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Andrea Mercatanti

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Via Cosimo Ridolfi, 10 – 56124 PISA – Tel. Segr. Amm. 050 2216231 Segr. Stud. 050 2216317 Fax 050 2216375
Cod. Fisc. 80003670504 - P. IVA 00286820501 - Web http://statmat.ec.unipi.it/ - E-mail: dipstat@ec.unipi.it

An ML procedure for partially identified causal models

Andrea Mercatanti
University of Pisa, Via C.Ridolfi 10, 56124 Pisa, Italy.
(mercatan@ec.unipi.it)

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Abstract

The full relaxation of the exclusion restriction in causal models yields a likelihood characterized by the presence of mixtures of distributions. This complicates a likelihood-based analysis because it implies only partially identified models and more than one maximum likelihood point. We propose a two step MLE when the outcome distributions of various compliance statuses are in the same class. In this case we do not need to impose any extra assumptions compared to those usually adopted for the instrumental variables technique.

Keywords: partially identified models, compliers, exclusion restriction, mixture distributions.

1 [1]Introduction

In spite of its importance, the exclusion restriction in causal inference can often be unrealistic in practice; however relaxing the assumption is not straightforward since it is directly related to the identifiability of the parametric models. Indeed, without the exclusion restriction, the parametric models do not have unique maximum likelihood points, but rather regions of values at which the likelihood function is maximized (Imbens and Rubin, 1997a;

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Hirano et al., 2000). Given this problem of identifiability, previous studies propose relaxing the assumption by relying on prior distributions in a Bayesian framework (Hirano et al., 2000), or by introducing auxiliary information from pretreatment variables in a likelihood-based context (Jo, 2002).

Nonparametric bounds, on the average treatment effects of a randomized experiment with imperfect compliance, over the whole population have been developed by Balke and Pearl (1997) under the exclusion restriction, and supposing a binary treatment and a binary outcome. Their paper was based on the general result of Manski (1990) for nonparametric bounds on treatment effects.

Subsequently, research in causal inference turned from the nonparametric instrumental variables method to parametric models. In particular with the contribution of Imbens and Rubin (1997a) who introduced a suitable likelihood function, and proposed also a weak version of the exclusion restriction requiring that the assignment to treatment has to be unrelated to potential outcomes but only for noncompliers, the individuals that would receive or would not receive the treatment regardless of whether it is offered.

The current study explores a new option, where we fully relax the exclusion restriction without introducing extra information compared to the usual set of conditions adopted to identify causal effect in the IV framework (Angrist et al., 1996). Supposing a binary treatment and outcome distributions of various compliance statuses in the same class, we show that relaxing the exclusion restriction introduce two mixtures of distributions in the parametric model. But the estimation of mixed distribution models implies analytical and computational difficulties due both to the singularities of the likelihood function and to the presence of several local maximum points (McLachlan and Peel, 2000). Moreover, here the analysis is complicated compared to usual studies on univariate finite mixtures models. This is principally due to the switching of mixture component indicators that we will see complicates the identification of causal effects. In order to resolve these complications, we propose a constrained maximization procedure, that can be performed by exploiting the information supplied by the usual IV set of assumptions.

This article is briefly organized as follows. Section 2 introduces the complications that relaxing the exclusion restriction produces on a likelihood-based analysis. In Section 3 we propose a two step restricted maximization procedure: it will be applied to a microeconomic dataset in Section 4. The application is suggested by a recent paper of Ichino and Winter-Ebmer (2004) who investigated the long run educational cost of World War II; the results obtained by applying the proposed procedure are compared to those obtained by the IV method.

2 Partial identifiability of ML causal analyses with same class outcome distributions

A remarkable contribution to the parametric formalization of the IV technique in identifying and estimating the causal effects is due to Imbens and Rubin (1997a). The authors based the resulting likelihood function on the concept of potential quantities: the concept of causality we want to adopt in this paper. Consequently, the population under study can be subdivided in four groups that are characterized by the way the individuals react, from a counterfactual point of view, to the assignment to treatment.

Let $Y_i(Z_i = z, D_i = d)$ with $z \in \{0, 1\}$ and $d \in \{0, 1\}$ be the potential outcome with respect to the assignment, z, and to the treatment, d. The exclusion restriction implies that $Y_i(Z_i = 1, D_i = d) = Y_i(Z_i = 0, D_i = d)$. In order to achieve a complete relaxation of the assumption, the current study employs a maximum likelihood estimation approach which is known to be often more efficient than the IV framework in the identification and estimation of causal effects for compliers (Imbens and Rubin, 1997a; Little and Yau, 1998; Jo, 2002). At these purposes let introduce this set of assumptions:

Assumption 1 : *S.U.T.V.A. (Stable Unit Treatment Value Assumption)* by which the potential quantities for each unit are unrelated to the treatment status of other units;

Assumption 2 : "*Random assignment to treatment*" by which the probability to be assigned to the treatment is the same for every unit;

Assumption 3 : *Nonzero average causal effect of Z_i on D_i*, imposing the presence of compliers;

Assumption 4 : *"Monotonicity"* imposing the absence of defiers;

Assumption 5 : the outcome distributions of various compliance statuses are in the same class.

Assumptions 1-4 are the necessary set of conditions for identifying the compliers average treatment effect by the IV method, apart from the exclusion restriction (Angrist et al., 1996).

