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Multiple attractor and non linear dynamics in an Overlapping Generations Model with Environment

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Multiple attractors and non linear dynamics in an Overlapping Generations Model with Environment

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Abstract

This paper develops a one-sector productive overlapping generations model with environment where a CES technology is assumed. Relying on numerical and geometrical approaches, various dynamic properties of the proposed model are explored: the existence of the phenomenon of multistability, or the coexistence of different attractors, is demonstrated. The final part is dedicated to a description of a non typical global bifurcation which determines the appearance of an attracting cycle.

1 Introduction

After the seminal contribution by John and Pecchenino (1994), the OLG specification has become a standard framework for the analysis of interplays between economic growth and environmental quality. During the last decade, in this field, two different lines of research have been developed, one is focussed on the role of environmental policies (taxation schemes, patents, etc.), considering the *static* structure of the models, that is when the variables are evaluated at steady states (see e.g. John et al.(1995), Ono (1996), Ono (2003), Yoshida (2002)); the other has investigated the possibility of *non linear dynamics* out of steady states (see e.g. Zhang (1999), Bhattacharya and Xue (2007), Antoci and Sodini (2009)).
From a general point of view, the OLG framework seems to provide a more appealing description of environmental dynamics, in contrast with the Solow or the Ramsey models: *a)* the discrete time allows temporal lags to be introduced between anthropic activities and their environmental impact; *b)* the demographic structure (that is the finite living agents assumption) creates a decisional mechanism for which the evolution of the environment is not internalized by agents and a cohort may generate environmental changes that outlive them and rebound over successive cohorts.

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We stress the fact that while the papers on the policy implication of environment are stated in a really general framework, to avoid analytical complexity, most of the studies on the dynamic aspects of such OLG models are based on the Cobb-Douglas specification of technology and on simplifying assumptions[1] on preferences.

The aim of this paper is to study the dynamics of a model in which a simple specification of utility is assumed, but a less restrictive specification of production function is introduced[2].

Apart from the more general assumptions on technology, the present paper is close to the analytical specification proposed in John and Pecchenino's work. We consider an overlapping generations model with the following characteristics: there exists a population of individuals whose welfare depends on the stock E_t of a free access environmental good and on the consumption c_t of a private good. We assume that E_t is negatively affected by private consumption and it is improved by specific environmental expenses. Differently from Zhang's model, where a social planner is introduced, but according to Antoci et al.(2009), we consider a decentralized solution: each economic agent can invest in environment and consider as given the allocations of the other agents of the same generation[3].

In this context, we describe conditions for which interesting dynamic phenomena arise and we show that coexistence of attractive steady states or coexistence of non trivial attractors may emerge for very different parameters configurations. According to works in continuous time (see for example Antoci et al.(2005), Wirl (2004)), these results suggest environmental externalities could be an engine for poverty traps[4] and/or complicated behaviour.

The paper is organized as follows. In the next section, we set out the model. In Section 3 and 4 the static and dynamic behaviours will be respectively studied.

2 Model set up

We assume that environmental quality at time t could be described by a positive synthetic index E_t. Without human activity, the dynamics are given by:

$$E_{t+1} = (1-b)E_t + b\overline{E}$$

with $b \in (0,1)$, $\overline{E} > 0$. It follows that such index tends asymptotically to the long run value $\overline{E}$. In order to introduce the influence of anthropic activity, we assume that the economy is populated by two-periods lived agents. At each date t, N identical persons are born. In the first period of their life (when young), the agents supply inelastically their time-endowment, normalized to

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[1] For example, the gross substitution between environment and private consumption.

[2] In a related paper, Antoci et al (2009), the role of assumptions on preferences and of heterogenity of agents are investigated.

[3] This implies that the choices of each agent generate negative externalities on the others and because the environment is a public good, its conservation is characterized by the standard free rider problem, intragenerational and intertemporal.

