Global optimization of a generalized linear program

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Abstract

In this paper a solution algorithm for a class of generalized linear programs having a polyhedral feasible region is proposed. The algorithm is based on the so called optimal level solutions method. Various global optimality conditions are discussed and implemented in order to improve the efficiency of the algorithm.

Key words: generalized linear programming, optimal level solutions, global optimization.
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1 Introduction

The aim of this paper is to study, from both a theoretical, an algorithmic and a computational point of view, a class of generalized linear programs, that is to say problems having a polyhedral feasible region and an objective function of the kind $f(x) = \phi(c^T x + c_0, d^T x + d_0)$.

Various particular problems belonging to this class have been studied in the literature of mathematical programming and global optimization, from both a theoretic and an applicative point of view ([2, 14, 15, 16, 17, 20, 25]). In particular, it is worth noticing that this class covers several multiplicative, fractional and d.c. problems (see for all [4, 13, 21]) which are very used in applications, such as location models and Data Envelopment Analysis (see for all [1, 11, 13, 19, 21]).

The solution method proposed to solve this class of problems is based on the so called "optimal level solutions" method (see [3, 4, 5, 6, 7, 8, 9, 10, 12, 18, 22, 23, 24]). It is known that this is a parametric method, which

Notice that functions f and ϕ share the same generalized convexity/concavity properties, as it is shown in the following general result.

Theorem 2.1 *Let $X \subseteq \Re^n$ be a convex set, $M \in \Re^{m\times n}$ be a matrix, $q \in \Re^m$ be a vector and $Y = \{y \in \Re^m : \ y = Mx + q, \ x \in X\}$. Let also $\phi : Y \longrightarrow \Re$ and $f : X \longrightarrow \Re$ be functions such that $f(x) = \phi(Mx + q)$. Then, the set Y is convex and function $\phi(y)$ is quasiconvex [convex, concave, quasiconcave, semistrictly quasiconvex] if and only if $f(x)$ is quasiconvex [convex, concave, quasiconcave, semistrictly quasiconvex].*

Proof The set Y is convex since it is an affine transformation of another convex set. Assume $\phi(y)$ to be quasiconvex and let $x_1, x_2 \in X$. Then, defining $y_1 = Mx_1 + q$ and $y_2 = Mx_2 + q$, for all $\lambda \in (0,1)$ it results:

$$\begin{aligned} f(\lambda x_1 + (1-\lambda)x_2) &= \phi(\lambda y_1 + (1-\lambda)y_2) \\ &\leq \max\{\phi(y_1), \phi(y_2)\} = \max\{f(x_1), f(x_2)\}. \end{aligned}$$

Assume now $f(x)$ to be quasiconvex and let $y_1, y_2 \in Y$. Then, for the definition of Y there exist $x_1, x_2 \in X$ such that $y_1 = Mx_1 + q$ and $y_2 = Mx_2 + q$. For all $\lambda \in (0,1)$ it yields:

$$\begin{aligned} \phi(\lambda y_1 + (1-\lambda)y_2) &= f(\lambda x_1 + (1-\lambda)x_2) \\ &\leq \max\{f(x_1), f(x_2)\} = \max\{\phi(y_1), \phi(y_2)\}. \end{aligned}$$

The whole result is then proved. The other cases are analogous. □

As a consequence, if function ϕ is semistrictly quasiconvex then problem P can be solved by means of any of the methods which looks for a local minimum point. On the other hand, if function ϕ is not semistrictly quasiconvex (that is to say that it has a more "concave" behaviour) then problem P allows the presence of many local minima and hence a specific solution method is needed in order to find a global minimum.

3 A parametric approach

In this section we show how problem P can be solved by means of the so called *optimal level solutions approach* (see for all [6, 7, 8, 22]). With this aim, let $\xi \in \Re$ be a real parameter and let us define the corresponding parametrical subset of X:

$$X_\xi = \{x \in \Re^n : \ Ax = b, \ x \geq 0, \ d^T x + d_0 = \xi\}$$

to move in the various steps through several optimal level solutions until the optimal solution is found (see [8]).

As preliminary results, let us now provide some conditions related to the existence of optimal level solutions.

