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A two sectors growth model with social capital

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Abstract

The present contribution proposes a dynamical model with private and social capital to shield light on the relations between growth and social development. We study whether these two processes move in the same direction or whether they conflict each other. We investigate the causes of both outcomes, depending on initial conditions and values of parameters. We concentrate especially on the relative weigh of social capital in market structure and social community that allow us to explain the rise of well-being traps. To clarify the results we present some numerical simulations of the model.

1 Introduction

The aim of this paper is to analyze some possible interactions between economic growth and social capital in an homogenous society. Recently, Antoci and Al(2001/2002/2002) have proposed several models in which social capital is a significative source for agents' well-being. We extend their analysis introducing the hypothesis that social capital matters in the productive sector. For instance, this possibility is empirically tested by Paldam(2001) Knack and Keefer (1997), Temple and Johnson (1998), Whiteley (2000). In Sodini(2006), we analyzed the same issue but in the present model we remove the limitative condition of no physical capital accumulation and we study this issue within a two-sector model of no-endogenous growth.
According to related papers, we focalize on the time allocation between private activity (production of material goods) and social activity[1].
In the present contribution, social capital assumes two different roles: it qualifies agents' leisure and contributes in economic activity as a no internalized

[1] We consider a very aggregate model and we do not distinguish between the different activities (game activity, political activity, voluntary work time spent with the family and so on) enjoyed by agents in a no market structure. However, following the approach of Antoci, no working time is assumed as the main mechanism to create and develop the social resources in a community.

productive factor[2]. With respect to works of Antoci et Al(2001/2002/2003), in which economic and social performances moves in opposite ways, the results could be more articulated[3]. Briefly, we describe the similarities and the differences between the mechanism arising in the models.
In common with their paper, a low level of social structure may induce to an under-investment in social capital[4] pushing to a self-enforcing shift from social activity to private one. As a consequence, this process encourages a rise of GNP[5], eventually featured by a low level of well-being for the negative impact of deteriorated social networks on utility. Anyway, in the present contribution, because of the weight of social capital in the productive sector, this process could also erode the trust of the society with a further restrictive effect in the economic performances. In such a case, we assist to an high working society with limited economic results. Although the overall effect depends on the values of parameters, strongly, especially for advanced economies, it emerges that private-oriented consumption patterns are not synonymous of high well-being for the agents.
Considering the analytical structure of the model, we point out that the equilibrium dynamics may exhibit two fixed points with different levels of output and social capital and, studying the transitional dynamics, we find out that the convergence to the steady states with positive social participation, usually, is not driven by the saddle path stability. We derive some conditions for which equilibrium can be indeterminate, or limit cycles emerge around the steady state. In contrast to the widespread view on the theme, we relate these dynamical results to the presence of no-market externalities. At our knowledge, this possibility is not been investigated before.
The article is so composed: in Section 2 we consider some theoretical and empirical aspects emerged in literature, in Section 3 we present some results in economic literature on endogenous labour supply, in Section 4,5,6 we study the tree models, 7 concludes.

2 Motivation

In the previous section we have used the term social capital. It may be useful to spend some words on this concept. In literature many definitions of social

[2] As argued by Fukuyama (1995), a developed social structure stimulates the density of trust in the community and it enables people to create more efficient organizations in productive sector.
On this issue some empirical works, for example Paldam and Svendsen (2000), confirm the role of social capital on the firms' size and the authors find out that countries with bigger and more competitive firms are characterized by high trust communities.

[3] In Antoci et Al(2003), neglecting the role of social capital in the productive sector, necessarily on the steady state with an higher social capital level , economic activity is lower.

[4] We argued below that the term investment is not really appropriated to describe the accumulation of social capital.

[5] The shift to a more private oriented consumption style is known in literature as defensive expenditure and it is the determinant key to explain the change of time allocation in these kinds of models.

capital have been proposed, and, consequently, many different studying strategies[6].
Nonetheless, the common characteristic in the various approaches is to consider it the set of social values, the productive assets, embedded in a group to reach common goals. The marriage between the two terms social and capital represents the focal aspect analyzed in the proposed analytical modelling. Social capital is social, so it rises up only if several individuals jointly act, for example in groups, families, associations, political parties, community and so on. Furthermore, it is capital, so it has a persistence and an evolution through the time, which justifies our choice of studying a dynamical model in spite of a static one.
In literature we can recognize two main thinking branches about the explanation on mechanism of social capital accumulation. The much part of economists considers social capital as an individual resource and focuses on individuals' investment decisions to make optimal the connections with others[7]. This approach is next to the framework elaborated in studying physical and human capital accumulation.
Anyway, the largest part of no economic literature on social capital underlines the public nature of social capital, and explains its accumulation as a residual effect of peculiar consume activities. Using economic terms, much part of social capital accumulation emerges as an externality[8].
Following the theoretical studies of Putnam and the analytical framework proposed by articles of Antoci at Al and Sodini(2006), we consider that the main mechanism of social capital evolution is due to individuals' participation in social activities. To explain why agents undertake social activities, we use the concept of relational goods introduced firstly by Ulhaner[9]. These kinds of goods are characterized by two main peculiarities: they cannot be enjoyed alone, and it is not possible to identify the two different stages of production and consume. Some examples are the friendship, the family, the enjoyment to be member of a group. The only mechanism for consuming and increase these goods is spending time in joint actions with several individuals.
It is undoubted that such goods are able to capture an important side of no-material individual well-being. The underlying idea has ancient roots and springs from Greek culture and humanist culture: the human being achieves his natural collocation in social structures because he finds inside of them part of his purposes and well-being. Nowadays this intuition is scientifically confirmed