[4] Poverty trap is a self-perpetuating condition where an economy, caught in a vicious condition, suffers from persistent underdevelopment.

one, to productive sector. Individuals born in t have preferences defined over consumption and an environmental index in old age[5], respectively c_{t+1} and E_{t+1}. They allocate their income between investments to preserve or improve the environmental quality[6], m_t, and saving, s_t.
We assume that the impact of consumption and environmental expenses is linear:

$$E_{t+1} = (1-b)E_t - \beta N c_t + \gamma N m_t + b\overline{E}$$

where $\beta > 0$ measures the degree of the impact of private consumption on environmental quality and $\gamma > 0$ measures the efficiency of environmental expenses.
The preferences of an individual are representable by an utility function $U(c_{t+1}, E_{t+1})$. U is assumed to be twice continuously differentiable and such that $U_c(\cdot) > 0$, $U_E(\cdot) > 0$; $U_{c,c}(\cdot) < 0$, $U_{E,E}(\cdot) < 0$; and $U_{c,E}() > 0$. We assume also that the *Inada* condition $\underset{c\to 0}{lim} U_c(c,E) = +\infty$ holds in order to avoid corner solutions with $c = 0$.

2.1 The productive sector

The economy is assumed to be perfectly competitive so we can introduce the representative firm producing the private good. We consider a CES-technology

$$Y = Af(k_t) = A(\alpha k_t^{-\rho} + 1 - \alpha)^{\frac{1}{-\rho}}$$

where k_t is physical capital at time t, $A > 0$ is a productive parameter, $\alpha \in (0,1)$ measures the degree of capital intensity of production, while $\theta = \frac{1}{1+\rho}$ (with $\rho > -1$, $\rho \neq 0$) is the elasticity of substitution between labour and capital. We assume that capital depreciates at rate δ. From the usual optimality conditions, equilibrium expressions of the wage and of the interest rate follow:

$$\begin{aligned} w_t &= A(1-\alpha)(\alpha k_t^{-\rho} + 1 - \alpha)^{-\frac{1+\rho}{\rho}} && (1) \\ r_t &= A\alpha k^{-1-\rho}(\alpha k_t^{-\rho} + 1 - \alpha)^{-\frac{1+\rho}{\rho}} && (2) \end{aligned}$$

2.2 Agent's problem

To describe the allocation problem of the resource, we consider a decentralized system of decisions where agents act strategically. Each individual takes as given the wage, w_t, the return on saving, r_{t+1}, environmental quality at the beginning of period t, E_t, and consumption of the old generation, c_t . Furthermore, he

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[5] This assumption is adopted in several overlapping generations models (see, among the others, Zhang (1999), Antoci el al (2009)). It simplifies our analysis by abstracting from the consumption-saving choices of agents.

[6] E_t may be interpreted as an index of the environmental amenity or as the stock of the free access environmental good at time t. It is common to all agents and could be regarded as a public good.

formulates expectations on the investment in environmental maintenance and improvement of the other agents:
Assuming that agents are identical, the problem faced by a generic agent born in t is to maximize with respect to c_{t+1} and m_{t+1} the objective function

$$\max_{c_{t+1},m_{t+1}} \; U(c_{t+1}, E^e_{t+1})$$

with the constraints

$$\begin{aligned} w_t &= s_t + m_t \\ c_{t+1} &= (1 + r_{t+1} - \delta)s_t \end{aligned}$$

where E^e_{t+1} is the expectation on environmental quality at the period $t+1$:

$$E^e_{t+1} = (1-b)E_t - \beta N c_t + \gamma(m^i_t + (N-1)m^e_t) + b\overline{E}$$

depending on m^e_t that is the expectations of agent i about the strategies of the other identical $N-1$ agents.

3 Equilibrium dynamics

We assume that the young agent, at time t, is able to perfectly foresee the values of others' protections (and consequently, the environmental index E_{t+1}). The equilibrium conditions $\forall t$ become:

$$\begin{cases} -U_{c_{t+1}}(\cdot)(1 + r_{t+1} - \delta) + \gamma \cdot U_{E^e_{t+1}}(\cdot) = 0 \\ m^e_t = m^*_t \\ E_{t+1} = (1-b)E_t - \beta N c^*_t + \gamma N m^*_t + b\overline{E} \\ k_{t+1} = s^{i*}_t \\ \text{equations (1) and (2).} \end{cases} \tag{3}$$

