for the composite estimator. Similarly, the $\overline{ARE}$ value for the Spatial EBLUP and EBLUP estimators is much smaller than the values for the post-stratified and composite estimators when the domain sample sizes are small ($n_i < 6$). For example, for $\rho = 0.75$ $\overline{ARE}$ is 13.34% (13.30%) for the Spatial EBLUP estimator compared to 20.05% for $\hat{Y}(pst)$ and 25.21% for $\hat{Y}(comp)$ with $n_i < 6$, and $\overline{ARE}$ is 11.62% (11.63%) for the Spatial EBLUP estimator compared to 13.95% for $\hat{Y}(pst)$ and 13.43% for $\hat{Y}(comp)$ with $n_i \geq 6$.

The overall conclusion from the conditional comparisons is that the Spatial EBLUP is significantly better than the EBLUP estimator when $n_i < 6$ for medium and high spatial correlation between small areas. This gain is reduced when $n_i \geq 6$, but the Spatial EBLUP estimator maintains its advantage with respect to the EBLUP estimator even in the case of moderate correlation.

5 Application: survey on Life Conditions in Tuscany

The region of Tuscany is divided into 10 provinces and 287 municipalities. Each province area is obtained by aggregating a varying number of municipalities. In order to analyze the local economic systems of the region, the territory has been officially divided into 43 sub-regions called Local Economy Systems (LESs). These areas are aggregations of municipalities but they are different from provinces. The main town of the region (Florence) is a separate LES. In this application the small area parameter of interest is the annual per-capita mean income for each LES in the year 2001. A map of the LESs of Tuscany and description of them can be found in Appendix E.

The primary source of data is the survey on Life Conditions (LC) in Tuscany, which provides the survey estimates of per-capita mean income at region level. The survey on Life Conditions was carried out in 2002 in order to measure the life conditions and to collect data relating to household budgets in Tuscany.

A total sample size of 2,612 households was obtained by stratified two-stage cluster sampling. A total of 45 municipalities (Primary Sampling Units - PSUs), grouped by population size, formed the strata. Of these, 22 were self-representing strata (main towns in the provinces and larger municipalities). From each of the other 23 strata, 3 PSUs were selected. At the second stage, a probability sample of households (Ultimate Units) was drawn from the PSUs. An average of 90 municipalities was represented in the sample. For details about the sampling design, see the technical report (IRPET, 2004). The distribution of the interviews by province is in Table 7.

We evaluated the performance of our proposed Spatial EBLUP estimator, comparing it with the corresponding EBLUP estimator under the Fay-Herriot model, and the direct estimator under the LC sampling design. In the sample 40 out of the 43 LESs are represented. The distribution of the sample by LES is reported in Appendix E.

There are three secondary sources of data which can be suitably used to arrive at the estimation of per-capita mean income at the LES level. The first source is the Italian Decennial Census of Population (ISTAT, 2001), which provides basic demographic information. Additional economic information wes obtained from the databases of the regional institute IRPET. Administrative records are the third source of data and offer other demographic and economic information at LES level (2000).

Our attempt to find a parsimonious model led to the choice of the following LES-level predictors: ageing index (x_1 - population over 64/ population under 15), the percentage of employees in industry (x_2) and current expenditure in municipality budgets at LES level (x_3). Moreover, the neighborhood structure $\mathbf{W}$ is defined as follows: spatial weight, w_{ij}, is 1 if LES i shares an edge with LES j and 0 otherwise. Then the general spatial weight matrix is defined in row standardized form, in which the row elements sum to one. In this case $\mathbf{W}$ is not symmetric, but it is row stochastic and ρ is called a spatial autocorrelation parameter. Regarding the sampling variances ψ_i, they are estimated through a

Jackknife procedure (Verma, 2004). The estimated variance $\hat{\psi}_i$ is then treated as a proxy to ψ_i. As result the $mse[\tilde{\theta}_i^S(\hat{\sigma}_u^2, \hat{\rho}, \hat{\psi}_i)]$ is greater than $mse[\tilde{\theta}_i^S(\hat{\sigma}_u^2, \hat{\rho}, \psi_i)]$.

In order to detect the spatial pattern (spatial association and spatial autocorrelation) of annual per-capita mean income, two standard global spatial statistics were calculated: Moran's I and Geary's C statistics (Cliff and Ord, 1981). Moran's I -if it is standardized- is analogous to the conventional correlation coefficient, and its values range from 1 (strong positive spatial autocorrelation) to -1 (strong negative spatial autocorrelation). Geary's C ranges between 0 and 2. Positive spatial autocorrelation is found with values ranging from 0 to 1 and negative spatial autocorrelation is found between 1 and 2. When the above indexes were calculated on our sample, they evidenced a positive spatial correlation for the per-capita mean income (see Table 8).

Using $\mathbf{x} = \{x_1, x_2, x_3\}$, assuming that the random effects are the result of simultaneous autoregressive process, ML and REML estimates of $\boldsymbol{\beta}$, σ_u^2 and ρ, the resulting Spatial EBLUP estimates and their estimated MSE were calculated. The estimated spatial autocorrelation coefficient $\hat{\rho}$ is 0.982 ($s.e. = 0.021$) with the ML procedure and 0.977 ($s.e. = 0.020$) with the REML method: this suggests the existence of a strong spatial relationship. For the non-sampled area, the Spatial EBLUP of θ_i is equal to $\mathbf{x}_i\hat{\boldsymbol{\beta}}$. The estimated variance components are applied to the complete neighborhood structure to calculate the mse estimator for each small area.

In order to appreciate the results obtained with the introduction of spatial information, the EBLUP estimates, using the same covariates, are also computed. Figure 1 displays the maps of the Spatial EBLUP and EBLUP estimates of annual per-capita mean income for the LESs in Tuscany. The maps give a visual representation of the different estimates. The territorial distribution of our estimates appears to be more variable than that obtained with the traditional EBLUP. At a larger range (5,054 Euros vs. 4,943) there is a wider diversification of annual per-capita mean income per LES. Given the same explanatory variables, the result is due to the additional spatial information inserted in our estimator. This moderates the smoothing effect resulting from the application of the traditional EBLUP and makes the specific characteristics of the LES evident. This happens without losing precision in the estimates. Our estimator, in fact, is less variable in each small area. The coefficient of variation (CV) per LES is mainly about $3-5\%$ for our estimator, while it is $5-7\%$ for the EBLUP. The results are clear from Table 9, which shows the distribution of the CV of small areas for the estimators.

The Spatial EBLUP maintains the behavior shown in the simulation study also in comparison with the direct estimator. Direct, EBLUP and Spatial EBLUP estimates of the annual per-capita mean income and their estimated standard errors are reported in Appendix E.

Inferences from model-based estimators refer to the distribution implied by the assumed model. Model selection and validation play an important role in model-based estimation; in fact if the assumed models do not perform a good fit to the data, the estimators will be model-biased and can lead to erroneous inferences.

An evaluation of our spatial model is performed by treating the standard residuals $r = \tilde{\theta}^S(\hat{\sigma}_u^2, \hat{\rho}) - \mathbf{X}\beta/(diag(\mathbf{V}))^{1/2}$ as iid $N(0,1)$. In particular, to check the normality of the standardized residuals r and to detect outlier r, a normal q-q plot is examined (Figure 2). It can be noted that there are few outliers r are few and that they correspond to neighboring low-income areas in the north-west of Tuscany. No other significant departures from the assumed model were observed. The Shapiro-Wilk W statistic gave a value of 0.979 for small area effects, yielding a p-value of 0.635, which suggests no evidence against the hypothesis of normality.

A simple exploratory device enabled another evaluation of the model: the plotting of estimated random effects $\hat{\mathbf{v}}$ against $\mathbf{W}\hat{\mathbf{v}}$, which corresponds to an average of the estimated random effects for nearest neighbors(Figure 3a). It is interesting to contrast this plot with an equivalent one (Figure 3b), where $\mathbf{W}\hat{\mathbf{v}}$ is defined as an average of "randomly chosen neighbors" (rather that near neighbors). If there is no spatial correlation in the area effects, then one would expect the slope of a least squares line fitted to the data not to be significantly different from zero. This is the case of the plots in Figure 3b: the comparison between plots (a) and (b) suggests that the random area effects are truly spatially correlated.