[6] See Paldam(2000) and Collier(1998) for theoretical studies on social capital.
[7] See Becker(1974) for pioneristic economic investigations on sociality and more recently Glaser et al(2002) for an intertemporal formulation of this kind of approach.
[8] For example Colllier (1998) uses these terms "Social capital is first a subset of the processes which generate externalities, namely, those which are generated by social interactions, [..] those which either are themselves durable or the effect of which are durable. Narayan defines social capital as "the norms and the social relations embedded in the social structures of society that enable people to coordinate action to achieve desired goals" and Putnam uses similar words in Putnam(1993). Other authors focus the analysis on trust. For instance, Paldam et Al(2000) consider this definition "Social capital is the density of trust existing within a group. The group might be extended to the whole of the society."
[9] Corneo et Jeanne(1999) call them "socially provided goods".

by many sociological and psychological studies about happiness. Experiences as volunteering, friendship, cultural clubs arise as consequences of no individualistic necessities.
However, the traditional economic view has neglected the investigation of these phenomena, considering them outside respect to economic studies, because driven by no economic needs[10]. For example Marshall affirms that the role of economy is to study the material side of well-being[11] and the market structure deriving.
Anyway, even according to this theory, modern economic structure has deeply modified the social endowments and we assist at the diffusion of market substitutes for social needs. Technological products as T.V., play-stations, chat lines and so on could be interpreted as a result of pressing constraints to built friendship[12]. For instance, to confirm this thesis, Putnam finds an important correlation between time spent watching T.V. and low social interactions[13]. So, the diffusion of economic substitutes for social needs could be interpreted as a signal of the increasing decline of traditional social networks of interactions.
To study this possible consequences of economy on social capital, according to Antoci's works and following Sodini(2006), we rather concentrate on the time allocation between social and economic activities.
Observe that, in Becker's view, if agents choose to increase the time devoted to economic activities, this fact implies that they find their returns better and this shifting makes them better. Anyway we introduce a wrong perception of agents on social capital evolution, considering that each agent perfect foresights its level for each date, but he considers his own contribute as negligible. That is we assume that individual is not able to intenalize the social capital dynamics in the allocative problem.
On the other side, in the economic literature the main part of researches has investigated the positive role of social capital on growth, and different explanations were considered. Paldam (2000), for example suggest three different, economically relevant, roles of social capital. It could be considered as a factor reducing monitoring costs: the trust in a group reduces free riding phenomena. It could reduce transaction costs or it could be the factor of production as phys-

[10] A need is economically relevant if it is characterized by the scarcity of the good, object of desire.

[11] See Bruni(1994).

[12] It is well-fitting the example proposed by Antoci et Al(2005) about the on-line announcement of brothers/sisters, friends sale.

[13] These kinds of consumes are known as defensive expenditures and they are object of specific econometric studies able to identify and to isolate their weight on economic indicators. In fact the economic activities could negatively influence the stock of common goods, usually no considered in GNP, and induce agents to shift their own consumption behavior to more private oriented one. In such a case, the growth of GNP could be a wrong indicator of the growth of well-being of people.
Even if the accounting of defensive behavior shows some difficulties, the role of these expenditures seems to be a growing phenomenon in advanced country. For example, Leipart (1989) finds that the ratio of defensive expenditure in GNP increased from 5.6 to 10 percent between 1970 and 1985.

ical capital, labour, knowledge and so on. We explore the case of social capital as a productive factor[14], no privately appropriable.

3 The literature on influence of labour supply on economic growth

The analytical framework of the model finds some similarities with endogenous labour supply models. Economic literature only recently have paid attention to the theme. Schematically we find two areas of investigations.
The first one is focalized on identifying properties of stability of the steady state: in his seminal work, Raichlin(1986) studies an overlapping generations model where the presence of low elasticity of substitution between labour and capital (Leontieff technology or CES technology "converging" to Leontieff type) could generate Hopf bifurcations and indeterminacy. In OLG or like-OLG structure, with a more complicated specifications, the same results are found by Nourry (2001), Nourry and Venditti (2001), Grandmont and Al(1998). In continuous time modelling, Benhabib and Farmer(1994) demonstrate that the contemporaneous presence of labour and capital externalities in productive sector could be a source of indeterminacy of equilibria.
The other area of investigation is centered to understand the mechanism trough which endogenous labour supply could influence the long time growth rate: in a OLG model, for the presence of endogenous labor supply, Duranton (2001) finds that the economy, can experience multiple equilibrium paths if leisure and consumption are substitutes, whereas if leisure and consumption are complements then production stays bounded even if endogenous growth is possible and socially desirable; Fisher and Hof (2000) show in a Ramsey type model that status[15] could create a too working and consuming society with respect to social optimum; Eriksson (1996) studies the long growth rate with diminishing labor supply; in a two sector model with physical and human capital, Landron-de-Guevara, Ortigueira, Santos (1999) show the possibility of multiple steady state of endogenous growth; Ortigueira (2000) considers an endogenous growth model in presence of qualified leisure (he captures the Becker's idea about the influence of human capital on the leisure value).
With respect to this literature, we consider a more complicated time-allocation framework because of the dynamics of social capital and we highlight its influences on no-market structure. Even considering a model with no endogenous growth and low externality in capital sector we find Pareto ranked equilibria and interesting dynamics like as the birth of limit cycles and indeterminacy of steady states.

[14] On this approach, see Bartolini's works.

[15] In agents' utility, the authors introduce the comparison of the agent's consumption with others' patterns.