The likelihood function for a randomized experiment with imperfect compliance, under the previous 1-5 assumptions, and adopting the parameter set proposed by Imbens and Rubin (1997a), can be written:

$$L(\boldsymbol{\theta}) \propto \prod_i f(y_i,\, d_i,\, z_i; \boldsymbol{\theta}) = \prod_i \Big\{ I(D_i = 1,\, Z_i = 0) \cdot \omega_a \cdot g^i_{a0} + I(D_i = 0,\, Z_i = 1) \cdot \omega_n \cdot g^i_{n1} +$$

$$+ I(D_i = 1,\, Z_i = 1) \cdot (\omega_a \cdot g^i_{a1} + \omega_c \cdot g^i_{c1}) + I(D_i = 0,\, Z_i = 0) \cdot (\omega_n \cdot g^i_{n0} + \omega_c \cdot g^i_{c0}) \Big\} =$$

$$= \prod_{i \in \varsigma(D_i=1, Z_i=0)} \omega_a \cdot g^i_{a0} \times \prod_{i \in \varsigma(D_i=0, Z_i=1)} \omega_n \cdot g^i_{n1} \times \prod_{i \in \varsigma(D_i=1, Z_i=1)} (\omega_a \cdot g^i_{a1} + \omega_c \cdot g^i_{c1}) \times$$

$$\times \prod_{i \in \varsigma(D_i=0, Z_i=0)} (\omega_n \cdot g^i_{n0} + \omega_c \cdot g^i_{c0}) \tag{1}$$

$$\Theta : \left\{ \boldsymbol{\theta} = (\omega_a, \omega_n, \omega_c, \boldsymbol{\eta}_{a0}, \boldsymbol{\eta}_{a1}, \boldsymbol{\eta}_{n0}, \boldsymbol{\eta}_{n1}, \boldsymbol{\eta}_{c0}, \boldsymbol{\eta}_{c1}) | \sum_{t=a,n,c} \omega_t = 1;\, \omega_t > 0\,, \forall t \right\} \tag{2}$$

where: $I(\cdot)$ is an indicator function; $\varsigma(D_i = d, Z_i = z)$ is the group of the units assuming treatment d and assigned to the treatment z; ω_t is the mixing probability, that is the probability of an individual being in the t group, $t = a$ *(always-takers)*, n *(never-takers)*, c *(compliers)*; the function $g^i_{tz} = g_{tz}(y_i; \boldsymbol{\eta}_{tz})$ is the same class outcome distribution for a unit in the t group and assigned to the treatment z.

Then (1) factors in four terms, where any term refers to a group $\varsigma(D_i = d, Z_i = z)$. In particular the units in group $\varsigma(D_i = 0, Z_i = 0)$ are a mixture of compliers and never-takers, and the units in group $\varsigma(D_i = 1, Z_i = 1)$ are a mixture of compliers and always-takers. The maximization of (1) faces both analytical and computational difficulties due to these two mixtures of distributions involved.

In order to explain the reasons of these difficulties, we will consider the general density for a mixture of distributions in the same class:

$$f(\mathbf{x};\boldsymbol{\theta}) = \sum_{h=1}^{T} \omega_h \cdot f_h(\mathbf{x};\boldsymbol{\eta}_h),$$

where

$$\Theta : \boldsymbol{\theta} = \left\{ (\omega_1, .., \omega_T, \boldsymbol{\eta}_1, .., \boldsymbol{\eta}_T) | \sum_{i=1}^{T} \omega_h = 1;\ \omega_h > 0,\ \forall h \right\},$$

and for which the corresponding likelihood is:

$$L(\boldsymbol{\theta}) = \prod_{i=1}^{n} \sum_{h=1}^{T} \omega_h \cdot f_h(\mathbf{x}_i;\boldsymbol{\eta}_h). \tag{3}$$

A first problem associated with maximum likelihood estimation arises from the possible unboundedness of (3) on Θ. In particular Day (1969) shows that, for f_h in the normal parametric family, a global maximum likelihood estimate does not exist, and moreover the unboundedness of (3) causes failures of optimization algorithms of both the EM and quasi-Newton types (Fowlkes, 1979; Hataway, 1985). Kiefer (1978) and Redner and Walker (1984) present the conditions for the existence of a strongly consistent, efficient and asymptotically normal local maximizer. However, with mixture models the likelihood function will generally have multiple roots. The local maximum points that do not correspond to the consistent maximizer are usually indicated as "spurious" maximum points. For f_h in the normal parametric family, $f_h = N\left(x_i\,;\mu_h,\,\sigma_h^2\right)$, the local maximum points corresponding to parameter points having at least one variance component, σ_h^2, very close to zero are generated by groups of few outliers (Day, 1969). Motivated by the presence of spurious maximum points, previous studies presented some alternative methods for identifying the consistent maximizer of (3); for example, Gan and Jiang (1999) who proposed a test based on the comparison between the information matrix and the negative of the Hessian of the log likelihood. In particular, for a mixture of normal densities, Hataway (1985) suggested a likelihood maximization restricted to appropriate parameter subspaces whose identification is supported by apriori information about the various variance components ratios. This approach suggests a maximization procedure restricted to the parameter subspace satisfying:

$$\forall\, h',\, h'' \in \{1, .., T\} :\ \sigma_{h'}/\sigma_{h''} \geq c > 0.$$

The global constrained maximizers shares all the good asymptotic proprieties of the consistent maximizers of (3); the only problem in practice is to choose a value for c for which the true parameter vector satisfies the constraints. For this reason, McLachlan and Peel (2000) proposed an approach based on running a sequence of unrestricted maximization procedures, followed by an analysis of the local maximum points located in order to detect the spurious ones. After these checks, the authors take the MLE of $\boldsymbol{\theta}$ to be the root of the likelihood function corresponding to the largest of the remaining local maximum points located. In order to obtain a bounded likelihood for a mixture of normal densities, an alternative and more recent method concerns the introduction of a penalized term, $p(\sigma_1, .., \sigma_T)$, in (3) (Ridolfi and Idier, 2002). The authors showed that if $p(\sigma_1, .., \sigma_T)$ is the product of T inverse Gamma distributions, the resulting penalized likelihood, $L^P(\boldsymbol{\theta}) \propto L(\boldsymbol{\theta})\ \ p(\sigma_1, .., \sigma_T)$, is bounded.