6 Final remarks

In this paper we have developed an indirect small area estimator based on a SAR spatial model. The proposed estimator performs very well for all the simulated populations and also when applied to the LS data.

Our results show that the Spatial EBLUP estimator based on a suitable spatial model (i.e. one that adequately reflects the between area variability in the population) works well. The results yield small area estimates that are better than the usual EBLUP estimates. Our estimator leads to a reduction in the bias of the traditional predictor and its MSE is smaller. The coverage of 95% confidence intervals is appreciable. It leads to intervals with coverage of less than 94% (but greater than 92%) only in case of moderate spatial correlation ($\rho = \pm 0.25$). From the conditional comparisons it seems that in the presence of medium and high spatial correlation between small areas, when the domain sample sizes, n_i, are small, the loss in efficiency from using the EBLUP estimator respect to the Spatial EBLUP estimator is high.

The main reason for using Spatial EBLUP is to make the best use of the available spatial auxiliary information in order to obtain the most efficient estimator possible. We have noted that the use of spatial auxiliary information can reduce both the bias and the sampling error in small area estimation. The spatial autoregressive coefficient and the contiguity matrix inserted in the variance-covariance matrix of random area effects give additional help in borrowing strength from related area in the estimation of small area parameters.

We believe that our approach can provide a useful general tool for methodologists faced with spatially correlated data in small area estimation. Questions beyond those discussed in this article need to be addressed to make the approach fully operational in a survey. Some of the issues mentioned in the paper require further theoretical work. For example we have considered only the estimation of population totals and means of continuous response variables, but most surveys involve other parameters of interest, such as counts and percentages when the response variable is discrete. In addition, what happens when random area effects follow a CAR process or when the set of auxiliary variables are also spatially correlated has to be explored.

Empirical studies are also important to gain further experience with the approach that we propose. The small area neighborhood structure should be based on knowledge of the case study. It would be useful to verify the performance of the methods using more complex spatial contiguity matrices (Cliff and Ord, 1981; Dacey, 1965; Getis and Aldstadt, 2004). In this context it is crucial to encourage the use of GIS, which would facilitate the extraction and use of spatial information. In some cases it has to be noted that spatial linkages can not always be properly defined by using geographical information. For these cases, the spatial weight matrices may be defined by using some social or economic indices. Since the analytical results may be sensitive to the specification of the spatial weight matrix, different spatial weight matrices may be needed for different kinds of studies (Bailey and Gatrell, 1995).

A final point relates to a drawback of the proposed estimator common to the EBLUP. We emphasize that both the Spatial EBLUP and the EBLUP are variable-specific since they depend on estimated variance components for a particular y. The problem of how to generalize the estimator in order to obtain a result useful for several study variables remains to be solved.

ρ	*Quality Measure*	$\hat{Y}(pst)$	$\hat{Y}(comp)$	$\tilde{\theta}(\hat{\sigma}^2_{u_{ML}})$	$\tilde{\theta}^S(\hat{\sigma}^2_{u_{ML}}, \hat{\rho}_{ML})$	$\tilde{\theta}(\hat{\sigma}^2_{u_R})$	$\tilde{\theta}^S(\hat{\sigma}^2_{u_R}, \hat{\rho}_R)$
$\rho = 0.75$	$\overline{\text{ARB}}$%	0.84	1.95	3.54	3.23	3.39	3.13
	$\overline{\text{EFF}}$%	100.00	110.15	122.02	155.68	122.18	155.77
	$\overline{\text{ARE}}$%	8.05	7.90	7.20	5.89	7.18	5.81
	$\overline{\text{RRMSE}}$%	10.09	9.91	8.84	7.07	8.83	7.09
$\rho = 0.5$	$\overline{\text{ARB}}$%	1.29	3.45	7.06	6.83	6.72	6.51
	$\overline{\text{EFF}}$%	100.00	109.54	130.59	137.83	130.81	138.50
	$\overline{\text{ARE}}$%	12.75	12.66	11.58	11.26	11.50	11.16
	$\overline{\text{RRMSE}}$%	16.11	15.99	14.32	13.94	14.26	13.85
$\rho = 0.25$	$\overline{\text{ARB}}$%	1.39	2.14	5.87	6.04	5.60	5.85
	$\overline{\text{EFF}}$%	100.00	110.29	137.40	135.23	137.74	135.59
	$\overline{\text{ARE}}$%	12.28	11.52	10.50	10.68	10.45	10.65
	$\overline{\text{RRMSE}}$%	15.45	14.39	12.77	12.95	12.77	12.96
$\rho = 0$	$\overline{\text{ARB}}$%	1.16	1.60	5.29	5.28	5.08	5.09
	$\overline{\text{EFF}}$%	100.00	121.11	159.84	156.69	159.98	157.63
	$\overline{\text{ARE}}$%	11.54	11.52	7.82	7.97	7.82	7.98
	$\overline{\text{RRMSE}}$%	14.58	13.48	9.33	9.57	9.40	9.63
$\rho = -0.25$	$\overline{\text{ARB}}$%	1.28	2.08	6.45	6.33	6.17	6.09
	$\overline{\text{EFF}}$%	100.00	110.86	121.34	121.70	122.32	122.39
	$\overline{\text{ARE}}$%	11.49	10.70	9.67	9.75	9.62	9.70
	$\overline{\text{RRMSE}}$%	14.38	13.29	11.52	11.60	11.50	11.58
$\rho = -0.5$	$\overline{\text{ARB}}$%	1.15	1.97	5.59	5.46	5.32	5.21
	$\overline{\text{EFF}}$%	100.00	110.08	124.51	133.32	125.25	133.27
	$\overline{\text{ARE}}$%	11.69	11.18	10.05	9.67	10.01	9.66
	$\overline{\text{RRMSE}}$%	14.78	14.03	12.24	11.85	12.24	11.88
$\rho = -0.75$	$\overline{\text{ARB}}$%	0.88	1.95	5.73	5.66	5.47	5.38
	$\overline{\text{EFF}}$%	100.00	110.20	126.82	132.04	127.19	132.45
	$\overline{\text{ARE}}$%	11.34	10.72	10.03	9.82	9.96	9.75
	$\overline{\text{RRMSE}}$%	14.39	13.33	12.30	11.96	12.27	11.93

Table 1: Unconditional comparison of small area estimators: $\overline{ARB}$, $\overline{EFF}$, $\overline{ARE}$ and $\overline{RRMSE}$.

