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# Pseudomonotonicity of an affine map and the two dimensional case.

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# Pseudomonotonicity of an affine map and the two dimensional case.

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**Abstract**

In this paper the pseudomonotonicity of the affine map $F(x) = Mx + q$ on the interior of the positive orthant of $\Re^n$ is studied. A new characterization is suggested involving the positive and the negative polar of the cone generated by the open cone $W^* = \{z = Mx+q,\ x \in int\Re^n_+\}$. The obtained results are applied to the two dimensional case in order to achieve a complete characterization of pseudomonotonicity in terms of the coefficients of $M$ and $q$.

**KeyWords** Pseudomonotonicity, pseudoconvexity.

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## 1 Introduction

Monotonicity and generalized monotonicity arise in complementarity problems, variational inequalities and more generally in equilibrium models [[8], [11], [13]]. In particular, pseudomonotonicity of an affine map is of particular interest for its relationships with pseudoconvexity and with the linear complementarity problem [[2], [7]]. This subject has been studied by several authors (see for instance [[5], [9], [10]] , and some sufficient and/or

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necessary conditions are given ([5]) but, unfortunately, the obtained results are not very useful in constructing or in testing pseudomonotonicity in a easily way.

The aim of this paper is to move a step in this direction; more precisely we suggest a new approach to the problem which allows us to find a new characterization of the pseudomonotonicity of an affine map on the interior of the positive orthant. By means of the obtained results, we are able to give a complete characterization, in the two dimensional case, in terms of the elements of the matrix $M$ and of the elements of the vector $q$ associated to the affine map $F(x) = Mx + q$.

In Section 2 and in Section 3 we establish a new characterization of pseudomonotonicity of the affine map $F(x) = Mx+q$ on the interior of the positive orthant, involving the positive and the negative polar of the cone generated by the set $W^* = \{z = Mx + q,\ x \in int\Re^n_+\}$.

In Section 4 we characterize the pseudomonotonicity of a bijective linear map $Mx$ in terms of the elements of the $2 \times 2$ matrix $M$; we find again, as a particular case, some results given in [Pini]. In Section 5 we extend the obtained results to a bijective affine map. At last in Section 6 we analyze the case where $M$ is a singular matrix.

## 2 Preliminary results

Let $H \subset \Re^n$ be a nonempty open convex set. In our approach to pseudomonotonicity we are interested to characterize the set $Z_H = \{z \in \Re^n : \exists a \in H, z^T a = 0\}$. With this aim, let $C_H$ be the open cone generated by $H$ that is $C_H = \{ta, t > 0, a \in H\}$.

The next theorem characterizes $Z_H$ by means of the polar cones

$C_H^+ = \{\alpha \in \Re^n : \alpha^T c \geq 0, \forall c \in C_H\}$ and $C_H^- = \{\alpha \in \Re^n : \alpha^T c \leq 0, \forall c \in C_H\}$.

More exactly, denoting with $clS$ and with $\partial S$ the closure and the boundary of the set $S$, respectively, we have the following theorems.

**Theorem 2.1**

$$Z_H = \{z \in \Re^n : \exists a \in H, z^T a = 0\} = (C_H^+ \cup C_H^-)^c. \tag{2.1}$$

*Proof* First of all we recall the following known properties regarding an open cone $C$ with vertex at the origin:

$c \in intC$ if and only if $\alpha^T c > 0$, $\forall \alpha \in C^+$; $c \in clC$ if and only if $\alpha^T c \geq 0$, $\forall \alpha \in C^+$.

In order to prove (2.1), let us note that $Z_H = \{z \in \Re^n : \exists c \in C_H, z^T c = 0\}$.

Let $z \in Z_H$ and assume $z \in C_H^+$. Since there exists $c \in C_H$ such that $z^T c = 0$, necessarily we have $c \in \partial C_H$ and this absurd since $C_H$ is an open set so that $\partial C_H = \emptyset$. Consequently $z \notin C_H^+$. In a similar way it can be proven $z \notin C_H^-$ and thus $z \in (C_H^+ \cup C_H^-)^c$.

Viceversa, let $z \in (C_H^+ \cup C_H^-)^c$, that is $z \notin C_H^+ \cup C_H^-$. Then there exist $c_1 \in C_H$, $c_2 \in C_H$ such that $z^T c_1 < 0, z^T c_2 > 0$. The continuity of the scalar product and the convexity of $C_H$ implies the existence of $c = \lambda c_1 + (1-\lambda) c_2, \lambda \in (0,1)$ with $c \in C_H$ and $z^T c = 0$. Consequently $z \in Z_H$ and the thesis is achieved. □

**Theorem 2.2**

*Let $W$ be an open convex cone with vertex at the origin and set $W^* = x_0 + W$. Then*

$$Z_W \subseteq Z_{W^*}. \tag{2.2}$$

*Proof* First of all we prove that $W = C_W \subseteq clC_{W^*}$. Let $w \in W$; we have $x_0 + nw \in x_0 + W$ for every positive integer $n$. Consequently $\frac{1}{n}(x_0 + nw) = \frac{1}{n}x_0 + w \in C_{W^*}$ and thus $w$ is an accumulation point for $C_{W^*}$ that is $w \in clC_{W^*}$.