4 The model

We consider a population of identical individuals, with size normalized to one, whose utility depends on two goods: private consumption good, C, and a relational good B[16]. Let us assume a CES instantaneous utility function, additively separable

$$U_i(C(t), B(t)) = \frac{C^{1-\theta}(t)}{1-\theta} + \frac{B(t)}{1-\varepsilon} \tag{1}$$

Socially provided good at time t is produced through the stock of social capital, K_s, the participation to social activity of agent i, s, and the average social participation, $\overline{s}$. This mechanism is described by the following equation:

$$B(t) = K_s^e \overline{s}(t)^\varepsilon s^{1-\varepsilon}(t), \tag{2}$$

As a consequence, we can reformulate (1) in this form[17]

$$U_i(t) = \frac{C^{1-\theta}(t)}{1-\theta} + Z(t)\frac{s^{1-\varepsilon}(t)}{1-\varepsilon} \tag{3}$$

with

$$Z(t) = K_s^e \overline{s}^\varepsilon(t), \tag{4}$$

where (4) captures the role of time allocation of others and social structure in the quality of no working time[18] of the single agent.
We assume a Cobb-Douglas specification in private production:

$$Y(t) = A(t)F(K(t), 1-s(t)) = A(t)K^\alpha(t)(1-s(t))^{1-\alpha} \tag{5}$$

in which

$$A(t) = \overline{K}^\beta(1-\overline{s}(t))^\eta K_s^p \tag{6}$$

A summaries three kinds of externality due to the average economy wide level of capital, labour and social capital. The aggregate production is then

[16] With respect to Antoci's framework, we prefer to do not introduce an explicit division between private consumption goods and private substitutes of socially goods.

[17] Benhabib,Farmer(1996) assume in a related model logaritmic utility on C,and CES form on disutility from labour, Ortigueira(2000) in a two sector endogenous model where education sector qualify, without externality, the leisure of agents assumes homogeneous Cobb-Douglas function, Ladron-de-Guevara and Al(1999)present mixed CES,Cobb-Douglas, specification. The prevalence of Cobb-Douglas formulation is due to the fact that this assumption, in endogenous growth models, generates a static labour supply on the balanced growth (see Barro-Sala-Martin(1995)). The unique exception in infinite living agents model time is due to Eriksson(1996) who, in a similar formulation (with labor disutility in spite of qualified utility), consider a separable CES. Even if in this article we consider steady states with growth rate equal to zero this change produces notable differences in the process of convergence to equilibrium.

[18] As noted by Becker, if I want to go to play tennis, I must find a partner and so it is necessary that even others have free time. A deep seated civic community (churches, voluntary associations, parties) encourages people to spend time in no working activity.

$$Y(t) = \overline{K}^{\beta}(1-\overline{s}(t))^{\eta}K_s^{p}K^{\alpha}(t)(1-s(t))^{1-\alpha} \tag{7}$$

According to Sodini(2006) we think of social capital as the residual product of other activities and the representative agent does not internalize its dynamics. While in the works of Antoci et Al(2001/2002/2003) and Bartolini(2004) the link between social capital and productive sector is from the first to the second, we consider a more general framework in which even private sector could influence the evolution of social capital.
Thus, we suppose that social norms, networks, trust are produced at every date by the interactions of the stock of social capital K_s, average participation to social activities $\overline{s}$, and private sector Y. Formally, the social accumulation dynamics are described[19] by

$$\dot{K}_s = g(Y, K_s, \overline{s}) - nK_s(t) \tag{8}$$

where we assume that g is a continuous function such that $\frac{\partial g}{\partial Y} > 0, \frac{\partial g}{\partial K_s} > 0$,and.$g$ is not increasing respect to $\overline{s}$ .To make the problem analytically tractable, we introduce the following functional form

$$g(Y, K_s, \overline{s}) = \begin{cases} \frac{Y^{\xi_1}(t)K_s^{\gamma_1}(t)}{(1-\overline{s}(t))^{\phi_1}} = \frac{(\overline{K}(t)^{\alpha+\beta}(1-\overline{s}(t))^{1-\alpha+\eta}K_s^{p}(t))^{\xi_1}K_s^{\gamma_1}(t)}{(1-\overline{s}(t))^{\phi_1}} = \frac{\overline{K}^{\xi}(t)K_s^{\gamma}(t)}{(1-\overline{s}(t))^{\phi}}, \ \overline{s} > 0 \\ T, \ \overline{s} = 0 \end{cases} \tag{9}$$

Where $\xi_1 > 0, \xi = (\alpha+\beta)\xi_1 > 0, \phi = \phi_1 - (1-\alpha+\eta)\xi_1 \lesseqgtr 0, \gamma = \gamma_1 + p\xi_1$.
The presence of the bound T of g respect to s precludes the occurrence of no well-defined functions and the possibility of results with no economic meaning: considering time allocation as the main mechanism of social norms formation, we exclude the possibility that social capital may increase in an economy with no social participation ($\overline{s} = 0$). Nonetheless, we can think of the upper bound definitely influenced by initial condition of private capital.

Expression (8) represents a natural extension of the one considered in Sodini(2006). Beside the stock of social capital, the evolution of social capital is due to the choices of agents about time allocation and the stock of private good

[19] In Antoci et Al(2003), the authors consider a simplified version of the "social capital productive function", assuming that, at each time, it coincides with the stock of rationally good. In order to consider a more articulated framework, we remove the hypothesis. It is quite obvious that the mechanisms of formation of norms, on one hand, and istantaneous forms of agent's utility, on the other hand, follows different paths. In fact a dinner with a friend could have a great impact on person's satisfaction but it is possible that it contributes to the creation of social capital, marginally. On the other side, other activities (for example political activity) could create a lower utility to the agent, contributing to the formation of social capital, deeply.

$\overline{K(t)}$ [20] high level of infrastructure such as pubs, theatres, schools, and so on are able to stimulate interactions. Moreover, some kinds of expenditures, such as travels, sport activities, may incorporate interactions[21].
To make some results stronger, we consider a inimical condition respect to result of undesirability of growth, allowing the economy to converge to a steady state with $K_s > 0$ even if social participation is zero[22]. Anyway, as we show below, even in presence of positive and permanent influences from private sector on social capital, low well-being traps could emerge.