The first equation in (3) is the *F.O.C.* for the generic agent and implicitly defines the solution for his maximization problem: m^*_t, s^*_t, c^*_t; the second group imposes the *ex-post perfect foresight* condition on m_t; the third and the fourth equations are the equilibrium dynamics equations for environment and physical capital; the equations (1) and (2) impose the market equilibrium conditions. Notice that each path followed by the economy represents a Nash equilibrium; that is, no agent has an incentive to modify his choices if the choices of the others are fixed. Analogously to the Zhang's model, to make the problem more handable and to reduce the dimension of the system, we introduce the following assumption:
Assumption *Let*

$$\eta_{c,E} \equiv \left| \frac{\Delta c}{c} / \frac{\Delta E}{E} \right| = \frac{\frac{\partial U}{\partial c}/c}{\frac{\partial U}{\partial E}/E} = \frac{E}{c}\frac{U_E}{U_c} > 0$$

the elasticity of private consumption of agent i with respect to environment. We assume that this value is constant[7]*.*

Notice that this parameter measures the reactivity of private consumption to environmental quality.

The following equilibrium relation holds

$$k_t = \frac{E_t}{\eta_{c,E}\gamma}$$

and the expressions of c_t^* and m_t^* in terms of E_t and E_{t+1} are:

$$\begin{aligned} c_t^* &= (1 + r_t - \delta)k_t = \\ &= A\alpha \left[\frac{E_t}{\gamma\eta_{c,E}}\right]^{-\rho} \left(\alpha \left[\frac{E_t}{\gamma\eta_{c,E}}\right]^{-\rho} + 1 - \alpha\right)^{-\frac{1+\rho}{\rho}} + (1-\delta)\left[\frac{E_t}{\gamma\eta_{c,E}}\right] \\ m_t^* &= w_t - s_t = \\ &= A(1-\alpha)(\alpha \left[\frac{E_t}{\gamma\eta_{c,E}}\right]^{-\rho} + 1 - \alpha)^{-\frac{1+\rho}{\rho}} - \left[\frac{E_{t+1}}{\gamma\eta_{c,E}}\right] \end{aligned}$$

Now, we can characterize the intertemporal equilibrium conditions through a single non linear difference equation in E_t:

$$E_{t+1} = \frac{\eta_{c,E}}{N + \eta_{c,E}} \left[\left(1 - b - \frac{N\beta(1-\delta)}{\gamma\eta_{c,E}}\right) E_t + NA\left[\gamma(1-\alpha) - \beta\alpha\left[\frac{E_t}{\gamma\eta_{c,E}}\right]^{-\rho}\right] \times \right. \tag{4}$$

$$\left. \times \left\{\alpha\left(\frac{E_t}{\gamma\eta_{c,E}}\right)^{-\rho} + 1 - \alpha\right\}^{-\frac{1+\rho}{\rho}} + b\overline{E}\right] \equiv G(E_t)$$

The dynamics are described by a very complicated nonlinear equation and we are not able to characterize analytically the steady states of the model. Nevertheless, some partial results can be derived.

Lemma 1

a) If $\rho < 0$ *then* $\lim_{E\to 0} G(E) = \frac{\eta_{c,E}}{N+\eta_{c,E}}\left(NA\gamma(1-\alpha)^{-\frac{1}{\rho}} + b\overline{E}\right) > 0$;

b) If $\rho > 0$ *then* $\lim_{E\to 0} G(E) = \frac{b\overline{E}\eta_{c,E}}{N+\eta_{c,E}} > 0$

From the definition of G, it follows that the positivity of G is a necessary condition to have a well defined evolution of E. Since $\lim_{E\to 0} G(E) > 0$ for every values of parameters, then G is positive on an interval $[0, E^*)$, with E^* possible infinite. Nevertheless, to have the dynamics defined for every initial condition on $[0, E^*)$ and for every t, more restrictive assumptions are required.

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[7] See Zhang for further comments. Nevertheless, notice that a large numbers of usual functional specifications of utility functions satisfy this property: logarithmic, Cobb-Douglas, CES.

Proposition 2 *Let* E_{sup} *be the superior extremum for* G *on* R^+ *and* E^* *be the first positive value of* E *such that* $G(E^*) = 0$. *The dynamics described by* G *are defined for every initial condition on* $[0, E^*)$ *and for every* t *if and only if* $G(E_{\text{sup}}) < E^*$.
If E^* *doesn't exist, then the map is well defined for each* t *and for every initial condition.*

We distinguish two cases with respect to the sign of ρ.