In spite of the existence of various alternative methods for a likelihood-based analysis of (3), an equivalent analysis of the function (1) is more complicated because of additional problems in the identification of the model. It is well known (McLachlan and Peel, 2000) that for a finite mixture of distributions in the same class, $f(\mathbf{x}; \boldsymbol{\theta}) = \sum_{h=1}^{T} \omega_h f_h(\mathbf{x}; \boldsymbol{\eta}_h)$, the parameter vector $\boldsymbol{\theta} = (\omega_1, ..., \omega_T, \boldsymbol{\eta}_1, ..., \boldsymbol{\eta}_T)$ is not identified. In general a parametric family of densities $\{f(\mathbf{x}; \boldsymbol{\theta}) : \boldsymbol{\theta} \in \Theta\}$ is identifiable if distinct members of the parameter vector $\boldsymbol{\theta}$ determine distinct members of the family. Since $f(\mathbf{x}; \boldsymbol{\theta})$ is invariant under the $T!$ permutations of the component labels h in $\boldsymbol{\theta}$, only a class of distributions is identified. For example, let $T = 2$ in (3) then the presence of two densities in the same class, $f_1(\mathbf{x}; \boldsymbol{\eta}_1)$ and $f_2(\mathbf{x}; \boldsymbol{\eta}_2)$, implies that $f(\mathbf{x}; \boldsymbol{\theta}) = f(\mathbf{x}; \boldsymbol{\theta}^*)$ if the component labels 1 and 2 are interchanged in $\boldsymbol{\theta}^*$ compared to $\boldsymbol{\theta}$. This means that only the set of parameter vectors invariant respect to the order of labelling the components is identified; however this is not a relevant problem in the maximum likelihood estimation of a same class components mixture model for cluster analysis purposes where, the components labelling does not matter.

In order to investigate the identifiability of $\boldsymbol{\theta}$ in (1), we have to take into account the possible consequences of a label components switching in one or both the two mixtures involved in the likelihood. First, let's consider the mixture of always-takers and compliers assigned to the treatment, $\varsigma(D_i = 1, Z_i = 1)$. A label components switching in $\varsigma(D_i = 1, Z_i = 1)$, is equivalent to interchange the value of $(\omega_a, \boldsymbol{\eta}_{a1})$ with $(\omega_c, \boldsymbol{\eta}_{c1})$ in $\boldsymbol{\theta}$. Consequently also the part of the likelihood regarding the not assigned always-takers, that is

$\prod_{i\in\varsigma(D_i=1,Z_i=0)}\omega_a\cdot g^i_{a0}=\omega_a^{\#\varsigma(D_i=1,Z_i=0)}\prod_{i\in\varsigma(D_i=1,Z_i=0)}g^i_{a0}$, will be affected by a permutation of the components labels in $\varsigma(D_i=1,Z_i=1)$. Likelihood (1) will be invariant, making the model not identified, only if:

$$\omega_a^{\#\varsigma(D_i=1,Z_i=0)}=\omega_c^{\#\varsigma(D_i=1,Z_i=0)}=(1-\omega_a-\omega_n)^{\#\varsigma(D_i=1,Z_i=0)}$$

that is, only if:

$$\omega_a=\frac{1}{2}-\frac{\omega_n}{2}=\omega_c.$$

Analogously, a label components switching in the mixture of never-takers and compliers not assigned to the treatment, $\varsigma(D_i=0,Z_i=0)$, will affect the part of the likelihood regarding the assigned never-takers, $\varsigma(D_i=0,Z_i=1)$. For the previous reasons, the likelihood will be invariant under a permutation of the components labels in $\varsigma(D_i=0,Z_i=0)$ only if:

$$\omega_n^{\#\varsigma(D_i=0,Z_i=1)}=\omega_c^{\#\varsigma(D_i=0,Z_i=1)}=(1-\omega_a-\omega_n)^{\#\varsigma(D_i=0,Z_i=1)}$$

that is, only if:

$$\omega_n=\frac{1}{2}-\frac{\omega_a}{2}=\omega_c.$$

Finally, a label components switching in both the mixtures will affect both the part of the likelihood regarding the not assigned always-takers, $\varsigma(D_i=1,Z_i=0)$, and the assigned never-takers, $\varsigma(D_i=0,Z_i=1)$, making the model not identified only if:

$$\omega_a^{\#\varsigma(D_i=1,Z_i=0)}\cdot\omega_n^{\#\varsigma(D_i=0,Z_i=1)}=\omega_c^{\#\varsigma(D_i=1,Z_i=0)+\#\varsigma(D_i=0,Z_i=1)}=$$

$$=(1-\omega_a-\omega_n)^{\#\varsigma(D_i=1,Z_i=0)+\#\varsigma(D_i=0,Z_i=1)}$$

that is, only if:

$$\omega_a=\left\{(\alpha-1)^{\frac{\#\varsigma(D_i=1,Z_i=0)+\#\varsigma(D_i=0,Z_i=1)}{\#\varsigma(D_i=0,Z_i=1)}}+\alpha\right\}^{-1} \tag{4}$$

$$\omega_n=1-\alpha\cdot\left\{(\alpha-1)^{\frac{\#\varsigma(D_i=1,Z_i=0)+\#\varsigma(D_i=0,Z_i=1)}{\#\varsigma(D_i=0,Z_i=1)}}+\alpha\right\}^{-1} \tag{5}$$

for any $\alpha > 1$ (same details are in the Appendix).

The parameter vector $\boldsymbol{\theta}$ is then partially identified for (1). However, and contrarily to a likelihood based analysis of (3) at cluster purposes, the components labelling matters for (1) at causal inference purposes in the region for which $\boldsymbol{\theta}$ is not identified.

In order to clarify, we shall introduce an alternative form of the parameter vector. The subvector $\boldsymbol{\omega}_t = (\omega_a, \omega_n, \omega_c)$ can be indeed decomposed and substituted with $\boldsymbol{\omega}_{tz} = (\omega_{a0}, \omega_{a1}, \omega_{n0}, \omega_{n1}, \omega_{c0}, \omega_{c1})$, where ω_{tz} is the probability of an individual being in the group of the units having compliance status t and assigned to z: $\upsilon(C_i = t,\, Z_i = z)$. The proposed decomposition is feasible if taking into account that $\omega_{tz} = \omega_t\, I(Z_i = 1)\, \pi + \omega_t\, I(Z_i = 0)\, (1 - \pi)$, and it produces the equivalent likelihood function[2]:

$$L(\boldsymbol{\theta}) = \prod_{i \in \varsigma(D_i=1, Z_i=0)} \omega_{a0} \cdot g^i_{a0} \times \prod_{i \in \varsigma(D_i=0, Z_i=1)} \omega_{n1} \cdot g^i_{n1} \times$$

$$\times \prod_{i \in \varsigma(D_i=1, Z_i=1)} (\omega_{a1} \cdot g^i_{a1} + \omega_{c1} \cdot g^i_{c1}) \times \prod_{i \in \varsigma(D_i=0, Z_i=0)} (\omega_{n0} \cdot g^i_{n0} + \omega_{c0} \cdot g^i_{c0}), \quad (6)$$

$$\Theta : \left\{ \boldsymbol{\theta} = (\boldsymbol{\omega}_{tz}, \boldsymbol{\eta}_{a0}, \boldsymbol{\eta}_{a1}, \boldsymbol{\eta}_{n0}, \boldsymbol{\eta}_{n1}, \boldsymbol{\eta}_{c0}, \boldsymbol{\eta}_{c1}) | \sum_{t=a,n,c} \sum_{z=0,1} \omega_{tz} = 1;\, \omega_{tz} > 0,\, \forall t,\, \forall z \right\}. \quad (7)$$