ρ	*Estimator*	A.E.mse	A.E.g_1	A.E.g_2	A.E.g_3	Coverage(%)
$\rho = 0.75$	$\tilde{\theta}(\hat{\sigma}^2_{u_{ML}})$	60.68	57.25	1.56	0.92	95%
	$\tilde{\theta}^S(\hat{\sigma}^2_{u_{ML}}, \hat{\rho}_{ML})$	38.57	33.67	2.54	1.15	96%
	$\tilde{\theta}(\hat{\sigma}^2_{u_R})$	61.88	58.45	1.52	0.96	95%
	$\tilde{\theta}^S(\hat{\sigma}^2_{u_R}, \hat{\rho}_R)$	39.91	34.87	2.51	1.27	96%
	$\hat{Y}(pst)$	90.05	–	–	–	94%
$\rho = 0.50$	$\tilde{\theta}(\hat{\sigma}^2_{u_{ML}})$	40.34	36.97	1.62	0.79	94%
	$\tilde{\theta}^S(\hat{\sigma}^2_{u_{ML}}, \hat{\rho}_{ML})$	38.59	33.91	1.78	1.52	95%
	$\tilde{\theta}(\hat{\sigma}^2_{u_R})$	41.53	38.13	1.60	0.83	95%
	$\tilde{\theta}^S(\hat{\sigma}^2_{u_R}, \hat{\rho}_R)$	39.72	34.74	1.75	1.69	95%
	$\hat{Y}(pst)$	78.10	–	–	–	95%
$\rho = 0.25$	$\tilde{\theta}(\hat{\sigma}^2_{u_{ML}})$	36.22	33.10	1.42	0.78	94%
	$\tilde{\theta}^S(\hat{\sigma}^2_{u_{ML}}, \hat{\rho}_{ML})$	37.18	32.00	1.53	1.87	94%
	$\tilde{\theta}(\hat{\sigma}^2_{u_R})$	37.47	34.34	1.35	0.82	94%
	$\tilde{\theta}^S(\hat{\sigma}^2_{u_R}, \hat{\rho}_R)$	38.54	33.04	1.49	2.08	94%
	$\hat{Y}(pst)$	72.18	–	–	–	94%
$\rho = 0$	$\tilde{\theta}(\hat{\sigma}^2_{u_{ML}})$	32.47	27.99	2.25	1.09	94%
	$\tilde{\theta}^S(\hat{\sigma}^2_{u_{ML}}, \hat{\rho}_{ML})$	35.76	26.93	2.93	3.24	94%
	$\tilde{\theta}(\hat{\sigma}^2_{u_R})$	34.42	29.82	2.21	1.19	94%
	$\tilde{\theta}^S(\hat{\sigma}^2_{u_R}, \hat{\rho}_R)$	38.34	28.71	2.20	3.71	94%
	$\hat{Y}(pst)$	94.53	–	–	–	93%
$\rho = -0.25$	$\tilde{\theta}(\hat{\sigma}^2_{u_{ML}})$	34.22	30.44	2.11	0.82	93%
	$\tilde{\theta}^S(\hat{\sigma}^2_{u_{ML}}, \hat{\rho}_{ML})$	35.32	28.41	2.52	2.18	92%
	$\tilde{\theta}(\hat{\sigma}^2_{u_R})$	35.74	31.92	2.08	0.87	93%
	$\tilde{\theta}^S(\hat{\sigma}^2_{u_R}, \hat{\rho}_R)$	37.38	29.99	2.43	2.42	93%
	$\hat{Y}(pst)$	83.18	–	–	–	93%
$\rho = -0.50$	$\tilde{\theta}(\hat{\sigma}^2_{u_{ML}})$	50.95	47.48	1.07	0.93	95%
	$\tilde{\theta}^S(\hat{\sigma}^2_{u_{ML}}, \hat{\rho}_{ML})$	46.01	38.64	2.32	0.93	95%
	$\tilde{\theta}(\hat{\sigma}^2_{u_R})$	52.74	49.68	1.13	0.93	95%
	$\tilde{\theta}^S(\hat{\sigma}^2_{u_R}, \hat{\rho}_R)$	48.02	41.08	2.70	0.94	95%
	$\hat{Y}(pst)$	100.50	–	–	–	95%
$\rho = -0.75$	$\tilde{\theta}(\hat{\sigma}^2_{u_{ML}})$	55.44	51.59	1.75	1.04	95%
	$\tilde{\theta}^S(\hat{\sigma}^2_{u_{ML}}, \hat{\rho}_{ML})$	50.39	42.66	2.92	2.39	95%
	$\tilde{\theta}(\hat{\sigma}^2_{u_R})$	57.03	53.15	1.70	1.09	95%
	$\tilde{\theta}^S(\hat{\sigma}^2_{u_R}, \hat{\rho}_R)$	52.42	44.32	2.83	2.63	95%
	$\hat{Y}(pst)$	98.46	–	–	–	95%

Table 2: Unconditional comparison of small area estimators: Average of the Estimated Mean Square Error (A.E.mse.) of Direct, EBLUP and Spatial EBLUP estimators.

ρ	*Quality Measure*	$\hat{Y}(pst)$	$\hat{Y}(comp)$	$\tilde{\theta}(\hat{\sigma}^2_{u_{ML}})$	$\tilde{\theta}^S(\hat{\sigma}^2_{u_{ML}}, \hat{\rho}_{ML})$	$\tilde{\theta}(\hat{\sigma}^2_{u_R})$	$\tilde{\theta}^S(\hat{\sigma}^2_{u_R}, \hat{\rho}_R)$
$\rho = 0.75$	**$\overline{\text{ARB}}$%**	1.59	4.96	6.42	6.03	6.18	5.83
	$\overline{\text{EFF}}$%	100.00	114.98	128.57	161.08	128.74	161.08
	$\overline{\text{ARE}}$%	15.41	14.64	13.43	12.03	13.42	12.02
	$\overline{\text{RRMSE}}$%	19.26	17.83	16.62	14.84	16.63	14.85
$\rho = 0.5$	**$\overline{\text{ARB}}$%**	1.26	4.71	6.61	6.43	6.33	6.15
	$\overline{\text{EFF}}$%	100.00	126.06	157.60	162.66	158.78	164.94
	$\overline{\text{ARE}}$%	12.08	11.13	9.31	9.22	9.23	9.12
	$\overline{\text{RRMSE}}$%	15.05	13.30	11.01	10.94	10.99	10.90
$\rho = 0.25$	**$\overline{\text{ARB}}$%**	1.33	3.29	6.84	6.91	6.55	6.65
	$\overline{\text{EFF}}$%	100.00	140.27	164.38	158.98	165.33	158.84
	$\overline{\text{ARE}}$%	13.72	10.72	9.82	9.96	9.76	9.90
	$\overline{\text{RRMSE}}$%	17.06	13.19	11.66	11.84	11.68	11.83
$\rho = 0$	**$\overline{\text{ARB}}$%**	1.61	4.30	6.24	6.93	5.97	6.46
	$\overline{\text{EFF}}$%	100.00	129.64	165.39	154.08	165.17	156.12
	$\overline{\text{ARE}}$%	16.54	13.81	11.89	12.46	11.93	12.38
	$\overline{\text{RRMSE}}$%	20.57	16.86	14.39	15.08	14.49	15.04
$\rho = -0.25$	**$\overline{\text{ARB}}$%**	2.07	4.52	8.12	7.62	7.71	7.35
	$\overline{\text{EFF}}$%	100.00	143.56	169.66	172.05	170.44	171.94
	$\overline{\text{ARE}}$%	15.75	12.12	11.23	11.04	11.14	11.02
	$\overline{\text{RRMSE}}$%	19.61	14.81	13.27	13.12	13.26	13.18
$\rho = -0.5$	**$\overline{\text{ARB}}$%**	1.27	3.00	6.12	5.47	5.88	5.30
	$\overline{\text{EFF}}$%	100.00	132.87	184.52	186.77	184.52	185.77
	$\overline{\text{ARE}}$%	14.38	11.45	9.45	9.28	9.48	9.32
	$\overline{\text{RRMSE}}$%	18.06	14.19	11.53	11.36	11.62	11.45
$\rho = -0.75$	**$\overline{\text{ARB}}$%**	1.75	13.18	10.27	11.18	9.86	10.85
	$\overline{\text{EFF}}$%	100.00	92.73	103.74	123.45	104.51	124.27
	$\overline{\text{ARE}}$%	17.22	19.65	17.32	15.00	17.17	14.90
	$\overline{\text{RRMSE}}$%	21.27	22.19	20.81	17.40	20.71	17.35

Table 3: Conditional comparison of small area estimators: $\overline{ARB}$, $\overline{EFF}$, $\overline{ARE}$ and $\overline{RRMSE}$.