It follows $(clC_{W^*})^+ = (C_{W^*})^+ \subseteq C_W^+$, $(clC_{W^*})^- = (C_{W^*})^- \subseteq C_W^-$ so that

$Z_{W^*} = (C_{W^*}^+ \cup C_{W^*}^-)^c \supseteq (C_W^+ \cup C_W^-)^c = Z_W$. □

Consider now the following two special cases:

$W = \{Mx, x \in int\Re_+^n\}$; $W^* = \{Mx + q, x \in int\Re_+^n\}$

where $M$ is a non singular matrix of order $n$.

Let us note that $W$ is the open polyhedral cone generated by the columns of the matrix $M$ while $W^* = W + q$. The following theorem characterizes the set $Z_W$.

**Theorem 2.3** *Consider the set $Z_W$. Then*

$$Z_W = \{z = (M^{-1})^T y, \quad y \notin \Re_+^n \cup \Re_-^n\}.$$

*Proof* Obviously we have $C_W = W$. It is known that the positive polar cone of $W$ is given by $W^+ = \{z : M^T z \geq 0\}$. Setting $M^T z = y$ we have $W^+ = \{z = (M^T)^{-1} y =$

$(M^{-1})^T y,\ y \geq 0\}$ and furthermore $W^- = \{z = (M^{-1})^T y,\ y \leq 0\}$. The thesis follows from Theorem 2.1. □

Regarding $W^* = W + q$, we have the following theorem:

**Theorem 2.4** *We have* $Z_{W^*} = (C^+_{W^*} \cup C^-_{W^*})^c$. *In particular:*

*i) if* $q \in W$ *then* $Z_{W^*} = Z_W$;

*ii) If* $-q \in W$ *then* $Z_{W^*} = \Re^n \setminus \{0\}$,

*Proof* The first statement follows from Theorem 2.1 setting $H = W^*$.
Statement $i$) follows by noting that $q \in W$ implies $C_{W^*} = C_W$.
Statement $ii$) follows by noting that $-q \in W$ implies $0 \in intC_{W^*}$. □

## 3 A new characterization of pseudomonotonicity of a bijective affine map on $int\Re^n_+$

In this section we will apply the results given in Section 2 in order to characterize the pseudomonotonicity of an affine map on $int\Re^n_+$.

We recall that a map $F : S \subseteq \Re^n \rightarrow \Re^n$ is said to be **pseudomonotone** on $S$ if for every $x, y \in S$ we have:

$$(y - x)^T F(x) \geq 0 \text{ implies } (y - x)^T F(y) \geq 0$$

or equivalently (see [1])

$$(y - x)^T F(x) > 0 \text{ implies } (y - x)^T F(y) > 0.$$

A very usefull characterization of pseudomonotonicity is given by Crouzeix in [10]; in the case $F(x) = Mx + q$ where $M$ is a matrix of order $n$ and $S = int\Re^n_+$ such a characterization assumes the form given in the following theorem.

**Theorem 3.1** $F(x) = Mx + q$ *is pseudomonotone on* $int\Re^n_+$ *if and only if for all* $x \in int\Re^n_+$ *and* $v \in \Re^n$,

$$v^T(Mx + q) = 0 \Longrightarrow v^T Mv \geq 0. \tag{3.3}$$

Let us note that (3.3) is obviously verified when $B = (M + M^T)/2$ is positive semidefinite, so that the main problem in characterizing the pseudomonotonicity of an affine map is related to matrices M for which $B$ is not positive semidefinite.
We say that the affine map $F$ is **merely pseudomonotone** if it is pseudomonotone and $B$ is not positive semidefinite.
The following theorem gives a new formulation of pseudomonotonicity.

**Theorem 3.2** *$F(x) = Mx + q$ is pseudomonotone on $int\Re^n_+$ if and only if*

$$v^T M v \geq 0 \quad \forall\, v \in Z_{W^*}. \tag{3.4}$$

*Proof* Taking into account Theorem 2.4, it is sufficient to note that the set of all elements $v$ such that $v^T(Mx + q) = 0$ is equal to $Z_{W^*}$. □

**Corollary 3.1** *The linear map $F(x) = Mx$ is pseudomonotone on $int\Re^n_+$ if and only if*

$$y^T (M^{-1})^T y \geq 0 \quad \forall\, y \notin \Re^n_+ \cup \Re^n_-. \tag{3.5}$$

*Proof* We have $Z_{W^*} = Z_W = \{z = (M^{-1})^T y, \ \ y \notin \Re^n_+ \cup \Re^n_-\}$ (see Theorem 2.3). The thesis is achieved taking into account that (3.3) becomes $v^T M v = y^T M^{-1} M (M^{-1})^T y = y^T (M^{-1})^T y$. □
A necessary condition for the affine map $F(x) = Mx + q$ to be pseudomonotone on $int\Re^n_+$ is given in the following theorem.

