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The Fragility Of Social Capital: An Analytical Approach

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Abstract

The present contribution proposes a model of growth with social capital to shed light on the relations between economic activity and social development. We study whether these two processes move in the same direction or whether they conflict each other. We investigate the causes of both outcomes, unveiling the role of the initial conditions and of some leading parameters.

We concentrate especially on the double role of social capital: in the market structure as a non internalized productive factor, in social community as a factor qualifying the agent's well-being. This modelling allows us to explain the rise of well-being traps or of increasing well being path. To clarify the results, with respect to significative parameters, we present several numerical simulations of the model.

1 Introduction

Although modern theories of growth are focused primely on technological changes and/or physical and human capital accumulation, many empirical studies relate the economic performances to cultural and sociological causes.

For instance, Knack and Keefer(1997) have empirically tested a positive impact of trust on aggregate economic activity. Halliwell et Al(1995) have found a significant role of social structure and economic growth in Italian regions. Reiser et Al(2001) show a strong positive correlation between social capital endowment and economic activities in ex-communist countries.

On the other side, many studies of sociologists, psychologists, but rarely of economists, confirm the intuition that modern economic organization and some consume-styles produce pressures on social structure: for example Putnam(1995) underlines the negative role of television in social networks, Shriff(1999) studies the negative role of labour mobility or migration on well-being.

By its nature, as we'll explain in the following section, social capital accumulation follows rules quite different from other kinds of capital, because of its public nature. Social capital is not privately appropriable and consequently, rational agents may not solve the maximization problem optimally. Because of the absence of coordination among individuals, a plausible result is that underinvestment¹ in social capital could arise with negative relapses on production and well-being.

The main novelty in this article is the studying of a dynamical model that considers the simultaneous role of social capital in economic activities and in the agents' utility. We investigate whether the economic process and social endowment move in the same direction, and whether these conflict each other. We analyze the role of cultural variables in the explanation of the possible different interactions between economic growth and social capital evolution and we find out probable obstacles for long run development.

In Section 2 we explain the motivations and the assumption beside the modelling; in Section 3 we present the model and we analyze the equilibrium dynamics; Section 4 explains short and long run phenomena through several numerical simulations; Section 5 concludes.

2 Social capital

In the last years sociologists, economists, psychologists, historicists have paid a great attention on phenomenon of social capital and its implication on economic development and on well-being.

In sociological literature, originally, this concept was introduced by Hanifan in 1916, using this terms:

"..those tangible substances that count for most in the daily lives of people: namely goodwill, fellowship, sympathy, and social intercourse among the

¹Below, we'll see that the term investment is not really suitable in describing social capital accumulation.

*individuals and families who make up a social unit...If an individual is helpless socially if left to himself..If an individual comes into contact with neighbor and they with other neighbors, there will be an accumulation of social capital, which may immediately satisfy his social needs and which may be a social potentially sufficient to the substantial improvement of living conditions in the whole community. The community as a whole will benefit by the cooperation of all its parts, while the individual will find in his associations the advantages of the help, the sympathy, and the fellowship of his neighbors"*².

The previous definition displays the core of many theoretical reflections considered in the latest studies, but only recently we attend to a really (re)birth of interest about social interactions, thanks to the works of sociologists as Fukuyama, Putnam, Granovetter and to Becker, the first economist who has introduced social relations in modern economic modelling.

2.1 Individual and social well-being

The concept of social capital is deeply related to the idea of well-being and happiness and tends to undermine various economic axioms on the theme. If we admit that agents have social needs and we insert other "objects" in utility function, different from consumption goods, the same economic growth, often considered as a panacea, could lose its fascinating idea about the growth of well-being. Economic development may be also accompanied by destruction of natural and social resources and in terms of agent's utility the whole effect could be negative, especially in advanced societies where basic needs are satisfied ³. With regard to this aspect, the idea of social capital takes place in a deep rooted tradition of thought which finds its origins in the Greek culture. In fact, according to suggestions in Bruni(2003), the core of Grecian⁴ anthropological studies⁵ was a man considered as a social being (homo sociologicus), who finds his natural collocation inside of the society, where he can flourish through the cooperation, and the no egoistic and no opportunistic relationships with Others⁶. Using more modern terms, with respect to this theory, the well-being of the single agent could not be considered isolated by the community in which he

²Cited in Putnam(2001).

³It is not our intention to be exhaustive in this part, anyway we think that it could be useful to underline some reflections, which have influenced the recent development of studies about social capital. Mainly, we limit ourselves to describe the more connected remarks with the analytical results shown in the paper.

⁴But, on the same tradition of thought, we find latin and medieval authors.

⁵Bruni underlines especially the importance and the actuality of the idea of social virtues in Aristotelian works. See the next footnote.

⁶It is interesting to note that a similar idea has been used recently by the philosopher Nussbaum to develop the concept of relational good used extensively in the present model. A good could be considered as relational, if a) it is created by the contribution of many people in a community; b) it is no privately appropriable; c) in contrast with private goods, we can't distinguish the stage of production and the stage of consumption, that is, like aristotelian virtues, it could be developed only trough its use.

As we'll show later, this kind of good, could be considered as a special case of spurious public good, and, analogously, it could be affected by the problem of free-riding behaviors and, consequently, by agents' under-investment.

is embedded.

On the contrary, the modern economic theory has found its fundamentals on the opposite approach of the methodological individualism, developed at the end of 15th century when thinkers and philosophers began to pay more and more attention to the single man and to his own possibilities to realize himself as an individual being, with private needs and goals.

Nonetheless, even limiting ourselves to economic literature, we assist to a slow sliding down of sensibility about sociality. If we compare the writings of the first generations⁷ of economists with the modern ones, we recognize that, even if they limited themselves to study the material aspect of well-being, the former were conscious about the existence of the no material well-being and about the importance of social networks⁸. In the neoclassical approach, which is the nowadays orthodoxy in economic analysis, these aspects are completely omitted and we find out that the whole life of a man consists in a private consumption problem, both in general equilibrium models and in economic growth ones.

