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Labour supply in a polluted world

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Abstract

We consider a simple two period overlapping generations model where agent's welfare depends on three goods: leisure in the first period of life, environmental quality and consumption in the second part.
We assume that the agent considers as negligible his own contribute to environmental deterioration and takes its level as esogenously given (even if he has perfect foresight about it).
Economic activity depletes the renewable natural resource. To react to such depletion, representative agent can increase the labour supply. The growth of economic activity generates a further decline of environmental quality which leads to a further increase of labour supply and so on. Thus, this process causes a change in the pattern of consumption from common free goods to private ones.
We point out that the equilibrium dynamic, described by the individual labor supply and market clearing conditions, may exhibit multiple fixed points corresponding to different level of output.
We analyze the stability condition of fixed points using the geometrical method developed by Grandmont, Pintus and de Vilder (1998), and we find that the equilibria may be indeterminate, and that invariant closed curve and cycle of period 2 may emerge around the steady state.
We also show that the welfare level associated to more productive economy can be Pareto dominated by economy with lower production but with more leisure and a better environmental quality.
Considering the optimal solution of the model, we find out that social planner can Pareto improve the agent's well-being associated to steady state setting labor supply esogenously to a lower level.

1 Introduction

The aim of this paper is to explore the role of environmental variables in the context of individual choices. We introduce a productive overlapping generation model in which many firms behave competitively and individuals live two periods. The agents' utility depends on leisure in the first part of their life, on consume (of a private consumption good) and on environmental quality when old. Environment is deteriorated by economic activity performed in the previous period and to make robust some considerations about well-being, we assume that environment is able to reproduce itself to its natural level every period[1].
In this model, environment is considered as a public good offering free disposal commodities which enter positively in the agents' utility. Some simple classroom examples are clean air, clean water, beaches, woods, etc.[2].
Nowadays, much part of literature on growth with environmental goods is centered to explore the case in which agents internalize the influences of their actions in environmental dynamics. For example, in their seminal paper[3] on the theme, John and Pecchenino (1994) (from now JP) consider an overlapping generations model where [4] the young agents solve their utility maximization problem allocating the earned resources between save and environmental conservative and/or improving expenditures.
Although in the decentralized economy no coordination among agents of different generations generates Pareto non optimal equilibria, the authors obtain a long run positive correlation between environmental quality and economic growth: the more the production rises up, the more agents allocate resources to make environment better.
In contrast with this approach, many empirical works (see later in the section for some references) underline that, it does not exist a specific market for many natural goods. Thus, if they are deteriorated by negative externalities, this event may lead the agents to rise up private consumption for substitute them[5].
In our paper, following Antoci's view, we introduce the next hypothesis on the agent's perception of environment: *a)* he can't influence its quality, alone, *b)* he overlooks the role of his choices on its dynamics. According to this modelling, in individual's allocation problem, each agent considers environmental development, as determined only by macroeconomic variables. Consequently, even perfect fore-

[1] This hypothesis neglects the possibility of cumulating pollution over the time.

[2] In what follows we use environmental quality and environmental good(s) as synonymous, because we consider the first one as an indicator about the amount of the second one.

[3] And on the same line we can find Jhon, Pecchenino,Schimmelpfenning and Schreft(1995), Ono(1996), Ono(2002).

[4] In their model, agents have preferences on consume and environmental quality in old age. Environment is negatively influenced by private consumption but even depends on the choices of previous generations.

[5] Phenomenon known in literature as "defensive expenditures".

seeing its level, he takes it as given[6].
In line with results found in Antoci and Bartolini (1999), this model shows that the scarcity of environmental quality could be an engine of private consumption growth. In fact, even if the steady state level of capital accumulation is esogenous respect agents' choices, considering explicitly the time allocation problem, a worse environmental quality could conduce to a more working and producing life style.
This result is due to the following self-enforcing mechanism: agents react to the scarcity of natural resources shifting to a more private consumption pattern. To make it possible, they allocate more time in working activities, but this reaction leads to a further worsening of environmental quality. Consequently, agents increase the labour supply and the economic activity still more, and the process starts again. This mechanism is empirical confirmed, especially in urban life, by the rise of health expenditures against pollution-related diseases (e.g. asthma and skin diseases), or by the intensive use of mineral water is spite of the tap one, of washing-machine also clothes-dryer (to preserve clothes from smog), the restoration expenses due to polluted air, and many other examples.
To confirm the role and the weight of defensive expenses, focusing attention on the Federal Republic of Germany, in a empirical work, Leipert (1989), finds that the ratio of defensive expenditures in GNP increased from 5.6 to 10 percent between 1970 and 1985[7].
Afterwards, we analyze the local dynamical properties of the model. As it presents a large number of technological, environmental and preferential parameters, to make clear the study, we use the geometrical-graphical method developed by Grandmont and DeVilder (1998) which allows us to characterize the stability properties of the (two dimensional discrete) dynamical system associated to the model.
Even if we consider competitive framework in which every generation consumes only when old, the study points out that the presence of endogenous labour supply and an environmental good create a large set of parameters for which steady state is indeterminate or limit cycles could emerge around steady state. At our knowledge this eventuality represents a new result, because the largest part of literature on business cycles and indeterminacy[8] has researched possible causes only inside of distortions of productive sector: no-perfect competition, scale externalities, and so on.
Finally, we consider well-being associated to the steady states. Analyzing the role of the environmental parameters, we show several significative differences among

[6]In our view, the main difference between JP model and this one is that, in the first, the no coordination happens among different generations, while in the last one is even interior to each single generation.

[7]See also Ostro and Chestnut(1998) and Zaim(1999) for a specific study of the correlation between pollution and healt costs.

[8]That is the presence of different(infinite) paths converging to the same steady state.

the decentralized and Pareto optimal economy. Then, studying a specification of the model in which, in decentralized economy, multiple equilibria exist, we confirm that, usually, well-being is not related to economic development.

The article is structured as follows. In Section 2 we describe the model and the role of environmental parameters on agents' well-being, in Section 3 we analyze the existence of the steady state and some of its properties, in Section 4 we consider its dynamical properties, in Section 5 we present a case of multiple steady states and in Section 6 we consider a possible modification of the description of the environmental dynamic.

2 Model set up

Let us consider a perfectly competitive economy populated by a continuum of size 1 of infinitely-lived atomistic firms and identical non-altruistic agents. Each agent lives for two periods and we assume, for the sake of simpleness, that there is not population-growth and we normalize the population-size to 1.
Each individual is endowed with $l^* > 1$ units of time that he partially supplies to firms during the first period. When old, he consumes the consumption good produced by the firms. The agent has preferences for his leisure in the first part of life and for consumption and environmental quality during the second part[9]. We assume that economic activity performed in the previous period reduces environmental quality[10].
The rest of this section gives a formally description of the model.

2.1 The firms' maximization problem

Considering many homogenous firms which behave competitively, we can consider a unique representative firm. So the gross production at time t, denoted by Y_t, is given by $Y_t = A_t F(L_t, K_t)$, where L_t and K_t are respectively the capital stock and the aggregate labour in the economy at time t. A_t is a productivity scaling parameter that we will use later to guaranty the existence of equilibrium.
We assume that $F(L_t, K_t)$ is homogeneous of degree one, and strictly concave respect to each argument. The production function in intensive form is:

$$y_t \equiv A_t \frac{Y_t}{L_t} = A_t \frac{F(L_t, K_t)}{L_t} = A_t F(\frac{L_t}{L_t}, \frac{K_t}{L_t}) = A_t F(1, \frac{K_t}{L_t}) \equiv A_t f(k_t) \quad (1)$$

where

$$k_t \equiv \frac{K_t}{L_t} \quad (2)$$

[9]There are several purposes for these assumptions. The first one is that many works on endogenous labor supply (Reichlin(1986),Grandmont(1998), Duranton(2001)), and on environmental quality (JP(1994)), developed in OLG model, consider the same framework where consumption (of private good and, eventually, of environmental asset) takes place only in the second part of the life. On the other side, the model seems enough general to understand the possible relations between economic growth and environment. Relaxing some assumptions, the analytical hardness is not accompanied by a true earn of interpretation of results. Anyway we can justify the model imagining that young agents have not money to spend or, in another hypothesis, they are endowed by a fix sum of money able only to a survival consumption. Such sum is given by old individuals so, even if young people are affected by (the scarcity of) environmental quality, they may only undergo it.

[10]See above for a formal definition of economic activity. In literature, other approaches consider degradation of environment as a result of production (Gutierrez (2000)), or consumption (JP(1994)). In the Appendix we develope the first case.

From the strictly concavity of F respect to each arguments it follows that

$$f'(k) < 0 \tag{3}$$

For analytic purposes, we also impose that production function satisfies the standard Inada conditions. Summing up, we consider the following properties fulfilled:

Condition 1 *f is strictly increasing with respect of its argument, C^r for enough high r, strictly concave and* $\lim_{k \to 0^+} f(k) = 0$, $\lim_{k \to 0^+} f'(k) = +\infty$, $\lim_{k \to +\infty} f'(k) = 0$

As we want to concentrate ourselves on the effect of environmental quality within the economy, we do not introduce any distortion on productive sector[11].
Let us indicate with w_t the wage due to workers at time t and R_t the interest rate on assets supplied. Assuming that capital depreciation is complete in a period, the conditions of perfect competition equilibrium are:

$$R_t \equiv \frac{\partial y_t}{\partial k_t} = A_t f'(k_t) \equiv R(k_t) \tag{4}$$

$$w_t \equiv y_t - R_t k_t = A_t(f(k_t) - kf'(k_t)) \equiv w(k_t) \tag{5}$$

and the equilibrium capital accumulation is

$$K_{t+1} = s_t \tag{6}$$

where s_t represents the level of save of young people at time t. Using notation introduced in (2) we can reformulate (6) as

$$k_{t+1} L_{t+1} = w(k_t) L_t \tag{7}$$

2.2 The agent's problem

Let $P_t \in (0, l^*)$ be the fraction of time endowment devoted to leisure at time t, c_t the consumption at time t, and E_t the stock of environmental quality at time t, we formalize the preferences of each agent introducing a utility function $U(P_t, c_{t+1}, E_{t+1})$ which satisfies the following properties:

Condition 2 *$U(P_t, c_{t+1}, E_{t+1})$ is a C^r function, with r sufficient large, time additive, i.e. $U(P_t, c_{t+1}, E_{t+1}) = u(P_t) + \frac{1}{1+\theta} v(\frac{c_{t+1}}{B}, E_{t+1})$, where $\frac{1}{(1+\theta)}$ is a discount factor and B is a scaling parameter. u and v are strictly increasing with respect*

[11] On this issue, Cazzavillan (1999) explores the role of externality in a private sector in a similar OLG-discrete model with an analysis on well-fare and stability properties; Duranton (2001) considers a model where the externality could be engine of positive long run growth.

to each argument (i.e. $u'(.) > 0$, $v_1'(.,.) > 0$, $v_2'(.,.) > 0$), and with strictly negative second order partial derivatives (i.e. $u''(.) < 0$, $v''_{1,1}(.,.) < 0, v''_{2,2}(.,.) < 0$, $v''_{1,2}(.,.) < 0$[12]).

Furthermore, we consider throughout the paper the condition of gross substitution between leisure and consumption fulfilled:

Condition 3 $\frac{v_{1,1}c}{v_1' l} > -1$

It is a plausible assumption[13] even straightened by econometric analysis (e.g. Maddison(1995)).

Each agent considers the wage w_t, the aggregate labour supply L_t as given and he perfectly foresees the interest factor R_{t+1}, and the environmental quality E_{t+1}. Let $l_t = l^* - P_t$ be the individual labour supply, the agent's problem could be formulated as follows:

$$\begin{cases} \max\limits_{s.t} U(l - l_t, c_{t+1}, E^e_{t+1}) \\ \quad c_{t+1} = l_t w_t R_{t+1} \\ \quad E^e_{t+1} = E_{t+1} \\ \quad c_{t+1} \geq 0 \\ \quad 0 \leq l_t \leq l^* \end{cases} \tag{8}$$

It is worth noting that differently from the JP model but according with papers of Antoci, each agent takes the environmental quality as a completely esogenous datum respect to his own allocation problem: even if he perfectly foresees its value (second constraint), he considers that his decisions don't influence it. Stressing the fact that environment is a public good without a market, he has not any direct instrument to improve its level.

Substituting the equality constraints in the utility function the problem could be reformulated as

$$\max_{0 \leq l_t \leq 1} U(l^* - l_t, l_t(f(k_t) - kf'(k_t))f'(k_{t+1}), E_{t+1}) \tag{9}$$

Under (2), there exists a unique solution of the problem and the optimality condition for the interior solution is[14]

$$-u'(l^* - l_t) + \frac{1}{1+\theta} v_1'(\frac{l_t w(k_t) R(k_{t+1}), E_{t+1}}{B}) \frac{w(k_t) R(k_{t+1})}{B} = 0 \tag{10}$$

[12]The assumption of negative mixed derivative allows us to formalize the fact that natural free good could be substituted by private consumption (see above).

[13]An equivalent assumption is used in Cazzavillan and al (1998) and in Cazzavillan and al (2001).

[14]Since now, when it is not necessary, we omit the arguments of functions.

Proposition 4 *Expression (10) implicitly defines a differentiable function* $l_t = l(w(k_t)R(k_{t+1}), E_{t+1})$*, increasing respect to* $z_{t+1} \equiv w(k_t)R(k_{t+1})$*, and decreasing respect to* E_{t+1}.

Proof. *Let us define* $D \equiv (l_t, z_{t+1}, E_{t+1}) = -u_{P_t}(l^* - l_t) + \frac{1}{1+\theta} v_1'(l_t z_{t+1}, E_{t+1}) z_{t+1}$.

$D(l_t, z_{t+1}) = 0$ *satisfies the hypothesis of the implicit function theorem, in fact we have that*

$$\Delta = \frac{\partial D(l_t, z_{t+1})}{\partial l_t} = u''_{P_t}(l^* - l_t) + \frac{1}{1+\theta} v_1''\left(\frac{l_t z_{t+1}, E_{t+1}}{B}\right) z_{t+1}^2 < 0, \forall l_t, z_{t+1}$$

So it is

$$\frac{\partial l_t}{\partial E_{t+1}} = \frac{1}{1+\theta} \frac{v''_{1,2}\left(\frac{l_t z_{t+1}, E_{t+1}}{B}\right) \frac{z_{t+1}}{B}}{-\Delta} < 0$$

$$\frac{\partial l_t}{\partial z_{t+1}} = \frac{1}{1+\theta} \frac{v''_{1,1}\left(\frac{l_t z_{t+1}, E_{t+1}}{B}\right) z_{t+1} + v_1'\left(\frac{l_t z_{t+1}, E_{t+1}}{B}\right)}{-\Delta} > 0$$

where numerator signs follow from condition (2). ■

The meaning of the previous proposition is that the shifting to a more labouring life-style could be caused by the increase of productivity parameter z_t, but even by the depletion of the environment which induces agents to work more to substitute the reduced natural resources with private consumption. In such a case environmental depletion represents a push factor respect to economic growth in contrast with the common idea for which this could cause a limit. The importance of this result will be cleared when we'll consider the implications on well-being and on the significance of growth.