5 Agent's problem

We can now formulate the agent's problem:

$$Max \int_0^{+\infty} U(t)e^{-rt} \tag{10}$$

$$\dot{K} = A(t)F(K(t), 1 - s(t)) - C(t) - mK \tag{11}$$

$$\dot{K}_s = g(Y, K_s, \overline{s}) - nK_s(t) \tag{12}$$

$$\lim_{t \to +\infty} K(t)\lambda_K(t)e^{-rt} = 0 \tag{13}$$

$$\lim_{t \to +\infty} K_s(t)\lambda_{K_s}(t)e^{-rt} = 0 \tag{14}$$

He maximizes his utility under constraints of motions (11),(12) where dynamic of K_s does not depend upon any control variables; (14), (13) are the usual transversality conditions.
The current value Hamiltonian function associated to problem is[23]

$$\begin{aligned} H(s(t), K(t), A(t), \lambda_K(t)) &= U_i(t) + \Lambda_K[A(t)F(K(t), 1 - s(t)) - C(t)] \\ &\quad + \Lambda_{K_s}(t)[g(Y(t), K_s(t), \overline{s}(t)) - n\dot{K}_s(t)] \end{aligned} \tag{15}$$

[20] As for the other variables involved, we consider that each agent neglects the influence of his decisions on social capital. Thus he does not consider the spillover of the stock of private production on social capital. and takes Y given for each date..

[21] In this model, differently from Sodini(2006), social capital could survives even if $s \to 0$.

[22] Even if social activity are reduced to zero, the on the job relations could make social capital positive, bounded at its lower level.

[23] In what follows we do not consider the motion of λ_{K_s} since steady state of other variables (interesting for our analysis) are not influenced by it. It is due primary to our assumption of scale functions to capture the influence of social capital stock on private sector and on utility. In modified versions we have only more complications to study nature of steady state without significative differences in qualitative results.

By applying the maximum principle[24] we obtain that the equilibrium dynamics[25] must satisfy the following conditions [26]:

$$\frac{\partial H}{\partial C} = \frac{1}{C^\theta} - \Lambda = 0$$
$$\frac{\partial H}{\partial s} = K_s^e(1-s)^{-\varepsilon}(1-\overline{s})^{\varepsilon} - (1-\alpha)K_s^p K^\alpha \overline{K}^\beta \lambda (1-s)^{-\alpha}(1-\overline{s})^\eta \leq 0;$$
$$\text{and} \quad \frac{\partial H}{\partial s} s = 0$$
$$\dot{K} = \frac{\partial H}{\partial \lambda} = \overline{K}^\beta (1-\overline{s})^\eta K_s^p K^\alpha(t)(1-s)^{1-\alpha} - mK - C$$
$$\dot{\Lambda} = r\Lambda - \frac{\partial H}{\partial k} = \Lambda((r+m) - \alpha \overline{K}^\beta (1-\overline{s})^\eta K_s^p K^{\alpha-1})(1-s)^{1-\alpha})$$

Imposing the symmetric equilibrium conditions, $s = \overline{s}, K = \overline{K}$, we get

$$\frac{\partial H}{\partial C} = \frac{1}{C^\theta} - \Lambda = 0 \tag{16}$$
$$\frac{\partial H}{\partial s} = K_s^e - (1-\alpha)K_s^p K^{\alpha+\beta}\Lambda(1-s)^{\eta-\alpha} \leq 0; \quad \frac{\partial H}{\partial s} s = 0 \tag{17}$$
$$\dot{K} = K^{\alpha+\beta}K_s^p(1-s)^{1-\alpha+\eta} - mK - C \tag{18}$$
$$\dot{\Lambda} = \Lambda((r+m) - \alpha K^{\alpha+\beta-1}K_s^p K^{\alpha-1}(1-s)^{1-\alpha+\eta}). \tag{19}$$

Notice that from (16), it follows that C cannot be chosen by the agent equal to zero. From (??), equilibrium time allocation for decentralized economy is:

$$(1-s^*) = \begin{cases} \left(\frac{1}{C^\theta}(1-\alpha)K^{\alpha+\beta}K_s^{p-e}\right)^{\frac{1}{\alpha-\eta}} & \text{if the expression is less than 1} \\ 1 & \text{otherwise} \end{cases} \tag{20}$$

We impose that leisure is a normal good, i.e. if cost of leisure growths the agent reduces his no working time. To do it, we introduce the following.

Condition 1 $\alpha > \eta$

[24] We are considering the problem faced by the representative agent who does not internalize the effect of various externalities. So the dynamic resulting does not coincide with the dynamic system deriving from the optimality conditions for the economy.

[25] The conditions are necessary and sufficient because of the concavity assumptions on functions. We face on case in which, in steady state, variables do not rise up, and so transversality conditions are fulfilled.

[26] We omit the indication of t for readable reason.

It means that externalities in the production due to interactions among workers in work places are low.
For the next analysis we formulate (20) as

$$(1-s) = \left(\Lambda(1-\alpha)K^{\alpha+\beta}K_s^{p-e}\right)^{\frac{1}{\alpha-\eta}} \tag{21}$$

According to Sodini(2006), notice that, the overall effect of an increase of social capital on labour supply is ambiguous and it depends on the sign of $p-e$. We remember that $p-e$ captures the difference of returns between leisure side and productive side of social capital. If it is positive, a growth of social capital makes trust up and induces people to work more to earn more, otherwise the same phenomenon increases the "value" of no working-time and induces to work less[27]. We consider both possibilities.