2.2 Modern literature on social relationships

This last drastic view strongly contrasts with the empirical results of psychological and sociological studies⁹ and the concept of social capital proposes a deep reflection in the economic idea of well-being.

Nowadays, even if there is no consensus about the exact definition of social capital, inside of the different views, the common characteristic is to consider it as the assets that rise in interactions among people and contribute to economic and social development.

In literature, we can recognize two main approaches in the studying about the theme. A first group of authors considers social capital as a private good. For its "historical" importance and for its weight in successive researches, we present the simplified model of Becker who was the first, in the modern economic literature, to face up to the problem of social interactions.

In Becker (1974) the author supposes that utility of each agent is affected by private consumption and by "characteristics of other persons that affect his output of commodities"¹⁰. The agent could affect these variables by his own effort. Using original symbolism, the agent's problem is

$$\begin{cases} \text{Max} U_i(x, R) \\ R = D_i + h \\ p_x x + p_r h = I_i \end{cases} \quad (1)$$

where I_i is the total amount of resources owned by the agent. He could allocate

⁷ Adam Smith himself, as Hirsh(1976) and Sen (1987) pointed out, was conscious of the role of informal and formal norms inside the social community to preserve economic and social life by degenerations due to the free market.

⁸ We can find this sensibility in the Cambridge tradition, in Malthus, Marshall and more recently Keynes and nowadays Sen.

⁹ But even isolated economic researches, e.g. Reich 2001.

¹⁰ In Becker's language commodity means need.

I_i among private consumption, x , and "expenses" to modify social contest, h . R represents the social side of the agent's utility and its value stands on the combination of agent and other's choices

So, with the standard first order conditions, Becker characterizes the allocation of resources of agent i . A similar approach is also used in Becker (1965) to study the time allocation between economic and no economic occupations: the agent knows the relapses on his own utility of all working and free time activities and he allocates his time endowment among them.

This brief description is sufficient to understand a possible limit of Becker framework. Even if he introduces forms of social interaction (even with no egoistic finalities), he approaches the theme as in the standard models, transfer in this new field the same tools of the neoclassical tradition. According to this mechanism, the individual studied by Becker is not something different from homo-economics, but an evolution of the former, who also uses rationality in private life.

An other important and more recent contribute about (private) social capital is due to Glaeser Laibson and Sacerdote (2002)¹¹. They have developed a model similar to traditional model of investment in human and physical capital in which finite living agents chose how much time invest in their own social capital¹². Even in this case, according to Becker's works, social capital is a private resource characterized by the common mechanism of consumption and investment.

Instead, the other trend of thought considers social capital, mainly, as an externality created by common activities, no privately appropriable. In this approach, social interactions produce no-planned "objects" (uses, norms, relationships), that persists through the time. According to this mechanism, social capital is inside of the web of relations among the agents and it is created at the same time in which is used.

On this theme, in his theoretical studies, Coleman introduces a complete and differentiated description of social capital depending on diverse mechanisms of formation and development. Nonetheless, he points out that the main side of social capital is due a by-product of activities with different aims, and, in our simplified analysis, we model only this aspect.

On the other side, despite the differences in the motivations and the mechanisms, the positive role of social capital in economic life seems enough accepted. For example Fukuyama (1995) stresses the importance of trust in economic growth and he argues that a society creates rational forms of cooperation which reduce costs, externalities and imperfections in the market.

Paldam and Svendsen (2000) classify three different kinds of positive effects of social capital on growth: a) social capital, as trust, enters directly in production function like physical and human capital; b) it can induce a reduction of trans-

¹¹They have adapted this model to find possible relations between social capital and labour mobility.

¹²Apart from technical aspects, our models differentiate from this one because the authors assume agents perfectly aware of the role of their decisions on the evolution of social capital (even if they admit the possibility that a kind of externality could rise at aggregate level).

action costs (on the same theme, Arrow); c) it could reduce monitoring costs, because of the reduction of free riding phenomena. Instead, for other authors, for example Glaeser, Becker, Coleman, Corneo, social capital have an indirect effect, enforcing human capital and creating efficient information networks. North (1990) underlines the importance of (formal and informal) institutions and organizations to understand the economic performance and develops a dynamical system of interactions between the former, centered to reduce the imperfections on markets and having a stabilizing function, and the latter, finalized to satisfy private interests and to use earning possibility offered by institutions. Putnam set his studies on the positive impacts of civic interactions on economic growth. In Putnam (1993) he argues that the differences in development between North and South Italy could be explained through the different civic communities characterizing their history. It is important to note that the author consider this phenomenon as a path dependent process, determined by long time traditions hardly to be changed in the short period¹³.

2.3 Fragility of social capital and our approach

In our model, we stay very close to the approach proposed by Antoci et Al in their works, that is considering possible conflicts between economic growth and social development. The arising trade-off with between social capital and economic performances are empirically tested by many studies. For example in Bowling alone (2000), facing on American society, Putnam highlights a generalized following down of the civic community, starting from the beginning of 60's, and a disintegration of spontaneous participation in group activities, without distinction of sex, economic status, religion. Related to this, he finds a great positive correlation between watching TV on one side, and the collapse of social capital, on the other side.

In the present paper, according to the Narayan(1999) and Coleman interpretation, we consider social capital accumulation as a secondary product of social activities¹⁴ and for the development of these last, we consider the time allocation of the agents as determinant¹⁵. We formalize this aspect considering the presence in the economy of a private consumption good and a relational good produced with the interactions of more agents (see Ulhaner (1989)).

Differently from Antoci's works, which are centered on the role of social capital in utility of the agents, we consider a more articulated framework in which social capital also produce direct influences in the productive sector. This issue is

¹³ Putnam suggests that governmental policies acted to stimulate civic community could lead to the opposite effect because of its nature of self enforcing phenomenon.