2.3 Environmental dynamics

Even if environment introduced in the last subsection is not influenced by the choices of the single agent, the economic activity performed in the previous period reduces its value. We assume the evolution of environment described by the following function

$$E_{t+1} = E(\overline{E}, L_t, f(k_t)) \tag{11}$$

with these properties:

Condition 5 *E is a* C^r *function with r sufficient large, which satisfies the following properties:*

1. $\frac{\partial E(\cdot)}{\partial L_t} < 0, \frac{\partial E(\cdot)}{\partial f(k_t)} < 0, \frac{\partial E(\cdot)}{\partial \bar{E}} > 0$;

2. $\lim_{L_t \to 0, f(k_t) \to 0} E(.) = \bar{E} > 1$, where $\bar{E}$ is the maximum value of E i.e. the natural endowment of the natural resource;

3. $\lim_{L_t \to l^*, f(k_t) \to +\infty} E(\cdot) = a$ (possible $-\infty$);

This way to describe the dynamics[15] of environmental quality is quite optimistic because it excludes the accumulation of the negative effects of economic activity on environment. Everyday, Nature is able to reproduce itself to its original level, and to supply the same flow of free disposal goods and services. A similar hypothesis is used also in Antoci et Al. (1999), and it is an interesting one, because inimical respect to results about undesirability of high economic performances shown in the Subsection 3.3.
Note that, in this model, in case of a general increase of labor supply, environment could represent a pressure mechanism for the individual to make his own working time higher. Substituting (11) into individual labour supply we obtain the following proposition.

Proposition 6 *Let the interior solution defined in proposition (4) holds, then* $\forall L_t, k_t$

$$\frac{\partial l_t}{\partial L_t} = \frac{\partial l_t}{\partial E_{t+1}} \frac{\partial E_{t+1}}{\partial L_t} > 0 \tag{12}$$

This result is similar to those found in models in which positional consumption (or relative income) is introduced: if many agents have high- working (and so high private consumption) life-style, this behavior creates a push effect on the single decision.

3 Equilibria and Steady states

3.1 Symmetric Nash Equilibrium conditions

We now turn to consider the problem of the existence and multiplicity of equilibrium. In line with much part of literature, we consider the Symmetric Nash Equilibrium (SNE) condition in which the choice about the time allocation of the agent is the same of the average one (Cazzavillan et Al(1998(, Duranold (2001)).

Definition 7 *A competitive perfect foresight (Symmetric Nash) equilibrium for the economy is an infinite sequence* $\{k_t, L_t\}_{t=0}^{\infty}$, *such that, at each date* $t = 1, 2, ..., +\infty$

[15]Examples of function of this kind are

$$E_{t+1} = \frac{\overline{E}}{k^{\alpha} + L}$$

or

$$E_{t+1} = \overline{E} - k^{\alpha} - L^{\beta}$$

These formulations could capture differentiated impact on environment by labour and capital. For example we may imagine two economies with the same level of output but a different production structure: the first, more capital intensive, the second, more labour intensive. In the first case, the impact of big infrastructures (highways, nuclear plants, etc.) may have a worse impact on nature.

$$l(R(k_{t+1})w(k_t), E(L_t, k_t)) = L_t \tag{13}$$

$$k_{t+1}L_{t+1} = w(k_t)L_t \tag{14}$$

Remark 8 *Without further restrictive conditions we can't exclude cases for which labour equilibrium condition (13) defines a corner solution. Anyway, for the problem of multiplicity of equilibria, the case $L_t = l(.,.) = 1$ could be assimilated to the case in which the interior first-order condition holds, and the existence of steady state is studied in (15).*
If $L_t = l(\cdot) = 0$ (14) is indeterminate (and no well defined considering that $k = \frac{K}{L}$). This case is not so interesting because it describes a situation in which agents don't work at all deriving their subsistence only by the consumption of natural resources. Such a case could be considered as a poverty due to a threshold to growth: agents are too lazy because of excessive generosity of Nature.

From (13) it is now possible to define the equilibrium labour supply.

Lemma 9 *If $\frac{\partial l}{\partial E}\frac{\partial E}{\partial L} \neq 1$ equation (13) implicitly defines a locally C^r function $L_t = \lambda(k_{t+1}, k_t)$*

Proof. Let $D(k_t, k_{t+1}, L_t) \equiv l(R(k_{t+1})w(k_t), E(L_t, k_t)) - L_t = 0$. From implicit function theorem, it follows that a sufficient condition to have the result is that
$\frac{\partial D(k_t,k_{t+1},L_t)}{\partial L_t} = \frac{\partial l}{\partial E}\frac{\partial E}{\partial L} - 1 \neq 0$ ■
As we are interested to work with "regular maps" we impose the following condition:

Condition 10 *Where explicitly it is not specified differently we consider the condition $\frac{\partial l}{\partial E}\frac{\partial E}{\partial L} \neq 1$ fulfilled in correspondence of steady state values.*

It is now possible to point out some properties of the equilibrium labour supply. To make clearer the results,in what follows, we consider explicitly the individual labor supply function defined in Proposition 4.

Proposition 11 *If $\frac{\partial l}{\partial E}\frac{\partial E}{\partial L} < 1$ then it is.*

$$\frac{\partial \lambda}{\partial z} > 0, \frac{\partial \lambda}{\partial \overline{\overline{E}}} < 0 \tag{15}$$

Instead if $\frac{\partial l}{\partial E}\frac{\partial E}{\partial L} > 1$ then it is

$$\frac{\partial \lambda}{\partial z} < 0, \frac{\partial \lambda}{\partial \overline{\overline{E}}} > 0 \tag{16}$$

The first part of the proposition describes the "normal" case in which the individual reactions have the same directions of macroeconomic tendencies. Instead, the second case arises if the reaction of the individual would be reversed, at macroeconomic level, by the change about expectations. For example an increase of the natural amenities induces the individual to work less, but the whole result may be reversed because of the falling down of the expected general labour supply.

3.2 Existence and multiplicity of steady states

In order to show the existence and multiplicity of steady state it is useful to express (13),(14) in a unique second-order non linear difference equation

$$k_{t+1}\lambda(k_{t+2},k_{t+1}) = w(k_t)\lambda(k_{t+1},k_t) \tag{17}$$

It is now natural step to introduce the following definition.

Definition 12 *A steady state equilibrium is a constant value k such that*

$$\phi(k) = k - w(k) = 0 \tag{18}$$

Remark 13 *In this model, as in Raichlin one, the existence and multiplicity of steady states (with positive labour supply) depends only by technological properties of production function. Anyway, differently by Raiclihn, the labour supply depends on environment too, which modifies the steady state gross production.*

Remark 14 *In this model we have to exclude the steady state without capital because if k=0 it follows that $w(0) = 0$[16]. In this case agents prefer not to work, but $L = 0$ is not allowed because k=$\frac{K}{L}$ is not defined.*

As in many OLG models (e.g. Nourry(2001)) we have to impose specific assumptions to obtain the existence of the steady state equilibrium. We state the following result.

Proposition 15 *Under Condition 1 on production function*
1) If $\lim_{k\to 0^+} \phi'(k) < 0$ then there exist interior steady states and generically the number of the steady-state is odd;
2) If $\lim_{k\to 0^+} \phi'(k) > 0$ then the number of interior steady state is generically odd and can be 0.

Proof. Let $\phi(k) \equiv k - w(k)$
Firstly, because of Condition 1, it is

[16] See the following proposition.

$$\lim_{k \to +\infty} \frac{f(k)}{k} = \lim_{k \to +\infty} f'(k) = 0; \tag{19}$$

and so
$\lim_{k \to +\infty} \phi(k) = \lim_{k \to +\infty} k(1 - \frac{f(k)-kf'(k)}{k}) = \lim_{k \to +\infty} k(1 - \frac{f(k)}{k} + f'(k)) = +\infty$
1) Remembering that $\phi(0) = 0$, if $\lim_{k \to 0^+} \phi'(k) < 0$, there exists k such that $\phi(k) = 0$ and generically the number of intersection is odd (we exclude the no robust cases in which $\phi(k)$ is tangent to k axis).
2)With similar arguments it is proved the second part of proposition. ∎

3.3 Cobb-Douglas example: part 1 (well-being of the agents associated to steady state solution)

We now present a really simple example that allows us to have the explicit solution of the model and that it will be considered even in the next section to show its stability properties.
We consider the following functional specifications

$$U(l^* - l_t, C_{t+1}, E_{t+1}) = \log(l^* - l_t) + \frac{1}{1+\theta} \log(\frac{c_{t+1}}{B} + E_{t+1}) \tag{20}$$
$$F(k_t, L_t) = ALk^\alpha, \alpha(0,1) \tag{21}$$
$$E_{t+1} = \overline{E} - \eta L_t - d(Ak_t^\alpha)^\beta \tag{22}$$

and to simplify calculation we suppose $d = 1/A^\beta$
As the model is not able to generate perpetual growth because of the diminishing return of the capital labour ratio, and because of the assumption of no population growth. A sufficient condition to have a well defined maximization problem for the agent is that $\overline{E}$ is enough large. In this specification $1/B$ is the marginal rate of substitution between C and E.
The competitive equilibrium conditions, the optimal allocation conditions and SNE conditions define the following equations (we are assuming that interior solution $l \in (0, l^*)$ holds for the agent's problem).

$$
\begin{aligned}
w_t &= A(1-\alpha)k_t^{\alpha} && (23)\\
R_t &= A\alpha k_t^{\alpha-1} && (24)\\
l_t &= (l^* - (1+\theta)\frac{E_{t+1}B}{A^2\alpha(1-\alpha)k_{t+1}^{\alpha-1}k_t^{\alpha}})\frac{1}{2+\theta} && (25)\\
L_t &= [l^* - (1+\theta)\frac{(\overline{E}-k_t^{\alpha\beta})B}{A^2\alpha(1-\alpha)k_{t+1}^{\alpha-1}k_t^{\alpha}}] \times && (26)\\
&\times \frac{A^2\alpha(1-\alpha)k_{t+1}^{\alpha-1}k_t^{\alpha}}{(2+\theta)A^2\alpha(1-\alpha)k_{t+1}^{\alpha-1}k_t^{\alpha} - \eta B(1+\theta)}
\end{aligned}
$$

And imposing the (interior) steady state condition we find

$$k = [A(1-\alpha)]^{\frac{1}{1-\alpha}}$$

$$
\begin{aligned}
L &= [l^* - (1+\theta)\frac{(\overline{E}-k^{\alpha\beta})B}{A^2\alpha(1-\alpha)k^{2\alpha-1}}] \times && (27)\\
&\times \left[\frac{A^2\alpha(1-\alpha)k^{2\alpha-1}}{(2+\theta)A^2\alpha(1-\alpha)k^{2\alpha-1} - \eta B(1+\theta)}\right]
\end{aligned}
$$

If the expressions in the brackets are both positive, then L react positively if E decreases (see figure).

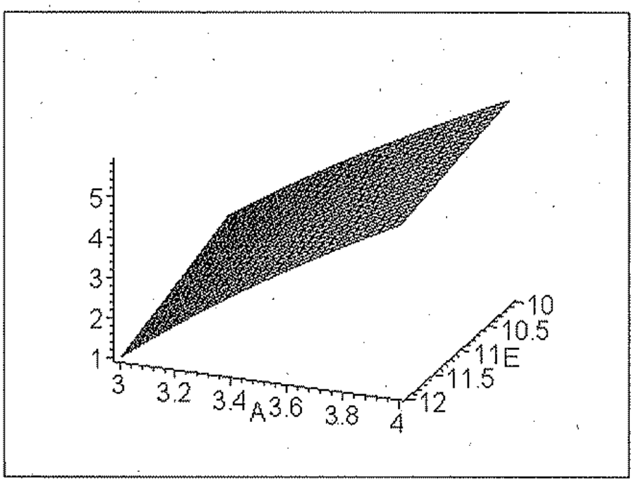

Figure 1: Steady state labour supply as a function of $\overline{E}$, and A, $\alpha = 1/3, \theta = 0.7, l^* = 7, B = 1$

In the next table we fix some economic parameters at a standard value and we show the role of η and β to define the steady state value of the model. In every cell it is reported a vector in which it is considered, respectively, the labour supply, the environmental quality, and the utility of the agent[17].
Let $A = B = 1, \alpha = (1/3), \theta = 0.012, l^* = 100, \beta = 1/2$.

$\overline{E}$	$\eta = 0.01, B = 0.5$	$\eta = 0.02, B = 0.5$	$\eta = 0.01, B = 0.6$	$\eta = 0.01, B = 0.9$
8	(43.54; 6.66; 7.40)	(43.95; 6.21; 7.39)	(42.30; 6.67; 7.26)	(38.35; 6.71; 6.97)
7	(44.47; 5.65; 7.37)	(44.89; 5.19; 7.35)	(43.42; 5.66; 7.23)	(40.23; 5.69; 6.92)
6	(45.41; 4.64; 7.33)	(45.83; 4.17; 7.32)	(44.54; 4.65; 7.19)	(41.92; 4.67; 6.87)
5	(46.34; 3.63; 7.30)	(46.78; 3.16; 7.28)	(45.66; 3.63; 7.14)	(43.61; 3.66; 6.81)
4	(47.27; 2.62; 7.27)	(47.72; 2.14; 7.25)	(46.78; 2.62; 7.10)	(45.30; 2.64; 6.76)
3	(48.21; 1.61; 7.23)	(48.66; 1.12; 7.21)	(47.90; 1.61; 7, 06)	(46.99; 1.62; 6.70)

This table shows, as mentioned in the first paragraph, the (no-intuitive) role of natural resources about the growth of economic activity: if the rate of substitution between private and environmental good is high, and/or the economic activity depletes heavily the environment, it is possible that the environmental scarcity pushes positively the economic growth. As far as the utility for the agents is concerned, this reaction is not able to counterbalance the loss of natural free disposal goods[18].

3.3.1 The centralized steady state solution

In this paragraph we introduce a central planner who wants to maximize the utility of the agents associated to steady state solution and we suppose that he knows the impact of labour on the environment. Furthermore, he has the possibility to impose the amount of weekly working hours (or more exactly the working hours for unity of time). Analytically, the central planner's problem is

$$\begin{cases} \max\limits_{s.t.} U(l^* - L, c, E) = \log(l^* - L) + \frac{1}{1+\theta}\log(\frac{c}{B} + E) \\ k = [A(1-\alpha)]^{\frac{1}{1-\alpha}} \\ E = \overline{E} - \eta L_t - (k_t^{\alpha})^{\beta} \end{cases}$$

Substituting the equality constraint in the utility function and imposing the optimality condition we find the optimal labour supply.

$$L^1 = \left[l^* - B(1+\theta)\left(\frac{E - [A(1-\alpha)]^{\frac{\alpha\beta}{1-\alpha}}}{A^2(1-\alpha)\alpha[A(1-\alpha)]^{\frac{2\alpha-1}{1-\alpha}} - B\eta}\right)\right] \frac{1}{2+\theta} \qquad (28)$$

[17] For all values considered the equilibrium interior solution of agent's problem is preferred to the corner one.

