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Conformism and Turnout [1] [2]

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Abstract

This paper proposes a model of turnout in which citizens have a preference for conformism, which adds to the instrumental preference for the electoral outcome. Under this environment multiple equilibria arise, some that generate a (more realistic) high level of turnout, for a wide rage of parameter values. It is also shown that high levels of turnout are robust to the introduction of asymmetry and heterogeneity in the parameter governing the preference for conformism and with respect to the reference group for conformism.

This model suggests that high turnout can only be achieved as the outcome of a particular coordination among citizens and, therefore, introduces a different perspective in understanding the citizens' decision to vote, which allows also to account for large shifts in turnout rates that are observed after compulsory laws have been introduced or abolished. Moreover, this set up proposes a theory for the D term used in rational theories of voting to account for high turnout rates.

JEL classification: D72, C72

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1 Introduction and literature review

The act of voting is of crucial importance as it is the means by which decisions are made in democracies. High turnout rates are considered desirable for many reasons, one of which is the legitimation of the democracy itself. Understanding the motives behind individuals' decision of voting is of tremendous relevance in the field of social sciences. The standard rational choice approach (see Downs, 1957; Myerson, 1998, 2000; Palfrey and Rosenthal, 1985; Riker and Ordeshook, 1968) produces a theory of voting in large populations that can hardly explain the high turnout rates observed in the data: since it is extremely unlikely that a voter can affect the electoral outcome, it is equally unlikely that, in equilibrium, the voter is willing to pay the cost of participating to the election, unless he or she gains a direct utility from it (the so called D term in Riker and Ordeshook (1968)) which is unrelated to the instrumental benefit, i.e. the policy, that follows from the electoral outcome. [1] Therefore it becomes crucial to understand the determinants of the term in the utility function that affects the willingness to vote (see Aldrich, 1993), if we want to avoid to fall into the trap of explaining any action "by making the appropriate assumptions *post hoc*" (Geys, 2006, p. 19). [2]

One justification for the existence of such a direct utility from voting comes from social elements, such as peer pressure or norms for voting. To better understand their role and power, we propose a model in which both pivot probabilities and social elements matter in the individuals' decision of voting. Citizens are motivated by the instrumental benefit from the election plus a direct benefit coming from acting along with the majority of a properly defined group. This second component of the utility function is defined as preference for conformism. The choice between voting or not becomes the outcome of weighting the private costs and benefits (which depend on the alternative that wins the election and the private cost of voting) and the social ones (which depend on conforming to the choice chosen by the majority). Contrary to models that assume an exogenous preference for the act of voting

[1] Rational choice theories of voting produce sensible predictions at the margin. It is well documented that citizens are less likely to vote if the cost of voting increases, in election where there is little at stake and where the expected margin is high. For a recent survey of rational choice models of voting, see Dhillon and Peralta (2002); Geys (2006).

[2] An alternative model that generates high turnout is given by obtaining the *minimax regret* criterion (see Ferejohn and Fiorina, 1974). The idea of regrets if failing to vote has recently been adopted by Li and Dipjyoti (2010).

(i.e. $D > 0$), in this context, the act of voting does not generate, necessarily, a positive utility. It does so only whenever the majority of the citizens vote. On the other hand, when the majority of citizens decides to abstain, going to vote will generate a disutility from not conforming.

As a result, the game so generated can have multiple equilibria: besides the low turnout one, there are also others where turnout can be significantly high. A characterization of these equilibria as a function of the intensity of the parameter governing the preference for conformism, the distribution of the private cost of voting, the reference group affecting the conformistic element of individuals' utility and the asymmetry or the heterogeneity in the parameter governing the preference for conformism is done. We find that multiple equilibria arise when conformism is relatively important in the citizens' preferences. Some of these equilibria involve high turnout rates (i.e. more than 50%); depending on the cost distribution, there can be multiple interior equilibria (i.e. where some vote and some abstain) with high turnout rates when preference for conformism is intermediate; multiple equilibria are robust with the introduction of asymmetry and heterogeneity in the preference for conformism and with respect to the reference group for conformism.

In addition, our paper suggests that the turnout decision might be the solution of a coordination problem among individual citizens and that the way this coordination is solved might depend on social factors, such as the reference group that determines turnout.