¹⁴ Theoretically, this approach creates a large break respect to works on private social capital in which the agents are able to internalize the effect of resource allocation even in social relations.

¹⁵ The importance of time allocation about formation and decline in social capital is empirically tested for example even in Costa and Khan (2001). In their study the authors find a great correlation between social capital decline and woman labour supply. Considering an homogeneous and asexual society, in our model, merely we study the rebounds of a general increase of working time on social capital.

considered in literature for example in Shriff (1994/2004), in which the author explores the assumption that social capital has a determinant role to create integration in labour sector.

With respect to the classification proposed by Paldam and Svendsen(2000) we consider social capital as a productive, no internalized factor. Similar frameworks are proposed, for example, in works of Bartolini et Al(1997/2004).

3 The model

In this section we consider a continuous time economy populated by a continuum of infinite living identical individuals, with size normalized to one. Well-being of each agent depends on the consumption of two goods: a private one, $C(t)$, produced in private sector, and a socially provided one, $B(t)$. According to the previous informal arguments, we assume that this last is produced by the joined action of the time devoted by the single agent i to social activities, by the average social participation $\bar{s}(t) = \int_0^1 s_i(t) di$, and by the stock of social capital, $K_s(t)$. Using different but significative terms, $B(t)$ captures the fact that leisure of the single agent is influenced by the social background and by the average no-working time in the community.

At every moment, each agent allocates his time endowment (normalized to one) between labour and social activities¹⁶ and, in the decisional process, he considers average social participation and the stock of social capital as exogenously given. To point out several relations between social capital and economic activity and to simplify the mathematical hardness we assume that $C(t)$ is not storable, that is all the private production is consumed at every moment¹⁷.

In addition, as suggested by Antoci et Al(2003) in their conclusions, we explore the possibility that social capital has a specific role¹⁸ in private sector. To reach this goal, we insert social capital in the private production function as a no internalized productive factor. Formally we have the following equations:

$$C(t) = C(K_s(t), 1 - s_i(t)) \quad (2)$$

$$B(t) = B(K_s, s_i(t), \bar{s}(t)) \quad (3)$$

The functions in (2) and (3) are assumed to be strictly increasing respect to each argument.

As underlined in the previous section, in our studying framework, social capital is basically an externality and the main engine of its evolution is the average participation to social activities. Nonetheless, social capital is not accumulated through a specific process of investment: agents don't spend time in no working

¹⁶We consider socially provided good, relational good, social activity as sinonymous.

¹⁷According to the traditional models on private consume, we consider that this activity doesn't require time.

¹⁸As argued by a number of empirical researches, in a community, an high quality of social capital stimulates the private sector and conduce to more efficient results. In what follows, for shortness, we refer to this aspect of social capital, using the term trust.

activities to create social capital but rather to enjoy the relational good and consequently its evolution is considered exogenous by each agent.

Since on the job interactions and some kinds of private goods may stimulate durable interactions¹⁹, we consider a positive spillover from private production on social capital creation²⁰.

Taking into account that social networks follow down over time if agents don't take care of them, we introduce that social capital depreciates at a rate $\eta > 0$. Formally, we can summarize the K_s evolution through the following equation

$$\dot{K}_s = G(\bar{s}(t), K_s(t), C(t)) - \eta K_s(t) \quad (4)$$

To make tractable the successive analysis, we consider specific functional forms for (2),(3),(4):

$$B(t) = K_s^\gamma(t) \bar{s}(t)^\epsilon s^{1-\epsilon} \quad (5)$$

$$C(t) = K_s^\alpha(t) (1 - s(t)) \quad (6)$$

$$G(\bar{s}(t), K_s(t), C(t)) = C(t)^\beta B(t)|_{s=\bar{s}} \quad (7)$$

Social capital accumulation²¹ becomes

$$\dot{K}_s = [(1 - \bar{s}(t)) K_s^\alpha]^\beta K_s^\gamma(t) \bar{s}(t) - \eta K_s(t) \quad (8)$$

with $\epsilon \in (0, 1)$, and $\alpha, \beta, \gamma > 0$ ²².

Finally, we assume that agent's instantaneous utility is

$$U_i(C(t), B(t)) = \ln(C(t)) + b \ln(dC(t) + B(t)) \quad (9)$$

where the first right term represents the utility deriving from private consumption, and the second one the utility due to the satisfaction of social needs²³

¹⁹For instance, guided tours, the diffusion of internet may stimulate durable interactions and the birth of new networks

²⁰It is quite natural to consider that such spillover is not internalized by individual.

²¹In this specification each social factor (i.e. social capital, time spent to work, time devoted to social activity) is essential in the production of relational good.

If $K_s = 0$ or if $\bar{s} = 0$, there do not exist any form of trust in private sector and so we can think that this occurrence neglects the possibility of working together.

On the other hand, if $\bar{s} = 1$ then $Y = 0$ and so the extreme poverty does not make social interactions possible: if the agent is in deep poverty he may lose his friends because he can't go out with them to have a dinner, to play football, to take a movie and so on.

This last observation means that even if socially provided goods are important to satisfy the agent's needs if nobody works, the basic goods (food, dress) are not produced so even social relations could not survive.

²²Even if we do not impose an upper bound to α, β, γ , for economic reason, it is natural considering them not too large.

²³Alternatively, as suggested before, we can decompose $B(t)$

$$B(t) = s_i(t)^\epsilon \bar{s}(t)^{1-\epsilon} K_s^\gamma(t)$$

(from the mix of socially provided good and defensive expenditures²⁴); $b > 0$ is the weight given by representative agent to social needs and $d > 0$ is a rate that captures the possibility to substitute relational good with private consumption.