**Theorem 3.3** *If the affine map $F(x) = Mx + q$ is pseudomonotone on $int\Re^n_+$, then the linear map $G(x) = Mx$ is pseudomonotone on $int\Re^n_+$.*

*Proof* It follows immediately from Theorem 2.2 and Theorem 3.2. □

As an application of the new characterization of pseudomonotonicity given in Theorem 3.2 and in Corollary 3.1, in the next two sections we express the pseudomonotonicity of a bijective map on $int\Re^2_+$ in terms of the elements of the $2 \times 2$ matrix $M$ in the linear case and in the affine case, respectively.

# 4 Pseudomonotonicity of a bijective linear map on $int\Re^2_+$

Consider the linear map $F(x) = Mx$ on the interior of the positive orthant, where $M$ is a $2 \times 2$ non singular matrix. In such a case, Corollary 3.1 assumes the following simple form:

$F(x) = Mx$ is pseudomonotone on $int\Re^2_+$ if and only if (4.6) holds:

$$y^T M^{-1} y \geq 0 \ \ \forall y = (y_1, y_2) \ \ such \ \ that \ \ y_1 y_2 < 0 \tag{4.6}$$

By means of (4.6) we are able to prove the following theorem.

**Theorem 4.1** *Let* $M = \begin{bmatrix} a & c \\ b & d \end{bmatrix}$ *be a nonsingular matrix. The linear map* $F(x) = Mx$ *is pseudomonotone on* $int\Re^2_+$ *if and only if one of the following conditions holds:*

$$\frac{a}{detM} \geq 0, \ \ \frac{d}{detM} \geq 0, \ \ \Delta = (b+c)^2 - 4ad \leq 0 \tag{4.7}$$

$$\frac{a}{detM} \geq 0, \ \ \frac{d}{detM} \geq 0, \ \ \frac{b+c}{detM} \geq 0, \ \ \Delta = (b+c)^2 - 4ad > 0. \tag{4.8}$$

*Proof* From (4.6), taking into account that $M^{-1} = \frac{1}{detM} \begin{bmatrix} d & -c \\ -b & a \end{bmatrix}$, $F(x)$ is pseudomonotone if and only if (4.9) holds:

$$\Psi(y_1, y_2) = \frac{d}{detM} y_1^2 - \frac{b+c}{detM} y_1 y_2 + \frac{a}{detM} y_2^2 \geq 0, \ \ y_1 y_2 < 0. \tag{4.9}$$

Let us note that $\Psi(y_1, y_2) \geq 0 \ \ \forall y_1, y_2 \in \Re$ if and only if the matrix associated to the quadratic form is positive semidefinite that is if and only if (4.7) holds.

Consider now the case $\Delta = (b+c)^2 - 4ad > 0$. Setting $m = \frac{y_1}{y_2}$, (4.9) is equivalent to

$$\Psi(m) = \frac{d}{detM} m^2 - \frac{b+c}{detM} m + \frac{a}{detM} \geq 0, \forall m < 0. \tag{4.10}$$

Assume the pseudomonotonicity of $F(x)$; the validity of (4.10) implies $\lim_{m \to -\infty} \Psi(m) \geq 0$ that is $\frac{d}{detM} \geq 0$ and $\lim_{m \to 0^-} \Psi(m) = \frac{a}{detM} \geq 0$. The assumption $\Delta > 0$ together with

(4.10) implies that $\Psi(m)$ must be two non negative roots that is $\frac{b+c}{detM} \geq 0$. Viceversa if (4.8) holds then $\Psi(m)$ has two nonnegative roots so that (4.10) is verified. $\square$

**Corollary 4.1** *Let* $M = \begin{bmatrix} a & c \\ b & d \end{bmatrix}$ *be a nonsingular matrix and consider the linear map* $F(x) = Mx$. *Then i) and ii) hold.*

*i)* $F(x)$ *is merely pseudomonotone on* $int\Re^2_+$ *if and only if (4.11) or (4.12) holds.*

$$detM > 0,\ a \geq 0,\ d \geq 0,\ b+c \geq 0,\ \Delta = (b+c)^2 - 4ad > 0 \tag{4.11}$$

$$detM < 0,\ a \leq 0,\ d \leq 0,\ b+c \leq 0. \tag{4.12}$$

*ii)* $F(x)$ *is pseudomonotone (not merely) on* $int\Re^2_+$ *if and only if (4.13) holds.*

$$a \geq 0,\ d \geq 0,\ \Delta = (b+c)^2 - 4ad \leq 0. \tag{4.13}$$

*Proof* It is sufficient to note that $detM < 0$ implies $\Delta = (b+c)^2 - 4ad > 0$. In fact $(b+c)^2 - 4ad \leq 0$, $ad - bc < 0$ implies $(b+c)^2 < ad < bc$ that is $(b-c)^2 < 0$ and this is absurd. $\square$