Compared to (2), the new parameter set allows to refer the mixing probabilities directly to the counterfactual groups, $\upsilon(C_i = t,\, Z_i = z)$, in which the population can be subdivided. The straightforward relations between the different patterns $\varsigma(D_i = d, Z_i = z)$ and $\upsilon(C_i = t,\, Z_i = z)$ are given by:

$$\varsigma(D_i = 1, Z_i = 0) = \upsilon(C_i = a,\, Z_i = 0),$$

$$\varsigma(D_i = 0, Z_i = 1) = \upsilon(C_i = n,\, Z_i = 1),$$

$$\varsigma(D_i = 0, Z_i = 0) = \upsilon(C_i = n,\, Z_i = 0) \cup \upsilon(C_i = c,\, Z_i = 0),$$

[2]The new likelihood function, (6), is equivalent to the previous, (1), for maximization purposes. Indeed, taking into account the specified restrictions, the reparameterized space, (7), is not greater than (2) and the invariance property of ML estimators hold.

$$\varsigma(D_i = 1, Z_i = 1) = \upsilon(C_i = a,\ Z_i = 1) \cup \upsilon(C_i = c,\ Z_i = 1).$$

The causal effects from a counterfactual point of view are defined by the three differences $\Delta_t = (\mu_{t1} - \mu_{t0})$, where $t = a, n, c$. Consequently, the right labelling of all the components now matters in order to identify Δ_t in the region where $\boldsymbol{\theta}$ is not identified. For example, let's consider a hypothetical local maximum likelihood point, $\hat{\boldsymbol{\theta}}$, for which the component labels of the mixture $\varsigma(D_i = 1, Z_i = 1)$, composed by assigned always-takers and assigned compliers, permute. In this case the causal effects of the assignment to treatment for always-takers and compliers are not identified because of the permutation of label components in $\hat{\boldsymbol{\theta}}$. Indeed, the causal effect for compliers Δ_c in $\hat{\boldsymbol{\theta}}$ would be wrongly identified by $(\mu_{a1} - \mu_{c0})$ instead of $(\mu_{c1} - \mu_{c0})$, and the causal effect for always-takers Δ_a would be wrongly identified by $(\mu_{c1} - \mu_{a0})$ instead of $(\mu_{a1} - \mu_{a0})$.

3 A two step ML approach

We have showed in Section 2 that in a likelihood based analysis of a randomized experiment without exclusion restriction the parameter vector $\boldsymbol{\theta}$ is only partially identified. In recent years, some methods for relaxing the exclusion restriction based on exploiting extra information compared to the assumptions 1-5 of Section 2 were proposed. For example, Hirano et al. (2000) that worked in a Bayesian context adopting a relatively diffuse but proper prior distribution, or more recently Jo (2002) that studied alternative model specifications allowing the identification of causal effects in the presence of observed pretreatment information. An alternative approach can be proposed considering that the mixing probabilities vectors $\boldsymbol{\omega}_t$ and $\boldsymbol{\omega}_{tz}$ are non-parametrically identified given the assumptions 1-5. In this Section we will see that these vectors can be easily estimated, and these estimates can be plugged into the likelihood function in order to maximize it over the remaining parameter set. We will also show how the EM algorithm can make the inference relatively straightforward.

As outlined by Imbens and Rubin (1997b), given the independence of assignment Z_i and compliance status C_i, the population proportions of type C_i, ϕ_t, are known in a large sample: $\phi_a = P(D_i = 1|Z_i = 0)$; $\phi_n = P(D_i = 0|Z_i = 1)$; $\phi_c = 1 - \phi_a - \phi_n$. These large sample proportions are equivalent to the three mixing probabilities $(\omega_a, \omega_n, \omega_c)$ from a frequentist point of view, and they can be estimated by[3] $\hat{\boldsymbol{\phi}}_t = (\hat{\phi}_a, \hat{\phi}_n, \hat{\phi}_c)$:

- the proportion of treated units in the group of not assigned units: $\hat{\phi}_a = \sum_i I(D_i = 1,\, Z_i = 0)/\sum_i I(Z_i = 0)$,
- the proportion of untreated units in the group of assigned units: $\hat{\phi}_n = \sum_i I(D_i = 0,\, Z_i = 1)/\sum_i I(Z_i = 1)$,
- the difference: $\hat{\phi}_c = 1 - \hat{\phi}_a - \hat{\phi}_n$;

where $I(\cdot)$ is an indicator function.

Analogously, the population proportions ϕ_{tz} of units in the group $\upsilon(C_i = t,\, Z_i = z)$ are known in a large sample, for example: $\phi_{a0} = P(D_i = 1,\, Z_i = 0)$. Consequently, in likelihood function (6), the subvector $\boldsymbol{\omega}_{tz}$ can be estimated by $\hat{\boldsymbol{\phi}}_{tz} = \left(\hat{\phi}_{a0}, \hat{\phi}_{a1}, \hat{\phi}_{n0}, \hat{\phi}_{n1}, \hat{\phi}_{c0}, \hat{\phi}_{c1}\right)$, that is a transformation of $\hat{\boldsymbol{\phi}}_t$:

$$\hat{\phi}_{a0} = \frac{\sum_i I(D_i = 1,\, Z_i = 0)}{n}, \hat{\phi}_{a1} = \hat{\phi}_a - \hat{\phi}_{a0},$$

$$\hat{\phi}_{n0} = \hat{\phi}_n - \hat{\phi}_{n1}, \hat{\phi}_{n1} = \frac{\sum_i I(D_i = 0,\, Z_i = 1)}{n},$$

$$\hat{\phi}_{c0} = \frac{\sum_i I(D_i = 0,\, Z_i = 0)}{n} - \hat{\phi}_{n0}, \hat{\phi}_{c1} = \frac{\sum_i I(D_i = 1,\, Z_i = 1)}{n} - \hat{\phi}_{a1},$$

where n is the sample size.