3.1 Agent's problem

Let r the discounting rate of future utility, the agent's problem is

$$\begin{aligned} & \max_{B(t), C(t)} \int_0^{+\infty} [\ln(C(t)) + b \ln(dC(t) + B(t))] e^{-rt} dt \\ & C(t) = K_s^\alpha (1 - s_i(t)) \\ & B(t) = K_s^\gamma \bar{s}(t)^\epsilon s^{1-\epsilon} \\ & K_s = (1 - \bar{s}(t))^\beta K_s^\gamma \bar{s}(t)^\epsilon - \eta K_s(t) \\ & s(t) \in [0, 1] \end{aligned} \quad (10)$$

Because K_s is completely exogenous with respect to allocative choices of agent i , we can solve the previous problem by means of an utility maximization point by point, with respect to the same control variables. Explicating the dependence of private consumption and social activities from the time allocation and substituting equality constraints we have the following simplified formulation of agent' problem:

$$\begin{aligned} & \max_{s(t)} U_i(K_s^\alpha (1 - s), K_s^\gamma \bar{s}(t)^\epsilon s^{1-\epsilon}) \\ & s(t) \in [0, 1] \end{aligned} \quad (11)$$

Proposition 1 *Problem (11) admits solution. Let $\tau = \gamma - \alpha$, and $A = d(b+1)$. We can classify the agent's time allocation as follows:*

1. if $\tau > 0$

$$s^*(t) = \begin{cases} 0, & K_s(t) < \widehat{K}_s \\ \frac{b\epsilon K_s^\tau(t) - A}{(1+b\epsilon)K_s^\tau(t) - A}, & K_s(t) \geq \widehat{K}_s \end{cases} \quad (12)$$

2. if $\tau < 0$

$$s^*(t) = \begin{cases} \frac{b\epsilon K_s^\tau(t) - A}{(1+b\epsilon)K_s^\tau(t) - A}, & K_s(t) < \widehat{K}_s \\ 0, & K_s(t) \geq \widehat{K}_s \end{cases} \quad (13)$$

where \widehat{K}_s is defined in proof²⁵.

and, using Becker's terminology, think of the whole right term as qualified leisure, where, in this case, the qualification is due to the time allocation done by the community and to the social framework.

²⁴In this context, the defensive expenditures are those consumptions done by the agents to substitute the poverty of rational goods. For example, if an individual has no time to foster friendships, he could substitute them with a less time intensive activity as watching TV.

²⁵In what follows, where it is not necessary, we omit the star to refer to optimal choice on s .

Proof. U_i is a continuous function and we can restrict the maximization problem on a subset $[0, a^*]$ with $a^* < 1$ because if $s \rightarrow 1$, then $U_i \rightarrow -\infty$. Hence, for Weierstrass theorem, optimal solution exists. Now, let us consider first derivative of $U_i(\cdot)$

$$\frac{\partial U(\cdot)}{\partial s} = -\frac{1}{1-s} + b \frac{-dK_s^\alpha + \varepsilon s^{-\varepsilon} \bar{s}^\varepsilon K_s^\gamma}{dK_s^\alpha(1-s) + s^{1-\varepsilon} \bar{s}^\varepsilon K_s^\gamma} \quad (14)$$

Imposing symmetric equilibrium conditions $s = \bar{s}$ we have that

$$\left. \frac{\partial U_i(\cdot)}{\partial s} \right|_{s=\bar{s}} \geq 0 \iff \quad (15)$$

$$b \frac{-dK_s^\alpha + \varepsilon K_s^\gamma}{dK_s^\alpha(1-s) + sK_s^\gamma} \geq \frac{1}{1-s} \iff \quad (16)$$

$$1-s \geq \frac{dK_s^\alpha(1-s) + sK_s^\gamma}{(-dK_s^\alpha + \varepsilon K_s^\gamma)b} \quad (17)$$

Dividing numerator and the denominator of the ratio on the right side of the previous expression by K_s^α we have

$$1-s \geq \frac{d(1-s) + sK_s^\tau}{(-d + \varepsilon K_s^\tau)b} \quad (18)$$

To obtain the optimality conditions we consider the following cases.

i) Let us consider $\tau > 0$ and $K_s > (\frac{d}{\varepsilon})^{\frac{1}{\tau}} \equiv K_s^*$; inequality (18) is fulfilled if and only if

$$s \leq \frac{b\varepsilon K_s^\tau - A}{(1+b\varepsilon)K_s^\tau - A} \equiv s^* \quad (19)$$

where $A = d(b+1)$, and $\tau = \gamma - \alpha$.

U_i increases until $s < s^*$ and decreases if $s > s^*$. Hence, $s = s^*$ is optimal solution if and only if $s^* \in (0, 1)$, otherwise the allocative solution is $s = 0$.

It is simple calculation to verify that

$$s^* \in (0, 1) \iff K_s > \left(\frac{A}{b\varepsilon}\right)^{\frac{1}{\tau}} \equiv \widehat{K}_s > K_s^* \quad (20)$$

ii) If $\tau > 0$ and $K_s < K_s^*$, then $\left. \frac{\partial U_i(\cdot)}{\partial s} \right|_{s=\bar{s}} < 0 \forall s \in (0, 1)$

iii) If $\tau < 0$ and $K_s < K_s^*$, then $\left. \frac{\partial U_i(\cdot)}{\partial s} \right|_{s=\bar{s}} \geq 0 \iff s \leq s^*$ and

$$s^* \in (0, 1) \iff K_s < \left(\frac{A}{b\varepsilon + 1}\right)^{\frac{1}{\tau}} \equiv \widehat{K}_s < K_s^* \quad (21)$$

iv) If $\tau < 0$ and $K_s < K_s^*$ we have the same result of case (ii). ■

The intuition behind the different results is straightforward, if we consider that social capital produces contrasting pressures on agents' choices, and the whole effect depends on magnitude of returns of social capital in the two sector.