Finally our model offers some interesting comparative statics. Marginal changes in the cost distributions might generate big changes in turnout rates by simply inducing voters to coordinate on a different equilibrium. This implication of the model can explain the massive decreases in turnout rates in all cases where mandatory voting laws were abolished. As it has been observed (e.g. Jackman, 2001) mandatory voting laws acted as social norms in enforcing high turnout rates. By imposing little or no penalties on the deviant, one can expect that their abolishment cannot have affected the private cost of voting too much. However we observe large shifts in turnout rates, which might be difficult to capture in a standard model unless we have a big mass of marginal voters.

Multiple equilibria with high turnout rates were initially found by Palfrey and Rosenthal (1983) in the standard pivot probability model, where citizens weigh the private cost and benefit from voting. However multiple equilibria disappear as soon as some uncertainty about fundamentals is introduced, and only the low turnout equilibrium remains. (Myerson, 1998, 2000; Palfrey and Rosenthal, 1985) [3] On the other hand, in our model the existence of multiple equilibria is due to the assumption that the aggregate behavior of citizens generates an externality to the individual voter's utility, and it is robust to the introduction of uncertainty in all the parameters (private cost of voting and intensity of preference for conformism).

The role of social preferences in generating an externality in individuals' utilities has a long and established tradition in the social sciences (see, e.g. Akerlof, 1997; Becker, 1974; Granovetter, 1978; Schelling, 1973, 1978). Also, the existence of multiple equilibria in a scenario where individual utilities are affected by aggregative behavior is well known (see Brock and Durlauf, 2001). [4] The contribution of our paper consists in analyzing such a setup in a pivot probability turnout model.

Multiplicity of equilibria is also investigated in the recent bounded rationality model of turnout, in which social dynamics are introduced to explain the possibility of convergence (and therefore selection) towards high turnout rates (see DeMichelis and Dhillon, 2009; Diermeier and Mieghem, 2008). While DeMichelis and Dhillon (2009) consider an evolutionary version of Palfrey and Rosenthal (1983), the multiplicity of equilibria generated in Diermeier and Mieghem (2008) is due to the existence of noisy public polls.

In recent attempts to rationalize higher levels of turnout, new approaches have been introduced in which citizens have ethical preference (Coate and Conlin, 2004; Feddersen and Sandroni, 2006). In these models, higher turnout rates are obtained because citizens act like rule utilitarian in the sense that they choose to vote or not in order to maximize the utility of a representative agent of their own group. Others have explored the role of altruism in fostering high turnout rates: Fowler (2006) tests in laboratory setting the extent to which altruism, in conjunction with party identification, can increase the propensity to vote. In both these approaches, turnout is higher because citizens internalize other citizens' benefit from the electoral outcome. In our model instead, high turnout is due to the fact that citizens internalize other citizens' action.

[3] Castanheira (Castanheira) assesses the impact of other maintained assumptions of the rational choice model, such as the electoral rule and the importance the mandate on the predicted levels of turnout.

[4] More recently, the literature on network games (see, e.g. Galeotti et al., 2010) has developed a framework to analyze social interaction games. In addition, Breton and Weber (2009) provide Nash equilibria existence result for a class of games where payoffs depend on three components: besides the intrinsic preference for the action taken, and the conformity component, they also consider the possibility of dyadic externalities exerted by peers. Manski (2000) provides a comprehensive overview of the economic analysis of social interactions.

The interpretation of turnout as a social phenomenon or the role of conformism in contributing to high voting rates is also widely proposed. Uhlaner (1986), for example, considers a model in which citizens belong to groups within which they share the same view towards politics. Group leaders will then mobilize their members towards the candidate they feel suites their group's needs the best. As a result, turnout rates are higher since group leaders can count on mass of votes, which then have a higher impact on the pivot probabilities. [5]

Coleman (2004) analyzes, in a decision theory framework, the effects of turnout induced by a norm of civic duty in the choice of the particular candidate. Moreover Coleman (2004) provides a wide discussion of the literature about conformism and voting. Of particular interest is the evidence found (Blais, 2000) in cross country studies that a high share of the adult population "believes that voting is a moral obligation and would feel guilty by not voting" (Coleman, 2004, p. 78). Another element of interest is the evidence found (Knack, 1992) of a correlation between norm induced behaviors such as voting and giving to charities or responding to census (op. cited in Coleman, 2004, p. 78). Similar correlations between a norm for voting and a norm against littering have been found by Cialdini et al. (1990).