ρ	*Estimator*	A.E.mse	A.E.g$_1$	A.E.g$_2$	A.E.g$_3$	Coverage(%)
$\rho = 0.75$	$\tilde{\theta}(\hat{\sigma}^2_{u_{ML}})$	78.74	72.72	3.68	1.15	95%
	$\tilde{\theta}^S(\hat{\sigma}^2_{u_{ML}}, \hat{\rho}_{ML})$	59.81	52.61	3.50	1.84	95%
	$\tilde{\theta}(\hat{\sigma}^2_{u_R})$	80.51	74.47	3.63	1.21	95%
	$\tilde{\theta}^S(\hat{\sigma}^2_{u_R}, \hat{\rho}_R)$	61.71	54.05	3.47	2.10	96%
	$\hat{Y}(pst)$	141.68	–	–	–	92%
$\rho = 0.50$	$\tilde{\theta}(\hat{\sigma}^2_{u_{ML}})$	42.43	37.19	3.04	1.08	93%
	$\tilde{\theta}^S(\hat{\sigma}^2_{u_{ML}}, \hat{\rho}_{ML})$	44.27	35.46	2.94	2.92	95%
	$\tilde{\theta}(\hat{\sigma}^2_{u_R})$	44.82	39.49	3.00	1.16	93%
	$\tilde{\theta}^S(\hat{\sigma}^2_{u_R}, \hat{\rho}_R)$	46.77	37.33	2.91	2.89	95%
	$\hat{Y}(pst)$	136.23	–	–	–	92%
$\rho = 0.25$	$\tilde{\theta}(\hat{\sigma}^2_{u_{ML}})$	47.32	41.07	3.41	1.39	92%
	$\tilde{\theta}^S(\hat{\sigma}^2_{u_{ML}}, \hat{\rho}_{ML})$	48.04	38.52	3.09	3.01	94%
	$\tilde{\theta}(\hat{\sigma}^2_{u_R})$	37.47	43.81	3.36	1.49	93%
	$\tilde{\theta}^S(\hat{\sigma}^2_{u_R}, \hat{\rho}_R)$	51.28	40.79	2.92	2.89	94%
	$\hat{Y}(pst)$	152.56	–	–	–	91%
$\rho = 0$	$\tilde{\theta}(\hat{\sigma}^2_{u_{ML}})$	57.41	51.52	3.12	1.35	93%
	$\tilde{\theta}^S(\hat{\sigma}^2_{u_{ML}}, \hat{\rho}_{ML})$	56.14	45.39	3.96	3.36	93%
	$\tilde{\theta}(\hat{\sigma}^2_{u_R})$	60.04	54.06	3.07	1.45	94%
	$\tilde{\theta}^S(\hat{\sigma}^2_{u_R}, \hat{\rho}_R)$	60.31	48.74	3.78	3.89	94%
	$\hat{Y}(pst)$	153.97	–	–	–	91%
$\rho = -0.25$	$\tilde{\theta}(\hat{\sigma}^2_{u_{ML}})$	63.04	55.46	3.84	1.83	93%
	$\tilde{\theta}^S(\hat{\sigma}^2_{u_{ML}}, \hat{\rho}_{ML})$	65.31	49.82	4.99	5.21	94%
	$\tilde{\theta}(\hat{\sigma}^2_{u_R})$	67.54	59.74	3.77	2.01	94%
	$\tilde{\theta}^S(\hat{\sigma}^2_{u_R}, \hat{\rho}_R)$	70.59	53.79	4.84	5.98	94%
	$\hat{Y}(pst)$	194.60	–	–	–	91%
$\rho = -0.50$	$\tilde{\theta}(\hat{\sigma}^2_{u_{ML}})$	69.46	62.20	3.18	2.00	95%
	$\tilde{\theta}^S(\hat{\sigma}^2_{u_{ML}}, \hat{\rho}_{ML})$	70.80	55.79	4.40	5.27	96%
	$\tilde{\theta}(\hat{\sigma}^2_{u_R})$	72.98	65.54	3.13	2.15	96%
	$\tilde{\theta}^S(\hat{\sigma}^2_{u_R}, \hat{\rho}_R)$	74.79	58.83	4.28	5.84	96%
	$\hat{Y}(pst)$	201.04	–	–	–	91%
$\rho = -0.75$	$\tilde{\theta}(\hat{\sigma}^2_{u_{ML}})$	117.81	113.24	1.84	1.34	94%
	$\tilde{\theta}^S(\hat{\sigma}^2_{u_{ML}}, \hat{\rho}_{ML})$	60.43	54.12	5.06	1.63	94%
	$\tilde{\theta}(\hat{\sigma}^2_{u_R})$	119.70	115.17	1.78	1.38	94%
	$\tilde{\theta}^S(\hat{\sigma}^2_{u_R}, \hat{\rho}_R)$	39.91	34.87	2.51	1.27	94%
	$\hat{Y}(pst)$	157.78	–	–	–	92%

Table 4: Conditional comparison of small area estimators: Average of the Estimated Mean Square Error (A.E.mse.) of Direct, EBLUP and Spatial EBLUP estimators.

ρ	*Quality Measure*	$\hat{Y}(pst)$	$\hat{Y}(comp)$	$\tilde{\theta}(\hat{\sigma}^2_{u_{ML}})$	$\tilde{\theta}^S(\hat{\sigma}^2_{u_{ML}}, \hat{\rho}_{ML})$	$\tilde{\theta}(\hat{\sigma}^2_{u_R})$	$\tilde{\theta}^S(\hat{\sigma}^2_{u_R}, \hat{\rho}_R)$
$\rho = 0.75$	$\overline{\textbf{ARB}}$%	2.48	13.41	10.15	9.40	9.88	9.16
	$\overline{\textbf{EFF}}$%	100.00	125.66	142.08	195.95	142.14	196.34
	$\overline{\textbf{ARE}}$%	20.05	18.54	15.90	13.34	15.89	13.30
	$\overline{\textbf{RRMSE}}$%	25.21	21.57	19.24	16.30	19.30	16.30
$\rho = 0.5$	$\overline{\textbf{ARB}}$%	1.77	13.89	13.01	12.11	12.48	11.47
	$\overline{\textbf{EFF}}$%	100.00	152.06	184.79	196.35	186.62	200.54
	$\overline{\textbf{ARE}}$%	20.12	17.89	14.79	14.42	14.52	14.10
	$\overline{\textbf{RRMSE}}$%	24.93	19.69	16.74	16.43	16.57	16.22
$\rho = 0.25$	$\overline{\textbf{ARB}}$%	1.68	7.04	6.70	7.76	6.51	8.17
	$\overline{\textbf{EFF}}$%	100.00	226.96	277.27	236.11	273.75	221.98
	$\overline{\textbf{ARE}}$%	23.17	11.40	10.17	11.10	10.29	11.55
	$\overline{\textbf{RRMSE}}$%	28.65	13.41	12.02	13.02	12.24	13.56
$\rho = 0$	$\overline{\textbf{ARB}}$%	2.82	12.70	11.46	13.46	11.17	12.68
	$\overline{\textbf{EFF}}$%	100.00	173.12	188.24	176.96	188.38	181.17
	$\overline{\textbf{ARE}}$%	26.56	17.30	15.64	17.51	15.71	17.16
	$\overline{\textbf{RRMSE}}$%	32.48	19.50	17.96	19.43	18.13	19.29
$\rho = -0.25$	$\overline{\textbf{ARB}}$%	3.57	10.13	11.00	10.04	10.69	9.94
	$\overline{\textbf{EFF}}$%	100.00	214.60	224.61	225.87	223.69	224.04
	$\overline{\textbf{ARE}}$%	23.92	12.17	11.95	11.68	11.91	11.74
	$\overline{\textbf{RRMSE}}$%	29.47	14.24	13.76	13.53	13.86	13.73
$\rho = -0.5$	$\overline{\textbf{ARB}}$%	1.90	6.07	6.79	6.57	6.65	6.34
	$\overline{\textbf{EFF}}$%	100.00	222.42	245.57	266.89	246.97	264.90
	$\overline{\textbf{ARE}}$%	23.43	10.92	10.22	10.85	10.33	10.84
	$\overline{\textbf{RRMSE}}$%	29.38	13.08	12.22	12.92	12.42	12.98
$\rho = -0.75$	$\overline{\textbf{ARB}}$%	2.96	34.76	19.27	16.06	18.64	15.68
	$\overline{\textbf{EFF}}$%	100.00	90.77	115.45	170.70	115.85	171.57
	$\overline{\textbf{ARE}}$%	31.15	37.16	28.39	19.67	28.20	19.52
	$\overline{\textbf{RRMSE}}$%	38.37	39.27	33.99	23.31	33.90	23.25

Table 5: Conditional comparison of small area estimators ($n_i < 6$): $\overline{ARB}$, $\overline{EFF}$, $\overline{ARE}$ and $\overline{RRMSE}$.