Theorem 3.1 *Consider problem P and let $\xi' \in \Lambda$ be a feasible level. The following properties hold:*

i) if $X_{\xi'}$ is compact then the sets X_ξ are compact $\forall \xi \in \Lambda$;

ii) if $S_{\xi'} = \emptyset$ then $S_\xi = \emptyset$ $\forall \xi \in \Lambda$;

iii) if $S_{\xi'} \neq \emptyset$ then $S_\xi \neq \emptyset$ $\forall \xi \in \Lambda$.

iv) if $S_{\xi'} \neq \emptyset$ and the set of feasible levels Λ is bounded, then problem P admits optimal solutions.

Proof *i)* Assume by contradiction that there exists $\tilde{\xi} \in \Lambda$ such that $X_{\tilde{\xi}}$ is not compact, so that there exists a half-line

$$r = \{x \in \Re^n : \; x = x_0 + tu, \; t \in \Re_+, \; x_0 \in X_{\tilde{\xi}}, \; u \in \Re^n\} \subseteq X_{\tilde{\xi}}$$

Hence, it is $\tilde{\xi} = d^T x_0 + t d^T u + d_0$ for all $t \geq 0$, which implies $d^T u = 0$. Let now be $x' \in X_{\xi'}$, for all $t \geq 0$ it results $d^T(x' + tu) + d_0 = d^T x' + t d^T u + d_0 = d^T x' + d_0 = \xi'$ and this contradicts the compactness of $X_{\xi'}$.

ii) Since $S_{\xi'} = \emptyset$ then $\inf_{x \in X_{\xi'}} \bar{c}^T x = -\infty$ and hence there exists a half-line $x' + tu$, $t \geq 0$, contained in $X_{\xi'}$ such that $d^T u = 0$ and $\bar{c}^T u < 0$. Given any feasible level $\xi \in \Lambda$ we can consider a half-line $x_\xi + tu$, $t \geq 0$, where $x_\xi \in X_\xi$; since $d^T u = 0$ this half-line belongs to X_ξ and from $\bar{c}^T u < 0$ it follows that $\lim_{t \to +\infty} \bar{c}^T (x_\xi + tu) = \lim_{t \to +\infty} (\bar{c}^T x_\xi + t \bar{c}^T u) = -\infty$ and hence $S_\xi = \emptyset$.

iii) Follows directly from *ii)*.

iv) From *iii)* it follows that for all feasible levels $\xi \in \Lambda$ there is $x_\xi \in S_\xi \neq \emptyset$. Given $h(\xi) = f(x_\xi)$ we have that the continuity of f and the convexity of Λ implies the continuity of h too, hence the result yields being Λ compact. □

The previous result points out that problem P admits no optimal level solutions if and only if for all feasible levels $\xi \in \Lambda$ the subproblems $\overline{P}_\xi$ have no finite optimum. In this case problem P can be studied as follows:

$$P \equiv \begin{cases} \inf_{\xi \in \Lambda} \phi(-\infty, \xi) & \text{if } \phi(y_1, y_2) \text{ is strictly increasing} \\ & \text{with respect to variable } y_1 \\ & \\ \inf_{\xi \in \Lambda} \phi(+\infty, \xi) & \text{if } \phi(y_1, y_2) \text{ is strictly decreasing} \\ & \text{with respect to variable } y_1 \end{cases}$$

4 Sensitivity analysis

In order to suggest a simplex-like method to solve problem P we need to characterize optimal level solutions and to determine some related properties. With this aim, in this section we assume that problem P admits optimal level solutions. Let $\xi \in \Lambda$ be a feasible level and let x' be the corresponding optimal level solution, that is the optimal solution for the linear programming problem $\overline{P}_\xi$:

$$\overline{P}_\xi : \begin{cases} \min \bar{c}^T x \\ Ax = b \\ d^T x = \xi - d_0 \\ x \geqq 0 \end{cases} \equiv \begin{cases} \min \bar{c}^T x \\ \tilde{A}x = \begin{pmatrix} b \\ \xi - d_0 \end{pmatrix} \\ x \geqq 0 \end{cases}$$

where $\tilde{A} = \begin{bmatrix} A \\ d^T \end{bmatrix}$. Let $\tilde{A}_B$ be the basis corresponding to the optimal solution $x' = (x'_B, 0)$, where $x'_B = \tilde{A}_B^{-1} \begin{pmatrix} b \\ \xi - d_0 \end{pmatrix}$; notice that the related reduced cost vector results to be nonnegative:

$$\tilde{c}_N^T = \bar{c}_N^T - \bar{c}_B^T \tilde{A}_B^{-1} \tilde{A}_N \geq 0$$