3.1 The case $\rho < 0$

In order to investigate the dynamics of the model under the assumption $\rho < 0$, we state the following lemma which gives some insight on the conditions in Proposition (2).

Lemma 3 *Let* $\rho < 0$, *then*
a) If $1 - b - \frac{N\beta(1-\delta)}{\gamma\eta_{c,E}} > \frac{NA\beta\alpha^2}{\gamma\eta_{c,E}}$ *then*
$\lim\limits_{E\to\infty} G(E) = \frac{\eta_{c,E}}{N+\eta_{c,E}}\left(NA\gamma(1-\alpha)(1-\alpha)^{-\frac{1+\rho}{\rho}} + b\overline{E}\right)$ *and the dynamics are defined for every* t
b) If $1 - b - \frac{N\beta(1-\delta)}{\gamma\eta_{c,E}} < \frac{NA\beta\alpha^2}{\gamma\eta_{c,E}}$ *then there exist* $\widehat{E} : G(\widehat{E}) = 0$.

The current case is easy enough to be studied because an unique steady state exists for all the configurations of parameters, as proved by the following proposition.

Proposition 4 *If* $\rho < 0$, *then there exists a unique positive steady state.*

Proof. To study the existence and the numerosity of the interior steady states of equation (4), we introduce the functions h and r, where

$$h(E) \equiv \frac{\eta_{c,E}}{N+\eta_{c,E}} NA\left[\gamma(1-\alpha) - \beta\alpha\left(\frac{E}{\gamma\eta_{c,E}}\right)^{-\rho}\right]\left\{\alpha\left(\frac{E}{\gamma\eta_{c,E}}\right)^{-\rho} + 1 - \alpha\right\}^{-\frac{1+\rho}{\rho}} + b\overline{E}$$

$$r(E) \equiv E\left[1 - \frac{\eta_{c,E}}{N+\eta_{c,E}}\left(1 - b - \frac{N\beta(1-\delta)}{\gamma\eta_{c,E}}\right)\right]$$

Steady states are given by the values of E such that $r(E) = h(E)$.
The graph of r is a line with positive slope. and

$$sign(h'(E)) = sign\left(\beta\rho + (1+\rho)\frac{\gamma(1-\alpha) - \beta\alpha\left(\frac{E_t}{\gamma\eta_{c,E}}\right)^{-\rho}}{\left(\alpha\left(\frac{E}{\gamma\eta_{c,E}}\right)^{-\rho} + 1 - \alpha\right)}\right)$$

It follows that h' becomes negative for high enough value of E.
With easy but long calculations it is possible to verify that h is concave in the

interval where h is increasing. It implies that h is bell shaped and an unique equilibrium (stable or unstable) exists. ■

Essentially, for $\rho < 0$, we have the same qualitative results of the Zhang's model: the unique steady state could be attractive or repelling and, in the second case, limit cycles or a chaotic attractor arise. However, differently from the Zhang's model and Antoci et al.(2009), the productivity parameter A affects the stability of the equilibrium.

Figure 1-a shows the case in which the interior steady state is globally stable, with $\alpha = 0.831$, $\beta = 0.55$, $\gamma = 1.56$, $\delta = 0.001$, $\eta_{c,E} = 5$, $\rho = -0.4$, $b = 0.52$, $A = 3$, $\overline{E} = 1$, $N = 2000$. Considering long run dynamics, it is plausible to assume that A may increase due to technological development. Starting from this configuration of parameters, if A increases, the equilibrium loses its stability and the map undergoes a period doubling route to chaos[8]. Figure 1-b shows the case in which for $A = 6$, the map generates chaotic dynamics.