ρ	*Quality Measure*	$\hat{Y}(pst)$	$\hat{Y}(comp)$	$\tilde{\theta}(\hat{\sigma}^2_{u_{ML}})$	$\tilde{\theta}^S(\hat{\sigma}^2_{u_{ML}}, \hat{\rho}_{ML})$	$\tilde{\theta}(\hat{\sigma}^2_{u_R})$	$\tilde{\theta}^S(\hat{\sigma}^2_{u_R}, \hat{\rho}_R)$
$\rho = 0.75$	$\overline{\text{ARB}}$%	1.32	2.32	5.25	4.98	5.03	4.79
	$\overline{\text{EFF}}$%	100.00	105.66	117.13	137.77	117.36	137.65
	$\overline{\text{ARE}}$%	13.95	13.43	12.66	11.62	12.65	11.63
	$\overline{\text{RRMSE}}$%	17.40	16.66	15.80	14.38	15.80	14.40
$\rho = 0.5$	$\overline{\text{ARB}}$%	1.12	2.20	4.87	4.88	4.65	4.70
	$\overline{\text{EFF}}$%	100.00	108.22	137.63	139.58	138.45	140.95
	$\overline{\text{ARE}}$%	9.89	9.29	7.82	7.80	7.79	7.77
	$\overline{\text{RRMSE}}$%	12.36	11.55	9.44	9.44	9.46	9.44
$\rho = 0.25$	$\overline{\text{ARB}}$%	1.25	2.41	6.87	6.71	6.56	6.30
	$\overline{\text{EFF}}$%	100.00	111.21	129.15	128.84	130.41	131.17
	$\overline{\text{ARE}}$%	11.50	10.56	9.73	9.69	9.64	9.51
	$\overline{\text{RRMSE}}$%	14.33	13.13	11.58	11.56	11.55	11.42
$\rho = 0$	$\overline{\text{ARB}}$%	1.28	2.01	4.82	5.17	4.55	4.76
	$\overline{\text{EFF}}$%	100.00	106.95	148.30	143.15	147.90	143.74
	$\overline{\text{ARE}}$%	13.81	12.86	10.87	11.09	10.90	11.07
	$\overline{\text{RRMSE}}$%	17.32	16.13	13.42	13.72	13.49	13.74
$\rho = -0.25$	$\overline{\text{ARB}}$%	1.54	2.53	7.09	6.76	6.65	6.43
	$\overline{\text{EFF}}$%	100.00	107.24	133.47	135.94	134.69	136.39
	$\overline{\text{ARE}}$%	12.85	12.11	10.98	10.81	10.87	10.77
	$\overline{\text{RRMSE}}$%	16.11	15.02	13.09	12.97	13.05	12.99
$\rho = -0.5$	$\overline{\text{ARB}}$%	1.13	2.28	5.96	5.21	5.70	5.06
	$\overline{\text{EFF}}$%	100.00	106.01	153.81	159.32	152.86	157.81
	$\overline{\text{ARE}}$%	12.25	11.58	9.27	8.82	9.28	8.96
	$\overline{\text{RRMSE}}$%	15.40	14.45	11.37	11.00	11.43	11.09
$\rho = -0.75$	$\overline{\text{ARB}}$%	1.37	6.44	9.65	7.46	9.34	7.12
	$\overline{\text{EFF}}$%	100.00	113.94	112.84	119.62	114.03	120.48
	$\overline{\text{ARE}}$%	12.86	14.18	13.85	13.54	13.73	13.46
	$\overline{\text{RRMSE}}$%	18.92	16.85	16.69	15.55	16.59	15.51

Table 6: Conditional comparison of small area estimators ($n_i \geq 6$): $\overline{ARB}$, $\overline{EFF}$, $\overline{ARE}$ and $\overline{RRMSE}$.

Province	**Interviews**	**Share**
Massa Carrara	188	0.60%
Lucca	259	1.11%
Pistoia	155	0.73%
Firenze	723	2.70%
Livorno	179	0.88%
Pisa	273	1.06%
Arezzo	277	0.97%
Siena	270	0.69%
Grosseto	221	0.59%
Prato	117	0.62%

Table 7: Number of interviews by province.

Index	*Value*	*p value*
Moran's I	0.11	0.08
Geary's C	0.75	0.05

Table 8: Value of Moran's I, Geary's C statistics.

Estimate	**Coefficients of Variation**			
	$3-5\%$	$5-7\%$	$>7\%$	**Total**
$\tilde{\theta}(\hat{\sigma}^2_{u_{ML}})$	0	37	6	43
$\tilde{\theta}^S(\hat{\sigma}^2_{u_{ML}}, \hat{\rho}_{ML})$	23	16	4	43
$\tilde{\theta}(\hat{\sigma}^2_{u_R})$	0	23	20	43
$\tilde{\theta}^S(\hat{\sigma}^2_{u_R}, \hat{\rho}_R)$	18	20	5	43

Table 9: Class distribution of coefficients of variation of small areas for EBLUP and Spatial EBLUP estimators.

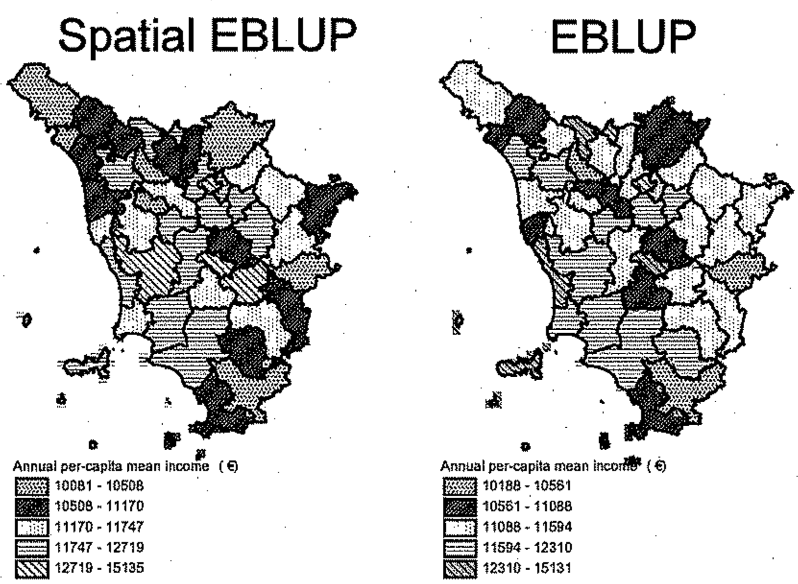

Figure 1: Map of the LESs of the region of Tuscany with Spatial EBLUP and EBLUP estimates for annual per-capita mean income.

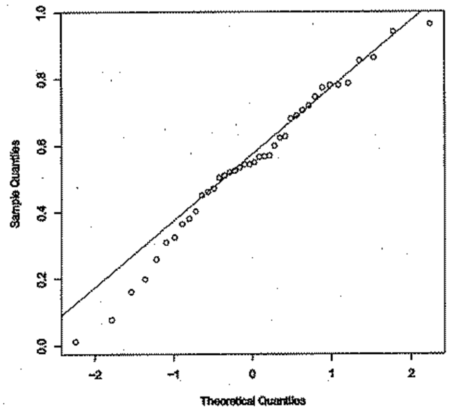

Figure 2: Normal q-q plot to check the normality of the standardized residuals r.

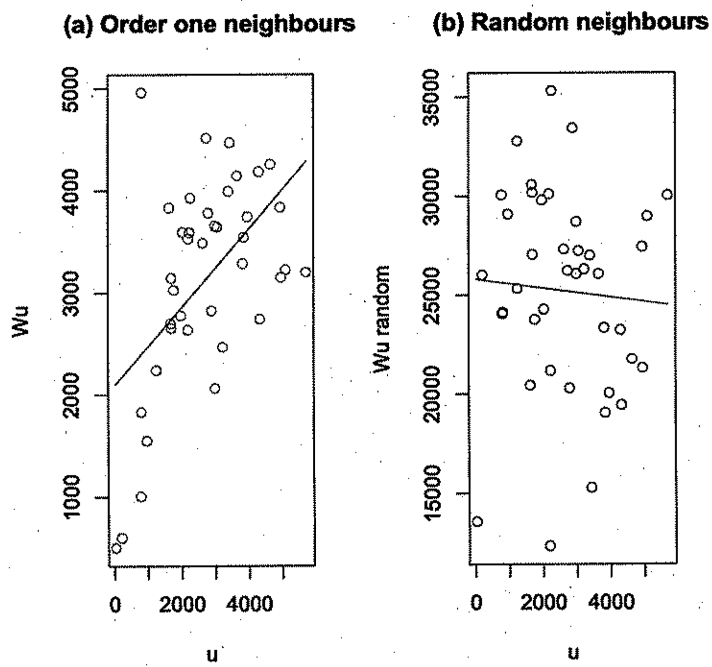

Figure 3: The estimated random effects $\hat{\mathbf{v}}$ against an average of estimated random effects for nearest neighbors $\mathbf{W}\hat{\mathbf{v}}$ (a) and an average of "randomly chosen neighbors" (b).