If $\tau > 0$, it means that intensity of social capital in the relational good production

is larger than in the private sector. For enough high value of social capital, it induces agents to allocate a positive amount of time in social activities, and if the process of increasement of the stock of social capital goes on, this process is accompanied by a decreasement of labour supply until a strictly positive limit²⁶. Instead, if social sector is under developed, agents prefer to allocate the whole amount of time in working activities to appropriate of the opportunities of private sector.

If $\tau < 0$ we assist to the opposite reaction of agents. They take advantage from the spillover of social interactions (through the creation of trust) on private sector and if the private production rises up, they offer more and more labor until they allocate the whole time in private good production.

Now we can turn to study the dynamics of social capital and well-being implications.

3.2 Dynamics of social capital accumulation and well-being analysis

Even if we have considered simple specification of involved functions, there exist multiple steady states and consequently poverty traps, even for positive and negative value of τ .

Let us state the following preliminary result.

Proposition 2 *Let two steady states $K'_s \geq 0$ and K''_s such that $K''_s > K'_s$, so we have that $U_i|_{K_s=K''_s} > U_i|_{K_s=K'_s}$.*

Proof. It is sufficient to notice that K_s enters positively in each member of utility function and consequently if K_s rises up, the possibility set of private and social production expands. ■

Proposition 3 *The social capital accumulation performed by (decentralized) economy is*

$$K_s = \begin{cases} -\eta K, & \text{if } K_s < \hat{K}_s \\ \left[\frac{K_s^\tau}{(1+b\epsilon)K_s^\tau - A} K_s^\alpha \right]^\beta K_s^\gamma \left(\frac{b\epsilon K_s^\tau - A}{(1+b\epsilon)K_s^\tau - A} \right) - \eta K_s, & \text{if } K_s > \hat{K}_s \end{cases} \quad \text{if } \tau > 0 \quad (22)$$

$$K_s = \begin{cases} \left[\frac{K_s^\tau}{(1+b\epsilon)K_s^\tau - A} K_s^\alpha \right]^\beta K_s^\gamma \left(\frac{b\epsilon K_s^\tau - A}{(1+b\epsilon)K_s^\tau - A} \right) - \eta K_s, & \text{if } K_s < \hat{K}_s \\ -\eta K, & \text{if } K_s > \hat{K}_s \end{cases} \quad \text{if } \tau < 0 \quad (23)$$

Proof. The expressions are obtained, respectively, with a substitution of (12),(13) into (8). ■

In the following proposition we classify the role of parameters on the numerosity of steady states and on their stability properties.

²⁶This result stands on the assumption of essentiality of private activity even for the development of social capital.

Proposition 4 *If $\tau > 0$ then:*

- i) $K_s = 0$ is attractive;
- ii) if $\alpha\beta + \gamma < 1$ then the number of (interior) steady state equilibria is generically even (possible 0), and all steady states with an odd index are repulsing, whereas the steady states with an even index are attractive;
- iii) if $\alpha\beta + \gamma > 1$ then the number of (interior) steady state equilibria is generically odd, and all steady states with an odd index are repulsing, whereas the steady states with an even index are attractive; if $K_s > K_s^*$, that is the highest steady state, there exists a perpetual endogenous growth path with increasing well-being with $s \rightarrow \frac{b\epsilon}{1+b\epsilon}$.

If $\tau < 0$ then:

- i) if $\alpha\beta + \gamma < 1$, then the number of (interior) steady state equilibria is generically odd and all steady states with an odd index are attractive, whereas the stationary states with an odd index are attractive, $K_s = 0$ is a repulsing point;
- ii) if $\alpha\beta + \gamma > 1$, then the number of (interior) steady state equilibria is generically even (possible 0), and all steady states with an odd index are repulsing, whereas the stationary states with an even index are attractive, $K_s = 0$ is an attractive point.

Proof. Let

$$P(K_s) = \left[\frac{K_s^\tau}{(1+b\epsilon)K_s^\tau - A} K_s^\alpha \right]^\beta K_s^\gamma \left(\frac{b\epsilon K_s^\tau - A}{(1+b\epsilon)K_s^\tau - A} \right) \quad (24)$$

We can express the dynamics of social capital in the case of positive time devoted to social activities as

$$K_s = P^*(K_s) \equiv P(K_s) - \eta K_s \quad (25)$$

Case $\tau > 0$

We have that $\lim_{K_s \rightarrow \hat{K}_s} P^*(K_s) = 0$ and if $\alpha\beta + \gamma > 1$ then $\lim_{K_s \rightarrow +\infty} P^*(K_s) = +\infty$, meanwhile, if $\alpha\beta + \gamma < 1$, then $\lim_{K_s \rightarrow +\infty} P^*(K_s) = -\infty$.

Case $\tau < 0$

We have to investigate the behavior of function $P^*(K_s)$ in the region $0 \leq K_s \leq \hat{K}_s$. It is easy to show that $\lim_{K_s \rightarrow 0} P^*(K_s) = 0$ and $\lim_{K_s \rightarrow \hat{K}_s} P^*(K_s) = 0$

To recognize the nature of point $K_s = 0$ and to infer on numerosity of steady states, we have to evaluate the curvature of $P^*(K_s)$ when $K_s \rightarrow 0$.

Focalizing to robust cases, that is neglecting cases of tangency between $P(K_s)$, and ηK_s , we get the classification of results about existence and stability. ■

Several examples of the previous classification are proposed in the following figures.

According to results found by Antoci et Al(2001), even introducing a productive social capital, we notice that we have the following property: a rise of η makes steady state $K_s = 0$ a probable result for the system (its attraction basis

²⁷ $P^*(K_s)$ is not defined in 0.

Figure 1: case $\tau > 0$ and $\alpha\beta + \gamma < 1$, no interior steady state.

Figure 2: case $\tau > 0$ and $\alpha\beta + \gamma < 1$, two positive steady states.

Figure 3: case $\tau > 0$ and $\alpha\beta + \gamma > 1$, one positive steady state.

Figure 4: case $\tau < 0$ and $\alpha\beta + \gamma < 1$, one positive steady state.