De Matos and Barros (2004) consider the role played by peer pressure in enforcing voting as a norm of behavior, in a network based representation of social interactions, and show that turnout can be the outcome of a contagion effect that starts from few highly motivated to vote individuals.

In all these cases however the element of strategic interaction is not accounted for and therefore the possibility of norms which induce no equilibria is not discussed.

The remainder of the paper is organized as follows. Section 2 describes the basic ingredients of the model and provides the main existence result. Section 3 characterize the equilibria of the game and analyzes the cases for multiplicity, while Section 4 provides few examples for different cost distributions. Sections 5 through 7 analyze the robustness of the results with respect to different reference group for conformism, asymmetric preferences for conformism, and heterogeneity in the preference for conformism. Last, Section 8 discusses an interpretation of mandatory voting laws as a coordinating device, and Section 9 concludes the paper. Proofs are relegated to the Appendix.

[5] Morton (1991) formalizes a rational choice model of turnout in which the behavior of groups is incorporated and shows that when positive turnout is observed, it will typically be higher than in the standard case.

2 The turnout model

Consider a large population of citizens who need to choose, via majority voting, between two exogenously given alternatives, R and L. The population is partitioned into two groups, one that prefers R to L (call them group R citizens) and one that prefers L to R (call them group L citizens). The utility received by citizens is set to 1 if their most preferred alternative is selected, or 0 if their least preferred alternative is selected. Voting is a costly activity. Let the cost of voting, c, be distributed over the interval $[0, 1]$ according to a continuous distribution $F(c)$ which admits density $f(c)$. Note that this specification of the cost distribution does not allow for any mass of voters with strictly negative costs. Hence there are no citizens that vote or abstain unconditionally.

In addition, citizens are endowed with a preference for conformism, in the sense that they gain utility any time they choose the same action (vote or not) of the majority of a reference group. We consider first the case where people care about the action taken by all the other citizens (population wide conformism). Later in the paper (section 5) we consider the other polar case where people care only about the action taken by their own group's members (group based conformism).

Each citizen simultaneously chooses whether to vote for R, or L or to stay home (abstain). The alternative that receives the majority of the votes is selected. A flip of a coin is used to break ties (so that eventually each alternative is equally likely to be selected).

Since there are only two alternatives, voting for the most preferred one weakly dominates the choice of voting for the least preferred. Therefore we can reduce the citizens' problem to either vote for the most preferred alternative or stay home. The payoff table can be simplified as follows:

	WIN	LOSE
VOTE	$1 - c + \delta x$	$-c + \delta x$
ABSTAIN	$1 + \delta(1 - x)$	$\delta(1 - x)$

where $\delta > 0$ denotes the relative importance of conformism and x denotes the share of citizens in the population who choose to vote. This specification is without loss of generality as it allows also for a utility that is a convex combination between the voting outcome and the choice of the other citizens.

The change in expected utility due to voting is given by the following equation

$$\Delta EU = \Delta P - c - \delta(1 - 2x)$$

where ΔP is the increase in the probability of winning induced by voting. Note that ΔP is non zero only when the voter is pivotal to the election. So its computation boils down to the calculation of the probability that the voter is pivotal, which is quite a small number for large elections.

Following Myerson (1998, 2000), the population size is uncertain and distributed according to a Poisson process with mean n, where n is a finite but large integer. Each citizen belongs to group R with probability θ. As we will see shortly, the use of a Poisson game allows for a tractable approximation of these pivot probabilities.

In this scenario a strategy can be simplified into a cutoff rule, $\bar{c}_j$, $j = R, L$ so that citizen i in group j with private cost c_{ij} votes if and only if $c_{ij} \leq \bar{c}_j$. By imposing the same cutoffs within each group, we are restricting our attention to semi-symmetric strategies.