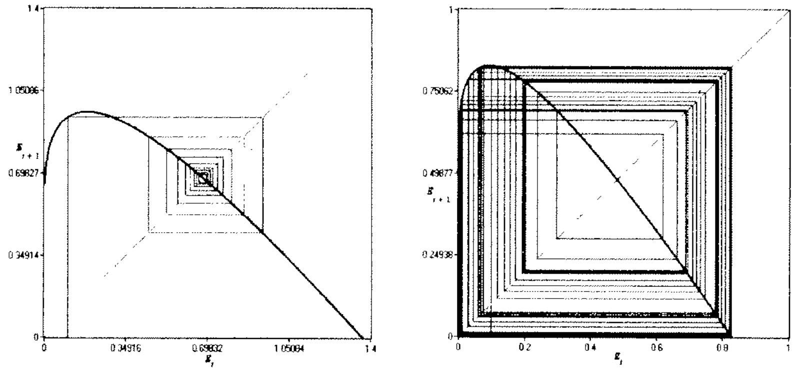

Figure 1: (a) Globally stable equilibrium; (b) Chaotic dynamics

3.2 The case $\rho > 0$

In the current case, more interesting phenomena are possible. Following the same steps as in case $\rho < 0$, we state first the following lemma.

Lemma 5 *Let* $\rho > 0$, *then*
a) if $1 - b < \frac{N\beta(1-\delta)}{\gamma\eta_{c,E}}$, *then there exist* $\widehat{E} : G(\widehat{E}) = 0$;
b) if $1 - b > \frac{N\beta(1-\delta)}{\gamma\eta_{c,E}}$ *then* $\lim_{E\to+\infty} G(E) = +\infty$ *and the dynamics are defined for every* t.

By using Lemma 5, it is easy to prove the next statement.

Proposition 6 *Let* $\rho > 0$, *then, generically, an odd number of steady states exists and steady states with an even index are unstable.*

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[8] The result is quite robust and holds for many different constellations of parameters.

The main difference from the case $\rho < 0$ is that multiple equilibria may exist. If this case occurs, then we can rank the equilibria according to the associated well-being.

Proposition 7 *The utility function value of an agent born in t and the index* E_{t+1} *are positively correlated. This implies that if there exist two steady states* E_1^* *and* E_2^* *such that* $E_2^* > E_1^*$, *then* E_2^* *Pareto-dominates* E_1^*; *that is,* E_1^* *is a poverty trap.*

Even if analytically we are not able to prove it, we have numerically verified that a maximum number of three steady states exists. It is interesting to note that a necessary condition to have multiple steady states is that ρ is enough far from 0. In fact if we consider $\rho \to 0^+$, then $f(k)$ tends to the Cobb-Douglas function and this case the map admits a unique steady state (see for example De la Croix and Michel (2002)). In the next Section we develop the analysis when multiple steady states occur.

4 The role of the increasement of productivity

In this Section, we will concentrate on the case $\rho > 0$ and we will investigate the role of productive parameter A.
Notice that a rise up of A affects in a non univocal way the points of the map. The points of the map on the left of $\widehat{E} \equiv \left(\frac{\gamma(1-\alpha)}{\beta\alpha}\right)^{-\frac{1}{\rho}}$ are translated upwards, the reverse occurs for the points on the right of $\widehat{E}$ and this stretch makes the two "humps" of the map more pronounced.
It follows that an increment of A could be the engine of a poverty trap. In order to understand the phenomenon, we provide a numerical example. Let $\alpha = .621$, $\beta = .1$, $\gamma = 1.12$, $\delta = 0.8$, $\eta_{c,E} = 1.03$, $\rho = 10.5$, $b = .7$, $A = 0.8$, $\overline{E} = 33$, $N = 100$. As shown in Figure 2-a, an unique attractive steady state E_1^* exists. If the value of productivity A grows until $A = 0.88$, two new steady states, E_2^*, E_3^* are born via a *fold bifurcation*, where $E_2^* < E_3^*$ is repelling and separates the basin of attractions of E_1^* and E_3^*. In Figure 2-b, we have considered $A = 0.92$, for which we have $E_1^* = 0.22$, $E_2^* = 1.25$, $E_3^* = 2.23$. Notice that the growth of productivity doesn't imply for an economy the convergence to the more developed state E_3^* : because of the presence of a threshold, economies with a low E at the moment of the bifurcation continue to converge to E_1^*.
This result contributes to the growing literature, initiated by Azariadis and Drazen (1990), on poverty traps. It provides a different explanation of their emergence that resides in the existence of a threshold in the relationship among productivity and environmental quality.