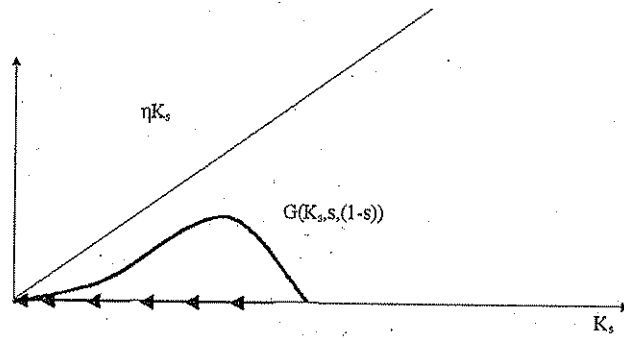


Figure 5: case $\tau < 0$ and $\alpha\beta + \gamma > 1$, no interior steady state.

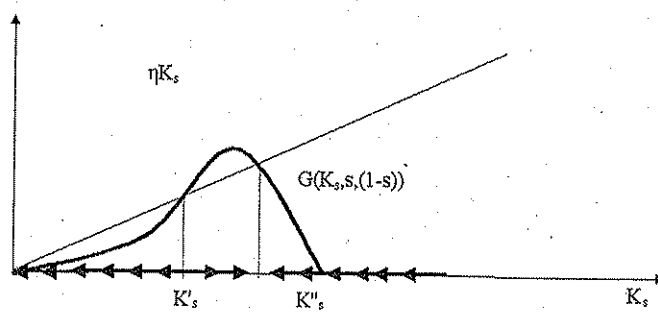


Figure 6: case $\tau < 0$ and $\alpha\beta + \gamma > 1$, two positive steady states.

enlarges). It is straightforward if we consider that a rise of depreciation rate of social capital could make too expensive its care. Anyway the presence of social capital even in production function makes more articulated the role of social development in an economy. If private sector is not enough trust intensive (that is $\tau > 0$)²⁸, then $K_s = 0$ is always an attractive fixed point for those economies poor of initial social capital. Anyway if γ is enough large ($\gamma > 1 - \alpha\beta$) if economies starts after the threshold \hat{K}_s , the mix of social networks and trust make possible sustained growth, even in private sector. This possibility is encouraged if the spillover of consumption in social capital creation is high. We can consider this case as the virtuous one.

In the opposite case ($\tau < 0$), the possibility of sustained endogenous growth is precluded for all initial conditions. Nonetheless, if $\alpha < \min(\gamma, \frac{1-\gamma}{\beta})$ the mix of parameters guarantees the escape from 0 social capital steady state. Otherwise, if $\alpha \not< \min(\gamma, \frac{1-\gamma}{\beta})$ the poverty trap $K_s = 0$ returns to be attractive, but in presence of multiple equilibria, even the well-being superior steady state. In such a case (see figure) the social capital initial condition becomes determinant.

4 Numerical simulation

To clarify the evolution of the variables of the model, according to the Proposition (4) and to possibilities proposed in the previous figures, we present several numerical simulations.

²⁸We are comparing the productivity of social capital in the two sectors.

Case $\tau > 0$ and $\alpha\beta + \gamma < 1$: no positive steady state

Parameters value: $\varepsilon = 0.12, \alpha = 0.01, \gamma = 0.82, \eta = 0.12, b = 4, d = 0.3, \beta = 0.4, r = 0.02$

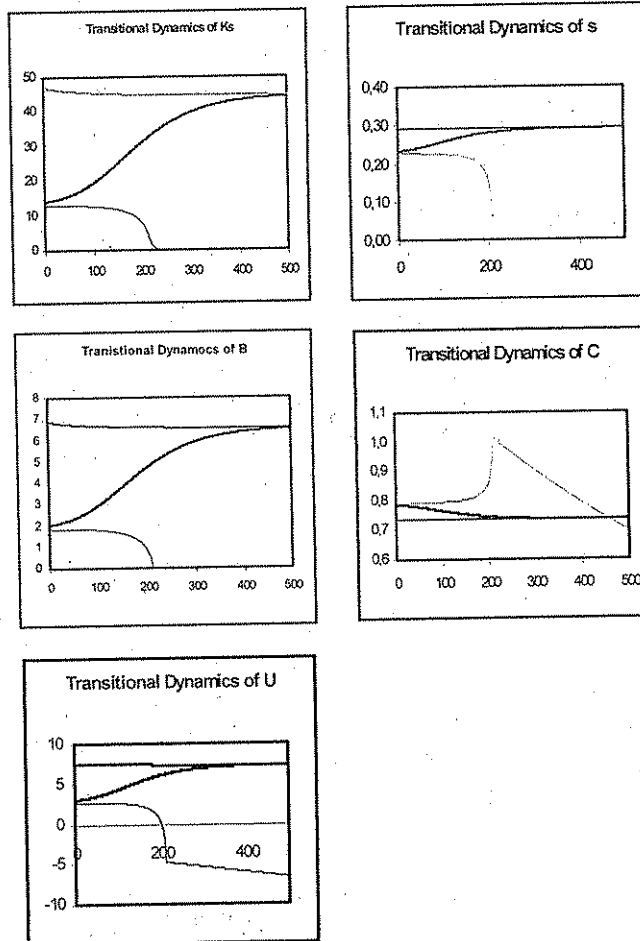
Steady state of the simulation: $K_s = 0$; Initial condition: $K_s(0) = 8$;

With this parametrization, economies converge to 0 social capital steady state for all initial conditions. During the transition we point out a temporary slow growth of private consumption, even if utility of agents decreases. The cause is due to different speeds of evolution of phenomena: the erosion of social capital is slower than the decline of social participation and, in the short run, that leads the agents to utilize and to wear out social endowment in the productive sector, causing a short-lived development of private sector. This example is able to capture the tendency of several mature economies which are receding, after a period of slow growth even if the economic activity (interpreted as labour time) is developed. Beside the role of declining private consumes, the well-being of the agent is deeply depressed by the erosion of social networks.