[18] The same result could be obtained varying β. For example, the decrease of β from 1/2 to 1/3, (getting $\eta = 1, B = 0.5$ and $\overline{E} = 8$),and the consequent worsening of environmental quality (it goes from 6.66 to 6.62) cause a rise of labour supply which goes from 43.54 to 43.57

For a large range of parameters it is $L^1 < L$ defined in (28). We state the following proposition about it

Proposition 16 *If the expressions in square brackets of (27) and (28) are positive and* $A > (\frac{B\eta}{\alpha(1-\alpha)^{\frac{\alpha}{1-\alpha}}})^{1-\alpha}$ *then* $L^1 < L$.

Proof. It is sufficient to introduce the following function

$$H(x) = \left[l^* - B(1+\theta)\left(\frac{E - [A(1-\alpha)]^{\frac{\alpha\beta}{1-\alpha}}}{A^2(1-\alpha)\alpha[A(1-\alpha)]^{\frac{2\alpha-1}{1-\alpha}} - B\eta x}\right)\right] \times \tag{29}$$

$$\frac{A^2(1-\alpha)\alpha[A(1-\alpha)]^{\frac{2\alpha-1}{1-\alpha}}}{A^2(1-\alpha)\alpha[A(1-\alpha)]^{\frac{2\alpha-1}{1-\alpha}}(2+\theta) - (1-x)\eta B(1+\theta)} \tag{30}$$

H coincides to L for $x = 0$, and to L^1 for $x = 1$ and it is decreasing with respect to x. It is in fact

$$H'_x = -\left[B(1+\theta)\left(\frac{(E - [A(1-\alpha)]^{\frac{\alpha\beta}{1-\alpha}})B\eta}{\left(A^2(1-\alpha)\alpha[A(1-\alpha)]^{\frac{2\alpha-1}{1-\alpha}} - B\eta x\right)^2}\right)\right] \times \tag{31}$$

$$\times \left[\frac{A^2(1-\alpha)\alpha[A(1-\alpha)]^{\frac{2\alpha-1}{1-\alpha}}}{A^2(1-\alpha)\alpha[A(1-\alpha)]^{\frac{2\alpha-1}{1-\alpha}}(2+\theta) - (1-x)\eta B(1+\theta)}\right] + \tag{32}$$

$$-\frac{A^2(1-\alpha)\alpha[A(1-\alpha)]^{\frac{2\alpha-1}{1-\alpha}}(\eta B(1+\theta))}{\left(A^2(1-\alpha)\alpha[A(1-\alpha)]^{\frac{2\alpha-1}{1-\alpha}}(2+\theta) - (1-x)\eta\beta(1+\theta)\right)^2} \times \tag{33}$$

$$\times \left[l^* - B(1+\theta)\left(\frac{E - [A(1-\alpha)]^{\frac{\alpha\beta}{1-\alpha}}}{A^2(1-\alpha)\alpha[A(1-\alpha)]^{\frac{2\alpha-1}{1-\alpha}} - B\eta x}\right)\right] \tag{34}$$

where all expressions in squared brackets are positive. ∎
If we calculate the optimal labour supply using the same values of the previous parametrization we find that it is ever $L_1 < L$[19].

[19] We have considered the value truncated at the second decimal digit, it follows that some values of the two tables seem the same. Anyway the environment and labour supply in decentralized economy are respectively lower and larger than in the centralized one

$\overline{E}$	$\eta = 0.01, B = 0.5$	$\eta = 0.02, B = 0.5$	$\eta = 0.01, B = 0.6$	$\eta = 0.01, B = 0.9$
8	$(43.02; 6.66; 7.40)$	$(42.89; 6.23; 7.39)$	$(41.65; 6.67; 7.26)$	$(37.49; 6.72; 6.99)$
7	$(43.96; 5.65; 7.37)$	$(43.83; 5.21; 7.35)$	$(42.78; 5.66; 7.23)$	$(39.21; 5.70; 6.93)$
6	$(44.90; 4.64; 7.33)$	$(44.81; 4.20; 7.32)$	$(43.92; 4.65; 7.19)$	$(40.93; 4.68; 6.88)$
5	$(45.84; 3.63; 7.30)$	$(45.77; 3.18; 7.28)$	$(45.07; 3.64; 7.15)$	$(42.65; 3.66; 6.82)$
4	$(46.73; 2.63; 7.27)$	$(46.73; 2.16; 7.25)$	$(46.19; 2.63; 7.10)$	$(44.37; 2.65; 6.76)$
3	$(47.72; 1.61; 7.23)$	$(47.69; 1.14; 7.21)$	$(47.34; 1.62; 7.06)$	$(46.09; 1.63; 6.70)$

(35)

4 Existence and dynamical properties of the normalized steady state

For the stability analysis of the model we find conditions guaranteeing that a normalized steady state exists and persists, varying locally some parameters of the model.
We precede as in Cazzavillan (2001): we fix a normalized steady state for which $(L, K, E) = (1, 1, 1)^{20}$ and, in a constructive way, we find some conditions on parameters to guarantee its existence.

Theorem 17 *Let $A = A^*$, $B = B^*$ (defined in the proof), if*
$\lim_{B \to +\infty} \frac{1}{1+\theta} v_1'(\frac{R(1)}{A^*B}, E(1,1,E^*))\frac{R(1)}{A^*B} < u'(l^*-1) < \lim_{B \to 0} \frac{1}{1+\theta} v_1'(\frac{R(1)}{A^*B}, E(1,1,E^*))\frac{R(1)}{A^*B}$
then there exists the normalized steady state $(L, K, E) = (1, 1, 1)$.

Proof We prove this assertion in three steps

1. In accumulation equilibrium condition valued at steady state, $Lw(k) - Lk = 0$, we fix $A = A^*$ to have $A(f(1) - f'(1)) - 1 = 0$, i.e.
$$A^* = \frac{1}{(f(1) - f'(1))} \tag{36}$$
2. In equation (48) we fix $\overline{E} = \overline{E}^*$, to have $E(1, 1, \overline{E}^*) = 1$.
3. The last step is to verify the existence of a particular value of B in correspondence of $L = K = E = 1$, such that the time allocation of agent is $l = 1$. Let us consider the (agent's) optimality condition:
$$\frac{1}{1+\theta} v_1'(\frac{R(1)}{A^*B}, E(1,1,E^*))\frac{R(1)}{A^*B} = u'(l^* - 1) \tag{37}$$

[20] Even if it would be sufficient to fix only k and leave (L, E) free, in order to give a clearer economical explanation of results, we prefer to have (L, K, E)=(1,1,1).

varying B, the right side of the equation (37) is constant and positive, while the left one is increasing respect to B (see condition (3)), then it is sufficient to impose the further condition

$$\begin{array}{l}\lim_{B\to+\infty}\frac{1}{1+\theta}\ v_1'(\frac{R(1)}{A^*B},E(1,1,E^*))\frac{R(1)}{A^*B}<u'(l^*-1)<\\ <\lim_{B\to 0}\frac{1}{1+\theta}\ v_1'(\frac{R(1)}{A^*B},E(1,1,E^*))\frac{R(1)}{A^*B}\end{array} \tag{38}$$

to have the result. ■.

4.1 Dynamics around the (normalized) steady state

Now we consider local dynamics around the (normalized) steady-state. Equation in (7) defines implicitly a two-dimensional discrete map.
In view of Hartman-Grobman theorem, to study the local stability properties, we linearize (17) in the neighborhood of normalized steady state[21]. To make economically clear the next analysis, we introduce the following notation: $\varepsilon_{l,z}, \varepsilon_{l,E}, \varepsilon_{E,k}, \alpha, \delta$, respectively the elasticity of the individual labour supply respect to z (i.e. the relative cost of leisure), the elasticity of the individual labour supply respect to the environment, the elasticity of environment respect to the capital/labour factor, the capital share respect to total income ($i.e.\ \alpha=\frac{f'k}{f}$),the elasticity of capital/labour substitution in the production function ($i.e.\ \delta=(1-\frac{f'k}{f})\frac{-f'}{f''k}$). We underline that we are referring to the individual labour supply instead of the aggregate labour supply, because we want to explicit the specific role of environment and private sector on stability properties[22].

Proposition 18 *Let $\varepsilon_{l,z}, \varepsilon_{l,E}, \varepsilon_{E,k}, \alpha, \delta$ all evaluated at an interior steady state value.*
The Jacobian matrix associated to (17) evaluated at an interior steady state (and so even at normalized one) is :

$$J=\begin{pmatrix}\frac{1}{1-\alpha}(1+\frac{(1-|\varepsilon_{l,E}||\varepsilon_{E,L}|)\delta}{|\varepsilon_{l,z}|})+\frac{\delta\alpha}{1-\alpha}\frac{|\varepsilon_{l,E}||\varepsilon_{E,f}|}{|\varepsilon_{l,z}|} & -\frac{\alpha}{1-\alpha}(1+\frac{1-|\varepsilon_{l,E}||\varepsilon_{E,L}|}{|\varepsilon_{l,z}|})-\frac{\alpha\delta}{1-\alpha}\frac{|\varepsilon_{l,E}||\varepsilon_{E,f}|}{|\varepsilon_{l,z}|}\\ 1 & 0\end{pmatrix} \tag{39}$$

with the following determinant and trace
$D(J)=D_1+\frac{\delta\alpha}{1-\alpha}\frac{|\varepsilon_{l,E}||\varepsilon_{E,f}|}{|\varepsilon_{l,z}|}$

[21]The study of the case $L>L^*$ is enough simple because the model collapses into a standard one sector model in which stability of equilibria could be showed in the plane k_t, k_{t+1}. Such stability depends on slope of w when it intersects the bisecting line of first quadrant.
[22]The expression of the aggregate labour supply does not clarify the different roles of environment and private sector on stability properties.

$T(J) = T_1 + \frac{\delta\alpha}{1-\alpha}\frac{|\varepsilon_{l,E}||\varepsilon_{E,f}|}{|\varepsilon_{l,z}|}$

where $D_1 = \frac{\alpha}{1-\alpha}(1 + \frac{1-|\varepsilon_{l,E}||\varepsilon_{E,L}|)}{|\varepsilon_{l,z}|})$, $T_1(\delta) = \frac{1}{1-\alpha}(1 + \frac{(1-|\varepsilon_{l,E}||\varepsilon_{E,L}|)\delta}{|\varepsilon_{l,z}|})$

Proof. Let $\omega \equiv -\frac{f'}{kf''}$ (and so $\delta = (1-\alpha)\omega$) the elasticity of the demand for capital respect to the rental rate evaluated at steady state. Note that in steady state, being all wage saved, it is that $f' = \frac{f'k}{k} = \frac{f'k}{w} = \frac{\alpha}{1-\alpha}$.
So considering $L_t = l_t(R(k_{t+1})w(k_t), E(L_t, f(k_t))$ it is

$\frac{\partial L_t}{\partial k_{t+1}} = \frac{\partial L_{t+1}}{\partial k_{t+2}} = -\frac{\frac{|\varepsilon_{l,z}|}{\omega}\frac{L}{K}}{1-|\varepsilon_{l,E}||\varepsilon_{E,L}|}$

$\frac{\partial L_t}{\partial k_t} = \frac{\partial L_{t+1}}{\partial k_{t+1}} = \frac{\alpha\left[\frac{|\varepsilon_{l,z}|}{(1-\alpha)\omega} + |\varepsilon_{l,E}||\varepsilon_{E,f}|\right]}{1-|\varepsilon_{l,E}||\varepsilon_{E,L}|}\frac{L}{k}$

By means of change of variable $Q_t = K_{t+1}$ the dynamical system becomes

$$\begin{array}{c} Q_t\lambda(Q_{t+1}, Q_t) - w(k_t)\lambda(Q_t, k_t)) = 0 \\ k_{t+1} = Q_t \end{array} \tag{40}$$

Let $H(Q_{t+1}, Q_t, k_t) \equiv Q_t\lambda(Q_{t+1}, Q_t) - w(k_t)\lambda(Q_t, k_t))$
The derivatives of Q_{t+1} respect to Q_t, and k_t , using implicit function theorem, are

$$\frac{dQ_{t+1}}{dQ_t} = -\frac{\frac{\partial H}{\partial Q_t}}{\frac{\partial H}{\partial Q_{t+1}}} \tag{41}$$

$$\frac{dQ_{t+1}}{dk_t} = -\frac{\frac{\partial H}{\partial k_t}}{\frac{\partial H}{\partial Q_{t+1}}} \tag{42}$$

Tedious but straightforward calculation and some algebraic artifices lead to the following expressions of partial derivatives (all evaluated at steady state)

$\frac{\partial H}{\partial Qt+1} = -k(\frac{\frac{|\varepsilon_{l,z}|}{\omega}\frac{L}{K}}{1-|\varepsilon_{l,E}||\varepsilon_{E,L}|})$

$\frac{\partial H}{\partial Qt} = L + k(\frac{\alpha\left[\frac{|\varepsilon_{l,z}|}{(1-\alpha)\omega} + |\varepsilon_{l,E}||\varepsilon_{E,f}|\right]\frac{L}{k}}{1-|\varepsilon_{l,E}||\varepsilon_{E,L}|}) + k\frac{\frac{|\varepsilon_{l,z}|}{\omega}\frac{L}{k}}{1-|\varepsilon_{l,E}||\varepsilon_{E,L}|}$

$\frac{\partial H}{\partial k_t} = -\frac{\alpha\left[\frac{|\varepsilon_{l,z}|}{(1-\alpha)\omega} + |\varepsilon_{l,E}||\varepsilon_{E,f}|\right]\frac{L}{k}}{1-|\varepsilon_{l,E}||\varepsilon_{E,L}|}k - \frac{\alpha}{\omega(1-\alpha)}L$

The substitution in (41) and (42) and a collection of the terms gives the result.
■

In line with other recent papers with endogenous labour supply, this model presents stability property of steady state which could be different respect to classical saddle path stability. In the economic models like this, where a unique

variable is predetermined (the initial condition $k = k_0$) and the other (l_0) is determined by agents' choices, the standard result is that convergence takes place on the stable (one dimensional) manifold, that is the eigenvalues of Jacobian matrix calculated at steady state values are, the Gauss-plane, respectively one inside and one outside of the circle centered in $(0, 0)$ with ray equal to 1.
Instead, in this model, the two eigenvalues could be both inside, and in such a case the equilibrium is a sink, or they could be outside and so the steady state is repulsive.
The first case, known in literature as indeterminate steady state, has an interesting economic implication: even if the model shows a unique attractive steady state and the initial conditions are the same, it follows that the convergence of different countries to equilibrium rises on many (infinite) paths characterized by different economic and well-being performances. This result is essentially due to the different expectations of the agents about the variables at the successive time[23].
A formal definition of such a case is (see Nourry and Venditti 2001).

Definition 19 *Let consider equation (17), and let* $\{k_t\}_{t=0}^{+\infty}$ *an equilibrium for the economy with a given initial condition* k_0.
It is locally indeterminate if for every $\varepsilon > 0$ *there exists an other sequence* $\{k'_t\}_{t=0}^{+\infty}$ *such that* $|k'_1 - k_1| < \varepsilon$ *and* $k'_0 = k_0$ *which is an equilibrium.*

Instead, the second case implies that even if the static formulation of the problem admits an equilibrium, it could not be attained for dynamical structure of the model.