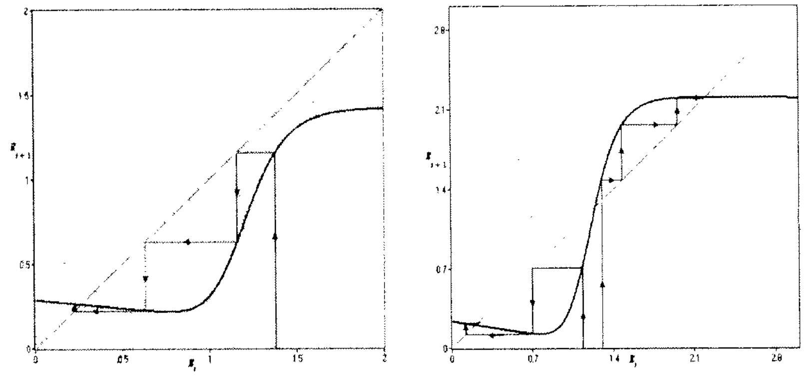

Figure 2: (a) Unique stable equilibrium; (b) Fold bifurcation and the birth of a poverty trap.

Differently from the case just described, we present an experiment that shows how changes in technology may be an engine of complex dynamics and bistable regime. To give some insight on these phenomena, we rely on numerical analysis. In particular, apart from the graphical analysis, we will use the so-called bifurcation diagrams, showing the possible long-term values (steady states, periodic or chaotic orbits) of the system as a function of a parameter. In what follows, we fix $\alpha = 0.07$, $\beta = 0.1$, $\gamma = 0.04$, $\delta = 0.4$, $\eta_{c,E} = 11$, $\rho = 12$, $b = 0.58$, $\overline{E} = 22$, $N = 100$ and we use A as the bifurcation parameter. Starting from $A = 0.179$, a unique unstable steady state E^1 exists, and this is enclosed by an attractive 2-period limit cycle. By increasing the value of A, the trajectories near to equilibrium follow to the classical period doubling route to chaos. For $A = 0.86$, via a fold bifurcation two other steady states E^3, E^2 arise with $E^3 < E^2 < E^1$ (the bifurcation value is $A = 0.82$); E^3 is attracting while E^2 is repelling and separates the basins of attraction of E^3 and the basin of the chaotic attractor. If we let A increase, then the point E_3 loses its stability trough a cascades of flip bifurcations (see Figure 3) and there exists a space of parameters for which the coexistence of two attractors occurs (see Figure 4-a). If we get A higher, the attractor dies ($A = 0.89$) and its points are attracted by the remaining attractor that lives at the right of the repelling steady state E_2^*. A further increase of A ($A > 1.18$) causes an enlargement of the attractor that invades the space before occupied by the other attractor (see Figure 4-b).

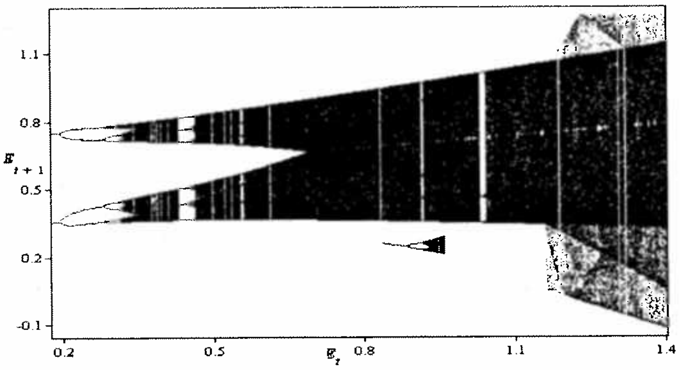

Figure 3: Overlapping bifurcation diagrams varying A.