**Example 4.1** *The following matrices are associated to merely pseudomonotone linear maps:*

$M = \begin{bmatrix} 0 & \alpha \\ \beta & \gamma \end{bmatrix}$, $\alpha\beta < 0,\ \alpha+\beta \geq 0,\ \gamma \geq 0$

$M = \begin{bmatrix} 0 & \alpha \\ \beta & \gamma \end{bmatrix}$, $\alpha\beta > 0,\ \alpha+\beta \leq 0,\ \gamma \leq 0$

$M = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$; $M = \begin{bmatrix} -1 & -2 \\ -1 & -1 \end{bmatrix}$.

# 5 Pseudomonotonicity of a bijective affine map on $int\Re^2_+$

Consider the affine map $F(x) = Mx + q$ where $M$ is a $2 \times 2$ non singular matrix; by means of Theorem 3.2 we are able to characterize the pseudomonotonicity of $F(x)$ on $int\Re^2_+$.

First of all we must characterize the set $Z_{W^*}$. Since $M$ is nonsingular, $q$ is a linear combination of the columns $m^{(1)}, m^{(2)}$ of $M$, that is there exist $\alpha, \beta \in \Re$ such that $q = \alpha m^{(1)} + \beta m^{(2)}$. In what follows we will use the following notations:

- $(m^{(1)})^\perp$, $(m^{(2)})^\perp$ are the orthogonal space of $m^{(1)}, m^{(2)}$, respectively;
- $M_1$ is the matrix having $q, m^{(1)}$ as columns;
- $M_2$ is the matrix having $q, m^{(2)}$ as columns.

We have the following theorem.

**Theorem 5.1** *It results:*

*(i) If* $\alpha \geq 0,\ \beta \geq 0$ *then* $Z_{W^*} = Z_W$.

*(ii) If* $\alpha < 0, \beta < 0$ *then* $Z_{W^*} = \Re^2$.

*(iii) If* $\alpha < 0, \beta = 0$ *then* $Z_{W^*} = \Re^2 \setminus (m^{(1)})^\perp$.

*(iv) If* $\alpha = 0, \beta < 0$ *then* $Z_{W^*} = \Re^2 \setminus (m^{(2)})^\perp$.

*(v) If* $\alpha > 0, \beta < 0$ *then* $Z_{W^*} = \{v = (M_2^{-1})^T y,\ \ y_1 y_2 < 0\}$.

*(vi) If* $\alpha < 0, \beta > 0$ *then* $Z_{W^*} = \{v = (M_1^{-1})^T y,\ \ y_1 y_2 < 0\}$.

*Proof* $(i)$ It follows by noting that $W^* = q + W \subset W$.
$(ii)$ It follows by noting that $0 \in intW^*$, so that $C_{W^*} = Z_{W^*} = \Re^2$.
$(iii)$ We have $C_{W^*} = \{z = t(x_1 + \alpha)m^{(1)} + tx_2 m^{(2)}, t > 0, x_1 > 0, x_2 > 0, \alpha < 0\}$. Set $\lambda = t(x_1 + \alpha)$ and $\mu = tx_2$; choosing $\epsilon > 0$ such that $x_1 = -\alpha \pm \epsilon > 0$ and $t = \frac{z}{\epsilon},\ z > 0$, we obtain $C_{W^*} = \{\lambda m^{(1)} + \mu m^{(2)}, \lambda \in \Re, \mu > 0\}$. It follows $Z_{W^*} = ((C_{W^*})^+ \cup (C_{W^*})^-)^c = \Re^2 \setminus (m^{(1)})^\perp$.
$(iv)$ The proof is similar to the one given in iii).
$(v)$ We have $C_{W^*} = \{z = t(x_1+\alpha)m^{(1)} + t(x_2+\beta)m^{(2)}, t > 0, x_1 > 0, x_2 > 0, \alpha > 0, \beta < 0\}$. Set $\lambda = t \cdot \frac{x_1+\alpha}{\alpha} > 0$ and $\mu = t \cdot \frac{\alpha x_2 - \beta x_1}{\alpha} > 0$; by simple calculations we obtain $C_{W^*} = \{z = t(x_1 + \alpha)m^{(1)} + t(x_2 + \beta)m^{(2)} = \alpha\lambda m^{(1)} + (\mu + \beta\lambda)m^{(2)} = \lambda q + \mu m^{(2)}, \lambda > 0, \mu > 0\}$. In other words, $C_{W^*}$ is the open convex cone generated by the columns of the matrix $M_2$, so that the thesis follows from Theorem 2.3 applied to the matrix $M_2$.