It is worth to note, that the previous estimated mixing probabilities vector $\hat{\boldsymbol{\phi}}_t$ (or $\hat{\boldsymbol{\phi}}_{tz}$) is exactly the same obtainable by maximizing the marginal likelihood $L(\boldsymbol{\omega}_t)$ [or $L(\boldsymbol{\omega}_{tz})$]:

$$L(\boldsymbol{\omega}_t) \propto \prod_i \int f(y_i,\, d_i,\, z_i\,; \boldsymbol{\omega}_t, \Psi)\, dy_i = \prod_{i \in \varsigma(D_i=1, Z_i=0)} \omega_a \times \prod_{i \in \varsigma(D_i=0, Z_i=1)} \omega_n \times$$

$$\times \prod_{i \in \varsigma(D_i=1, Z_i=1)} (\omega_a + \omega_c) \times \prod_{i \in \varsigma(D_i=0, Z_i=0)} (\omega_n + \omega_c)$$

[3]Let indicate $\hat{\phi}_t$ the estimated probability being compliance status t on the analogy of the Imbens and Rubin (1997b) notation.

$$L(\boldsymbol{\omega}_{tz}) \propto \prod_i \int f(y_i,\, d_i,\, z_i\,; \boldsymbol{\omega}_{tz},\, \Psi)\, dy_i = \prod_{i \in \varsigma(D_i=1,Z_i=0)} \omega_{a0} \times \prod_{i \in \varsigma(D_i=0,Z_i=1)} \omega_{n1} \times$$

$$\times \prod_{i \in \varsigma(D_i=1,Z_i=1)} (\omega_{a1} + \omega_{c1}) \times \prod_{i \in \varsigma(D_i=0,Z_i=0)} (\omega_{n0} + \omega_{c0})$$

where $\Psi = (\boldsymbol{\eta}_{a0}, \boldsymbol{\eta}_{a1}, \boldsymbol{\eta}_{n0}, \boldsymbol{\eta}_{n1}, \boldsymbol{\eta}_{c0}, \boldsymbol{\eta}_{c1})$; the mixing probabilities are then marginally identifiable.

Now an approach to maximize the likelihood function relaxing the exclusion restriction and exploiting the information about the estimated mixing probabilities, can be proposed by constraining the maximization of likelihood functions (1) or (6) to $\hat{\boldsymbol{\phi}}_t$ or $\hat{\boldsymbol{\phi}}_{tz}$ respectively.. The remaining parameter set $\{\Psi\}$ will be again partially identified, given the resulting constrained likelihood functions:

$$L\,(\Psi) \propto \prod_i f(y_i,\, d_i,\, z_i\,; \Psi,\, \hat{\boldsymbol{\phi}}_t) \propto \prod_i f(y_i,\, d_i,\, z_i\,; \Psi,\, \hat{\boldsymbol{\phi}}_{tz})$$

where:

$$\prod_i f(y_i,\, d_i,\, z_i\,; \Psi,\, \hat{\boldsymbol{\phi}}_t) = \prod_{i \in \varsigma(D_i=1,Z_i=0)} \hat{\phi}_a \cdot g^i_{a0} \times \prod_{i \in \varsigma(D_i=0,Z_i=1)} \hat{\phi}_n \cdot g^i_{n1} \times$$

$$\times \prod_{i \in \varsigma(D_i=1,Z_i=1)} (\hat{\phi}_a \cdot g^i_{a1} + \hat{\phi}_c \cdot g^i_{c1}) \times \prod_{i \in \varsigma(D_i=0,Z_i=0)} (\hat{\phi}_n \cdot g^i_{n0} + \hat{\phi}_c \cdot g^i_{c0})\ ,$$

and

$$\prod_i f(y_i,\, d_i,\, z_i\,; \Psi,\, \hat{\boldsymbol{\phi}}_{tz}) = \prod_{i \in \varsigma(D_i=1,Z_i=0)} \hat{\phi}_{a0} \cdot g^i_{a0} \times \prod_{i \in \varsigma(D_i=0,Z_i=1)} \hat{\phi}_{n1} \cdot g^i_{n1} \times$$

$$\times \prod_{i \in \varsigma(D_i=1,Z_i=1)} (\hat{\phi}_{a1} \cdot g^i_{a1} + \hat{\phi}_{c1} \cdot g^i_{c1}) \times \prod_{i \in \varsigma(D_i=0,Z_i=0)} (\hat{\phi}_{n0} \cdot g^i_{n0} + \hat{\phi}_{c0} \cdot g^i_{c0})\ ,$$

are invariant under the permutations of the component labels in one or both the two mixtures if:

$$\hat{\phi}_a = \frac{1}{2} - \frac{\hat{\phi}_n}{2} = \hat{\phi}_c \tag{8}$$

and/or

$$\hat{\phi}_n = \frac{1}{2} - \frac{\hat{\phi}_a}{2} = \hat{\phi}_c \tag{9}$$

(or equivalently if $\hat{\phi}_{n0} = \hat{\phi}_{c0}$ and/or $\hat{\phi}_{a1} = \hat{\phi}_{c1}$).

However the partial identifiability of the model now depends on weaker and easier testable conditions compared to the unconstrained maximization of likelihood (1) we have showed in the previous section. In particular the proposed constrained maximization eliminates the nonlinear identifiability conditions defined by the two equalities (4) and (5).