An interior equilibrium is represented by a pair of cutoffs $(\bar{c}_R, \bar{c}_L)$ for which

$$P_R(\rho, \lambda, \theta, n) = \bar{c}_R + \delta\left[1 - 2\theta\rho - 2(1-\theta)\lambda\right] \quad (1)$$

$$P_L(\rho, \lambda, \theta, n) = \bar{c}_L + \delta\left[1 - 2\theta\rho - 2(1-\theta)\lambda\right] \quad (2)$$

and $\rho = F(\bar{c}_R)$, $\lambda = F(\bar{c}_L)$, where $P_R(\cdot)$, $P_L(\cdot)$ denote the probabilities that a group R, L, citizen is pivotal to the election.

Furthermore, cutoffs can be at a corner (so that either nobody or everybody vote). In that case the equilibrium condition becomes one of the following

$$P_R(0, \lambda, \theta, n) \leq \delta\left[1 - 2(1-\theta)\lambda\right] \quad (3)$$

$$P_L(\rho, 0, \theta, n) \leq \delta\left[1 - 2\theta\rho\right] \quad (4)$$

$$P_R(1, \lambda, \theta, n) \geq 1 + \delta\left[1 - 2\theta - 2(1-\theta)\lambda\right] \quad (5)$$

$$P_L(\rho, 1, \theta, n) \geq 1 + \delta\left[2\theta(1-\rho) - 1\right] \quad (6)$$

Note that the role of preferences for conformism is to (endogenously) increase or decrease the cost of voting. We should therefore expect that the voting game might have multiple equilibria: when there is enough mass of citizens who chose to vote, and the preference for conformism is sufficiently strong, more find it optimal to turn out. Hence a high turnout equilibrium should be expected. When, on the other hand, the expectation is that few will vote, then in equilibrium very few or even nobody will vote, depending on how strong the preference for conformism is.

Since ties are broken via a flip of a coin, a player's vote matters when it allows to force a winning tie, or to break a losing tie. Therefore the pivot probabilities of the Poisson game are given by

$$P_R(\rho,\lambda,\theta,n) = \frac{1}{2}\sum_{k=0}^{\infty}\frac{e^{-\rho\theta n}(\rho\theta n)^k}{k!}\frac{e^{-\lambda(1-\theta)n}(\lambda(1-\theta)n)^k}{k!}\left[1+\frac{\lambda(1-\theta)n}{k+1}\right]$$
$$= \frac{1}{2}e^{-n(\rho\theta+\lambda(1-\theta))}\left[I_0(2n\sqrt{\rho\lambda\theta(1-\theta)}) + \sqrt{\frac{\lambda(1-\theta)}{\rho\theta}}I_1(2n\sqrt{\rho\lambda\theta(1-\theta)})\right] \quad (7)$$

and

$$P_L(\rho,\lambda,\theta,n) = \frac{1}{2}\sum_{k=0}^{\infty}\frac{e^{-\rho\theta n}(\rho\theta n)^k}{k!}\frac{e^{-\lambda(1-\theta)n}(\lambda(1-\theta)n)^k}{k!}\left[1+\frac{\rho\theta n}{k+1}\right]$$
$$= \frac{1}{2}e^{-n(\rho\theta+\lambda(1-\theta))}\left[I_0(2n\sqrt{\rho\lambda\theta(1-\theta)}) + \sqrt{\frac{\rho\theta}{\lambda(1-\theta)}}I_1(2n\sqrt{\rho\lambda\theta(1-\theta)})\right] \quad (8)$$

where $I_a(b)$ denotes a modified Bessel function of the first order of parameters a and b (Abramowitz and Stegun, 1965). Note that when $\rho = \lambda = 0$ these equations boil down to $1/2$.

We can provide an equilibrium existence result, for any $\theta \in (0,1)$ and $\delta \geq 0$, by means of a standard fixed point argument.

Proposition 1. *There exists at least one equilibrium in cutoff points* $(\bar{c}_R, \bar{c}_L)$ *for any value of* $\delta \geq 0$ *and* $\theta \in (0,1)$.

It is now of interest to characterize the equilibria of the game as a function of δ and the cost distribution $F(c)$.