$(vi)$ The proof is similar to the one given in $v$). $\square$

Now we are able to characterize the pseudomonotonicity of an affine map on the interior of the positive orthant.

**Theorem 5.2** *Let* $M = \begin{bmatrix} a & c \\ b & d \end{bmatrix}$ *be a nonsingular matrix and consider the affine map* $F(x) = Mx + q$. *Set* $(\alpha, \beta)^T = M^{-1}q$. *Then* $F(x)$ *is merely pseudomonotone on* $int\Re^2_+$ *if and only if one of the following cases is verified.*

*i)* $\alpha \geq 0,\ \beta \geq 0$ *and one of the following conditions holds:*

$$detM > 0,\ a \geq 0,\ d \geq 0,\ b + c \geq 0,\ \Delta = (b+c)^2 - 4ad > 0 \tag{5.14}$$

$$detM < 0,\ a \leq 0,\ d \leq 0,\ b + c \leq 0,\ \Delta = (b+c)^2 - 4ad > 0 \tag{5.15}$$

*ii)* $\alpha > 0,\ \beta < 0$ *and one of the following conditions holds:*

$$detM > 0,\ a > 0,\ d = 0,\ b + c > 0,\ \ -\frac{\beta}{\alpha} \leq \frac{a}{b+c} \tag{5.16}$$

$$detM > 0,\ a > 0,\ d > 0,\ b + c > 0,\ \Delta = (b+c)^2 - 4ad > 0,\ \ -\frac{\beta}{\alpha} \leq \frac{b+c-\sqrt{\Delta}}{2d} \tag{5.17}$$

$$detM < 0,\ a < 0,\ d = 0,\ b + c < 0,\ \ -\frac{\beta}{\alpha} \leq \frac{a}{b+c}. \tag{5.18}$$

$$detM < 0,\ a < 0,\ d < 0,\ b + c < 0,\ \Delta = (b+c)^2 - 4ad > 0,\ \ -\frac{\beta}{\alpha} \leq \frac{b+c+\sqrt{\Delta}}{2d}. \tag{5.19}$$

*iii)* $\alpha < 0,\ \beta > 0$ *and one of the following conditions holds:*

$$detM > 0,\ a = 0,\ d > 0,\ b + c > 0,\ \ -\frac{\alpha}{\beta} \leq \frac{d}{b+c} \tag{5.20}$$

$$detM > 0,\ a > 0,\ d > 0,\ b + c > 0,\ \Delta = (b+c)^2 - 4ad > 0,\ \ -\frac{\alpha}{\beta} \leq \frac{b+c-\sqrt{\Delta}}{2a} \tag{5.21}$$

$$detM < 0,\ a = 0,\ d < 0,\ b + c < 0,\ \ -\frac{\alpha}{\beta} \leq \frac{d}{b+c}. \tag{5.22}$$

$$detM < 0,\ a < 0,\ d < 0,\ b + c < 0,\ \Delta = (b+c)^2 - 4ad > 0,\ \ -\frac{\alpha}{\beta} \leq \frac{b+c+\sqrt{\Delta}}{2a}. \tag{5.23}$$

*Furthermore,* $F(x)$ *is pseudomonotone (not merely) on* $int\Re^2_+$ *if and only if (5.24) holds.*

$$a \geq 0,\ d \geq 0,\ \Delta = (b+c)^2 - 4ad \leq 0. \tag{5.24}$$

*Proof* From Theorem 3.2 the affine map $F(x)$ is pseudomonotone on $int\Re^2_+$ if and only if $v^T M v \geq 0 \ \ \forall v \in Z_{W^*}$.

• If $\alpha \geq 0, \ \beta \geq 0$, from i) of Theorem 5.1 we have $Z_{W^*} = Z_W$ so that the pseudomonotonicity of $F(x)$ reduces to the pseudomonotonicity of the linear map $G(x) = Mx$ which is equivalent to (5.14), (5.15) (see Corollary 4.1).

• If $\alpha < 0, \ \beta < 0$, from ii) of Theorem 5.1 we have $Z_{W^*} = \Re^2$ so that $F(x)$ is pseudomonotone if and only if $\frac{M+M^T}{2}$ is positive semidefinite, that is if and only if (5.24) holds.

• If $\alpha < 0, \ \beta = 0$ or $\alpha = 0, \ \beta < 0$, from iii), iv) of Theorem 5.1 $Z_{W^*}$ is the union of two open halfspaces so that $F(x)$ is pseudomonotone if and only if $\frac{M+M^T}{2}$ is positive semidefinite, that is if and only if (5.24) holds.