6 Steady state and local stability property

By differentiating each equilibrium differential equation with respect to k, λ,k_s, and evaluating it at the steady state points, we obtain the coefficients of the matrix of the linearized system around a steady state. The eigenvalues of this matrix completely determine the local stability of the steady state. In the present model two variables (k_s, k) are predetermined, so if two of the eigenvalues have negative real part and the other is positive we have the standard result of saddle path stability and the dynamics take place on stable manifold and the system converges monotonically to the steady state thanks to allocative choices of agents. If all eigenvalues have negative real parts the steady state is locally indeterminate and the convergence could run on different paths. If two or three eigenvalues have positive real parts the steady state is unstable and even if economy starts in a position very close to the steady state it is repulsed and tends to go away. Before turning to ate analysis of dynamics, it may be useful to illustrate some resolutes about the stability property of steady state in a three dimensional dynamical system.

6.1 Relations between invariants of 3x3 matrix, eigenvalues signs and stability property of a steady state

Consider a three dimensional dynamical system in normal form:

$$\begin{cases} \dot{x}(t) = F(x(t), y(t), z(t)) \\ \dot{y}(t) = G(x(t), y(t), z(t)) \\ \dot{z}(t) = H(x(t), y(t), z(t)) \end{cases} \tag{22}$$

the Jacobian associated is

[27] See Sodini(2006A) for a similar result.

$$\widehat{J} = \begin{pmatrix} \frac{\partial \dot{x}(t)}{\partial x(t)} & \frac{\partial \dot{x}(t)}{\partial y(t)} & \frac{\partial \dot{x}(t)}{\partial z(t)} \\ \frac{\partial \dot{y}(t)}{\partial x(t)} & \frac{\partial \dot{y}(t)}{\partial y(t)} & \frac{\partial \dot{y}(t)}{\partial z(t)} \\ \frac{\partial \dot{z}(t)}{\partial x(t)} & \frac{\partial \dot{z}(t)}{\partial y(t)} & \frac{\partial \dot{z}(t)}{\partial z(t)} \end{pmatrix} \tag{23}$$

and its characteristic equation is

$$X^3 + AX^2 + BX + C = 0 \tag{24}$$

where

$A = -trace(\widehat{J})$

$B = \det \begin{pmatrix} \frac{\partial \dot{x}(t)}{\partial x(t)} & \frac{\partial \dot{x}(t)}{\partial y(t)} \\ \frac{\partial \dot{y}(t)}{\partial x(t)} & \frac{\partial \dot{y}(t)}{\partial y(t)} \end{pmatrix} + \det \begin{pmatrix} \frac{\partial \dot{y}(t)}{\partial y(t)} & \frac{\partial \dot{y}(t)}{\partial z(t)} \\ \frac{\partial \dot{z}(t)}{\partial y(t)} & \frac{\partial \dot{z}(t)}{\partial z(t)} \end{pmatrix} + \det \begin{pmatrix} \frac{\partial \dot{x}(t)}{\partial x(t)} & \frac{\partial \dot{x}(t)}{\partial z(t)} \\ \frac{\partial \dot{z}(t)}{\partial x(t)} & \frac{\partial \dot{z}(t)}{\partial y(t)} \end{pmatrix} =$

$C = -\det(J)$

We recall some results about signs of eigenvalues of $\widehat{J}$ and locally properties of steady state, referring to the case in which two variables are predetermined.

Proposition 2 *All real eigenvalues and all real part of the complex conjugate eigenvalues are negative if and only if $-A, B, -C$ and $AB - C$, are positive. In this case steady state is indeterminate.*

Proposition 3 *A negative C and a negative B are sufficient for saddlepoint stability, that is a unique two-dimensional manifold determines the set of stable flows that converge to steady state.*

Proposition 4 *A negative C and a positive A are sufficient for saddlepoint stability, that is a unique two-dimensional manifold determines the set of stable flows that converge to steady state.*

Proposition 5 *A positive C is sufficient to have two positive eigenvalues. In this case steady state is repulsive.*

Conditions in (2) derive directly from Routh-Hurwitz criterion (see for example Lorenz(1994), Gandolfo(1995) and Wirl(1997) for a specific study of no internalized sector).
The proof of the other two lemmas is straightforward if we consider the following relations between the eigenvalues and the coefficient of characteristic polynomial.
If all real eigenvalues are real then we have that:

$$A = -(\lambda_1 + \lambda_2 + \lambda_3), B = (\lambda_1(\lambda_2 + \lambda_3) + \lambda_2\lambda_3), C = -\lambda_1\lambda_2\lambda_3 \tag{25}$$

If λ_1, is real and $\lambda_{2,3}$ are conjugate complex eigenvalues ($\lambda_{2,3} = a \pm ib$),then we have that:

$$A = -(\lambda_1 + 2a), B = 2a\lambda_1 + (a^2 + b^2), C = -\lambda_1(a^2 + b^2) \tag{26}$$

6.2 Regime s=0

Fixed Points: existence

Consider now the system determined by the choice s=0:

$$\begin{aligned} \dot{K} &= K^{\alpha}K_s^{p} - mK - C && (27)\\ \dot{\Lambda} &= \Lambda((r+m) - \alpha K^{\alpha+\beta-1}K_s^{p}) && (28)\\ \dot{K_s} &= T - nk_s && (29) \end{aligned}$$

performing the transformation

$$k_s = \log K_s, k = \log K, \lambda = \log \Lambda \qquad (30)$$

the system becomes

$$\begin{aligned} \dot{k} &= e^{(\alpha+\beta-1)k+pk_s} - m - e^{-\frac{1}{\theta}\lambda-k} && (31)\\ \dot{\lambda} &= (r+m) - \alpha e^{(\alpha+\beta-1)k+pk_s} && (32)\\ \dot{k_s} &= Te^{-k_s} - n && (33) \end{aligned}$$