A Appendix

The $MSE[\tilde{\theta}_i^S(\sigma_u^2,\rho)]$, depending on two parameters (σ_u^2,ρ), can be expressed as:

$$MSE[\tilde{\theta}_i^S(\sigma_u^2,\rho)] = g_{1i}(\sigma_u^2,\rho) + g_{2i}(\sigma_u^2,\rho) \tag{A-1}$$

with

$$g_{1i}(\sigma_u^2,\rho) = \mathbf{b}_i^T\{\sigma_u^2[(\mathbf{I}-\rho\mathbf{W})(\mathbf{I}-\rho\mathbf{W}^T)]^{-1} - \sigma_u^2[(\mathbf{I}-\rho\mathbf{W})(\mathbf{I}-\rho\mathbf{W}^T)]^{-1}\mathbf{Z}^T\times$$

$$\times\{diag(\psi_i)+\mathbf{Z}\sigma_u^2[(\mathbf{I}-\rho\mathbf{W})(\mathbf{I}-\rho\mathbf{W}^T)]^{-1}\mathbf{Z}^T\}^{-1}\mathbf{Z}\sigma_u^2[(\mathbf{I}-\rho\mathbf{W})(\mathbf{I}-\rho\mathbf{W}^T)]^{-1}\}\mathbf{b}_i \tag{A-2}$$

and

$$\begin{aligned} g_{2i}(\sigma_u^2,\rho) &= (\mathbf{x}_i - \mathbf{b}_i^T\sigma_u^2[(\mathbf{I}-\rho\mathbf{W})(\mathbf{I}-\rho\mathbf{W}^T)]^{-1}\mathbf{Z}^T\times \\ &\times\{diag(\psi_i)+\mathbf{Z}\sigma_u^2[(\mathbf{I}-\rho\mathbf{W})(\mathbf{I}-\rho\mathbf{W}^T)]^{-1}\mathbf{Z}^T\}^{-1}\mathbf{X})\times \\ &\times(\mathbf{X}^T\{diag(\psi_i)+\mathbf{Z}\sigma_u^2[(\mathbf{I}-\rho\mathbf{W})(\mathbf{I}-\rho\mathbf{W}^T)]^{-1}\mathbf{Z}^T\}^{-1}\mathbf{X})^{-1}\times \\ &\times(\mathbf{x}_i - \mathbf{b}_i^T\sigma_u^2[(\mathbf{I}-\rho\mathbf{W})(\mathbf{I}-\rho\mathbf{W}^T)]^{-1}\mathbf{Z}^T\times \\ &\times\{diag(\psi_i)+\mathbf{Z}\sigma_u^2[(\mathbf{I}-\rho\mathbf{W})(\mathbf{I}-\rho\mathbf{W}^T)]^{-1}\mathbf{Z}^T\}^{-1}\mathbf{X})^T. \end{aligned} \tag{A-3}$$

B Appendix

The log-likelihood function is:

$$l(\boldsymbol{\beta},\sigma_u^2,\rho) = -\frac{1}{2}m\log 2\pi - \frac{1}{2}\log|\mathbf{V}| - \frac{1}{2}(\hat{\boldsymbol{\theta}}-\mathbf{X}\boldsymbol{\beta})^T\mathbf{V}^{-1}(\hat{\boldsymbol{\theta}}-\mathbf{X}\boldsymbol{\beta}) \tag{B-1}$$

with $\mathbf{V}$ as represented in (7) and the partial derivatives of $l(\boldsymbol{\beta},\sigma_u^2,\rho)$ with respect to σ_u^2 and ρ given by

$$\begin{aligned} s_{\sigma_u^2}(\boldsymbol{\beta},\sigma_u^2,\rho) = \frac{\partial l}{\partial\sigma_u^2} = &-\tfrac{1}{2}tr\{\mathbf{V}^{-1}\mathbf{Z}\mathbf{C}^{-1}\mathbf{Z}^T\} - \tfrac{1}{2}(\hat{\boldsymbol{\theta}}-\mathbf{X}\boldsymbol{\beta})^T(-\mathbf{V}^{-1}\mathbf{Z}\mathbf{C}^{-1}\mathbf{Z}^T\mathbf{V}^{-1})(\hat{\boldsymbol{\theta}}-\mathbf{X}\boldsymbol{\beta}) \\ s_{\rho}(\boldsymbol{\beta},\sigma_u^2,\rho) = \frac{\partial l}{\partial\rho} = &-\tfrac{1}{2}tr\{\mathbf{V}^{-1}\mathbf{Z}\sigma_u^2[-\mathbf{C}^{-1}(2\rho\mathbf{W}\mathbf{W}^T-2\mathbf{W})\mathbf{C}^{-1}]\mathbf{Z}^T\} - \\ &-\tfrac{1}{2}(\hat{\boldsymbol{\theta}}-\mathbf{X}\boldsymbol{\beta})^T(-\mathbf{V}^{-1}\mathbf{Z}\sigma_u^2[-\mathbf{C}^{-1}(2\rho\mathbf{W}\mathbf{W}^T-2\mathbf{W})\mathbf{C}^{-1}]\mathbf{Z}^T\mathbf{V}^{-1})(\hat{\boldsymbol{\theta}}-\mathbf{X}\boldsymbol{\beta}) \end{aligned} \tag{B-2}$$

with $\mathbf{C} = [(\mathbf{I}-\rho\mathbf{W})(\mathbf{I}-\rho\mathbf{W}^T)]$. The matrix of expected second derivatives of $-l(\boldsymbol{\beta},\sigma_u^2,\rho)$ with respect to σ_u^2 and ρ is given by

$$\mathcal{I}(\sigma_u^2,\rho) = \begin{bmatrix} \frac{1}{2}tr\{\mathbf{V}^{-1}\mathbf{Z}\mathbf{C}^{-1}\mathbf{Z}^T\mathbf{V}^{-1}\mathbf{Z}\mathbf{C}^{-1}\mathbf{Z}^T\} & \frac{1}{2}tr\{\mathbf{V}^{-1}\mathbf{Z}\mathbf{C}^{-1}\mathbf{Z}^T\mathbf{V}^{-1}\mathbf{Z}\mathbf{A}\mathbf{Z}^T\} \\ \frac{1}{2}tr\{\mathbf{V}^{-1}\mathbf{Z}\mathbf{A}\mathbf{Z}^T V^{-1}\mathbf{Z}\mathbf{C}^{-1}\mathbf{Z}^T\} & \frac{1}{2}tr\{\mathbf{V}^{-1}\mathbf{Z}\mathbf{A}\mathbf{Z}^T\mathbf{V}^{-1}\mathbf{Z}\mathbf{A}\mathbf{Z}^T\} \end{bmatrix} \tag{B-3}$$

with $\mathbf{A} = \sigma_u^2[-\mathbf{C}^{-1}(2\rho\mathbf{W}\mathbf{W}^T - 2\mathbf{W})\mathbf{C}^{-1}]$. The ML estimators $\hat{\sigma}^2_{u_{ML}}$ and $\hat{\rho}_{ML}$ can be obtained iteratively using the "scoring" algorithm:

$$\begin{bmatrix} \sigma_u^2 \\ \rho \end{bmatrix}^{(n+1)} = \begin{bmatrix} \sigma_u^2 \\ \rho \end{bmatrix}^{(n)} + [\mathcal{I}(\sigma_u^{2^{(n)}}, \rho^{(n)})]^{-1} \cdot s\left[\hat{\boldsymbol{\beta}}(\sigma_u^{2^{(n)}}, \rho^{(n)}), \sigma_u^{2^{(n)}}, \rho^{(n)}\right] \quad \text{(B-4)}$$

where n indicates the number of iteration.