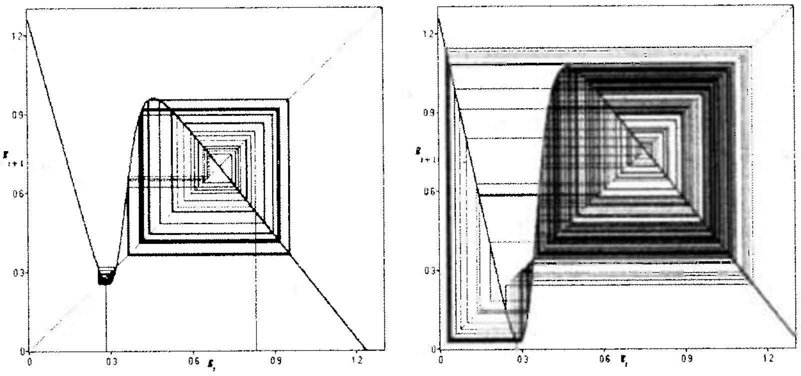

Figure 4: (a) Two chaotic trajectories starting from 0.83 and 0.28 ($A = 0.86$); (b) Enlargement of the attractor ($A = 1.29$)

4.1 The emergence of a non local bifurcation

In the present paragraph we describe an uncommon bifurcation arising for a particular set of parameters. To illustrate such scenario we fix $\alpha = 0.07$, $\beta = 0.1$, $\gamma = 0.04$, $\delta = 0.4$, $\eta_{c,E} = 11$, $\rho = 12$, $b = 0.58$, $\overline{E} = 22$, $N = 100$ and we let A vary. In Figure 5 we show the evolution of the second iterate of the map at the increase of A.

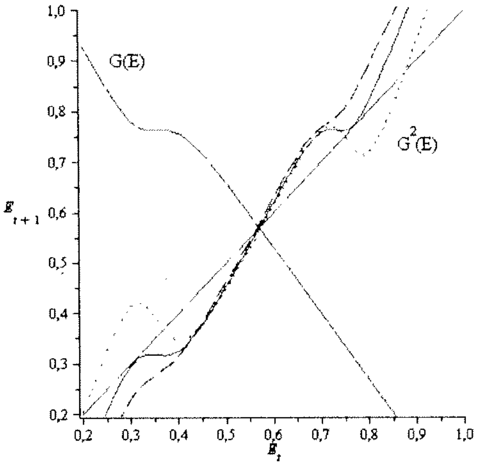

Figure 5: Evolution of the second iterate (G is drawn at the bifurcation value $A = 0.132$);

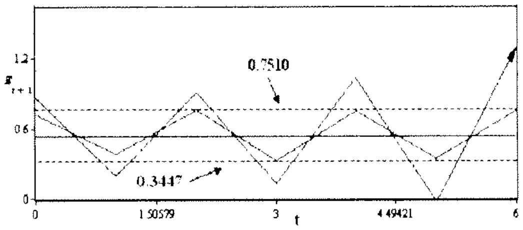

Figure 6: Time evolution of E with three different initial conditions: $E(0) = 0.548$ (the unstable steady state), $E(0) = .74$ (trajectory converging to the 2-period stable limit cycle), $E(0) = 0.77$ (trajectory diverging from the unstable 2 period limit cycle).

We can note that second iterate is characterized by 2 humps, or more exactly by a maximum and a minimum point. If we let A increase, such humps become more pronounced: starting from $A = 0.10$, G^2 has no intersection point with the $45° - line$ (the dashed line). For $A = 0.132$, the second iterate creates 2 tangent points (the solid line), and for $A = 0.16$, two fixed points of the second iterate are born (the dotted line). In this process, driven by the increase of A, we assist to a *fold* bifurcation of G^2 inducing a 2-period stable cycle and a 2-period unstable cycle. We underline that this is a global bifurcation and not a local one: it arises "far" from the fixed point that holds its instability for the whole process (in Figure 5 we have drawn G when the bifurcation occurs). In particular, as shown by the simulations in Figure 6 (in which $A = 0.17$), the interior 2-period cycle is stable, while the external is unstable.

5 Conclusions

The local and global dynamics of an overlapping generations model with environment have been investigated assuming a CES production function. Despite its simplicity, this model displays very rich dynamics. First of all, differently from other works inspired by John and Pecchenino's paper, multistability can occur even if a strictly positive environmental maintenance is assumed. The emergence of complex dynamics when productivity increases is both showed and analytically proved: apart from the classical period-doubling cascades to chaos, non local bifurcations can occur, hence drastically changing the global behaviour of the model.

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