It has a stationary equilibrium with coordinates

$$\begin{aligned} k_s^* &= \log(\frac{T}{n}),\ k^* = \log(\frac{T^p\alpha}{(1+m)n^p})^{1-(\alpha+\beta)},\\ \lambda^* &= \log(\frac{((1+m)n^p)^{1-(\alpha+\beta)}\alpha^{\alpha+\beta}}{(T^{p(1-(\alpha+\beta))}(r+m(1-\alpha))})^{\theta} && (34) \end{aligned}$$

for which the following steady state relations are fulfilled

$$\frac{(r+m)}{\alpha} - m = e^{-\frac{1}{\theta}\lambda-k} \qquad (35)$$

$$\frac{(r+m)}{\alpha} = e^{(\alpha+\beta-1)k+pk_s} \qquad (36)$$

$$\frac{T}{\eta} = e^{k_s} \qquad (37)$$

Concerning on the role of parameters related to social capital we not that the steady state value of k_s depends positively on value of T (that is the bound of social endowment without participation in social activities) and negatively on η. Their ratio influences positively even the state value of k. The interpretation of this result is enough straightforward: if on the job place, the workers share the same social values and behaviors, the productivity is larger. Differences of

languages and traditions could create obstacles in development of co-ordination of labour.
To have the compatibility of the previous steady state and the agents' allocations the following condition must be satisfied.

Proposition 6 *(k^*, λ^*, k_s^*) is attainable if and only if the following relation is fulfilled:*

$$\left(\frac{T^p \alpha^{\frac{\alpha+\beta+\theta}{(\alpha+\beta-\theta)}}}{n^p(1+m)}\right)^{(1-(\alpha+\beta))(\alpha+\beta-\theta)} \left(\frac{T}{n}\right)^{p-e} \left(\frac{(1-\alpha)^{1/\theta}}{r+m(1-\alpha)}\right)^{\theta} > 1 \tag{38}$$

Proof. A triplet (K, Λ, K_s) is compatible with the choice $s = 0$ if and only if

$$\left(\Lambda(1-\alpha)(K)^{\alpha+\beta} K_s^{p-e}\right)^{\frac{1}{\alpha-\eta}} > 1 \tag{39}$$

Substituting steady state values (K^*, Λ^*, K_s^*) we have

$$\begin{aligned} &((\frac{T^p \alpha}{(1+m)n^p})^{1-(\alpha+\beta)})^{\alpha+\beta} (\frac{T}{n})^{p-e}(1-\alpha)\times \\ &\times(\frac{((1+m)n^p)^{1-(\alpha+\beta)}\alpha^{\alpha+\beta}}{(T^{p(1-(\alpha+\beta)}(r+m(1-\alpha))})^{\theta} > 1 \end{aligned} \tag{40}$$

Rearranging (40) and collecting we have the result. ■
The inequality (38) is quite difficult to be analyzed completely. Anyway, we can obtain several interpretations if we concentrate on a subset of parameters, and we fix the others in a convenient way. We have classified the following cases:
The condition (40) is fulfilled if, ceteris paribus, r is enough low. Being enough patient, agents give an high weight to allocate their own time endowment in productive sector because they want to increase their utility by future consumes[28].
If $\alpha + \beta > \theta$ then condition (40) is fulfilled if m is low;
If $\alpha + \beta < \theta$ then condition (40) is fulfilled if m is large;

To understand the role of the parameters of social capital we have to consider the sign of $(p-e) + (1-(\alpha+\beta))(\alpha+\beta-\theta)$: if it is positive, condition (40) is fulfilled if $\frac{T}{n}$ is quite large, the opposite if it is negative. The interpretation of these results is straightforward. If the role of social capital in utility function is enough low the agents are induced to spend their time in working activities especially if the lower bound of sociality is high. Notice that considering the usual calibration of economic side of the model, the aggregate $(1-(\alpha+\beta))(\alpha+\beta-\theta)$ is positive. This makes enough probable the existence of low social capital steady state even if $p - e < 0$[29].

[28] Becuse of our assumptions, agents do not internalize the future effects of time allocated in social activities.

[29] This result contrasts the analogous one found in Sodini(2006). However, in the present model, we have introduced a lower bound that makes the allocation of the whole time endowment in labouring activities more attracting.

Dynamical properties of the steady state
The Jacobian matrix associated to the dynamical system is

$$J = \begin{bmatrix} \frac{(r+m)}{\alpha}(\alpha+\beta) - m & (\frac{(r+m)}{\alpha} - m)/\theta & (\frac{(r+m)}{\alpha} - m)p \\ -(r+m)(\alpha+\beta-1) & 0 & -(r+m)p \\ 0 & 0 & -K_s^l \end{bmatrix} \tag{41}$$

J is a upper block triangle matrix: an eigenvalue is read in the last term on diagonal and it is $-K_s^l < 0$,. The other two are determined by the sub-matrix

$$J_1 = \begin{bmatrix} \frac{(r+m)}{\alpha}(\alpha+\beta) - m & (\frac{(r+m)}{\alpha} - m)/\theta \\ -(r+m)(\alpha+\beta-1) & 0 \end{bmatrix} \tag{42}$$

that has $trace(J_1) > 0$ and $\det(j) < 0$

It follows that the two eigenvalues have opposite real part and the stationary equilibrium (k^*, λ^*, k_s^*) is saddle point if we exclude β very high. Note that the parameters of social capital do not enter in the determination of stability conditions of steady state, but influence its existence.