Let us now study the parametric problem $\overline{P}_{\xi+\theta}$ with $\theta \in \Re$ such that the corresponding optimal level solutions have the same basis $\tilde{A}_B$:

$$\overline{P}_{\xi+\theta} : \begin{cases} \min \bar{c}^T x \\ Ax = b \\ d^T x = \xi - d_0 + \theta \\ x \geqq 0 \end{cases} \equiv \begin{cases} \min \bar{c}^T x \\ \tilde{A}x = \begin{pmatrix} b \\ \xi - d_0 + \theta \end{pmatrix} \\ x \geqq 0 \end{cases}$$

The optimal solutions of $\overline{P}_{\xi+\theta}$ are given by $x'(\theta) = (x'_B(\theta), 0)$ where:

$$x'_B(\theta) = x'_B + \theta \tilde{A}_B^{-1} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

while the reduced cost vector is $\tilde{c}_N$ as defined above. Vector $\tilde{c}_N$ results to be independent to the parameter θ, hence the level optimality of x' (that is to say the nonnegativeness of $\tilde{c}_N$) implies the level optimality of all the vectors $x'(\theta)$ which are feasible for X, given by the values $\theta \in \Re$ such that $x'_B(\theta) \geqq 0$. In this light it is worth defining the following values:

$$F_L := \inf\{\theta \in \Re : x'_B(\theta) \geqq 0\} \quad , \quad F_R := \sup\{\theta \in \Re : x'_B(\theta) \geqq 0\}$$

A basis of this problem has a structure of the kind:

$$\begin{bmatrix} A_B & A_{N_k} \\ d_B^T & d_{N_k} \end{bmatrix}$$

where $d_{N_k} \neq 0$ to guarantee its nonsingularity. The optimality of x' with respect to problem $\overline{P}_\xi$ can then be characterized by:

$$\hat{c}_{N_j} - [\hat{c}_B^T, \hat{c}_{N_k}] \begin{bmatrix} A_B & A_{N_k} \\ d_B^T & d_{N_k} \end{bmatrix}^{-1} \begin{bmatrix} A_{N_j} \\ d_{N_j} \end{bmatrix} \geq 0 \quad \forall N_j \neq N_k$$

which, by means of simple calculations, is equivalent to:

$$\hat{c}_{N_j} \geq \frac{\hat{c}_{N_k} \hat{d}_{N_j}}{\hat{d}_{N_k}} \quad \forall N_j \neq N_k$$

This condition holds if and only if the three following conditions are all verified:

$$\hat{c}_{N_j} \geq 0 \quad \forall N_j \neq N_k \text{ such that } \hat{d}_{N_j} = 0$$

$$\frac{\hat{c}_{N_k}}{\hat{d}_{N_k}} \leq \frac{\hat{c}_{N_j}}{\hat{d}_{N_j}} \quad \forall N_j \neq N_k \text{ such that } \hat{d}_{N_j} > 0$$

$$\frac{\hat{c}_{N_k}}{\hat{d}_{N_k}} \geq \frac{\hat{c}_{N_j}}{\hat{d}_{N_j}} \quad \forall N_j \neq N_k \text{ such that } \hat{d}_{N_j} < 0$$

5 Solution algorithm

In order to find a global minimum (assuming that one exists) it would be necessary to solve problems $\overline{P}_\xi$ for all the feasible levels. In this section we will show that this can be done by means of a finite number of iterations, using the results of the previous section.

The method scans all the feasible levels starting from a certain feasible level ξ'; the levels $\xi > \xi'$ are visited in increasing order, while the levels $\xi < \xi'$ are visited in decreasing order. For the sake of convenience, the described algorithm visits the feasible levels only in increasing order, hence the levels $\xi < \xi'$ can be analyzed by solving the following problem which is equivalent to P:

$$P \equiv \tilde{P} : \begin{cases} \inf f(x) = \tilde{\phi}(c^T x + c_0, \tilde{d}^T x + \tilde{d}_0) \\ x \in X \end{cases}$$

where $\tilde{\phi}(y_1, y_2) = \phi(y_1, -y_2)$, $\tilde{d} = -d$ and $\tilde{d}_0 = -d_0$. In this light, the decreaseness of the feasible levels of P corresponds to the increaseness of the feasible levels of $\tilde{P}$.