From a computational point of view, the EM algorithm can make the inference relatively straightforward. The EM algorithm is indeed attractive in making maximum likelihood inference because if the compliance status C_i was known for all units, the likelihood would not involve mixtures. The compliance status of the units in any of the two mixtures can be indeed considered as a missing information whose imputation produces the so-called augmented likelihood. Moreover, in our context the augmented log-likelihood function is linear in the missing information, so the EM algorithm corresponds to fill-in missing data and then updating parameter estimates. The imputation of the unobserved compliance status is handled by the E-step; it requires the calculation of the conditional expectation of C_i given the observed data and the current fit for $\mathbf{\Psi}$. The compliance status C_i can be represented by a three component indicator $t = c$ *(complier)*, n *(never-taker)*, a *(always taker)*. At the k-iteration, the conditional probability of subject i being type t given the observed data and a current value of the vector $\mathbf{\Psi}$, $\tau_{it}^{(k)}(\hat{\mathbf{\Psi}}^{(k-1)})$, is obtainable by a ratio of two quantities. The numerator of the ratio is the corresponding Table 1 (or Table 2) entry and the denominator is the corresponding row total, where $\hat{g}_{tz}^{i\,(k-1)}$ is the outcome distribution for a unit in the t group and assigned to the treatment z, based on the estimated parameter vector updated at the $(k-1)$ iteration, $\hat{\mathbf{\Psi}}^{(k-1)}$.

Table 1. *Inputs for calculating the conditional probabilities* $\tau_{it}^{(k)}(\hat{\Psi}^{(k-1)})$ *by using* $\hat{\phi}_t$.

D_i	Z_i	Subject type t		
		$t = a$	$t = n$	$t = c$
0	0	0	$\hat{\phi}_n \cdot \hat{g}_{n0}^{i\,(k-1)}$	$\hat{\phi}_c \cdot \hat{g}_{c0}^{i\,(k-1)}$
0	1	0	1	0
1	0	1	0	0
1	1	$\hat{\phi}_a \cdot \hat{g}_{a1}^{i\,(k-1)}$	0	$\hat{\phi}_c \cdot \hat{g}_{c1}^{i\,(k-1)}$

Table 2. *Inputs for calculating the conditional probabilities* $\tau_{it}^{(k)}(\hat{\Psi}^{(k-1)})$ *by using* $\hat{\phi}_{tz}$.

D_i	Z_i	Subject type t		
		$t = a$	$t = n$	$t = c$
0	0	0	$\hat{\phi}_{n0} \cdot \hat{g}_{n0}^{i\,(k-1)}$	$\hat{\phi}_{c0} \cdot \hat{g}_{c0}^{i\,(k-1)}$
0	1	0	1	0
1	0	1	0	0
1	1	$\hat{\phi}_{a1} \cdot \hat{g}_{a1}^{i\,(k-1)}$	0	$\hat{\phi}_{c1} \cdot \hat{g}_{c1}^{i\,(k-1)}$

The subsequent M-step then maximizes the log-likelihood function based on the augmented dataset, that is the dataset created by merging the observed and the imputed data. This is equivalent to a weighted maximization of the log-likelihood function, where subjects are differently classified in the different compliance groups, t, with weights equal to the conditional probabilities of being in t calculated in the E-step. The output is the update estimated vector $\hat{\Psi}^{(k)}$.

In particular, for the normal distributions case the updates of the component means, $\hat{\mu}_{tz}^{(k)}$, and component variances, $(\hat{\sigma}_{tz}^{(k)})^2$, are given by:

$$\hat{\mu}_{tz}^{(k)} = \sum_{i=1}^{n} \left\{\tau_{it}^{(k)}(\hat{\Psi}^{(k-1)}) \cdot y_i \cdot I(Z_i = z)\right\} / \sum_{i=1}^{n} \left\{\tau_{it}^{(k)}(\hat{\Psi}^{(k-1)}) \cdot I(Z_i = z)\right\},$$

$$(\hat{\sigma}_{tz}^{(k)})^2 = \sum_{i=1}^{n} \left\{\tau_{it}^{(k)}(\hat{\Psi}^{(k-1)}) \cdot (y_i - \hat{\mu}_{tz}^{(k)})^2 \cdot I(Z_i = z)\right\} / \sum_{i=1}^{n} \left\{\tau_{it}^{(k)}(\hat{\Psi}^{(k-1)}) \cdot I(Z_i = z)\right\}.$$

4 An illustrative example

In microeconomic literature, the IV method has been widely used in evaluating return to schooling. In particular, two remarkable studies have been recently proposed by Ichino and Winter-Ebmer (IW henceforth) in 1999 and 2004. In both papers the authors investigated the causal effect of education on earnings: the first paper (1999) intended for estimating lower and upper bounds of returns to schooling in Germany, the second (2004) for quantifying the long run educational cost of World War Two in Germany and Austria. In particular the basic idea characterizing the IW 2004 paper relies on the fact that individuals who were about ten years old during or immediately after the war, were damaged in their educational choices compared to individuals in the immediately previous or subsequent cohorts. War physical disruptions and related consequences indeed made harder to achieve the desired level of education for most of the schooling age population in these two countries. Moreover the authors show, using the IV method, that individuals whose education was affected by the war (compliers) suffered a significant earning loss about forty years after the end of the war. For this purpose the IW causal analysis was supported by several instruments; in particular, given the date of birth can be reasonably supposed to be a random event, cohort of birth was adopted as an instrumental variable for both countries. The authors had to assume the exclusion restriction, other than the assumptions 1-4 of Section 2, for identifying and evaluating the average causal effect for compliers by the IV method.

In order to show an example of fully relaxing the exclusion restriction and consequently estimating causal effects also for non-compliers, the previously proposed constrained ML procedure will be here applied to the same economic context of the IW (2004) paper. The data are from Mikrozensus 1981 for Austria (a 1% sample of the Austrian population), and from the Socio-Economic Panel, wave 1986, for Germany. We are considering males born between 1925 and 1949 for both countries.

Log hourly earnings for employed workers are observed about 40 years after the end of the war. Like IW, and in order to consider the increasing trend of individual earnings respect to age, the outcome Y_i is defined as the residual of a regression of log hourly earnings on a cubic polynomial in age. An increasing trend respect to age also characterized the candidate treatment, that is the individual years of education; for this reason the residuals of a regression of years of education on a cubic polynomial in age are calculated.