6.3 Case $s \in (0,1)$

If $s \in (0,1)$ the dynamical system that describes the evolution of the model is

$$\dot{K} = (1-\alpha)^\chi K_s^{\widehat{p}} \lambda^\chi K^{(\chi+1)\vartheta} - \frac{1}{\Lambda^{1/\theta}} - mK \tag{43}$$

$$\dot{\Lambda} = \Lambda((r+m) - (1-\alpha)^\chi K_s^{\widehat{p}} \alpha K^{(\chi+1)\vartheta-1} \lambda^\chi) \tag{44}$$

$$\dot{K}_s = \frac{K(t)^\xi K_s^\gamma(t)}{\left(\Lambda(1-\alpha)K^\vartheta K_s^{p-e}\right)^{\frac{\varphi}{\alpha-\eta}}} - nk_s(t) \tag{45}$$

whit $\chi = \frac{1-\alpha+\eta}{\alpha-\eta} > 0, \vartheta = (\alpha+\beta) > 0$, and $(p-e) \lesseqgtr 0$, $\widehat{p} = p + (p-e)\chi \lesseqgtr 0$
We consider the transformation

$$k_s = \log K_s, k = \log K, \lambda = \log \Lambda \tag{46}$$

The system could be expressed as

$$\dot{k} = e^{D_0 + \widehat{p}k_s + \chi\lambda + ((\chi+1)\vartheta - 1)k} - e^{-\frac{1}{\theta}\lambda - k} - m \tag{47}$$

$$\dot{\lambda} = (r+m) - \alpha e^{D_0 + \widehat{p}k_s + \chi\lambda + ((\chi+1)\vartheta-1)k} \tag{48}$$

$$\dot{k}_s = e^{D_1 + D_2 k_s + D_3 \lambda + D_4 k} - \eta \tag{49}$$

where $D_0 = \log(1-\alpha)^\chi < 0, D_1 = \log(1-\alpha)^{\frac{1}{\eta-\alpha}} > 0, D_2 = \gamma - 1 - \frac{\varphi(p-e)}{\alpha-\eta}$, $D_3 = -\frac{\varphi}{\alpha-\eta}$, $D_4 = \xi - \frac{\vartheta\varphi}{\alpha-\eta}$

It is routine calculation to find the steady state of the model[30]: we have that in steady state the following expressions are valid:

$$e^{-\frac{1}{\vartheta}\lambda-k} = \frac{(r+m)}{\alpha} - m \tag{50}$$

$$e^{D_0+\widehat{p}k_s+\chi\lambda+((\chi+1)\vartheta-1)k} = \frac{(r+m)}{\alpha} \tag{51}$$

$$e^{D_1+D_2k_s+D_3\lambda+D_4k} = \eta \tag{52}$$

and taking logs, we obtain steady state values of (k,λ,k_s):

$$\begin{aligned} k_s^{**} = &-\left\{[(\vartheta+\chi\vartheta-1-D_0\theta)D_3+(D_0-\chi)D_4]\left(\ln\tfrac{r+m-m\alpha}{\alpha}\right)\theta+\right.\\ &+(\vartheta-\eta+\chi\vartheta)\ln\eta-[\vartheta(1+\chi(1-\theta))+1]D_1+\\ &\left.+\left(\ln\tfrac{r+m}{\alpha}\right)(D_3\theta-D_4))\right\}*D^{-1}\\ k^{**} = &\tfrac{-D_1+D_2D_0+(D_3\theta-D_2\chi)\left(\ln\frac{r+m-m\alpha}{\alpha}\right)\theta-D_2\ln\frac{r+m}{\alpha}}{D}\\ \lambda^{**} = &(-\theta(-D_1+D_2D_0-D_2\ln\tfrac{r+m}{\alpha}+\ln\eta+\\ &-\left(\ln\tfrac{r+m-m\alpha}{\alpha}\right)[(D_2(\chi\vartheta+1)-D_2-D_4)]*D^{-1} \end{aligned}$$

with the condition that $D = D_2\chi\theta - D_2\chi\vartheta - D_3\theta + D_4 \neq 0$

The Jacobian matrix is

$$J = \begin{bmatrix} [(\chi+1)\vartheta]\frac{(r+m)}{\alpha} - m & \chi\frac{(r+m)}{\alpha} + \frac{1}{\theta}(\frac{(r+m)}{\alpha} - m) & \widehat{p}\frac{(r+m)}{\theta\alpha} \\ -[(\chi+1)\vartheta-1](r+m) & -\chi(r+m) & -\widehat{p}(r+m) \\ D_4\eta & D_3\eta & D_2\eta \end{bmatrix} \tag{53}$$

To make clearer the role of social capital sector on stability property we impose the following conditions:

Condition 7 *$m = 0$ i. e. we ignore (physical) capital depreciation and $\beta = 0$, i.e. we consider that there is not capital externality[31] and so $\vartheta = \alpha$.*

The Jacobian matrix then turns out to be

$$J = \begin{bmatrix} (\chi+1)r & \frac{\chi r}{\alpha} + \frac{r}{\theta\alpha} & \frac{\widehat{p}r}{\theta\alpha} \\ -((\chi+1)\alpha-1)r & -\chi r & -\widehat{p}r \\ D_4\eta & D_3\eta & D_2\eta \end{bmatrix} \tag{54}$$

Consider the characteristic equation

$$X^3 + AX^2 + BX + C = 0 \tag{55}$$

where

$A = -trace(J) = -(r + D_2\eta)$

[30] We do not give the expression for the existence of interior steady state because it is to hard to analise for the number of parameters.