C Appendix

The partial derivatives of the restricted log-likelihood function $l_R(\sigma_u^2, \rho)$ with respect to variance components are:

$$\begin{aligned} s_{R_{\sigma_u^2}}(\sigma_u^2, \rho) = \frac{\partial l_R}{\partial \sigma_u^2} = & -\tfrac{1}{2} tr\{\mathbf{PZC}^{-1}\mathbf{Z}^T\} + \tfrac{1}{2}\hat{\theta}^T \mathbf{PZC}^{-1}\mathbf{Z}^T\mathbf{P}\hat{\theta} \\ s_{R_\rho}(\sigma_u^2, \rho) = \frac{\partial l_R}{\partial \rho} = & -\tfrac{1}{2} tr\{\mathbf{PZ}\sigma_u^2[-\mathbf{C}^{-1}(2\rho\mathbf{W}\mathbf{W}^T - 2\mathbf{W})\mathbf{C}^{-1}]\mathbf{Z}^T\} + \\ & + \tfrac{1}{2}\hat{\boldsymbol{\theta}}^T \mathbf{PZ}\sigma_u^2[-\mathbf{C}^{-1}(2\rho\mathbf{W}\mathbf{W}^T - 2\mathbf{W})\mathbf{C}^{-1}]\mathbf{Z}^T\mathbf{P}\hat{\boldsymbol{\theta}} \end{aligned} \quad \text{(C-1)}$$

with $\mathbf{P} = \mathbf{V}^{-1} - \mathbf{V}^{-1}\mathbf{X}(\mathbf{X}^T\mathbf{V}^{-1}\mathbf{X})^{-1}\mathbf{X}^T\mathbf{V}^{-1}$ and $\mathbf{C} = [(\mathbf{I} - \rho\mathbf{W})(\mathbf{I} - \rho\mathbf{W}^T)]$.

The $\mathcal{I}_R(\sigma_u^2, \rho)$ matrix assumes the form:

$$\mathcal{I}_R(\sigma_u^2, \rho) = \begin{bmatrix} \tfrac{1}{2} tr\{\mathbf{PZC}^{-1}\mathbf{Z}^T\mathbf{PZC}^{-1}\mathbf{Z}^T\} & \tfrac{1}{2} tr\{\mathbf{PZC}^{-1}\mathbf{Z}^T\mathbf{PZAZ}^T\} \\ \tfrac{1}{2} tr\{\mathbf{PZAZ}^T\mathbf{PZC}^{-1}\mathbf{Z}^T\} & \tfrac{1}{2} tr\{\mathbf{PZAZ}^T\mathbf{PZAZ}^T\} \end{bmatrix} \quad \text{(C-2)}$$

with $\mathbf{A} = \sigma_u^2[-\mathbf{C}^{-1}(2\rho\mathbf{W}\mathbf{W}^T - 2\mathbf{W})\mathbf{C}^{-1}]$. Then the "Nelder-Mead" method and the "scoring" algorithm (B-4) are used and at convergence the REML estimators are obtained. The asymptotic covariance matrix of $\hat{\boldsymbol{\beta}}_R$, $\hat{\sigma}^2_{u_R}$ and $\hat{\rho}_R$ has a diagonal structure $diag\left[\bar{\mathbf{V}}(\hat{\boldsymbol{\beta}}_R), \bar{\mathbf{V}}(\hat{\sigma}^2_{u_R}, \hat{\rho}_R)\right] \approx diag\left[\bar{\mathbf{V}}(\hat{\boldsymbol{\beta}}_{ML}), \bar{\mathbf{V}}(\hat{\sigma}^2_{u_{ML}}, \hat{\rho}_{ML})\right]$ with

$$\bar{\mathbf{V}}(\hat{\boldsymbol{\beta}}_R) \approx \bar{\mathbf{V}}(\hat{\boldsymbol{\beta}}_{ML}) = (\mathbf{X}^T\mathbf{V}^{-1}\mathbf{X})^{-1}$$

$$\bar{\mathbf{V}}(\hat{\sigma}^2_{u_R}, \hat{\rho}_R) \approx \bar{\mathbf{V}}(\hat{\sigma}^2_{u_{ML}}, \hat{\rho}_{ML}) = \mathcal{I}^{-1}(\sigma_u^2, \rho). \quad \text{(C-3)}$$

D Appendix

If the ML procedure is used to estimate the variance components the term $\mathbf{b}^T_{ML}(\hat{\sigma}_u^2, \hat{\rho}) \bigtriangledown g_{1i}(\hat{\sigma}_u^2, \hat{\rho})$ of the MSE estimator of Spatial EBLUP is given by:

$$\bigtriangledown g_{1i}(\sigma_u^2, \rho) = \mathbf{b}_i^T \left\{ \begin{array}{l} (\mathbf{C}^{-1} - [\mathbf{C}^{-1}\mathbf{Z}^T\mathbf{V}^{-1}\mathbf{Z}\sigma_u^2\mathbf{C}^{-1} + \sigma_u^2\mathbf{C}^{-1}\mathbf{Z}^T(-\mathbf{V}^{-1}\mathbf{Z}\mathbf{C}^{-1}\mathbf{Z}^T\mathbf{V}^{-1})\mathbf{Z}\sigma_u^2\mathbf{C}^{-1} + \\ (\mathbf{A} - [\mathbf{A}\mathbf{Z}^T\mathbf{V}^{-1}\mathbf{Z}\sigma_u^2\mathbf{C}^{-1} + \sigma_u^2\mathbf{C}^{-1}\mathbf{Z}^T(-\mathbf{V}^{-1}\mathbf{Z}\mathbf{A}\mathbf{Z}^T\mathbf{V}^{-1})\mathbf{Z}\sigma_u^2\mathbf{C}^{-1} + \\ \quad + \sigma_u^2\mathbf{C}^{-1}\mathbf{Z}^T\mathbf{V}^{-1}\mathbf{Z}\mathbf{C}^{-1}]) \\ \quad + \sigma_u^2\mathbf{C}^{-1}\mathbf{Z}^T\mathbf{V}^{-1}\mathbf{Z}\mathbf{A}]) \end{array} \right\} \mathbf{b}_i \quad \text{(D-1)}$$

and

$$\mathbf{b}_{ML}^T(\sigma_u^2,\rho) = \frac{1}{2m}\left\{\mathcal{I}^{-1}(\sigma_u^2,\rho)\begin{bmatrix} tr[(\mathbf{X}^T\mathbf{V}^{-1}\mathbf{X})^{-1}\mathbf{X}^T(-\mathbf{V}^{-1}\mathbf{Z}\mathbf{C}^{-1}\mathbf{Z}^T\mathbf{V}^{-1})\mathbf{X}] \\ tr[(\mathbf{X}^T\mathbf{V}^{-1}\mathbf{X})^{-1}\mathbf{X}^T(-\mathbf{V}^{-1}\mathbf{Z}\mathbf{A}\mathbf{Z}^T\mathbf{V}^{-1})\mathbf{X}] \end{bmatrix}\right\} \tag{D-2}$$

where $\mathcal{I}(\sigma_u^2,\rho)$ is given by (B-3), $\mathbf{C} = [(\mathbf{I}-\rho\mathbf{W})(\mathbf{I}-\rho\mathbf{W}^T)]$ and $\mathbf{A} = \sigma_u^2[-\mathbf{C}^{-1}(2\rho\mathbf{W}\mathbf{W}^T - 2\mathbf{W})\mathbf{C}^{-1}]$.

E Appendix

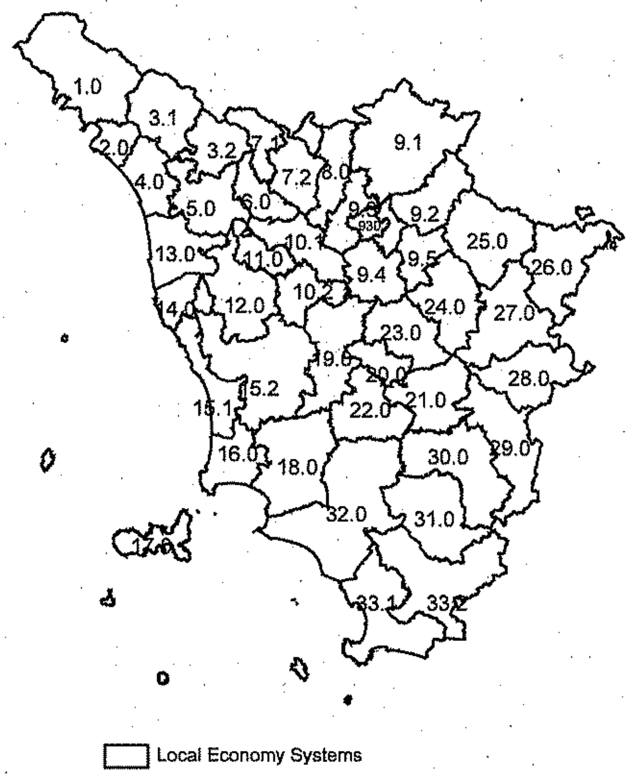

Figure E-1: Map of the LES of Tuscany.