4.2 Geometrical method and stability properties of the (normalized) steady state

It is now possible to study local stability properties of steady state using the geometrical method developed by Grandmont et Al (1998).
It allows us to investigate through a simple graphical analysis in the plane, $(Trace, Determinant)$ the role of the significative parameters of the model about the stability of steady state.
The procedure aims at studying the variation of the trace and the determinant i.e., respectively, the sum and the product of the roots of characteristic polynomial, $P(x) = x^2 - Tx + D$, varying continuously a significative parameter, said variational parameter.
Considering $|\varepsilon_{E,f}|$ as the variational parameter and varying it from 0 to infinity, $\Delta \equiv (T(J), D(J))$ defines a positively sloped half line (the slope of Δ is $+1$) starting from (T_1, D_1) when $|\varepsilon_{E,f}| \to 0$, and it approaches to $(+\infty, +\infty)$ when

[23] See Farmer (1993) for the possible links of these results with the Keynesian theory of the animal spirits.

$|\varepsilon_{E,f}| \rightarrow +\infty$.
The planar expression of half-line is

$$D = \frac{\alpha}{1-\alpha}\left(1 + \frac{1 - |\varepsilon_{l,E}|\,|\varepsilon_{E,L}|)}{|\varepsilon_{l,z}|\,(1-\alpha)}\right) + (T - T_1) \quad (43)$$

$$T \geq T_1 \quad (44)$$

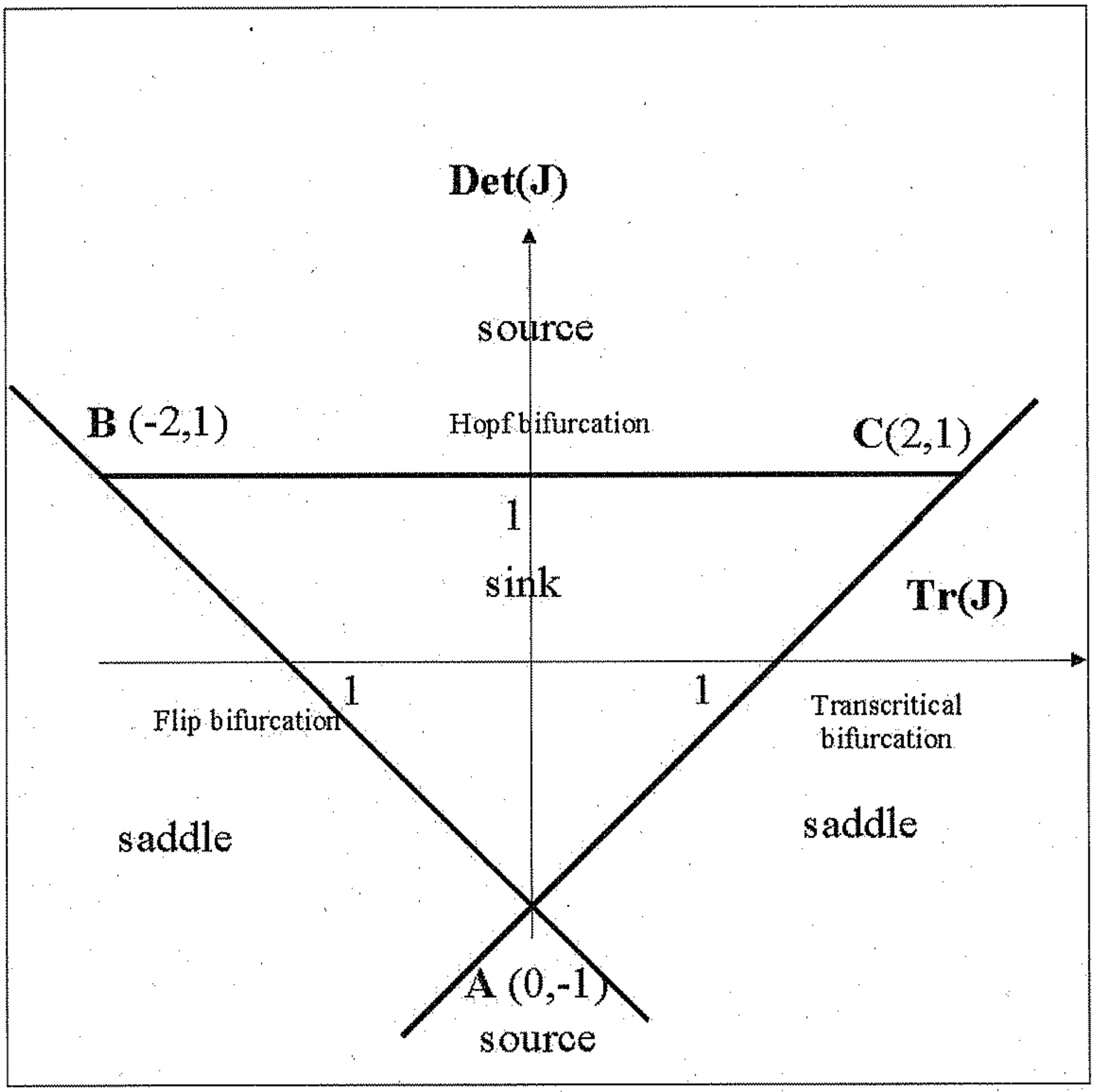

Figure 2: The Geometrical Method

On the line AC defined as $1 - T + D = 0$ an eigenvalue is equal to 1; on the line AB defined as $1 + T + D = 0$ an eigenvalue is equal to -1; on the side BC

defined as $D = 1$ and $T \in (-2, 2)$ eigenvalues are complex with modulus equal to 1.
Inside the triangle ABC with vertices $A(0, -1), B(-2, 1), C(2, 1)$ eigenvalues have modulus less than one, and so the steady state is attractive (sink). On the right of the line AC and on the left of the line AB an eigenvalue is less than one and the other is higher and so the steady state is saddle-stable; in the other two open regions, both eigenvalues have modulus higher than one and so the steady state is repulsive (source).
When the point of line Δ intersects the line AB the dynamical system undergoes a flip bifurcation and a two-period cycle emerges. When the point of half-line Δ intersects the segment BC, the map, generically undergoes a Neimark-Hopf bifurcation and an invariant closed curve appears. Finally, if the half line Δ intersects the line AC, generically, the system has an exchange of stability between the analyzed steady state (in our case the normalized steady state $(1, 1)$) which persists varying the parameter (according to our assumptions), and the other one, through a transcritical bifurcation.
In our case where a unique variable is predetermined inside the triangle ABC the steady state is indeterminate and endogenous stochastic fluctuations can emerge around the steady state and around cycle generated by flip or Hopf bifurcation when attractive[24].
The locoes of points $\Delta_1 = (T_1(|\varepsilon_{l,z}|), D_1(|\varepsilon_{l,z}|))$ is an half-line with $slope(\Delta_1) = \frac{\alpha}{\delta} \in (0, +\infty))$ that starts (getting $|\varepsilon_{l,z}| \to +\infty$) from a point H located in the first quadrant on the line $T - D - 1 = 0$ and goes to $(+\infty, +\infty)$ (respectively $(-\infty, -\infty)$) if $|\varepsilon_{l,E}|\,|\varepsilon_{E,L}| < 1$ (if $|\varepsilon_{l,E}|\,|\varepsilon_{E,L}| > 1$).
To make more suitable the study of local properties of the steady state, according to empirical studies we do the following assumption (see Seegmuller(2001)) on capital share.

Condition 20 $\alpha < \frac{1}{2}$

By this assumption H is on to section of the segment AC in the first quadrant. With respect to the value of $slope(\Delta_1)$ we find these cases:

Lemma 21 *Let consider* $|\varepsilon_{l,E}|\,|\varepsilon_{E,L}| < 1$,

1. If $\delta > \alpha$ the slope of half-line $\Delta_1, \frac{\alpha}{\delta}$, is less then the half-line AC and so Δ_1 is located outside the triangle ABC;
2. if $\delta = \alpha$ then $Slope(\Delta_1) = Slope(AB) = 1$ and so the two half-lines have the same direction;
3. if $0 < \delta < \alpha$ then $Slope(\Delta_1) > 1$ and so Δ_1 cuts the segment BC for $|\varepsilon_{l,z}| = \left|\varepsilon_{l,z}^{BC}\right|$

[24]See Grandmont (1998).

Lemma 22 *Let consider* $|\varepsilon_{l,E}|\,|\varepsilon_{E,L}| > 1$ *these cases are possible*

1. If $\delta > \alpha$ the slope of half-line $\Delta_1, \frac{\alpha}{\delta}$, is less then the half-line AC and so Δ_1 crosses the triangle ABC and it cuts the line AB; for $|\varepsilon_{l,z}| = |\varepsilon_{l,z}^{AB}|$

2. if $\delta = \alpha$ then $Slope(\Delta_1) = Slope(AB) = 1$ and so the two half-lines have the same direction;

3. if $0 < \delta < \alpha$ then $Slope(\Delta_1) > 1$ and Δ_1 lies outside the triangle and cuts the line AB for $|\varepsilon_{l,z}| = |\varepsilon_{l,z}^{AB}|$

Using geometrical considerations, we obtain the following results summarizing the main properties of normalized steady state:

Theorem 23 *Indicate with* $|\varepsilon_{E,f}^{AC}|, |\varepsilon_{E,f}^{BC}|$ *the value of* $|\varepsilon_{E,f}|$ *for which* Δ *respectively intersects the line* AC *and the late* BC*. and let* $|\varepsilon_{l,E}|\,|\varepsilon_{E,L}| < 1$

1. Assume that $\delta > \alpha$, then the steady state is a saddle for every value of $|\varepsilon_{l,z}|$ and $|\varepsilon_{E,f}|$

2. Assume that $\delta < \alpha$
   if $|\varepsilon_{l,z}| > |\varepsilon_{l,z}^{BC}|$then for $|\varepsilon_{l,f}| < |\varepsilon_{l,f}^{BC}|$ the steady state is a sink (indeterminate) and for $|\varepsilon_{l,f}| > |\varepsilon_{l,f}^{BC}|$ it is a source; if $|\varepsilon_{l,z}| < |\varepsilon_{l,z}^{BC}|$ then for every value of $|\varepsilon_{E,f}|$ the steady state is a source. For $|\varepsilon_{l,f}| = |\varepsilon_{l,f}^{BC}|$ the steady state undergoes a Niemark-Hopf bifurcation and, generically a non-periodic limit cycle arises.

Theorem 24 *Indicate with* $|\varepsilon_{E,f}^{AC}|, |\varepsilon_{E,f}^{BC}|$ *the value of* $|\varepsilon_{E,f}|$ *for which* Δ *respectively intersects the line* AC *and the side* BC*. and let* $|\varepsilon_{l,E}|\,|\varepsilon_{E,L}| > 1$

1. Assume that $\delta < \alpha$, if $|\varepsilon_{l,z}| < |\varepsilon_{l,z}^{AB}|$ then for $|\varepsilon_{l,f}| < |\varepsilon_{l,f}^{AB}|$ the steady state is a source and for $|\varepsilon_{l,f}| > |\varepsilon_{l,f}^{AB}|$ it is a saddle; if $|\varepsilon_{l,z}| > |\varepsilon_{l,z}^{AB}|$ the steady state is a saddle;

2. Assume that $\delta > \alpha$
   if $|\varepsilon_{l,z}| > |\varepsilon_{l,z}^{AB}|$ then for $|\varepsilon_{l,f}| < |\varepsilon_{l,f}^{BC}|$ the steady state is a sink (indeterminate) and for $|\varepsilon_{l,f}| > |\varepsilon_{l,f}^{BC}|$ it is a saddle. For $|\varepsilon_{l,f}| = |\varepsilon_{l,f}^{BC}|$ the steady state undergoes a Niemark-Hopf bifurcation and, generically a non-periodic limit cycle arises;.
   if $|\varepsilon_{l,z}| < |\varepsilon_{l,z}^{AB}|$ then for $|\varepsilon_{l,f}| < |\varepsilon_{l,f}^{AB}|$ the steady state is a saddle; for