[31] Bhenabib and Farmer(1994) show in their one sector model with endogenous labour supply that this hypotesis rule out the possibility of indeterminacy so that (interior) steady state could be or saddle point or repulsive. We show below that if we consider the Social Capital sector the indeterminacy could come back.

$$B = (-r^2 + r^2\alpha\chi + \theta\alpha\widehat{p}fnr - D_2n\widehat{p}r\theta - \theta\chi r^2 + \theta r D_2 n\alpha + r^2\alpha)/(\alpha\theta)$$

$$C = -\det(J) = -r^2\eta * (D_2\alpha\chi + D_2\alpha - D_2 - \widehat{p}D_4 - \chi\theta D_2 + \widehat{p}f\theta)/(\alpha\theta)$$

Proposition 8 *Necessary condition to have indeterminate steady state is that* $\gamma > (\frac{\varphi(p-e)}{\alpha-\eta} + 1 - r)\frac{1}{\eta}$

As well as many standard economic models, it is difficult to have a clear explanation about the causes of indeterminacy. Anyway the previous proposition suggests that in a decentralized economy for which there is not a perfect perception of the mechanism of formation of social capital if the same has a relative high weight on g it could create indeterminacy. Notice that it is a no really restrictive condition because it is fulfilled if $p < e$, *i.e.* if labour supply is a decreasing function respect to social capital.
For example with the parametrization $\alpha = 1/3, \eta = 1/3, r = 0.02, n = 0.8, \chi = 0.9$,$p = 0.28$, $e = 0.01$, $\xi = 0.02$, δ=0.4, $\varphi = 0.1, \gamma = 0.4$ the steady stale values are $k_s = .6555948124$, $k = .2179303353$, $\lambda = .2690790561$, and $10 - s = 7.981858376$, and eigenvalues are $\lambda_1 = -.08980911310 + .3320726491I$, $\lambda_2 = -.08980911310 - .3320726491I$, $\lambda_3 = -.02768177329.1$.
In such a case, economies with the same parameters, converge to the same steady state but on different paths as showed in figure.

It's interesting to note that social sector could become for specific values of parameters an engine of cycle around steady state.

Proposition 9 *Necessary conditions to have a stable limit cycle is that the following conditions hold:*

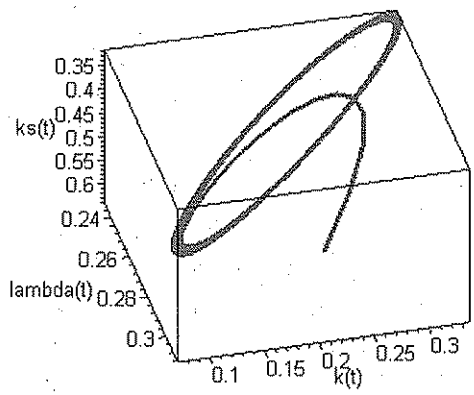
$$r + D_2\eta < 0 \tag{56}$$

$$(-r^2 + r^2\alpha\chi + \theta\alpha\widehat{p}fnr - D_2n\widehat{p}r\theta - \theta\chi r^2 + \theta r D_2 n\alpha + r^2\alpha) > 0 \tag{57}$$

$$(D_2\alpha\chi + D_2\alpha - D_2 - \widehat{p}D_4 - \chi\theta D_2 + \widehat{p}f\theta) > 0 \tag{58}$$

$$(r + D_2\eta)(-r^2 + r^2\alpha\chi + \theta\alpha\widehat{p}fnr - D_2n\widehat{p}r\theta - \theta\chi r^2 + \theta r D_2 n\alpha + r^2\alpha) = \tag{59}$$

$$= (D_2\alpha\chi + D_2\alpha - D_2 - \widehat{p}D_4 - \chi\theta D_2 + \widehat{p}f\theta) \tag{60}$$



Stable limit cycle. Parameters value: α=1/3, η=1/3, r=0.02, n=0.17, ρ=0.20, p=0.9, e=0.77, χ=0.02, δ=0.2, θ=0.1, $\xi = 0.02$, $\gamma = 0.4$

We present the following theorem that characterizes the local properties of steady state (k^*, λ, k_s^*)

Proposition 10 *If $D_4 < -\frac{r}{\eta}$, then we can't tule rule out the possibility of indeterminacy even if repulsive and saddle path behavior are possible.*
If $D_4 > -\frac{r}{\eta}$ then the steady state can't be everything (repulsive and saddle path stable) but on indeterminate.

7 Well-being

The presence of multiple steady states creates a big interest for the possibility of occurrence of poverty traps. If in standard economic models the term poverty is univocally associated with the idea of "material richness", Antoci shows how the presence of different terms in utility function could create "paradoxical results" in which economic system converges to an high consumption state meanwhile the well-being is Pareto dominated by the other fixed points. This is not too paradoxical if we think to workalcoholic society populated by reach but unsatisfied men. The results in Antoci are enforced in this model for the repercussion of low social capital to production. It means that a high social capital level with a positive participation to social activities could be more productive of an economy with the same parameters but captured by fixed point with low social capital and all time devoted to work. Unfortunately the model doesn't allow general results on Pareto dominance of the two steady states, anyway we have show numerically a large set of parameters for which the steady state with high social capital dominates the second one, but it is repulsive or only locally attractive.

8 Conclusion

We have explored a three dimensional model of probable interactions between social capital and private sector. According to Antoci's framework, we have considered models with constant infinitely lived homogeneous population in which the time spent in social activities plays the determinant role to explain the social development of a community and each agent doesn't consider the effect of his own time allocation on the evolution of social capital.

Our study has introduced a certain number of innovative features: we have considered, at the same time, the effects of social capital on private production and on utility of the agents. This possibility has created a more articulated description of the phenomenon and the weight and the direction of the contribute of social capital on economic growth is based on a complex mix of interplays between social structure, individual preferences and productive sector.
Respect to Sodini (2006), we have introduced the possibility of capital accumulation and, to make the problem analytically tractable, we have imposed more restrictive hypothesis. Anyway some interesting results emerge: the model, if some technical conditions are fulfilled, exhibits two fixed points: the first one with no social capital and a second one with a positive stock of social capital. Even if the Pareto dominance depends on the magnitude of the parameters it emerges that the presence of a low stock of social resources and on coordination among agents could create well-being trap characterized by too working life style with possible negative effects even in material welfare. It is interesting to note that the study of the stability of the steady state with positive stock of social capital have shown that this last could be a source of no saddle point stability allowing a social based explanation of economic fluctuations and long run differences in economic performances.

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