3 Equilibrium Analysis

The characterization of the equilibria as a function of δ and $F(c)$ proceeds along the following lines: first corner equilibria are analyzed for any value of θ and any continuous cost distribution; then existence and uniqueness of interior equilibria are studied, under the assumption of $\theta = 1/2$ and $F(c)$ is C^1 over $(0,1)$. As it turns out, the choice of $\theta = 1/2$ allows us to characterize the equilibria by means of graphical analysis, once $F(c)$ is specified.

Proposition 2. *The corner equilibrium* $(0,0)$ *exists if and only if* $\delta \geq 1/2$. *The corner equilibrium* $(1,1)$ *exists if and only if* $\delta \geq 1 - P_j(1,1,\theta,n)$ *for all* j.
In addition, when $\theta = 1/2$ *there are no other corner equilibria.*

Proposition 2 establishes the cutoffs values of δ for which full abstention and full turnout are equilibria. As the preference for conformism is positively correlated with the incentives coming from the pivot probability at low turnout rates, the full abstention equilibrium arises for small values of δ. Much higher preference for conformism is needed to overcome the incentive coming from the low pivot probability to induce a full turnout equilibrium.
We now focus on the study of the interior equilibria. We are able to show that, depending on the values of δ, we might have either an odd (and possibly unique) or an even (and possibly zero) number of interior equilibria. As stated, we restrict our analysis to the case of $\theta = 1/2$ and we pose the restriction that $F(c)$ is C^1 in $(0,1)$. Write c_m to indicate the median of $F(c)$, i.e. that value of c for which $F(c) = 1/2$, and M to indicate the peak (or mode) of $f(c)$, whenever it exists. If the distribution has more than one mode, we let M indicate the global maximizer of $f(c)$. Note that, by definition of p.d.f., it follows that $f(M) \geq 1$.
The first result shows that the interior equilibria of the game are symmetric when $\theta = 1/2$.

Proposition 3. *When* $\theta = 1/2$, *all the equilibria of the game are symmetric, i.e.* $\bar{c}_j = c$ *for all* j.

Proposition 3 implies that the equilibria of the game can be characterized by studying the equation

$$P_j(F(c), F(c), 1/2, n) = c + \delta(1 - 2F(c)) \tag{9}$$

as c and δ vary. By evaluating the equation for the pivot probability at $\theta = 1/2$ and $\lambda = \rho = F(c)$, we then get

$$l(c) \equiv \frac{e^{-nF(c)}}{2}\left[I_0(nF(c)) + I_1(nF(c))\right] = c + \delta\left[1 - 2F(c)\right] \equiv r(c) \tag{10}$$

The following characterization of the number of interior equilibria applies.

Proposition 4. *Let* $\Omega = \{\delta \in \mathbb{R}_+ | \ \delta \leq 1/2 \cup \delta \geq 1 - (2n\pi)^{-1/2}\}$. *There exists (generically) an odd number of interior equilibria if and only if* $\delta \in \Omega$.
There exists (generically) an even number of interior equilibria, and possibly zero, if and only if $\delta \in \mathbb{R}_+ \setminus \Omega$.

Observe that when $\theta = 1/2$, $P(1,1,1/2,n) \approx (2n\pi)^{-1/2}$ (see equations (A-7) and (A-8) in the Appendix). Thus this value of θ sets the lower bound of the corner equilibrium $(1,1)$ to exist. Proposition 4 tells us that the number of interior equilibria just depends on whether the corner equilibria $(0,0)$ and $(1,1)$ exist.

In addition, when δ is sufficiently small there is only one interior equilibrium, which is associated to a low turnout.

Proposition 5. *Suppose $\theta = 1/2$ and $f(c) < \infty$ for all $c \in [0,1]$. There exists a unique interior equilibrium whenever $\delta < [2f(M)]^{-1}$. If in addition $n > [\pi c_m^2]^{-1}$ the unique equilibrium induces a turnout rate which is below 50% and which decreases with δ.*

The condition on n is needed to ensure that the cutoff is to the left of the median. However it is quite mild. When, for example, the cost distribution has a median equal to 0.01, we just need a population of size at least 318,310 for the inequality to hold.