Procedure Visit(*inputs*: P, ξ', ξ_{max}, x', $\bar{x}$, UB; *outputs*: Opt, $OptVal$)
let $\delta > 0$ be the step parameter;
while $\xi' < \xi_{max}$
set $\Delta_x := \frac{x'_\delta - x'}{\delta}$ where $x'_\delta := \arg\min\{\overline{P}_{\xi'+\delta}\}$;
set $F_R := \sup\{\theta \geq 0 : x' + \theta\Delta_x \geq 0\}$;
let $z(\theta) = \phi\left(\theta c^T\Delta_x + c^T x' + c_0, \xi' + \theta\right)$;
set $[\overline{\theta}, z_{inf}] := MinRestriction(z(\theta), [0, F_R])$;
if $z_{inf} = -\infty$ then $\overline{x} := [\,]$; $UB := -\infty$; $\xi' := +\infty$ *else*
if $z_{inf} < UB$ *then*
$UB := z_{inf}$;
if $\overline{\theta} = +\infty$ *then* $\overline{x} := [\,]$ *else* $\overline{x} := x' + \overline{\theta}\Delta_x$ *end if*;
end if;
set $\xi' := \xi' + F_R$ and $x' := x' + F_R\Delta_x$;
end if;
# Optional : $[\xi', x'] := ImplicitVisit(\xi', x', \Delta_x, \xi_{max})$;
end while;
$Opt := \overline{x}$; $OptVal := UB$;
end proc.

It remains to verify the convergence (finiteness), that is to say that the procedure stops after a finite number of steps. First note that, at every iterative step of the proposed algorithm, an edge of the feasible region is fully visited; note also that the level is increased from ξ' to $\xi' + F_R > \xi'$, so that it is not possible to consider an already visited edge; the convergence then follows since in a polyhedron there is a finite number of possible edges.

Remark 5.1 Let us point out that problems $\overline{P}_\xi$ are independent of the function ϕ. This means that problems having the same feasible region, the same c and d, but different function ϕ, they share the same set of optimal level solutions. As a consequence, when procedure "*Main*()" explicitly visits all the feasible levels, these different problems are solved by means of the same iterations of the while cycle in procedure "*Visit*()".

6 Solving more general problems

The problem discussed so far assume function $\phi(y_1, y_2)$ to be strictly monotone with respect to variable y_1. It is worth pointing out that this assumption is not restrictive, in the sense that it may be possible to partition the feasible region X in subsets where $\phi(y_1, y_2)$ is either strictly monotone or constant with respect to variable y_1.

7 Algorithm improvements

In this section we aim to discuss how the proposed algorithm can be improved in the visit of the feasible levels.

7.1 Underestimation functions

Some feasible levels can be implicitly visited by using an underestimation function, that is a function $\psi(\xi)$ which verifies the following property for all the feasible levels ξ:

$$\min_{x \in X_\xi} f(x) \geq \psi(\xi)$$

In fact, given $\xi_a \in [\xi', \xi_{max}]$ it can be easily seen that:

$$\psi(\xi) \geq UB \ \forall \xi \in [\xi', \xi_a] \quad \Rightarrow \quad \min_{x \in X_{[\xi', \xi_{max}]}} f(x) = \min_{x \in X_{[\xi_a, \xi_{max}]}} f(x)$$

This property suggests a way to improve the algorithm by reducing in the various iterations of procedure "*Visit*()" the set of feasible levels to be scanned, that is to say by implicitly visiting some of the feasible levels.