Case $\tau > 0$ and $\alpha\beta + \gamma < 1$: two positive steady states

Parameters value: $\varepsilon = 0.12, \alpha = 0.01, \gamma = 0.82, \eta = 0.12, b = 4, d = 0.3, \beta = 0.4, \tau = 0.02$

Steady state of the simulation: $K_s = 0, K_s = 13.04, K_s = 44.76$; Initial conditions²⁹: $K_s(0) = 13, K_s = 14, K_s(0) = 47$.



With respect to the previous case, we have considered a lower depreciation rate of social capital. According to this parametrization we have two positive steady state, $K_s = 13.04, K_s = 44.76$. The first is repulsing while the second one is attractive. Economies converges to 0 social capital steady state or to the positive attracting one, according to the initial conditions. It is interesting to note the dynamics of private consumption for $K_s = 13.04$. At the beginning, it increases quickly, while social capital moves down. But this behavior is a short run tendency, driven by the increasing of working time, and when the upper

²⁹The thin line indicates the simulations related to $K_s(0) = 12$; the medium one, $K_s(0) = 13$; the large one, $K_s(0) = 47$.

bound ($s = 0$) is reached, private consumption rapidly decreases. This case has an interesting relevance in the analysis of data. If we recognize a rapid growth of private sector, but a decrement of social participation, in the long run, the tendency of the first one could be reversed by the second one. During the transition we point out a continuous falling down of the utility of the agents, with a speeding up when the whole time endowment is devoted to working activities.

Case $\tau > 0$ and $\alpha\beta + \gamma > 1$: no positive steady state

Parameters value: $\varepsilon = 0.5, \alpha = 0.9, \gamma = 0.7, \eta = 0.12, b = 1, d = 0.4, \beta = 0.4, r = 0.02$

Steady state of the simulation³⁰: $K_s = 0, Ks = 0.55$; Initial condition $K_s(0) = 0.5$;

³⁰Because of scala reason, according to different initial conditions, we present two simulations separately.

Initial condition $K_s(0) = 0.6$;

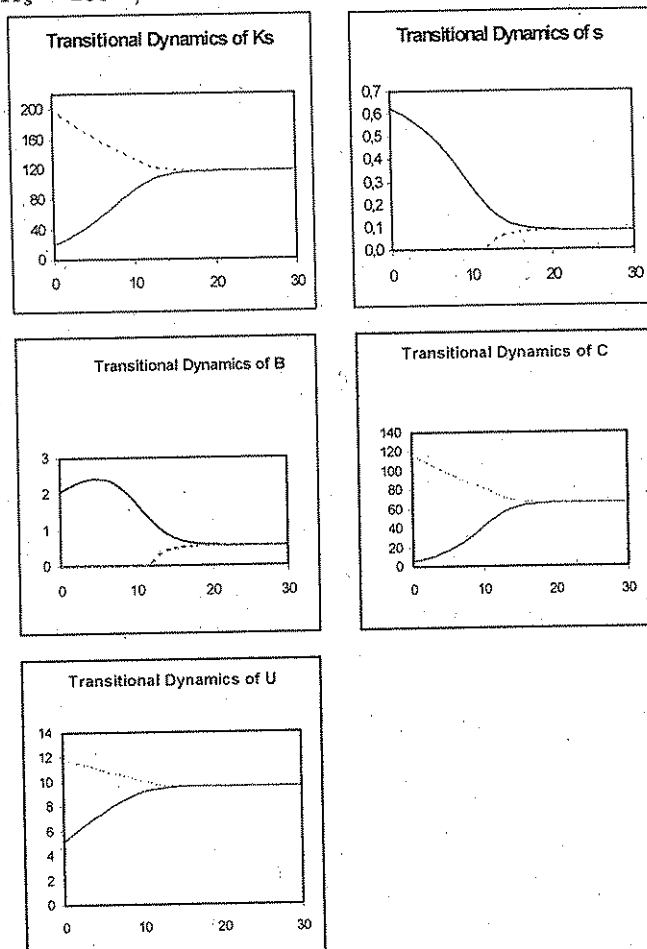
In this case, if the economy starts below the positive steady state, the system converges to $K_s = 0$. Because agents choose to allocate the whole time endowment in working activities, during the transition, B stay at 0. Instead, if economy starts above the steady state, agents spend more and more time in social activities attaining a mix that allows a long run growth of private consumption and a perpetual social development. It is important to note how initial conditions matter with this parametrization: a slight difference could generate opposite results.

Case $\tau < 0$ and $\alpha\beta + \gamma < 1$: one positive steady state

Parameters value:

$\varepsilon = 0.7, \alpha = 0.9, \gamma = 0.4, \eta = 0.04, b = 4, d = 0.05, \beta = 0.5, r = 0.02.$

Steady state of the simulation: $K_s = 117, 20$; Initial conditions: $K_s(0) = 20$ and $K_s = 200$ ³¹;



In this case, even if economies start very far each other, they are driven to the unique positive steady state. While other variables have monotonic behavior, in the transition B and C could experience variations of tendencies because of the opposite dynamics of the variables implicated in their production. In this simulation, if the economy starts below the steady state we register a firstly increasing and after decreasing movement of B . It is possible because the temporary and positive role of private sector in social development. This case could have an interesting role in empirical researches. If we recognize a growth of social capital, but a decrement of social participation, the tendency of social

³¹The dashed lines indicate simulations with initial condition $K_s = 200$

capital may be a short run condition that tends to dissolve.

Case $\tau < 0$ and $\alpha\beta + \gamma > 1$: no positive steady state

Parameters value: $\varepsilon = 0.5, \alpha = 0.9, \gamma = 0.7, \eta = 0.04, b = 1, d = 0.4, \beta = 0.8, r = 0.02$.