But in order to apply the previously proposed procedure, the treatment has to be a binary variable. Then we define the treatment, D_i, equal to one if the individual residual is smaller than the residuals sample average and equal to zero if the individual residual is greater than the residuals sample average. In this way we are considering individuals having $D_i = 1$ as low educated, and individuals having $D_i = 0$ as high educated. The cohort of birth is used as an instrumental variable, Z_i, having the role of a random assignment to treatment. For this purpose, Z_i has to be necessarily equal to one for people assigned to being low educated, and equal to zero for people assigned to being high educated. Table 3 shows that both the estimated mean years of education and the estimated mean residuals of the years of education are smaller for individuals in the cohort 1930-39 than for people in the cohort obtained merging 1925-29 and 1940-49 cohorts. These results suggest defining $Z_i = 1$ for individuals born during 1930-39, and $Z_i = 0$ for individuals born during 1925-29 or 1940-49.

Table 3. *Estimated mean years of education and estimated mean residual of years of education per country and cohort of birth.*

Country	Cohort of birth	Num. observ.	Years of education	Residuals of years of educ.
Germany	1930-39	633	11.36 (0.091)	-0.243 (0.091)
	1925-29 ∪ 1940-49	893	11.86 (0.084)	0.099 (0.083)
Austria	1930-39	11765	9.18 (0.017)	-0.134 (0.017)
	1925-29 ∪ 1940-49	17383	9.49 (0.015)	0.073 (0.015)

Standard errors in parenthesis.

In order to apply the constrained likelihood maximization presented in Section 2, we assume normality for the outcome distributions This assumption is made accordingly to Imbens and Rubin (1997b) who estimated the return to high school in the United States with quarter to birth as an instrumental variable. Normality for the log of weekly earning was there assumed in order to present a parametric MLE alternative to the standard IV method. Other than the exclusion restriction, the authors imposed also that the variance for not assigned compliers equals that for never-takers and the variance for assigned compliers equals that for always-takers. Table 4 presents the values of the estimated mixing probabilities for the two countries $\hat{\boldsymbol{\phi}}_t = (\hat{\phi}_a, \hat{\phi}_n, \hat{\phi}_c)$, on which the analysis has to be restricted. Units having missing values in the years of education and/or in the hourly earning have been dropped. The resulting sample size is 15434 individuals for Austria, and 1160 for Germany.

Table 4. *Estimated mixing probabilities $\hat{\phi}_t$ per country, $t = a, n, c$.*

Country	$\hat{\phi}_a$	$\hat{\phi}_n$	$\hat{\phi}_c$
Germany	0.7309 (0.0171)	0.2219 (0.0187)	0.0470 (0.0254)
Austria	0.7798 (0.0043)	0.1519 (0.0044)	0.0682 (0.0062)

Standard errors in parenthesis.

The value $\hat{\phi}_c$ in Table 4, estimating the probability of an individual being in the group of compliers, can also be obtained as the difference between the average treatment under $Z_i = 1$ and $Z_i = 0$. A simple t-test on $\hat{\phi}_c$ informs about the causal effect of the supposed randomized instrument on the treatment; we obtain a highly significant result for the t-test on $\hat{\phi}_c$ for Austria (t: 10.58, *s.e.*: 0.0062, p-value: 0.000); for Germany the t-test on $\hat{\phi}_c$ assumes a value of 1.83 corresponding to a p-value of 0.067 (s.e.: 0.0254), then a significant effect but at a level of at least 6.7%.

We have seen in the previous Section that the parameter vector $\mathbf{\Psi}$, in the constrained likelihood function, is identified unless $\hat{\phi}_a = \hat{\phi}_c$ and/or $\hat{\phi}_n = \hat{\phi}_c$; these trivial conditions on the mixing probabilities has been largely refused by likelihood ratio tests for both the countries.

Table 5 presents the results of MLE constrained on $\hat{\phi}_t$; calculations are based on the EM algorithm (Dempster et al., 1977). For Germany the constrained likelihood maximization produces a non-spurious solution whose elements are all significantly different from zero apart from the outcome means for compliers, $\hat{\mu}_{c0}$ and $\hat{\mu}_{c1}$. For Austria, we obtain a non-spurious solution for which all the parameters are significantly different from zero apart from the outcome mean for assigned compliers, $\hat{\mu}_{c1}$.

Table 6 presents the estimated causal effect for each compliance status compared to the estimated causal effect for compliers obtained by applying the IV method under the exclusion restriction (LATE: Local Average Treatment Effect).

For Germany, the estimated LATE assumes a value of -0.1538 but not significantly different from zero (s.e.: 0.6565). Relaxing the exclusion restriction is not sufficient to obtain a significant compliers average causal effect, but produces significant effects for both the non-compliers types; in particular we observe a negative effect for always-takers (-0.0614), and a positive effect for never-takers (+0.1527).

Table 5. *Constrained MLE results per country.*

$\hat{\Psi}$	Germany	Austria
$\hat{\mu}_{a0}$	-0.0872 (0.0317)	-0.0740 (0.0032)
$\hat{\mu}_{a1}$	-0.1487 (0.0154)	-0.0800 (0.0041)
$\hat{\mu}_{n0}$	0.2233 (0.0253)	0.2819 (0.0128)
$\hat{\mu}_{n1}$	0.3761 (0.0514)	0.3501 (0.0123)
$\hat{\mu}_{c0}$	0.3751 (0.2520)	0.3407 (0.0297)
$\hat{\mu}_{c1}$	0.3125 (0.3034)	-0.0446 (0.0344)
$\hat{\sigma}_{a0}$	0.5324 (0.0083)	0.2780 (0.0019)
$\hat{\sigma}_{a1}$	0.2734 (0.0114)	0.2476 (0.0031)
$\hat{\sigma}_{n0}$	0.2692 (0.0202)	0.2914 (0.0094)
$\hat{\sigma}_{n1}$	0.4653 (0.0219)	0.3779 (0.0080)
$\hat{\sigma}_{c0}$	1.0254 (0.1816)	0.4729 (0.0180)
$\hat{\sigma}_{c1}$	1.4665 (0.3747)	0.5114 (0.0220)
# Obs.	1160	15434
LogLik.	-2140.759	-20798.523

Standard errors in parenthesis.