• If $\alpha > 0, \ \beta < 0$, taking into account v) of Theorem 5.1, $F(x)$ is pseudomonotone if and only if (5.25) holds:

$$y^T M_2^{-1} M (M_2^{-1})^T y \geq 0, \quad \forall \ y = (y_1, y_2)^T \ \ with \ \ y_1 y_2 < 0. \tag{5.25}$$

Consequently (see (4.6)), the pseudomonotonicity of $F(x)$ is equivalent to the pseudomonotonicity of the linear map $H(x) = M_2^* x$ where $M_2^* = [M_2^{-1} M (M_2^{-1})^T]^{-1} = M_2^T M^{-1} (M_2)$.

Setting $M_2 = \begin{bmatrix} \alpha a + \beta c & c \\ \alpha b + \beta d & d \end{bmatrix}$, by simple calculations we have

$M_2^* = \begin{bmatrix} a\alpha^2 + (b+c)\alpha\beta + d\beta^2 & \alpha b + \beta d \\ \alpha c + \beta d & d \end{bmatrix}$.

From Corollary 4.1, taking into account that $detM_2^* = \alpha^2 detM$ and $\Delta^* = ((b+c)\alpha + 2\beta \cdot d)^2 - 4d(a\alpha^2 + (b+c)\alpha\beta + d\beta^2) = \alpha^2[(b+c)^2 - 4ad] = \alpha^2 \Delta$, $F(x)$ is merely pseudomonotone if and only if (5.26) or (5.27) holds:

$$detM > 0, \ \Delta > 0, \ \ d \geq 0, \ \ (b+c)\alpha + 2\beta d \geq 0, \ \ a\alpha^2 + (b+c)\alpha\beta + d\beta^2 \geq 0 \tag{5.26}$$

$$detM < 0, \ d \leq 0, \ \ (b+c)\alpha + 2\beta d \leq 0, \ \ a\alpha^2 + (b+c)\alpha\beta + d\beta^2 \leq 0. \tag{5.27}$$

Consider the case $detM > 0$. Taking into account the necessary pseudomonotonicity condition stated in Theorem 3.3, $F(x)$ is merely pseudomonotone if and only if the conditions

$detM > 0, a \geq 0,\ d \geq 0,\ b+c \geq 0,\ \Delta > 0$ are verified and furthermore the following two inequalities hold:

$$2d\,\frac{\beta}{\alpha} + b + c \geq 0 \tag{5.28}$$

$$d\,(\frac{\beta}{\alpha})^2 + (b+c)\,\frac{\beta}{\alpha} + a \geq 0 \tag{5.29}$$

Let us note that the case $a = 0$ cannot occur. In fact if $a = 0$ and $d = 0$, the conditions $\Delta > 0$ and $b + c \geq 0$ imply $b + c > 0$ so that (5.29) is not verified since $\beta < 0$; if $a = 0$ and $d > 0$, (5.29) is verified for $\frac{\beta}{\alpha} \geq 0$ (and this is a contradiction) or for $\frac{\beta}{\alpha} \leq \frac{-(b+c)}{d}$ while (5.28) is verified for $\frac{\beta}{\alpha} \geq \frac{-(b+c)}{2d} > \frac{-(b+c)}{d}$ and this is a contradiction.

Consider the case $a > 0$ and $d = 0$; since $\Delta > 0$ we have $b + c > 0$ so that (5.28) is satisfied. Condition (5.29) is verified if and only if $-\frac{\beta}{\alpha} \leq \frac{a}{b+c}$, that is if and only if (5.16) holds.

When $a > 0$ and $d > 0$, we have $\gamma_1 < \gamma_2 < 0$ where $\gamma_1 = \frac{-(b+c)-\sqrt{\Delta}}{2d}$, $\gamma_2 = \frac{-(b+c)+\sqrt{\Delta}}{2d}$. Condition (5.28) is verified if and only if $\frac{\beta}{\alpha} \geq \frac{-(b+c)}{2d}$ while (5.29) is verified if and only if $\frac{\beta}{\alpha} \geq \gamma_2$ or $\frac{\beta}{\alpha} \leq \gamma_1$. Consequently (5.28) and (5.29) hold if and only if $\frac{\beta}{\alpha} \geq \gamma_2$ that is if and only if (5.17) holds.

Consider now the case $detM < 0$. Let us note that (5.27 ) is equivalent to (5.26 ) changing $a, d, b+c$ with $-a, -d, -(b+c)$, respectively. Applying these changes to (5.16) and (5.17) we obtain (5.18) and (5.19), respectively.

• At last, consider the case $\alpha < 0,\ \beta > 0$. Taking into account $vi)$ of Theorem 5.1, $F(x)$ is pseudomonotone if and only if $y^T M_1^{-1} M (M_1^{-1})^T y \geq 0,\ \forall y = (y_1, y_2)^T$ with $y_1 y_2 < 0$, where $M_1 = \begin{bmatrix} \alpha a + \beta c & a \\ \alpha b + \beta d & b \end{bmatrix}$.