Proposition 5 allows us to tell even more: since the equilibrium cutoff c is a continuous function of δ, and since it decreases with δ, we know that for values of δ, equilibrium turnout rates are even lower than when conformism does not matter. As δ approaches 1/2, the low turnout equilibrium c approaches 0, i.e. the extreme case where nobody votes. This is not surprising: when conformism does not matter (i.e. $\delta = 0$), citizens vote with very small probability because the chance they affect the outcome is virtually zero. As conformism starts to kick in, this effect is reinforced and therefore turnout decreases further up to the point where everybody stays home ($\delta \geq 1/2$).

Note also that $f(M) < \infty$ is sufficient but not necessary for an interior equilibrium, as the example in Proposition 8 shows. Finally note that as a corollary we have that a necessary condition for high turnout equilibria to arise is that $\delta > [2f(M)]^{-1}$.

Moving on, for sufficiently large δ other equilibria appears. Since when $\theta = 1/2$, $P(1,1,1/2,n) \approx (2n\pi)^{-1/2}$, we know that when $\delta \geq 1 - (2\pi n)^{-1/2}$ there is a corner equilibrium where everybody votes (Proposition 2), and that there is also at least one interior equilibrium (Proposition 4). We now show that for sufficiently large n this equilibrium has high turnout, since the cutoff value is larger than the median, i.e. $c > c_m$.

Proposition 6. *Suppose $\theta = 1/2$, $\delta \geq 1 - (2\pi n)^{-1/2}$ and $n > [\pi c_m^2]^{-1}$. There exists at least one interior equilibrium whose turnout rate is higher than 50%. In addition, the smallest and the largest of these equilibria generates a turnout rate that decreases with δ.*

Proposition 6 implies that we can always find a sufficiently large δ, after which there is always a unique interior equilibrium.

To wrap things up we have found that: for low values of δ a unique equilibrium with low turnout is observed. This result maps the standard pivot probability model (which corresponds to $\delta = 0$ in our case) where low turnout is observed, and is reinforced by the preference for conformism. As δ increases such a reinforcement effect becomes stronger and therefore an equilibrium whereby no one votes is also present. However, equilibria where a significant fraction of the population vote now tend to appear, along with the one where everybody votes. Hence high turnout equilibria require a sufficiently large weight on the preference for conformism. Note that such a large weight on the presence of conformism also implies that a no vote equilibrium is also possible.

Because of this multiplicity, which equilibrium is eventually selected becomes then crucial for predicting the correct level of turnout. This question is specifically analyzed in a companion paper (Landi and Sodini, 2010). In the following sections, instead, we focus on the equilibrium characterization for specific specifications of the cost distribution (Section 4) and on the analysis of robustness of our results with respect to changes in parametrization of the model. As we shall see, multiple equilibria with high turnout rates are still observable.

4 Some examples

Our analysis shows that a full characterization of the interior equilibria can be done via simple study of the functions $l(c)$ and $r(c)$ in equation (10) whose specific form depends on the the parametrization of $F(c)$. In this section we provide some examples.

Suppose first that costs are uniformly distributed, so that $F(c) = c$. We have that $l(c)$ is decreasing and convex, while $r(c)$ is linear. In fact,

$$r(c) = c(1 - 2\delta) + \delta$$

Thus $r(c)$ is strictly increasing for any $\delta < 1/2$, flat at $\delta = 1/2$, and strictly decreasing for any $\delta > 1/2$. The equilibria of the voting game can be characterized by inspecting figure 1.

[Figure 1 about here.]

The red lines indicate $r(c)$ for values of $\delta \leq 1/2$ while the dashed blue lines indicate $r(c)$ for values of $\delta > 1/2$. Under this configuration of costs we always get either a unique interior equilibrium, or no interior equilibria. The following proposition summarizes the characterization of the equilibria of the game.