LES code	*LES description*	Sample size
1	Lunigiana	44
2	Area di Massa-Carrara	144
3.1	Valle del Serchio - Quadrante Garfagnana	24
3.2	Valle del Serchio - Quadrante Media Valle	23
4	Versilia	99
5	Area Lucchese	113
6	Val di Nievole	109
7.1	Area Pistoiese - Quadrante Montano	–
7.2	Area Pistoiese - Quadrante Metropolitano	46
8	Area Pratese	117
9.1	Area Fiorentina - Quadrante Mugello	13
9.2	Area Fiorentina - Quadrante Val di sieve	10
9.3	Area Fiorentina - Quadrante Centrale	173
9.4	Area Fiorentina - Quadrante Chianti	76
9.5	Area Fiorentina - Quadrante Valdarno Superiore Nord	34
10.1	Circondario di Empoli - Quadrante empolese	81
10.2	Circondario di Empoli - Quadrante Valdelsano	26
11	Valdarno Inferiore	57
12	Val d'Era	61
13	Area Pisana	104
14	Area Livornese	69
15.1	Val di Cecina - Quadrante Costiero	35
15.2	Val di Cecina - Quadrante Interno	51
16	Val di Cornia	75
17	Arcipelago	–
18	Colline Metallifere	29
19	Alta Val d'Elsa	25
20	Area Urbana	47
21	Crete Senesi - Val d'Arbia	19
22	Val di Merse	16
23	Chianti	10
24	Valdarno Superiore Sud	43
25	Casentino	10
26	Alta Val Tiberina	56
27	Area Aretina	115
28	Val di Chiana Aretina	53
29	Val di Chiana Senese	53
30	Amiata - Val d'Orcia	50
31	Amiata Grossetano	–
32	Area Grossetana	108
33.1	Albegna-Fiora - Quadrante Costa d'Argento	39
33.2	Albegna-Fiora - Quadrante Colline Interne	45
930	Firenze	310

Table E-1: LES code and sample size, Tuscany.

LES code	Direct	*s.e.*	$\tilde{\theta}_{ML}$	*s.e.*	$\tilde{\theta}^S_{ML}$	*s.e.*	$\tilde{\theta}_{REML}$	*s.e.*	$\tilde{\theta}^S_{REML}$	*s.e.*
1	10167.49	1371.97	11194.05	784.90	10401.01	727.30	11088.30	874.83	10358.63	740.87
2	9938.32	741.38	10999.28	653.15	10521.63	570.22	10800.37	672.06	10476.05	587.80
3.1	8580.21	1596.80	11157.80	639.30	10751.14	658.30	10982.65	772.89	10679.80	680.44
3.2	8677.20	1763.12	11630.66	697.54	10910.04	532.23	11445.09	825.89	10862.19	545.51
4	11774.57	1201.40	11721.31	686.41	11174.40	513.67	11759.04	784.71	11147.86	525.35
5	11691.50	1084.32	11892.84	654.66	12231.04	654.54	11881.41	745.84	12192.25	665.72
6	9531.66	844.43	10393.04	651.79	10098.32	460.57	10247.00	699.45	10081.44	482.46
7.1	–	2965.51	12794.53	931.26	11885.00	858.21	12761.70	1054.76	11870.78	875.73
7.2	11899.10	1604.70	11463.95	644.70	10808.12	440.63	11502.15	777.89	10807.42	460.66
8	11074.33	952.47	11342.46	711.76	11071.00	554.07	11309.37	765.49	11077.76	569.56
9.1	11532.61	2904.06	10925.62	629.46	10491.21	511.18	10956.77	785.63	10507.96	526.30
9.2	10600.48	3012.66	11255.15	621.97	11278.87	489.77	11250.21	778.34	11297.28	508.10
9.3	12063.35	808.79	11126.42	655.04	11888.74	673.38	11316.72	692.90	11935.68	698.21
9.4	13646.16	1406.75	11529.23	663.67	12030.66	464.99	11720.52	782.79	12055.17	480.46
9.5	12587.11	1980.34	11651.35	684.18	11887.41	541.26	11713.98	821.66	11908.45	557.89
10.1	12240.36	1240.74	10774.84	728.00	11273.61	597.26	10920.52	820.08	11277.48	608.75
10.2	12223.22	2284.77	11871.42	751.60	11967.64	565.73	11875.97	883.94	11963.19	579.75
11	11579.70	1359.23	11303.17	794.76	10412.23	699.70	11324.50	884.20	10394.24	711.21
12	10859.71	1309.80	11446.04	695.80	11427.90	555.89	11391.23	800.66	11415.30	577.69
13	13550.35	1297.51	11351.03	696.69	10998.65	535.33	11593.90	802.09	11008.13	552.30
14	10067.99	1079.06	11095.47	678.09	11280.68	521.56	10983.15	762.68	11294.25	547.67
15.1	15723.00	2433.36	12165.74	627.56	11864.43	470.08	12310.13	781.14	11882.62	497.38
15.2	13007.86	1701.97	12192.50	695.15	13218.54	808.78	12237.68	821.51	13228.62	840.18
16	12022.66	1239.00	11813.52	739.17	11518.08	593.19	11821.06	826.19	11520.63	610.79
17	–	2189.14	12980.03	953.44	12416.99	825.73	13068.16	1070.27	12446.20	834.69
18	12330.66	2119.80	11678.91	676.39	12227.29	522.62	11713.24	815.87	12242.85	538.63
19	10343.28	1805.38	11255.14	654.44	12307.23	569.67	11210.87	792.45	12321.36	590.05
20	17720.15	2288.80	13869.08	802.97	14635.55	684.26	14044.72	939.09	14667.00	698.69
21	12075.41	2410.07	11256.83	634.03	12693.66	788.65	11280.44	783.90	12719.25	815.08
22	10856.30	2450.98	10915.69	637.81	11624.14	541.89	10914.21	788.28	11636.61	555.98
23	18190.08	5459.31	10732.53	658.94	11081.40	499.82	10813.49	817.27	11112.07	518.38
24	10323.60	1436.21	11467.94	670.54	12301.43	570.10	11378.02	790.09	12319.25	590.97
25	11039.84	3131.57	11300.76	634.32	11726.02	584.95	11292.95	788.21	11746.65	612.10
26	11047.62	1334.14	11456.81	642.56	11026.63	502.94	11420.29	760.86	11036.50	519.03
27	12227.06	1034.10	10997.84	661.85	11138.63	596.81	11178.65	744.29	11170.25	624.97
28	9434.48	1168.40	10279.17	671.37	10081.14	552.79	10188.58	770.02	10093.99	569.25
29	11829.12	1462.18	11163.48	673.90	10960.02	508.93	11214.72	793.17	10971.58	524.29
30	11747.44	1471.12	11412.04	756.42	11577.69	683.55	11419.90	858.61	11579.27	694.58
31	–	2672.13	11880.27	1005.91	11049.77	901.01	11842.08	1116.03	11043.88	908.81
32	12559.21	1090.79	11763.56	694.62	12277.11	870.67	11893.28	776.97	12286.07	892.79
33.1	9951.94	1438.63	11056.24	765.47	10893.45	659.17	11002.23	869.14	10901.17	672.61
33.2	9802.54	1329.82	10657.24	848.40	10423.29	797.73	10560.67	924.06	10414.03	811.54
930	15144.66	784.65	15097.81	832.88	15120.78	727.18	15131.13	824.09	15135.77	731.30

Table E-2: Direct, EBLUP and Spatial EBLUP estimates for annual per-capita mean income for each LES of the Tuscany region.

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Anno: **1997**

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Anno: **1998**

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Anno: **1999**

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Anno: **2000**

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Anno: **2001**

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Anno: **2002**

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Anno: **2003**

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Anno: **2004**

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Anno: **2005**

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