Steady state of the simulation: $K_s = 0$; Initial condition $K_s(0) = 0.2$;

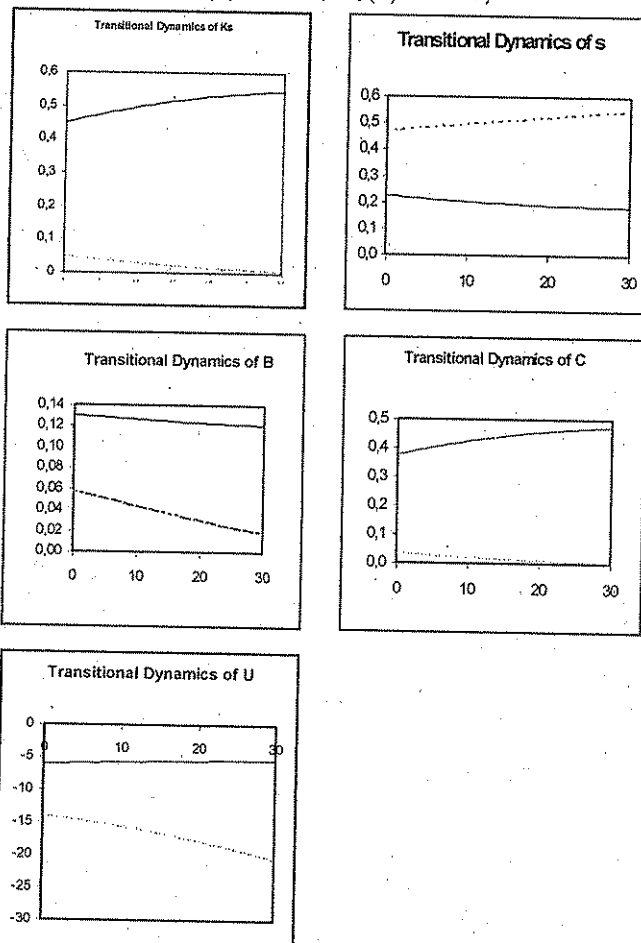
With this parametrization we assist to a poverty situation due to the excessive time spent in social activities. Social organization is too oppressive and creates an obstacle for the growth of private sector in the community. In such a case social capital could be an engine of under-development: because the low productivity of private sector, agents are induced to spend more and more time to enjoy relational goods. But this behavior is accompanied by a continuous decline of social capital and, as time goes on, the productivity of time spent in social activities decreases and, observing utility dynamics, the whole effect is negative. This case may capture some peculiarities of really backward countries entrapped in vicious circles of poverty.

Case $\tau < 0$ and $\alpha\beta + \gamma > 1$: no positive steady state³²

Parameters value: $\varepsilon = 0.5, \alpha = 0.9, \gamma = 0.7, \eta = 0.12, b = 4, d = 0.4, \beta = 0.8, r = 0.02$.

Steady state of the simulation: $K_s = 0, K_s = 0.54$;

Initial conditions: $K_s(0) = 0.05, K_s(0) = 0.45$;



If the economy starts below the repulsing steady state, the dynamics are quite similar to the previous one and K_s approach to 0. Instead, if $K_s(0)$ is above the repulsing steady state, agents reduce social oriented time and increase the private consumption. In contrast to the cases in which $\tau > 0$, this behavior is able to balance the negative relapses on the production of relational good, and, as time goes on, well-being increases. In this case the transition to a more private oriented society is a positive phenomenon. However, even approaching to the "new" steady state, the time devoted to economic activity stays positive and social endowment becomes higher for the spillover from private sector.

5 Conclusions

We have explored a no static model of interactions between social capital and private production with three main characteristics:

- i) the time spent by the community in social activities plays a determinant role to explain the social development;
- ii) each agent doesn't internalize the effect of his own time allocation on the evolution of social capital;
- iii) social capital is a no internalized productive factor in private sector and a qualifying factor in agent's well-being.

With respect to the literature on the theme, our analysis has introduced a certain number of innovative features and, as argued in the last section, its results may be useful to understand and to discriminate several empirical situations. The model suggests that the comparison of social capital in social and economic sector is the main explanatory key to understand economic and social dynamics. We can recognize if the phenomena move in the same direction, and whether these two processes conflict each other, if they are short run tendencies or long run ones. Specifically, if private sector is trust intensive, no perpetual growth could arise. In such a case, the possibility of multiple steady states depends on returns of social capital in social sector: if they are enough high dynamics admit a unique globally stable positive equilibrium. Otherwise, the depreciation of social capital plays a determinant role: if they are enough high, all economies converge to no social capital equilibrium, even if this tendency makes agent's utility lower; if it is enough low dynamics admit multiple attracting equilibria. IN the simulation we have centred on the case of two fixed points: one in which social capital is absent, the other in which social capital endowment is strictly positive. The possibility to converge to Pareto-superior steady state stands on initial social capital endowment. We have interpreted these cases, as possible explanations about tendencies of country with private oriented social values. The model suggests that values sympathetic to economic life create favorable substrate for the economic development, even for initially poor country, allowing the persistence of social capital, but preventing the possibility of perpetual growth³³. On the other side we argue that a community more devoted to no market activities experiences better economic performances, if endowed with a sufficient high initial stock of social resources.

The investigation could be developed through different ways. First of all it could be interesting to introduce private capital accumulation and to see how this process is influenced by social capital development. On the other side it is possible to introduce more articulated allocative framework and to differentiate inside the different forms of social capital accumulation.

³³This point is stressed by Weber(1947), who underlines the role of Protestantism in the birth of capitalism. In the same terms, more recently, many authors propose the importance of Confucianism to explain the take off of Asian tigers.

However, it is to be noted that the religious-cultural resources present some limits if extensively used in economic life. For example, in Kim (2003), analyzing economic and social data of Korea, the author points out that the negative relapses of economic growth on social structure of the country based on family are becoming a constraint for long run economic growth.

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