Table 6. *Estimated causal effects for each compliance status from the constrained MLE, and estimated LATE per country.*

	Germany	Austria
$\hat{\mu}_{a1} - \hat{\mu}_{a0}$	-0.0614 (0.0302)	-0.0059 (0.0053)
$\hat{\mu}_{n1} - \hat{\mu}_{n0}$	+0.1527 (0.0573)	+0.0682 (0.0177)
$\hat{\mu}_{c1} - \hat{\mu}_{c0}$	-0.0625 (0.3947)	-0.3853 (0.0456)
LATE	-0.1538 (0.6565)	-0.3006 (0.0720)

Standard errors in parenthesis.

For Austria, the estimated non-parametric LATE assumes a significantly different from zero value of -0.3006 (s.e.: 0.0720). Relaxing the exclusion restriction produces a non-spurious solution that is characterized by a more pronounced significant estimated causal effect for compliers ($\hat{\mu}_{c1} - \hat{\mu}_{c0}$: -0.3853) compared to the LATE, and by a significant positive effect for never-takers ($\hat{\mu}_{n1}-\hat{\mu}_{n0}$: +0.0682). The resulting significant effects for non-compliers can be explained by general equilibrium considerations. In a recent remarkable paper Card and Lemieux (2001), using a model with imperfect substitution between similarly educated workers in different cohort of birth, argued that shifts in the college-high school wage gap reflect changes in the relative supply of highly educated workers across cohorts. The authors argued that the increase in wage gap for younger men in U.S.A., U.K. and Canada in the past two decades is due to the rising of relative demand for college educated labor, coupled with the slowdown in the rate of growth of the relative supply of college educated workers.

Even if Card and Lemieux's (2001) conclusions do not regard causal relationships but only observed wage gap between cohorts, these general equilibrium considerations can justify the violation of the exclusion restriction in our cases. The lower average education in the 1930-39 cohort, as indicated in Table 3, can indeed explain both the positive return to education for never-takers, individuals always high educated under the two different assignments, and the negative return to education for always-takers, individuals always low educated under the two different assignments. Indeed, the exclusion restriction states the instrumental variable has to have only a treatment mediated effect. But given our definition of the variables Z_i and D_i, we know that the different educational levels between cohorts are due only to the compliers behavior. Consequently the value of the instrumental variable, other than providing information regarding the compliers educational choices, also gives information on the relative supplies of differently educated workers in different cohorts. For example considering the individuals born in the 1930-39 period, we know that compliers born in that cohort will be low educated. Therefore, given the invariant educational behaviors of non-compliers, it is reasonable to suppose a decrease in the relative supply of high educated workers compared to the other cohort (1925-29 $\cup$ 1940-49). Consequently it is reasonable to think never-takers would exploit less competitive labor market conditions then increasing their mean outcome, and on the contrary always-takers would experience worse labor market conditions then decreasing their mean outcome.

5 Conclusions

Identifiability problems in ML causal analyses when outcome distributions of various compliance statuses are in the same class have been considered. The main difficulties in this task is due to the presence of mixtures of distributions implying partially identified models. Furthermore, contrary to the traditional mixtures analyses for cluster purposes, a causal likelihood-based analysis suffers from the switching of the mixture component indicators.

We propose to restrict the likelihood maximization to a suitable parameter subspace, in order to exploit the information provided by the set of assumptions usually adopted when identifying causal effects by the IV method. In particular, the proposed constraining subspace is identified by the marginal maximum likelihood estimates of the mixing probabilities. Moreover, for computational purposes and for exploiting the particular incomplete structure of the likelihood a constrained EM algorithm can be easily developed.

An empirical microeconomic example has also been proposed. Supposing normal distributions for the outcome, we estimate the non-compliers cohort of birth effects on earnings (other than the compliers average causal effect) for individuals born in Germany and Austria between 1925 and 1949. The microeconomic context has been suggested by a recent paper of Ichino and Winter-Ebmer (2004).

6 Appendix

The subspace of the bi-dimensional space $(\omega_a,\, \omega_n)$ for which

$$\omega_a^{\#\varsigma(D_i=1,Z_i=0)} \cdot \omega_n^{\#\varsigma(D_i=0,Z_i=1)} = (1-\omega_a-\omega_n)^{\#\varsigma(D_i=1,Z_i=0)\,+\,\#\varsigma(D_i=0,Z_i=1)},$$

can be identified if considering that $\omega_n = 1-\alpha\,\omega_a$, where $\alpha > 1$ in order to satisfy the constraints:

$$0<\omega_n<1;\ 0<\omega_a<1;\ \text{and}\ 0<(1-\omega_a-\omega_n)<1.$$

Consequently:

$$\omega_a^{\#\varsigma(D_i=1,Z_i=0)}.(1-\alpha\,\omega_a)^{\#\varsigma(D_i=0,Z_i=1)} = (1-\omega_a-1+\alpha\,\omega_a)^{\#\varsigma(D_i=1,Z_i=0)\,+\,\#\varsigma(D_i=0,Z_i=1)},$$

$$\left(\frac{1}{\omega_a}-\alpha\right)^{\#\varsigma(D_i=0,Z_i=1)} = (\alpha-1)^{\#\varsigma(D_i=1,Z_i=0)+\#\varsigma(D_i=0,Z_i=1)},$$

and finally

$$\frac{1}{\omega_a} - \alpha = (\alpha - 1)^{\frac{\#\varsigma(D_i=1,Z_i=0)+\#\varsigma(D_i=0,Z_i=1)}{\#\varsigma(D_i=0,Z_i=1)}}.$$

Then the solving subspace is defined by:

$$\omega_a = \left\{(\alpha - 1)^{\frac{\#\varsigma(D_i=1,Z_i=0)+\#\varsigma(D_i=0,Z_i=1)}{\#\varsigma(D_i=0,Z_i=1)}} + \alpha\right\}^{-1},$$

and

$$\omega_n = 1 - \alpha \cdot \left\{(\alpha - 1)^{\frac{\#\varsigma(D_i=1,Z_i=0)+\#\varsigma(D_i=0,Z_i=1)}{\#\varsigma(D_i=0,Z_i=1)}} + \alpha\right\}^{-1},$$

for any $\alpha > 1$.

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