Proposition 7. *Suppose $\theta = 1/2$ and $F(c) = c$. There is a unique interior equilibrium $(, c_L)$, with $c_L < 1/2$ for $\delta < 1/2$. There is a unique corner equilibrium $(0,0)$ for $\delta \in [1/2, 1-(2\pi n)^{-1/2})$. There are two corner equilibria, $(0,0)$ and $(1,1)$, and a unique interior equilibrium (c_H, c_H), with $c_H > c_m$ for $\delta \geq 1-(2\pi n)^{-1/2}$.*

Things are similar when costs are given by $F(c) = c^k$ with $k \geq 2$. In this case we have that $M = 1$ and $f(M) = k$, therefore $r(c)$ is convex and strictly increasing for $\delta < (2k)^{-1}$ and with a unique interior maximum otherwise. In fact,

$$r(c) = c + \delta(1 - 2c^k)$$

and the interior maximum is given by

$$c^\star = \left(\frac{1}{2k\delta}\right)^{\frac{1}{k-1}}$$

On the other hand, $l(c)$ has an inflexion point. Figure 2 allows to provide the characterization of the equilibria.

[Figure 2 about here.]

The red curves indicate $r(c)$ for $\delta < 1/2$, whereas the dashed blue curves indicate $r(c)$ for $\delta > 1/2$. Again, there is at most a unique interior equilibrium and the complete characterization follows Proposition 7

Multiple interior equilibria do appear when the cost distribution is strictly concave, at least locally. Suppose for example that costs are distributed according to a power distribution with parameters 1 and $1/k$, so that $F(c) = c^{1/k}$, with $k \geq 2$. We still have that $l(c)$ is decreasing and convex, whereas now $r(c)$ is concave and has an interior minimum. In fact

$$r(c) = c + \delta(1 - 2c^{1/k})$$

and the interior minimum is given by

$$c^\star = \left(\frac{2\delta}{k}\right)^{\frac{k}{k-1}}$$

Note that $M = 0$ and $f(c)$ is discontinuous at the origin. Therefore there are no values of $\delta \geq 0$ for which $r(c)$ is strictly increasing. Under this configuration of costs we have one, two, three, or no interior equilibria. The equilibria of the game can be characterized by looking at figure 3

[Figure 3 about here.]

The black curve represents $l(c)$. It starts at 1/2 and decreases monotonically. The red curves represent $r(c)$ for values of δ less than 1/2 while the dashed blue curves represent $r(c)$ for values of δ larger than 1/2. First observe that given the parameter specifications we have that $n\pi > c_m^{-2}$. The low turnout equilibrium decreases with δ as can be seen by looking at the red curves. Interestingly enough multiple high turnout equilibria arise even when $\delta < 1/2$. Finally when $\delta > 1-(2n\pi)^{-1/2}$ (see the two lowest of the blue curves) we have only one interior equilibrium which again decreases with δ. The following proposition reports the complete characterization of the equilibria.

Proposition 8. *Suppose $\theta = 1/2$ and $F(c) = c^{1/k}$, with $k \geq 2$. There is a unique interior equilibrium (c_L, c_L), with $c_L < c_m$ for $\delta \in [0, \delta_1)$ with $\delta_1 < 1/2$. There are three interior equilibria, (c_L, c_L), (c_{H1}, c_{H1}), and (c_{H2}, c_{H2}), with $c_L < c_m < c_{H1} < c_{H2}$ for $\delta \in (\delta_1, 1/2)$. There are two interior equilibria, (c_{H1}, c_{H1}), and (c_{H2}, c_{H2}), with $c_{H2} > c_{H1} > c_m$ and a corner equilibrium $(0,0)$ for $\delta \in (1/2, 1-(2\pi n)^{-1/2})$. There is one interior equilibrium (c_H, c_H), with $c_H > c_m$ and two corner equilibria, $(0,0)$ and $(1,1)$ for $\delta \geq 1-(2\pi n)^{-1/2}$.*

Another concave cost distribution is given by $F(c) = 1-(1-c)^k$ with $k \geq 2$. [6] Figure 4 reports the plots of $l(c)$ and $r(c)$ as a function of δ.

[Figure 4 about here.]

We see a similar pattern with two main differences. First, for intermediate values of δ it is possible that no interior equilibria arise. Second, interior high turnout equilibria cannot coexist with interior low turnout ones.