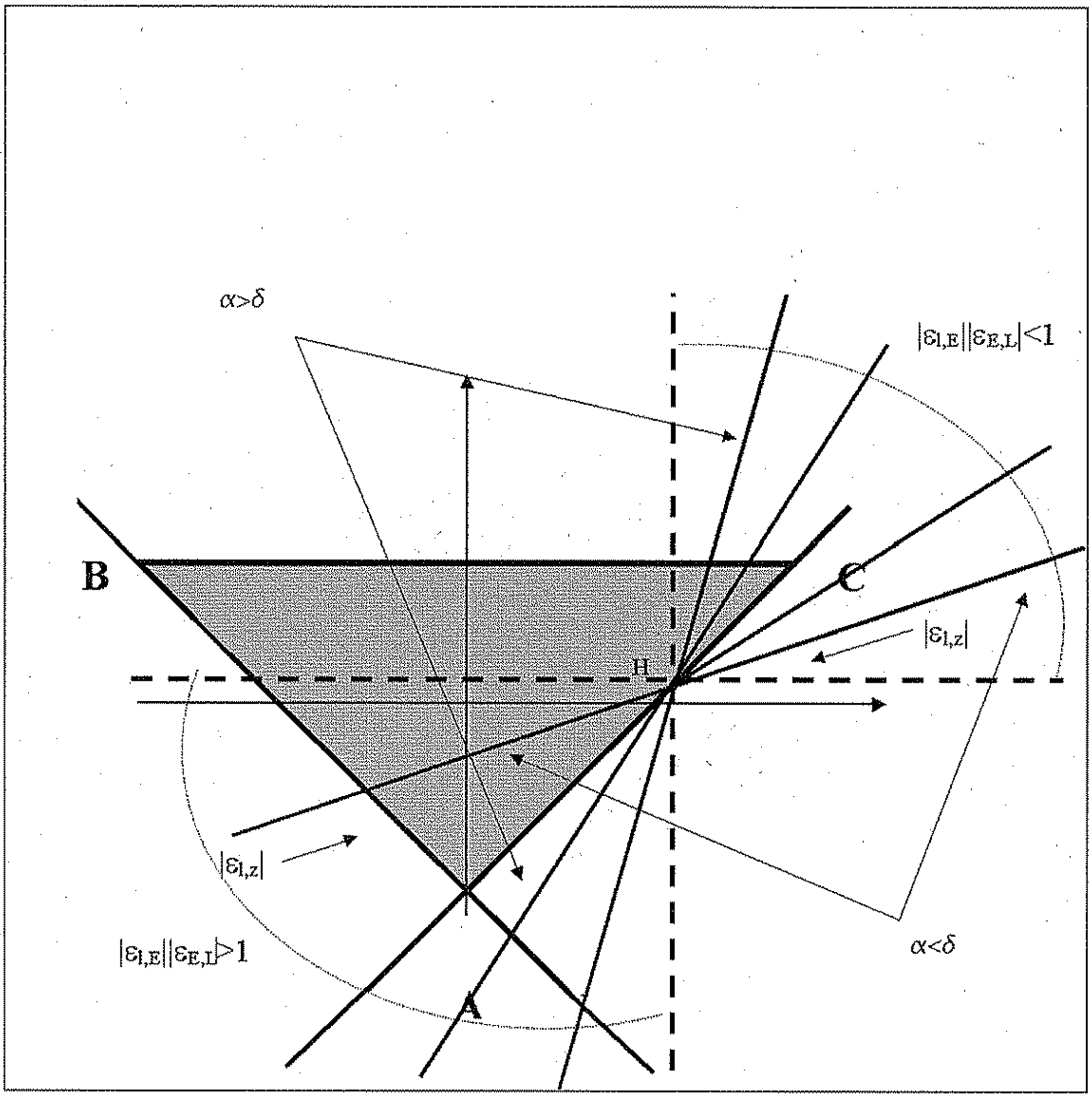

Figure 3: the half-line Δ_1 in the (T, D) plane

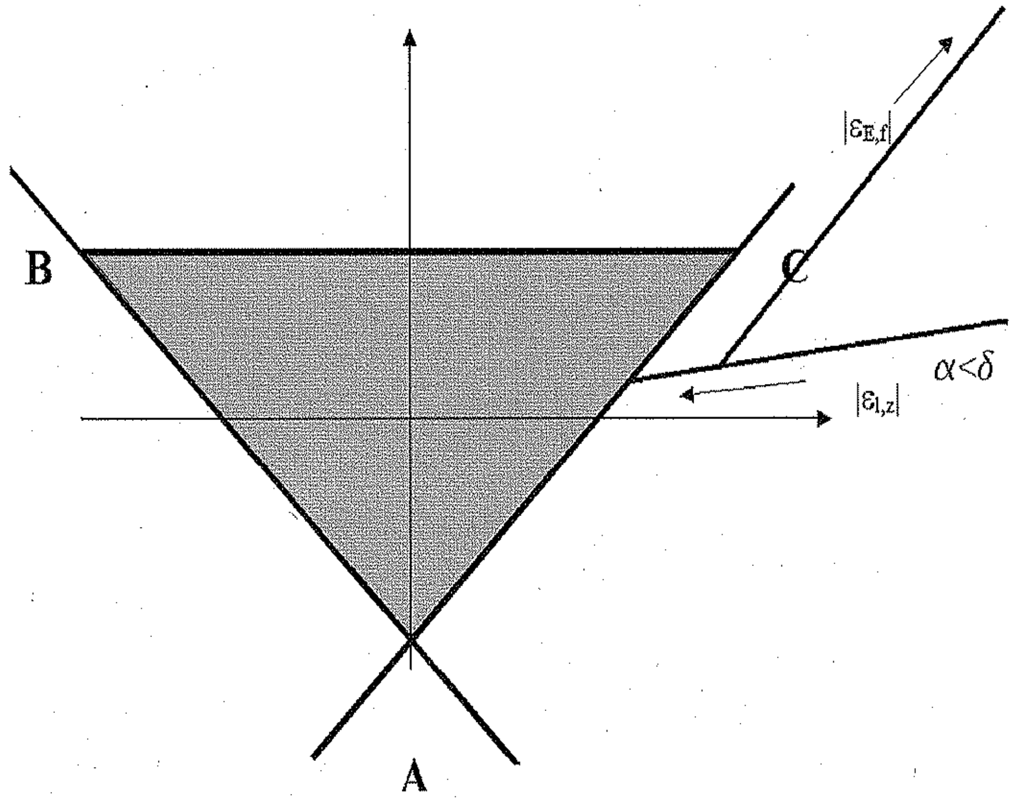

Figure 4: Case $|\varepsilon_{l,E}|\,|\varepsilon_{E,l}| < 1$ and $\alpha < \delta$

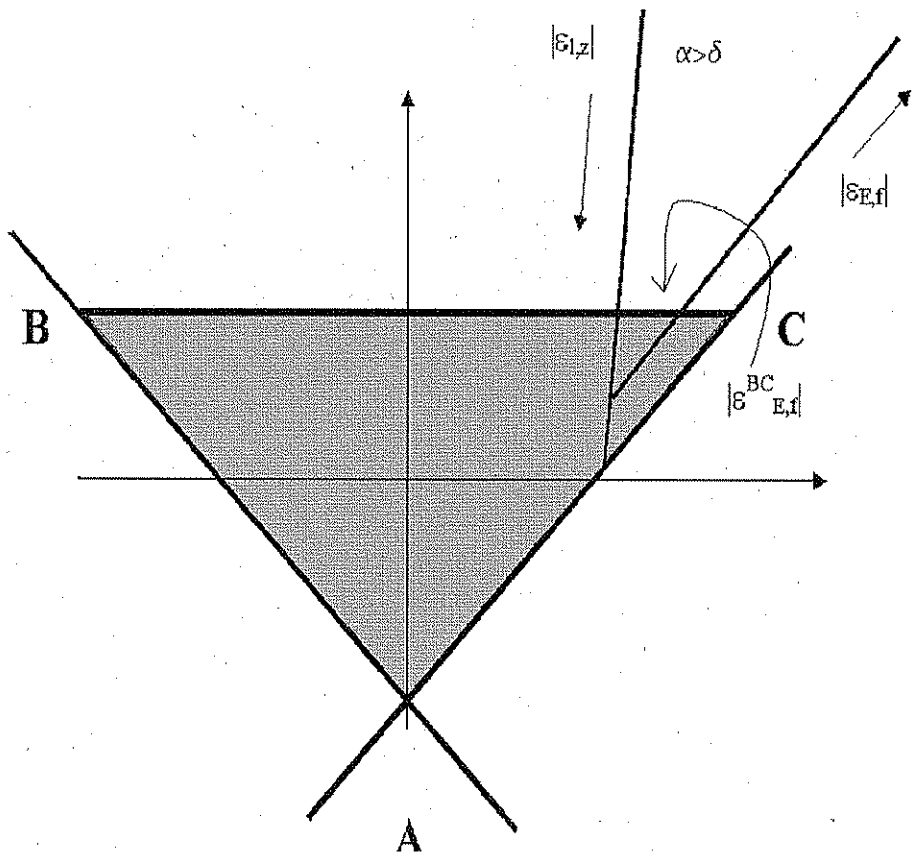

Figure 5: Case $|\varepsilon_{l,E}|\,|\varepsilon_{E,l}| < 1$ and $\alpha > \delta$

$\left|\varepsilon_{l,f}^{AB}\right|<|\varepsilon_{l,f}| < \left|\varepsilon_{l,f}^{BC}\right|$ it is a sink (indeterminate); for $|\varepsilon_{l,f}| < \left|\varepsilon_{l,f}^{BC}\right|$ it is a source. For $|\varepsilon_{l,f}| = \left|\varepsilon_{l,f}^{AB}\right|$the steady state undergoes a flip bifurcation and, generically, a two-period cycle arises, and for $|\varepsilon_{l,f}| = \left|\varepsilon_{l,f}^{BC}\right|$ the steady state undergoes a Niemark-Hopf bifurcation and, generically a non-periodic limit cycle arises;

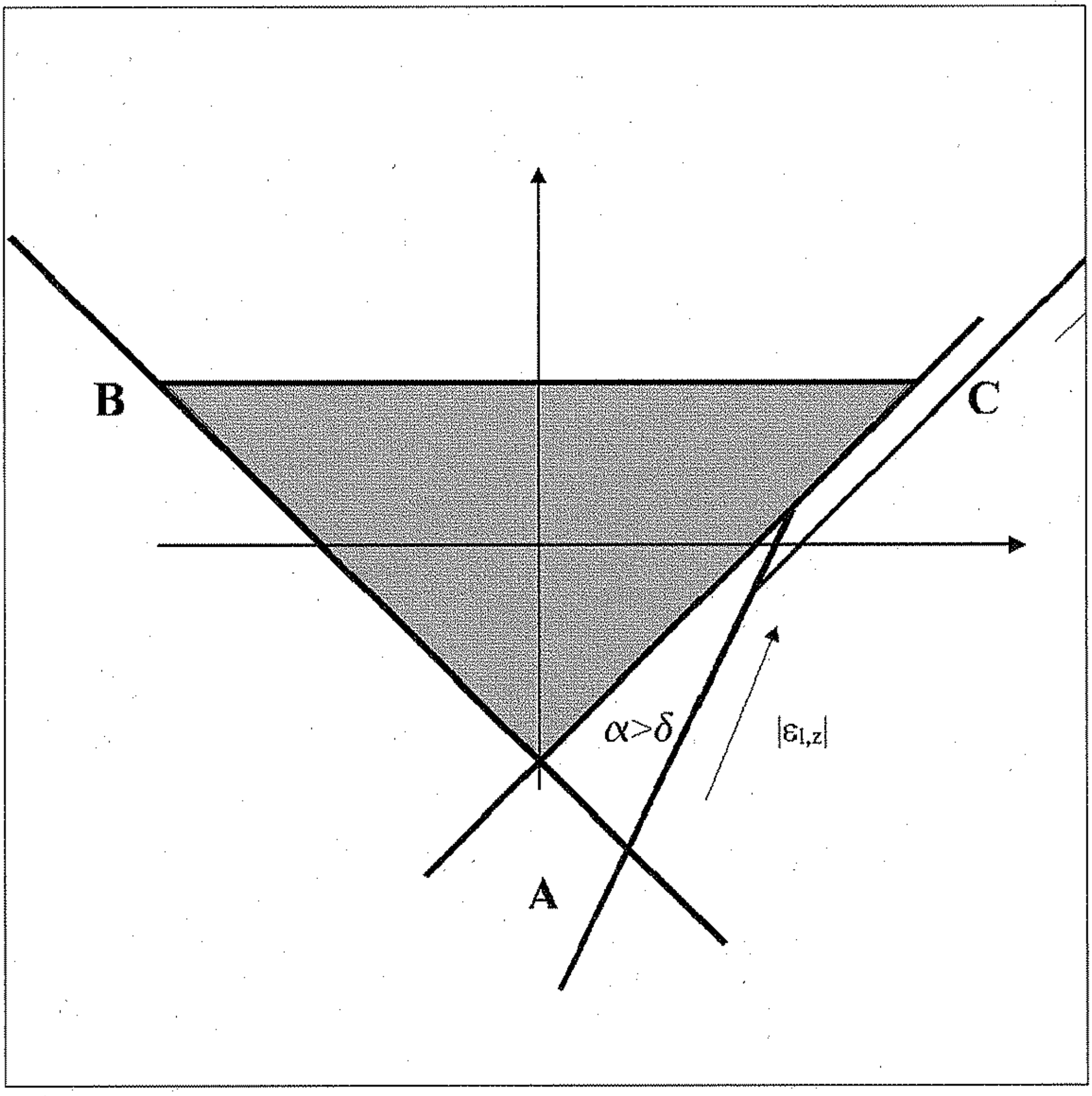

Figure 6:

Remark 25 *What we have shown in this analysis is that if the agents are influenced by environment, then interesting dynamics take place and that we cannot*

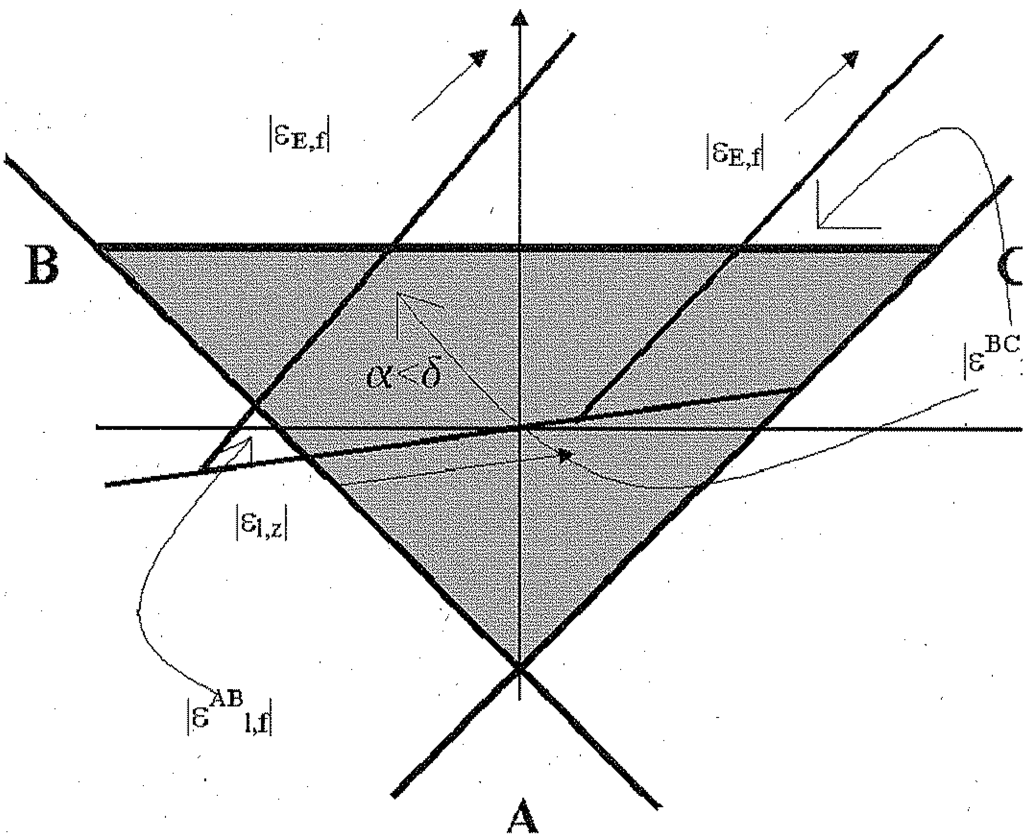

Figure 7:

exclude cases in which (deterministic or stochastic) oscillations around steady state or different paths of convergence (indeterminacy) could emerge (for a large set of technological parameters values). In particular, even if there is a high substitution between productive factors, $\delta > \alpha$, the presence of a moderate elasticity of environmental quality respect to (intensive) production could create closed invariant curve around steady state.

4.3 Cobb-Douglas example: part 2

We show the procedure exposed on Cobb-Douglas example.