By simple calculations we have $M_1^* = M_1^T M^{-1} M_1 = \begin{bmatrix} a\alpha^2 + (b+c)\alpha\beta + d\beta^2 & \alpha a + \beta c \\ \alpha a + \beta b & a \end{bmatrix}$.

It follows that $M_1^*$ can be obtained from $M_2^*$ changing $a, \alpha$ with $d, \beta$, respectively. Applying these changes in $ii)$ we obtain $iii)$.

The proof is complete □

# 6 The singular case

In this section we will characterize the pseudomonotonicity of the affine map $F(x) = Mx + q$, $M = \begin{bmatrix} a & c \\ b & d \end{bmatrix}$, $rank(M) = 1$, on the interior of the positive orthant.

Set $x^T = (x_1, x_2)$, $q^T = (q_1, q_2)$ and denote by $m^{(1)}$, $m^{(2)}$ the first and the second column of $M$, respectively. Since $rank(M) = 1$ at least one column is not the null vector. Without loss of generality assume the existence of $\alpha \in \Re$ such that $m^{(2)} = \alpha m^{(1)}$.

Two cases arise: $q$ is proportional to $m^{(1)}$, that is there exists $\beta \in \Re$ such that $q = \beta m^{(1)}$, or $q$ is not proportional to $m^{(1)}$ that is $rank[q, m^{(1)}] = 2$. In the first case we have the following theorem:

**Theorem 6.1** *Consider the affine map $F(x) = Mx + q$, $q = \beta m^{(1)}$, $\beta \in \Re$. Then $F(x)$ is pseudomonotone on $int\Re^2_+$ if and only if one of the following conditions holds:*

*i) $\alpha \geq 0$, $\beta \geq 0$;*

*ii) $a > 0$, $b = \alpha a$.*

*Proof* It results $v^T(Mx + q) = (x_1 + \alpha x_2 + \beta)v^T m^{(1)}$.

If for every $(x_1, x_2) \in int\Re^2_+$ it results $x_1 + \alpha x_2 + \beta \neq 0$, then $v^T(Mx+q) = 0 \Leftrightarrow v^T m^{(1)} = 0$; on the other hand $v^T m^{(1)} = 0$ implies $v^T Mv = 0$ and thus $F(x)$ is pseudomonotone. Let us note that $x_1 + \alpha x_2 + \beta \neq 0$, $\forall (x_1, x_2) \in int\Re^2_+$ if and only if $\alpha \geq 0$, $\beta \geq 0$, so that condition $i)$ is verified.

If there exists $(x_1, x_2) \in int\Re^2_+$ such that $x_1 + \alpha x_2 + \beta = 0$ then it results $v^T(Mx + q) = 0, \forall v \in \Re^2$, so that $v^T Mv \geq 0$ if and only if $B = \frac{M+M^T}{2}$ is positive semidefinite. Let us note that $B$ is positive semidefinite if and only if $a \geq 0$, $\alpha b \geq 0$ and $detB = -(\alpha a - b)^2 \geq 0$ that is, taking into account that $m^{(1)} \neq 0$, if and only if $a > 0$, $b = \alpha a$, so that condition $ii)$ holds. □

**Corollary 6.1** *Consider the affine map $F(x) = Mx + q$, $q = \beta m^{(1)}$, $\beta \in \Re$. Then:*

*i) $F(x)$ is merely pseudomonotone if and only if*

$$M = \begin{bmatrix} a & \alpha a \\ b & \alpha b \end{bmatrix}, q = \begin{pmatrix} \beta a \\ \beta b \end{pmatrix}, \alpha \geq 0,\ \beta \geq 0,\ b \neq \alpha \cdot a \tag{6.30}$$

*ii) $F(x)$ is pseudomonotone (not merely) if and only if*

$$M = a \begin{bmatrix} 1 & \alpha \\ \alpha & \alpha^2 \end{bmatrix}, a > 0,\ \alpha \in \Re. \tag{6.31}$$