A simple underestimation function, given by the strict monotonicity of $\phi(y_1, y_2)$ with respect to variable y_1, is:

$$\psi(\xi) = \phi(c_0 + \bar{k}, \xi) \quad \text{where} \quad \bar{k} = \begin{cases} \inf_{x \in X} \bar{c}^T x & \text{if } \bar{c} = c \\ -\inf_{x \in X} \bar{c}^T x & \text{otherwise} \end{cases}$$

Another useful underestimation function can be determined within the various iterations of the "while" cycle in procedure "*Visit*()". Let x' and Δ_x be the values obtained in the single iteration. The points in $\{x = x' + (\xi - \xi')\Delta_x, \ \xi' \leq \xi \leq \xi_{max}\}$ have the same reduced cost vector; as a consequence, they are optimal level solutions of problem P whenever they are feasible, while if the feasibility is lost they are optimal level solutions of a problem having a wider feasible region. For this very reason, the following function is another underestimation function:

$$\psi(\xi) = \phi\left(c_0 + c^T[x' + (\xi - \xi')\Delta_x], \xi\right) \quad ; \quad \xi' \leq \xi \leq \xi_{max}$$

An overall underestimation function, which combines the previously proposed ones, is then given by:

$$\psi(\xi) = \max\left\{\phi\left(c_0 + \bar{k}, \xi\right), \phi\left(c_0 + c^T x' + (\xi - \xi')c^T \Delta_x, \xi\right)\right\} \ , \ \xi' \leq \xi \leq \xi_{max}$$

As a consequence, the following procedure "*ImplicitVisit*()" can be proposed in order to improve the visit of the feasible levels.

Procedure **ImproveStartingValues**(*outputs*: x', ξ', $\bar{x}$, UB)

if $|\xi_{min}\xi_{min}| < +\infty$ *then* $\xi_1 := \xi_{min}$ and $\xi_2 := \xi_{max}$

else $\xi_{big} >> 0$, $\xi_1 := \max\{\xi_{min}; -\xi_{big}\}$ and $\xi_2 := \min\{\xi_{max}; \xi_{big}\}$

end if;

Let $x_1 := \arg\min_{x \in X_{\xi_1}} \bar{c}^T x$ and $x_2 := \arg\min_{x \in X_{\xi_2}} \bar{c}^T x$;

if $f(x_1) < UB$ *then* $\bar{x} := x_1$ and $UB := f(\bar{x})$ *end if*;

if $f(x_2) < UB$ *then* $\bar{x} := x_2$ and $UB := f(\bar{x})$ *end if*;

if $\inf_{x \in X} \bar{c}^T x > -\infty$ *then*

Let $x_3 := \arg\min_{x \in X} \bar{c}^T x$;

if $f(x_3) < UB$ *then* $\bar{x} := x_3$ and $UB := f(\bar{x})$ *end if*;

end if;

$x' := \bar{x}$ and $\xi' := d^T \bar{x} + d_0$;

end proc.

8 Computational results

The previously described procedures have been fully implemented with the software MatLab 7.5 R2007b on a computer having 2 Gb RAM and two Xeon dual core processors at 2.66 GHz. The following four different objective functions have been used in the computational test:

	$\phi(y_1, y_2)$	$f(x)$
P_1	$y_1 - y_2^6$	$\left(c^T x + c_0\right) - \left(d^T x + d_0\right)^6$
P_2	$y_1 y_2^3 - y_2^6$	$\left(c^T x + c_0\right)\left(d^T x + d_0\right)^3 - \left(d^T x + d_0\right)^6$
P_3	$y_1 / y_2^2 - y_2^2$	$\left(c^T x + c_0\right) / \left(d^T x + d_0\right)^2 - \left(d^T x + d_0\right)^2$
P_4	$y_2^2 log(y_1) - y_2^4$	$\left(d^T x + d_0\right)^2 log\left(c^T x + c_0\right) - \left(d^T x + d_0\right)^4$

Notice that the functions considered in these problems are not quasi-convex and hence allow the presence of many local minima. We considered problems with a number of variables from n = 20 to n = 200, while the number of equality constraints has been fixed to $m = n/5$.

The problems have been randomly created; in particular, matrices and vectors $c, d \in \Re^n$, $A \in \Re^{m \times n}$ and $b \in \Re^m$ have been generated with components in the interval [-10,10] by using the "rand()" MatLab function (numbers generated with uniform distribution). In P_2 and P_3 the value $d_0 \in \Re$ has been chosen in order to have function $d^T x + d_0$ positive over the feasible region, while in P_4 the value $c_0 \in \Re$ has been chosen in order to have function $c^T x + c_0$ positive over the feasible region. In all of the other cases these

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