4.3.1 Steady state normalization

From $w(k) - k = 0$ we find that the equation is fulfilled for $k = 1$ if and only if $A = A^* = \frac{1}{1-\alpha}$. In this case $wR = \frac{\alpha}{1-\alpha}$, $\frac{kf'}{f} = \alpha$, $\delta = 1$. For $E = K = 1$ the agent chooses $l = 1$ (see expression (25)) if and only if $B = B^* = (l^* - (2 + \theta))\frac{\alpha}{1-\alpha}\frac{1}{1+\theta} > 0$; for $L = K = 1$ it is that $E = 1$ (see equation (22))if and only if $\overline{E} = \overline{E}^* = 2 + \eta$. It is simple calculation to find that the associated elasticities are $|\varepsilon_{E,f}| = \beta$, $|\varepsilon_{E,L}| = \eta$, $|\varepsilon_{l,E}| = \frac{(l^*-(2+\theta))}{2+\theta}$, $|\varepsilon_{l,z}| = (l^* - (2 + \theta))\frac{1-\alpha}{\alpha(2+\theta)}$. The Jacobian matrix and determinant and trace are

$$J = \begin{pmatrix} \frac{1}{1-\alpha}\left(1 + \frac{(1-\eta\frac{(l^*-(2+\theta))}{2+\theta})}{(l^*-(2+\theta))\frac{1-\alpha}{\alpha(2+\theta)}}\right) + \frac{\alpha}{1-\alpha}\beta\frac{(l^*-(2+\theta))}{2+\theta} & \frac{-\alpha}{1-\alpha}\left(1 + \frac{(1-\eta\frac{(l^*-(2+\theta))}{2+\theta})}{(l^*-(2+\theta))\frac{1-\alpha}{\alpha(2+\theta)}}\right) - \frac{\alpha}{1-\alpha}\beta\frac{(l^*-(2+\theta))}{2+\theta} \\ 1 & 0 \end{pmatrix} \quad (45)$$

$$D(J) = \frac{\alpha}{1-\alpha}\left(1 + \frac{(1-\eta\frac{(l^*-(2+\theta))}{2+\theta})}{(l^*-(2+\theta))\frac{1-\alpha}{\alpha(2+\theta)}}\right) + \frac{\alpha}{1-\alpha}\beta\frac{(l^*-(2+\theta))}{2+\theta}$$

$$T(J) = \frac{1}{1-\alpha}\left(1 + \frac{(1-\eta\frac{(l^*-(2+\theta))}{2+\theta})}{(l^*-(2+\theta))\frac{1-\alpha}{\alpha(2+\theta)}}\right) + \frac{\alpha}{1-\alpha}\beta\frac{(l^*-(2+\theta))}{2+\theta}$$

So, varying β while all other parameters are fixed, we find the stability of the steady state and the bifurcation value: for example let $\alpha = 1/4$, $\theta = 0.7$, $l^* = 5$, $\eta = 1.5$ we find that for $\beta = 2.4754$ the two eigenvalues are complex conjugate and they have modulus equal to one.

5 Pareto ranked equilibria

The possibility of multiple steady state, and the dynamical properties in the decentralized economy have a great impact on the well-being analysis of the model because it creates Pareto ranked equilibria, and the convergence to the better one could be not guaranteed. In fact, two possibilities could arise: in the first the best steady state could be repulsive; in the second one, starting enough

close to a worst attractive equilibrium, the economy could be trapped by it. Typically, when endogenous labour supply is considered, the result is that the steady state with lower working time is Pareto dominated by the other one (see Duranton, Cazzavillan). On the contrary, Antoci et Al(1999) present cases in which the result is reversed because of the environmental externality.
In line with these works, we present an example to show that high economic performance is not synonymous of high well-being condition.
In order to have the possibility of multiple steady state, we introduce a more general production function. We then consider the same specification of utility function and of environmental dynamic of the previous example.
In this section we always consider

$$
\begin{aligned}
U(l^* - l_t, C_{t+1}, E_{t+1}) &= \log(l^* - l_t) + \frac{1}{1+\theta}\log(\frac{C_{t+1}}{B} + E_{t+1}) && (46)\\
f(k) &= A(\theta^1 k^{-\gamma} + 1 - \theta^1)^{-\frac{1}{\gamma}} && (47)\\
E_{t+1} &= \overline{E} - \eta L_t - ((\theta^1 k^{-\gamma} + 1 - \theta^1)^{-\frac{1}{\gamma}})^{\beta} && (48)
\end{aligned}
$$

with $\gamma > -1, \theta^1 \in (0,1)$.
From (48) we find that

$$
\begin{aligned}
w(k_t) &= A(1-\theta^1)(\theta^1 k_t^{-\gamma} + 1 - \theta^1)^{-\frac{1+\gamma}{\gamma}} && (49)\\
R(k_{t+1}) &= A\theta^1 k_{t+1}^{-1-\gamma}(\theta^1 k_t^{-\gamma} + 1 - \theta^1)^{-\frac{1+\gamma}{\gamma}} && (50)
\end{aligned}
$$

We want the triplet $(k, L, E) = (1,1,1)$ to be a steady state. Imposing the normalization procedure we have that $A = A^* \equiv \frac{1}{1-\theta^1}$,and $w(1) = 1, R(1) = \frac{\theta^1}{1-\theta^1}, \overline{E} = 2 + \eta$, $B = B^* =(l^* - (2+\theta))\frac{\theta^1}{(1-\theta^1)(1+\theta)} > 0$

Theorem 26 *Consider 49.and A=A**
i) There exists one and only one steady state if and only if $\gamma \leq 0$
ii)For $\gamma > 0$ *there exist generically two distinct steady states:* $k = 1$ *and* $k = k_1$*.If* $\gamma \in (0, \gamma^*)$ *then* $k_1 < 1$*, if* $\gamma >, \gamma^*$ *then* $k_1 > 1$

Proof. Consider the function $X(k) = (\theta^1 k^{-\gamma} + 1 - \theta^1)^{-\frac{1+\gamma}{\gamma}} - k$, its zeros define k values of steady state. For the previous analysis we know that $k = 1$ is a zero
It is convenient to rewrite $X(k)$ in this way
$X(k) = k(\frac{k^\gamma}{(\theta^1+(1-\theta^1)k^\gamma)^{\frac{1+\gamma}{\gamma}}} - 1)$
and consider $\widetilde{X(k)} = \frac{k^\gamma}{(\theta^1+(1-\theta^1)k^\gamma)^{\frac{1+\gamma}{\gamma}}} - 1$ and its derivative
$\widetilde{X'(k)} = \frac{\gamma k^{\gamma-1}(\theta^1+(1-\theta^1)k^\gamma)^{\frac{1+\gamma}{\gamma}} - (1+\gamma)(1-\theta^1)(\theta^1+(1-\theta^1)k^\gamma)^{\frac{1}{\gamma}}k^{2\gamma-1}}{\left((\theta^1+(1-\theta^1)k^\gamma)^{\frac{1+\gamma}{\gamma}}\right)^2}$;

if $\gamma \leq 0$, then $X(k)$ is a monotone function and $k = 1$ is the unique zero;
if $\gamma > 0$ then $\lim_{k \to +\infty} \widetilde{X(k)} = -1$, and $\lim_{k \to +0} \widetilde{X(k)} = -1$. It follows that $X(k)$ is first decreasing and then increasing, so we have generically two distinct steady states. Because $\widetilde{X'(1)} = 1 - \theta^1 - \gamma\theta^1 > 0 \iff \gamma < \frac{1-\theta^1}{\theta^1} = \gamma^*$ it follows the second part of the theorem. ■

To avoid cases in which, for high accumulation of capital, the possibility of consumption of agents falls down[25],we impose the next assumption:

Condition 27 $\frac{\partial w(r)R(k)}{\partial k} > 0$.

Let consider the case *ii)* in(26). It is interesting to calculate the well-being of the two steady states. The following graphic shows that there isn't a positive link between the high economic performance and well-being: in the gray-area the steady state with more capital accumulation and gross production is dominated by the other one.

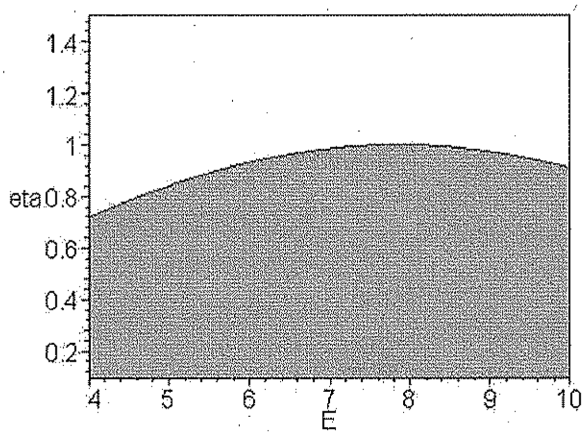

Figure 8: $\theta_1 = 0.2, \theta = 0.3, \gamma = 1.3, l^* = 100, \eta = 0.5\ \beta = 0.5, E = 10$

[25]It is due to the specification of model in which consumption is done only in old age and the total quantity of consumption is given by $w(k)R(k)$, where $w(k)$ is increasing in k, while $R(k)$ is decreasing.

6 Conclusions

The present contribution sheds light on the interplay between economic growth and environmental resources, in a scenario in which agents overlook the weight of their decisions. In contrast to the enormous literature subsequent to JP work, the model shows that environmental quality and economic growth may have opposite directions. In fact, agents may defend themselves from the scarcity of environmental resources, by shifting to a more producing life-style. Because utility of the agents depends on both environmental and private consume, the whole effect on agents' well-being is not obvious, and the Pareto criterion may recommend to decrease the economic activity.
For the same reasons, considering the case in which multiple steady states arise in decentralized economy, we may be witness of cases in which the more producing steady state is Pareto-inferior.
We have also shown that the presence of environmental variable in the agent's time allocation may generate interesting dynamical results. Specifically, the model present a large set of parameters for which environment is engine of indeterminacy and limit cycles, and so it creates different growth paths and fluctuations in the economy.
The model could be extended in different ways. Obviously, consumption and environmental quality could be introduced in both stages of agents' life. Preliminary studies seems to show that the multiplicity of interplays with different directions make the study too hard to be considered in its generality, and numerical simulations are necessary.
We think that the most interested analysis could be done, considering the environment even in production function. In such a case the scarcity of natural resources may have negative consequences in private sector and it may generate a more intensive shift to labouring life style because of the necessity to increase the human side of production.

7 Appendix

In this section we consider a modification of the model in which the level of environmental quality is determined by the production function. So it is

$$E_{t+1} = E(\overline{E}, F(K_t, L)) \tag{51}$$

The construction of the model is very similar to the previous one, and so we consider directly its dynamical properties. We have the following proposition:

Proposition 28 *Let $z, |\varepsilon_{l,z}|, |\varepsilon_{l,F,}|, \alpha, \delta$ respectively $w(k)R(k)$, the elasticity of the labour supply respect to z, the elasticity of the labour supply respect to production function (in no-intensive expression), the capital share,elasticity of substitution among productive factors, where all parameters are evaluated at steady state value.*
The Jacobian matrix associated to the dynamical system and valued at the steady state is

$$J = \begin{matrix} \left(\frac{1}{1-\alpha} + \frac{\delta}{(1-\alpha)|\varepsilon_{l,z}|}\right) - \delta\frac{|\varepsilon_{l,F}|}{|\varepsilon_{l,z}|} & \left(-\frac{\alpha}{1-\alpha} - \frac{\alpha}{|\varepsilon_{l,z}|(1-\alpha)}\right) + \alpha\left(\frac{1-\delta}{1-\alpha}\right)\frac{|\varepsilon_{l,F}|}{|\varepsilon_{l,z}|} \\ 1 & 0 \end{matrix}$$

with the following determinant and trace[26]*.*

$D(J) = D_1 - \alpha\left(\frac{1-\delta}{1-\alpha}\right)\frac{|\varepsilon_{l,F}|}{|\varepsilon_{l,z}|}$

$T(J) = T_1 - \delta\frac{|\varepsilon_{l,F}|}{|\varepsilon_{l,z}|}$

where $D_1 = \left(\frac{\alpha}{1-\alpha} + \frac{\alpha}{|\varepsilon_{l,z}|(1-\alpha)}\right), T_1(\delta) = \left(\frac{1}{1-\alpha} + \frac{\delta}{(1-\alpha)|\varepsilon_{l,z}|}\right)$

We consider the half-line $\Delta = (T(J), D(J))$ and we denote with

$$S(\delta) = \frac{\alpha}{1-\alpha}\frac{1-\delta}{\delta} \tag{52}$$

its slope.
The locus of points $\Delta_1 = (D_1(|\varepsilon_{l,z}|), T_1(|\varepsilon_{l,z}|))$ is a half-line positive sloped (analytic value is $\frac{\alpha}{\delta} \in (0, +\infty)$) that starts from a point H located in the first quadrant on the line $T - D - 1 = 0$.
We consider the gross substitution condition fulfilled. We divide analysis in subcases respect to value of δ.
It follows immediately comparing the two values of slope this preliminary result:

Lemma 29 *1. If $\delta < \alpha$ then the slope of half-line Δ_1, $\frac{\alpha}{\delta}$, is less then the slope of half-line Δ, $\frac{\alpha}{\delta}\frac{1-\delta}{1-\alpha}$;*

2. if $\delta = \alpha$,then $Slope(\Delta_1) = Slope(\Delta) = 1$

[26]Likewise to the first model, we have to impose that $|\varepsilon_{l,F}| \neq 1$.

3. *if* $\alpha < \delta < 1$, *then* $Slopen(\Delta_1) > Slope(\Delta) > 0$

4. *if* $\delta = 1$ *then* $Slope(\Delta) = 0$

5. *if* $\delta > 1$,*then* $-1 < Slope(\Delta) < 0$

Using geometrical properties we obtain next results that summarize main properties of normalized steady state:

Theorem 30 *Indicate with* $\left|\varepsilon_{l,f}^{AC}\right|, \left|\varepsilon_{l,F}^{AB}\right|, \left|\varepsilon_{l,F}^{BC}\right|$ *the value of* $|\varepsilon_{l,F}|$ *for which* Δ *respectively intersects the line AC, AB, and side BC.*

1. *Assume that* $\alpha = \delta$. *The half-line belongs on half-line AC and for* $|e_{l,f}| \in (0, \varepsilon_{l,F}^{AB})$ *the steady state is a saddle; for* $|e_{l,F}| > \left|\varepsilon_{l,F}^{AB}\right|$ *it is a source; for* $|e_{l,f}| = \left|\varepsilon_{l,f}^{AB}\right|$ *it undergoes a flip bifurcation;*