*Proof* We have $v^T(Mx+q) = (x_1 + \alpha x_2 + \beta)v^T m^{(1)}$.
If $\alpha < 0$ it is possible to choose $x_2^* > 0$ large enough such that $x_1^* = -\alpha x_2^* - \beta > 0$; it follows that $(x_1^* + \alpha x_2^* + \beta)v^T m^{(1)} = 0$ is verified $\forall v \in \Re^2$. Consequently, in the case $\alpha < 0$, $f(x)$ is pseudomonotone on $int\Re^2_+$ if and only if the matrix $\frac{M+M^T}{2}$ is positive semidefinite (in such a case it is easy to prove that it assumes the form (6.33)).
If $\alpha \geq 0,\ \beta < 0$ it is possible to choose $x_2^* > 0$ close to zero such that $x_1^* = -\alpha x_2^* - \beta > 0$; it follows that $(x_1^* + \alpha x_2^* + \beta)v^T m^{(1)} = 0$ is verified $\forall v \in \Re^2$. Consequently, also in the case $\alpha \geq 0,\ \beta < 0$, $f(x)$ is pseudomonotone on $int\Re^2_+$ if and only if the matrix $\frac{M+M^T}{2}$ is positive semidefinite.
In the case $\alpha \geq 0,\ \beta \geq 0$, $v^T(Mx+q) = (x_1 + \alpha x_2 + \beta)v^T m^{(1)} = 0, x_1 > 0, x_2 > 0$ if and only if $v^T m^{(1)} = 0$, so that $v^T M v = 0$ and $f(x)$ is pseudomonotone.
The proof is complete. □

**Corollary 6.2** *Consider the linear map $F(x) = Mx$. Then:*
*i) $F(x)$ is merely pseudomonotone if and only if*

$$M = \begin{bmatrix} a & \alpha a \\ b & \alpha b \end{bmatrix}, \alpha \geq 0,\ b \neq \alpha a \tag{6.32}$$

*ii) $F(x)$ is pseudomonotone (not merely) if and only if*

$$M = a \begin{bmatrix} 1 & \alpha \\ \alpha & \alpha^2 \end{bmatrix}, a > 0,\ \alpha \in \Re. \tag{6.33}$$

The following theorem characterizes the pseudomonotonicity of the affine map $F(x)$ when $rank[q, m^{(1)}] = 2$.

**Theorem 6.2** *Consider the affine map $F(x) = Mx + q$ with $rank[q, m^{(1)}] = 2$. Then:*
*i) $F(x)$ is merely pseudomonotone if and only if one of the following conditions holds:*

$$M = \begin{bmatrix} a & \alpha a \\ b & \alpha b \end{bmatrix}, \alpha \geq 0, \; bq_1 - aq_2 > 0, \; q_2 \leq \alpha q_1, \; b < \alpha a \tag{6.34}$$

$$M = \begin{bmatrix} a & \alpha a \\ b & \alpha b \end{bmatrix}, \alpha \geq 0, \; bq_1 - aq_2 < 0, \; q_2 > \alpha q_1, \; b > \alpha a \tag{6.35}$$

*ii) $F(x)$ is pseudomonotone (not merely) if and only if (6.33) holds.*

*Proof* Let $C_{W^*}$ be the cone generated by $W^* = \{Mx + q, \; x \in int\Re^2_+\}$. It is easy to prove that:

- if $\alpha < 0$ then $C_{W^*}$ is the half-plane containing $q$ and having $r = \{x = tm^{(1)}, t \in \Re\}$ as supporting line;
- if $\alpha \geq 0$ then $C_{W^*}$ is the open convex cone generated by the vectors $m^{(1)}$ and $q$.

In the first case, that is $\alpha < 0$, $Z_{W^*} = ((m^{(1)})^{\perp})^c$ (see Theorem 2.1); when $\alpha \geq 0$, we have $Z_{W^*} = \{v = (A^{-1})^T y, \; y \notin \Re^2_+ \cup \Re^2_-\}$ where $A$ is the matrix having $m^{(1)}$ and $q$ as columns (see Theorem 2.3).
From Theorem 3.2, $F(x)$ is pseudomonotone if and only $v^T M v \geq 0, \; \forall v \in Z_{W^*}$. Consequently:

- if $\alpha < 0$ we have $v^T M v \geq 0, \; \forall v \in ((m^{(1)})^{\perp})^c$, that is $v^T M v \geq 0, \; \forall v \in \Re^2$, so that $F(x)$ is pseudomonotone if and only if $\frac{M+M^T}{2}$ is positive semidefinite;
- if $\alpha \geq 0$, $F(x)$ is pseudomonotone if and only if

$$y^T A^{-1} M (A^{-1})^T y \geq 0 \;\; \forall \, y = (y_1, y_2)^T, \;\; y_1 y_2 < 0 \tag{6.36}$$

By simple calculations we find

$$A^{-1}M(A^{-1})^T = \frac{1}{bq_1 - aq_2}\begin{bmatrix} 0 & 0 \\ b - \alpha a & -q_2 + \alpha q_1 \end{bmatrix} \tag{6.37}$$

so that (6.36) becomes

$$\frac{-q_2 + \alpha q_1}{bq_1 - aq_2}\, y_2^2 - \frac{(\alpha a - b)}{bq_1 - aq_2}\, y_1 y_2 \geq 0 \;\; \forall y_1 y_2 < 0. \tag{6.38}$$

Following the same lines given in the proof of Theorem 4.1, the thesis is achieved.

□

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