2. *Assume that* $\alpha > \delta$
i) Consider $|\varepsilon_{l,z}|$ *large enough, for* $|e_{l,F}| \in (0, \varepsilon_{l,f}^{AC})$ *the steady state is a sink (indeterminate); for* $|e_{l,F}| \in (\left|\varepsilon_{l,F}^{AC}\right|, \left|\varepsilon_{l,F}^{BC}\right|)$ *it is a saddle (and so the steady state is determinate); for* $|e_{l,F}| > \left|\varepsilon_{l,f}^{AC}\right|$ *it is a source. For* $|e_{l,F}| = \left|\varepsilon_{l,F}^{AB}\right|$ *it undergoes a flip bifurcation, for* $|e_{l,F}| = \left|\varepsilon_{l,F}^{AC}\right|$ *there is a change of stability of the steady state*[27]*;*
ii) Consider $|\varepsilon_{l,z}|$ *small enough, for* $|e_{l,F}| \in (0, \left|\varepsilon_{l,F}^{BC}\right|)$ *it is a source; for* $|e_{l,F}| \in (\left|\varepsilon_{l,F}^{BC}\right|, \varepsilon_{l,F}^{AC})$ *the steady state is a sink (and so the steady state is indeterminate); for* $|e_{l,f}| \in (\left|\varepsilon_{l,f}^{AC}\right|, \left|\varepsilon_{l,f}^{BC}\right|)$ *it is a saddle; and for* $|e_{l,F}| > \left|\varepsilon_{l,F}^{AC}\right|$ *it is a source; for* $|e_{l,F}| = \left|\varepsilon_{l,f}^{AB}\right|$ *it undergoes a flip bifurcation; for* $|e_{l,F}| = \left|\varepsilon_{l,F}^{AC}\right|$ *there is a change of stability of steady state; for* $|e_{l,F}| = \left|\varepsilon_{l,F}^{BC}\right|$ *it undergoes a Neimark-Hopf bifurcation;*
iii) Consider $|\varepsilon_{l,z}|$ *very small, for* $|e_{l,F}| \in (0, \varepsilon_{l,f}^{AC})$ *the steady state is a source (indeterminate) for* $|e_{l,F}| \in (\left|\varepsilon_{l,F}^{AC}\right|, \left|\varepsilon_{l,F}^{BC}\right|)$ *it is a saddle; for* $|e_{l,F}| > \left|\varepsilon_{l,F}^{AC}\right|$ *it is a source; for* $|e_{l,F}| = \left|\varepsilon_{l,F}^{AB}\right|$ *it undergoes a flip bifurcation, for* $|e_{l,f}| = \left|\varepsilon_{l,f}^{AC}\right|$ *there is a change of stability of steady state.*

3. *Assume that* $\alpha < \delta < 1$,
i) if $|\varepsilon_{l,z}|$ *is high enough for* $|e_{l,F}| \in (0, \varepsilon_{l,f}^{AC})$ *the steady state is a saddle; for* $\left|\varepsilon_{l,F}^{AC}\right| < |e_{l,F}| < \left|\varepsilon_{l,F}^{AB}\right|$ *it is a sink; for* $|e_{l,F}| = \left|\varepsilon_{l,F}^{AB}\right|$ *it undergoes a flip bifurcation; for* $|e_{l,F}| > \left|\varepsilon_{l,F}^{AB}\right|$ *it is again a saddle;*
ii) if $|\varepsilon_{l,z}|$ *is low enough for* $|e_{l,F}| \in (0, \varepsilon_{l,f}^{AC})$ *the steady state is a saddle; for* $\left|\varepsilon_{l,F}^{AC}\right| < |e_{l,F}| < \left|\varepsilon_{l,F}^{BC}\right|$ *it is a source; for* $|e_{l,F}| = \left|\varepsilon_{l,F}^{BC}\right|$ *it undergoes a Hopf bifurcation; for* $\left|\varepsilon_{l,F}^{BC}\right| < |e_{l,F}| < \left|\varepsilon_{l,F}^{AB}\right|$ *it is a sink; for* $|e_{l,F}| = \left|\varepsilon_{l,F}^{AB}\right|$ *it undergoes a flip bifurcation; for* $|e_{l,F}| > \left|\varepsilon_{l,F}^{AB}\right|$ *it is again a saddle;*

[27] We can't assert that the steady state undergoes to a transcritical bifurcation because the map for that value is not smooth.

iii) if $|\varepsilon_{l,z}|$ *is very low for* $|e_{l,F}| \in (0, \varepsilon_{l,f}^{AC})$ *the steady state is a saddle; for* $|\varepsilon_{l,F}^{AC}| < |e_{l,F}| < |\varepsilon_{l,F}^{AB}|$ *it is a source, for* $|e_{l,F}| = |\varepsilon_{l,F}^{AB}|$ *it undergoes a flip bifurcation; for* $|e_{l,F}| > |\varepsilon_{l,F}^{AB}|$ *it is again a saddle;*

4. *Let* $\delta = 1$
   i) if $|\varepsilon_{l,z}|$ *is high enough for* $|e_{l,F}| \in (0, \varepsilon_{l,f}^{AC})$ *the steady state is a saddle; for* $|\varepsilon_{l,F}^{AC}| < |e_{l,F}| < |\varepsilon_{l,F}^{AB}|$ *it is a sink; for* $|e_{l,F}| = |\varepsilon_{l,F}^{AB}|$ *it undergoes a flip bifurcation; for* $|e_{l,F}| > |\varepsilon_{l,F}^{AB}|$ *it is again a saddle;*
   ii) if $|\varepsilon_{l,z}|$ *is low for* $|e_{l,F}| \in (0, \varepsilon_{l,f}^{AC})$ *the steady state is a saddle; for* $|\varepsilon_{l,F}^{AC}| < |e_{l,F}| < |\varepsilon_{l,F}^{AB}|$ *it is a source, for* $|e_{l,F}| = |\varepsilon_{l,F}^{AB}|$ *it undergoes a flip bifurcation, for* $|e_{l,F}| > |\varepsilon_{l,F}^{AB}|$ *it is again a saddle;*

5. *Let* $1 < \delta < \delta^*$ *or* $\alpha < \alpha^*$
   i) if $|\varepsilon_{l,z}|$ *is high enough for* $|e_{l,F}| \in (0, \varepsilon_{l,f}^{AC})$ *the steady state is a saddle; for* $|\varepsilon_{l,F}^{AC}| < |e_{l,F}| < |\varepsilon_{l,F}^{AB}|$ *it is a sink; for* $|e_{l,F}| = |\varepsilon_{l,F}^{AB}|$ *it undergoes a flip bifurcation; for* $|e_{l,F}| > |\varepsilon_{l,F}^{AB}|$ *it is a saddle;*
   ii) if $|\varepsilon_{l,z}|$ *is low for* $|e_{l,F}| \in (0, \varepsilon_{l,f}^{AC})$ *the steady state is a saddle; for* $|\varepsilon_{l,F}^{AC}| < |e_{l,F}| < |\varepsilon_{l,F}^{AB}|$ *is a source and for* $|e_{l,F}| > |\varepsilon_{l,F}^{AB}|$ *saddle;*

6. *Let* $\delta > \delta^*$ *and* $\alpha > \alpha^*$
   i) if $|\varepsilon_{l,z}|$ *is high enough for* $|e_{l,F}| \in (0, \varepsilon_{l,f}^{AC})$ *the steady state is a saddle; for* $|\varepsilon_{l,F}^{AC}| < |e_{l,F}| < |\varepsilon_{l,F}^{BC}|$ *it is a sink; for* $|e_{l,F}| = |\varepsilon_{l,F}^{BC}|$ *it undergoes a Neimark-Hopf bifurcation, for* $|\varepsilon_{l,F}^{BC}| < |e_{l,F}| < |\varepsilon_{l,F}^{AB}|$ *is a source, for* $|e_{l,F}| > |\varepsilon_{l,F}^{AB}|$ *it is a saddle;*
   ii) if $|\varepsilon_{l,z}|$ *is low for* $|e_{l,F}| \in (0, \varepsilon_{l,f}^{AC})$ *the steady state is a saddle; for* $|\varepsilon_{l,F}^{AC}| < |e_{l,F}| < |\varepsilon_{l,F}^{AB}|$ *it is a source, for* $|e_{l,F}| > |\varepsilon_{l,F}^{AB}|$ *,it is a saddle;*

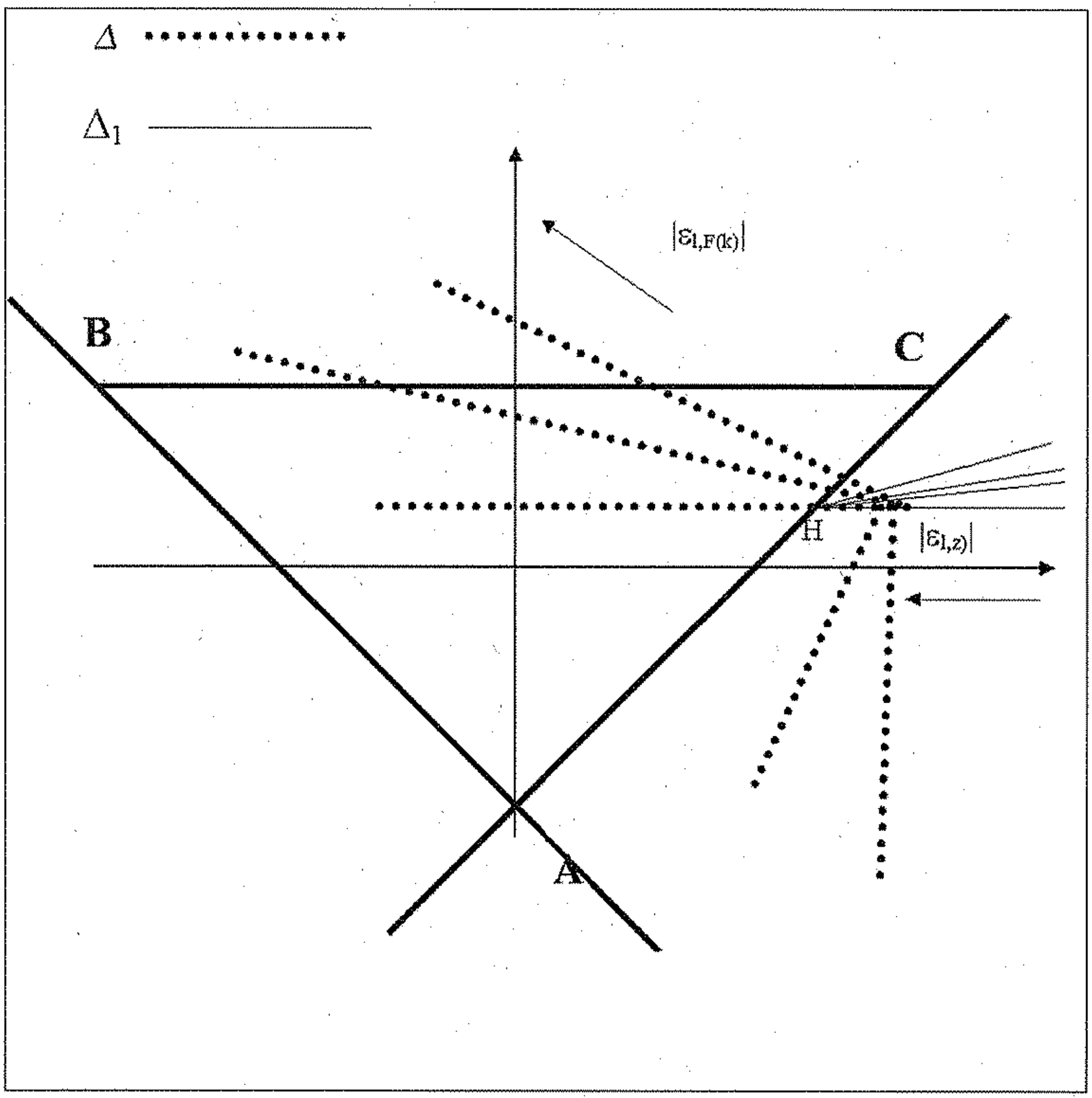

Figure 9:

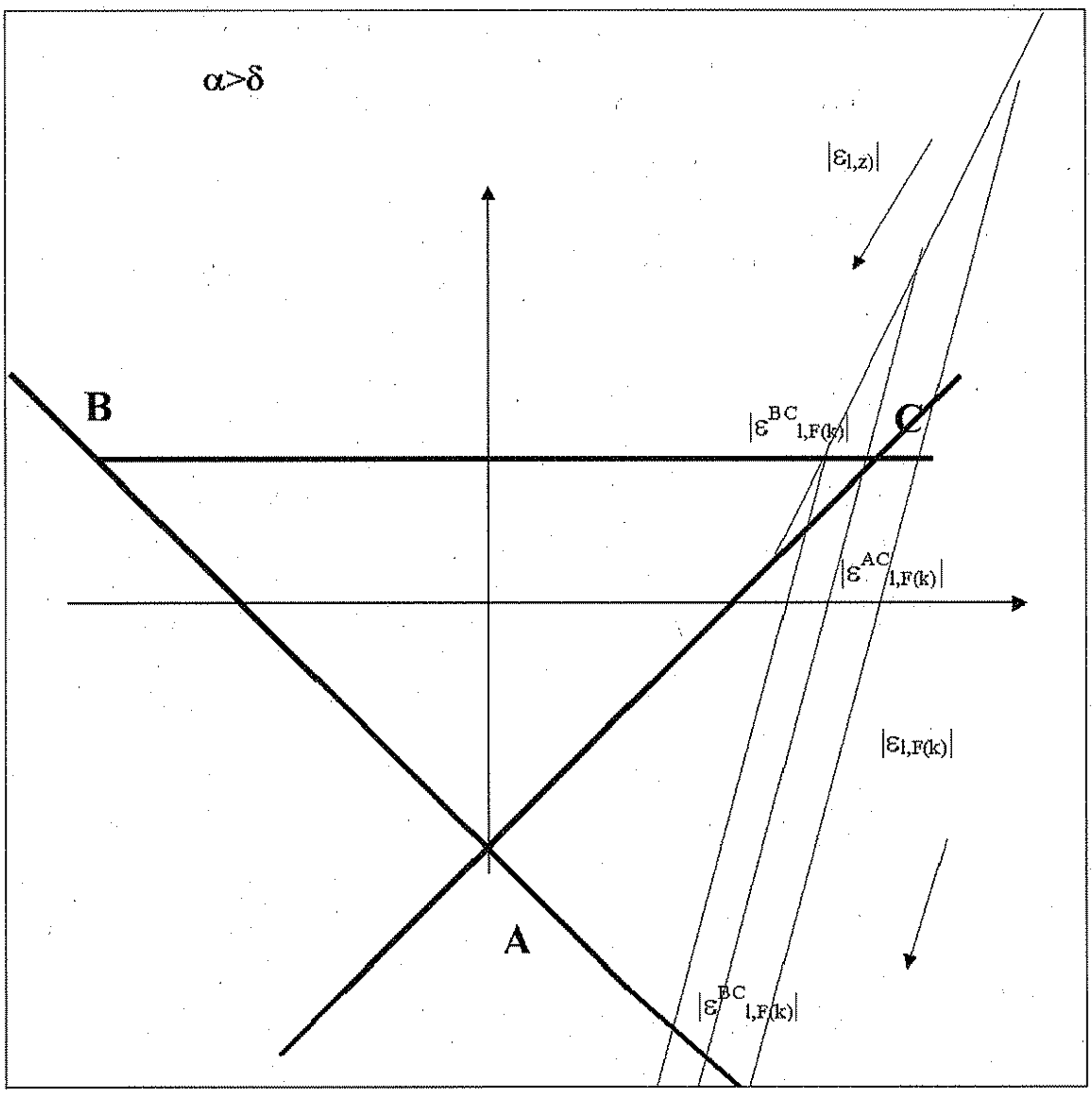

Figure 10: Case $\alpha > \delta$

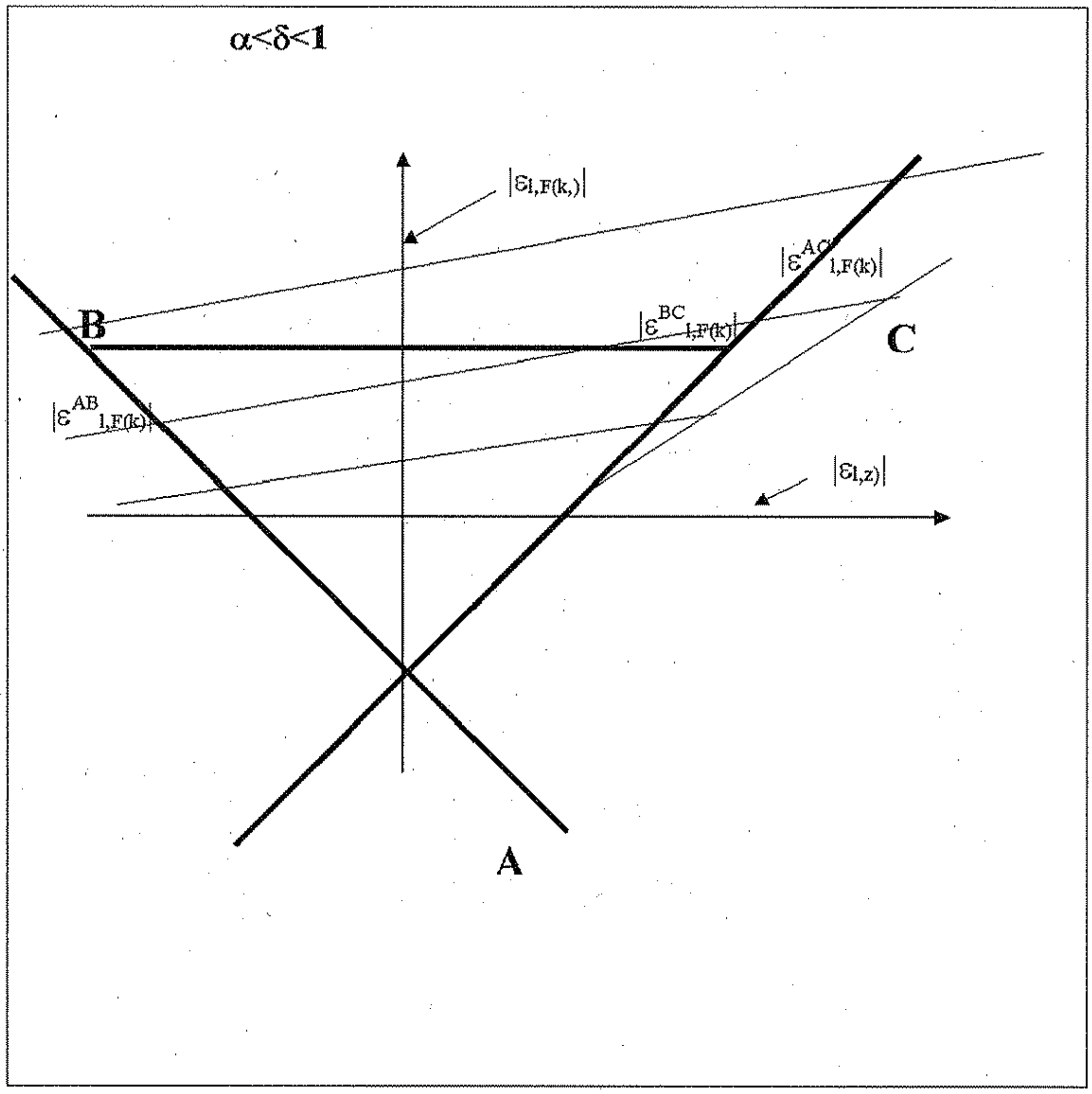

Figure 11: Case $1 < \delta < \alpha$

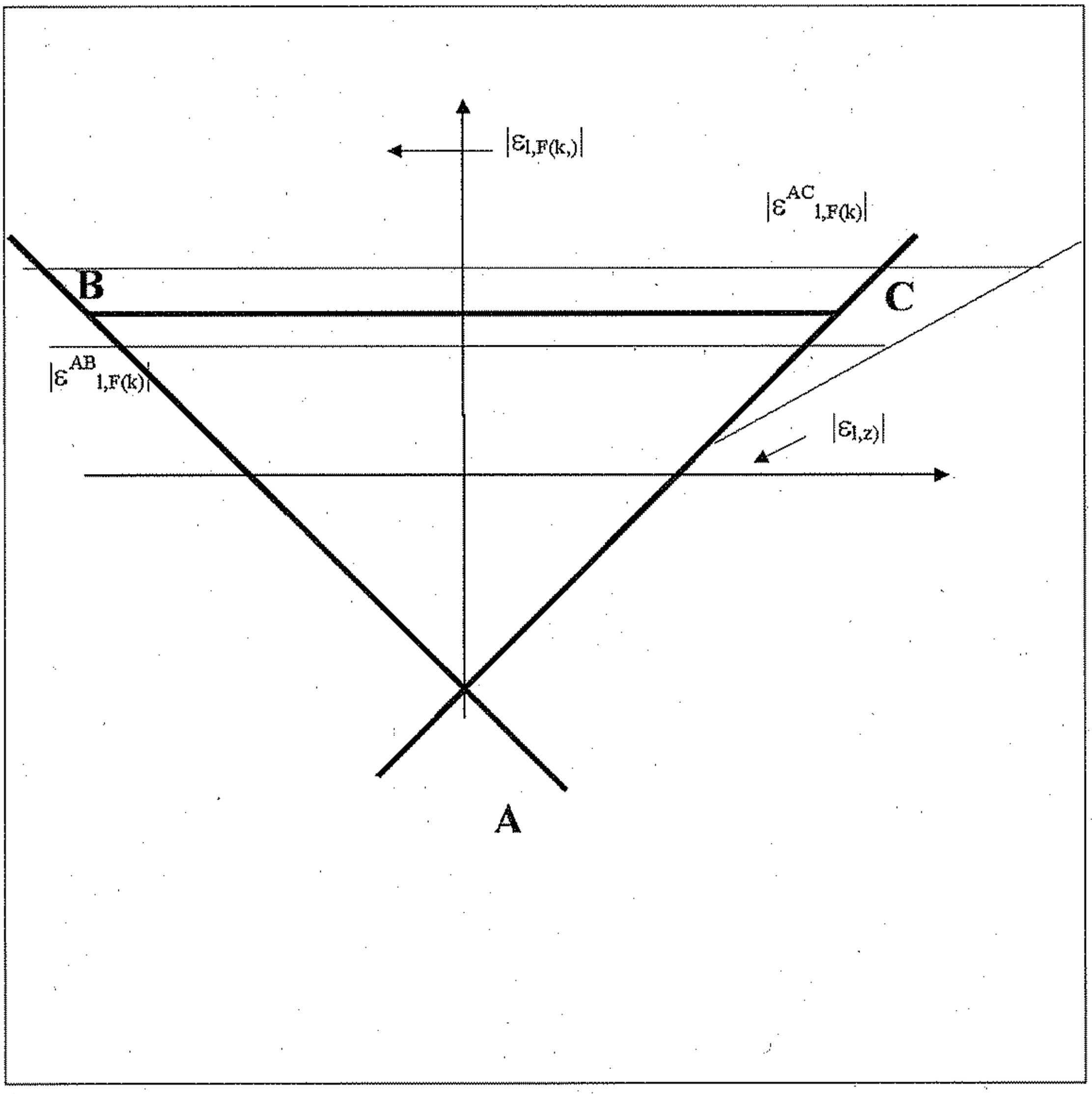

Figure 12: Case $\delta = 1$

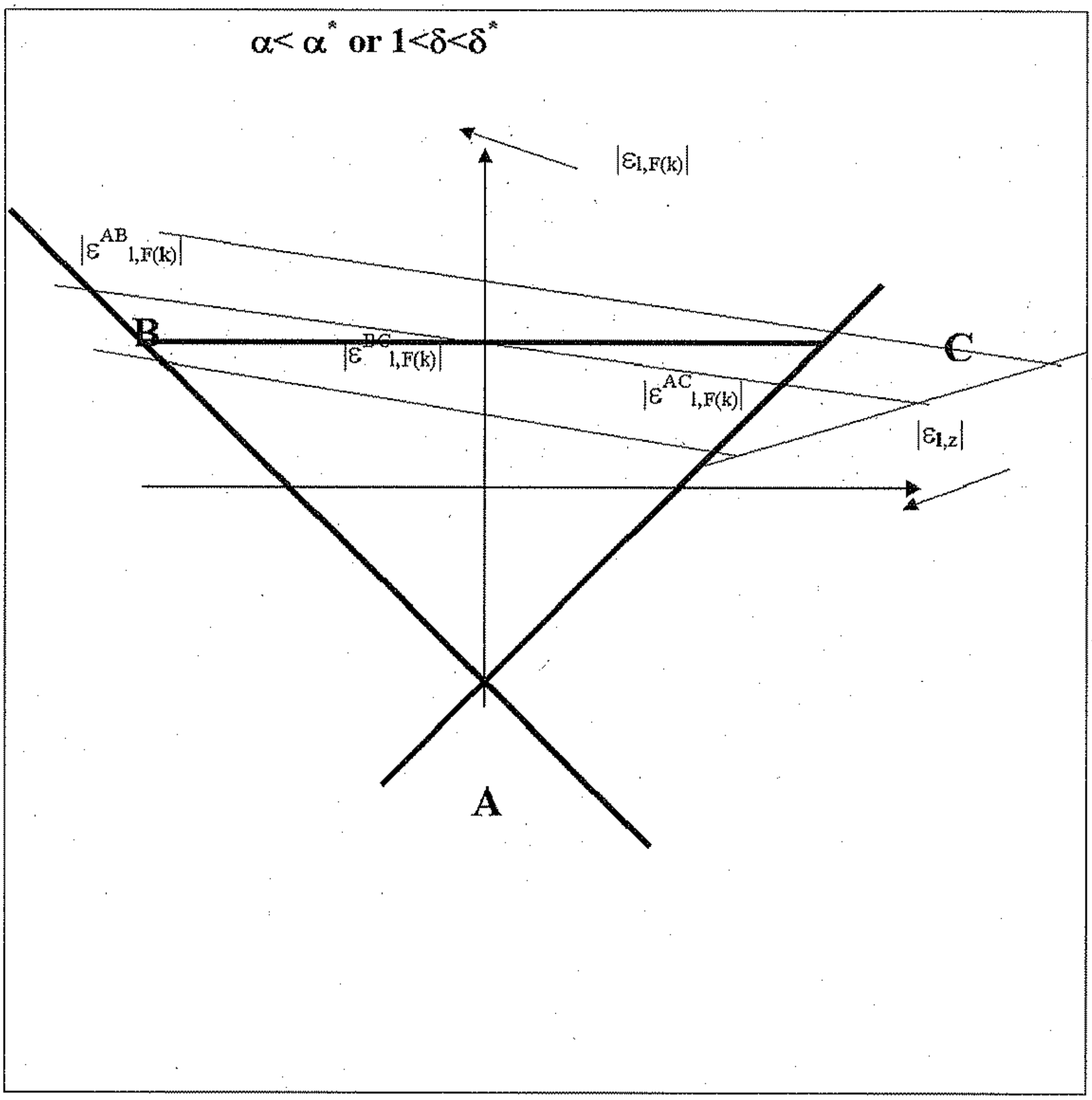

Figure 13: Case $\alpha < \alpha^*$ or $1 < \delta < \delta^*$

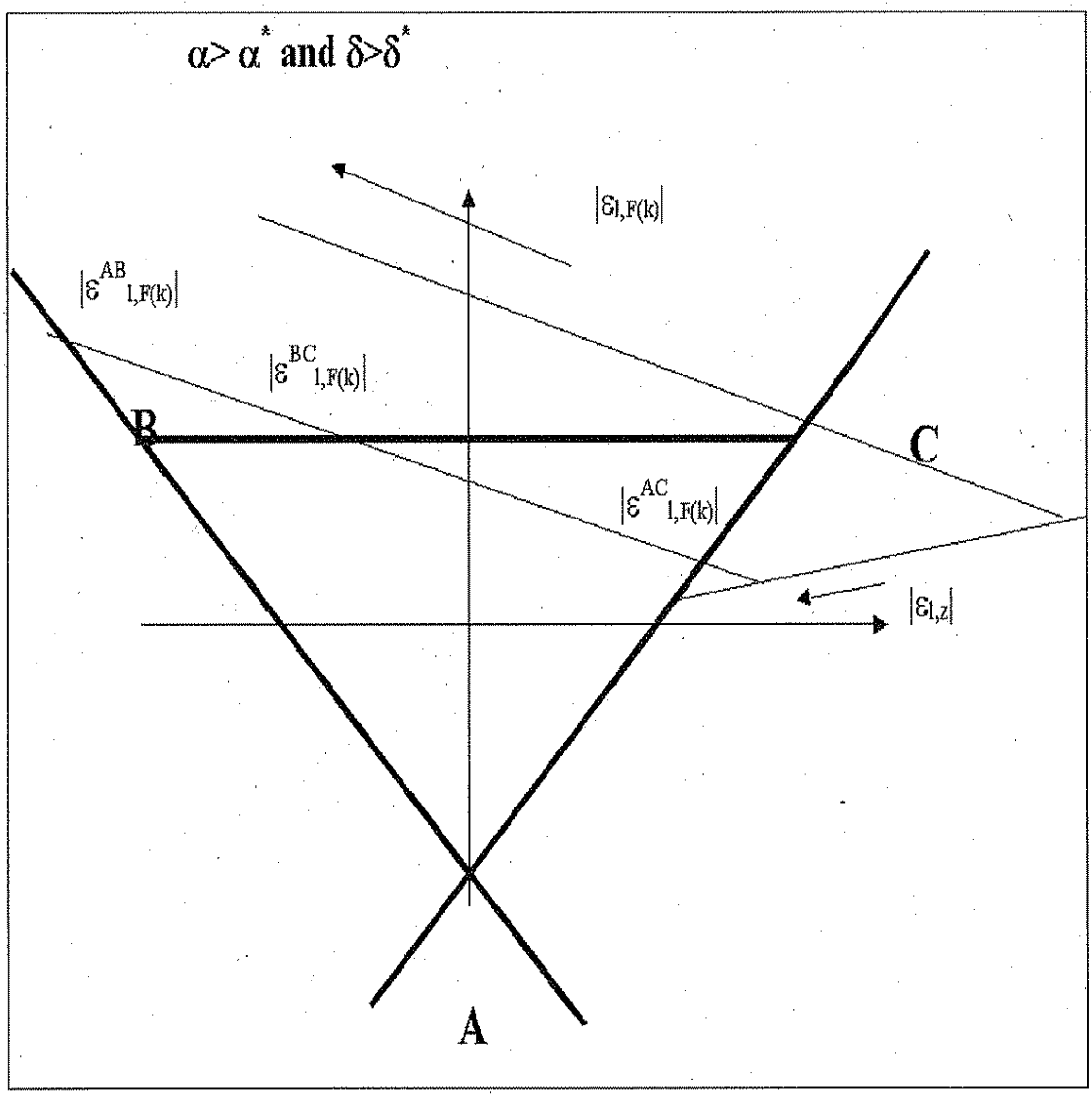

Figure 14: Case $\alpha > \alpha^*$ and $\delta > \delta^*$

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