Proposition 9. *Suppose $\theta = 1/2$ and $F(c) = 1-(1-c)^k$, with $k \geq 2$. There is a unique interior equilibrium (c_L, c_L), with $c_L < c_m$ for $\delta < 1/2$. There is only a corner equilibrium $(0,0)$ for $\delta \in [1/2, \delta_1)$, with $\delta_1 < 1-(2\pi n)^{-1/2}$. There are two interior equilibria*

[6]This is the parametrization used in Landi and Sodini (2010).

(c_{H1}, c_{H1}) *and* (c_{H2}, c_{H2}), *with* $c_{H2} > c_{H1} > c_m$ *and a corner equilibrium* $(0,0)$ *for* $\delta \in (\delta_1, 1 - (2\pi n)^{-1/2})$. *There is a unique interior equilibrium* (c_H, c_H), *with* $c_H > c_m$ *and two corner equilibria* $(0,0)$ *and* $(1,1)$ *for* $\delta \geq 1 - (2\pi n)^{-1/2}$.

Finally we look at bimodal distributions. Consider two Beta distributions with mode symmetrically distributed about 1/2 and sufficiently distant. If we take a simple average of the two we obtain a bimodal distribution. Figure 5 reports the equilibria when the modes are given by $M_1 = 0.1$ and $M_2 = 0.9$.

[Figure 5 about here.]

From the above examples we observe a pattern. A necessary condition condition for multiple interior equilibria is that the cost distribution is concave at least locally. In addition, whether the cost distribution is single peaked might not affect much the pattern of the interior equilibria. This is an interesting feature of the model, because smooth distributions associated to costs that are mainly small are strictly concave. Hence, multiple equilibria should be expected to be the norm whenever costs are small.

We now consider another possible extension of the model, in which conformism is group based, in the sense that it depends on the choice made by the majority of the citizens within the same group. We obtain the same characterization of the interior equilibria, but now some more corner equilibria appear.

5 Groups' related conformism

When players care only about the voting decision of their own group, the cutoff equilibrium costs are obtained from

$$P_R(\rho, \lambda, \theta, n) = \bar{c}_R + \delta [1 - 2\rho] \quad (11)$$

$$P_L(\rho, \lambda, \theta, n) = \bar{c}_L + \delta [1 - 2\lambda] \quad (12)$$

where $\rho = F(\bar{c}_R)$ and $\lambda = F(\bar{c}_L)$. (Note how now the above conditions do not depend of θ.) We can parallel the logic in the proof of Proposition 3: by taking the ratio between equations (11) and (12) we obtain

$$\sqrt{\frac{F(\bar{c}_L)}{F(\bar{c}_R)}} = \frac{\bar{c}_R + \delta [1 - 2F(\bar{c}_R)]}{\bar{c}_L + \delta [1 - 2F(\bar{c}_L)]}$$

Thus a unique interior symmetric solution is $\bar{c}_R = \bar{c}_L = c$ and the interior equilibria are still characterized by studying the equation (9). Therefore the interior equilibria are the same as with population wide conformism.

However in this simple set up, the two models differ in the type of corner equilibria they generate. In fact, with group based conformism we have many more of them, as summarized by the following proposition.

Proposition 10. *If conformism depends on the behavior of one player's group, all the corner equilibria* $(0,0)$, $(0,1)$, $(1,0)$, *and* $(1,1)$ *are possible for suitable parameter values. Side equilibria of the form* $(0,\lambda)$, $(\rho,0)$, $(1,\lambda)$ *and* $(\rho,1)$ *are also possible.*

Few comments are in order. As the interior solutions are obtained from conditions (11) and (12), we see that they do not depend on θ. Yet when conformism is population wide, the interior equilibria do depend on θ. Therefore the equivalence of the set of the interior equilibria with two different types of conformism depends crucially on the assumption about $\theta = 1/2$. [7]

Moreover, new corner equilibria arise because when conformism is group based the trade off between conformism and free riding on own members is weaker. In fact, when preference for conformism is sufficiently strong, pivot probability considerations become irrelevant relative to acting according to the majority of one's own group.

We now proceed into analyzing the role of δ. We first consider the importance of having a symmetric δ (i.e. citizens are neutral to their choice of voting or not with respect to conformism), and then consider the role of homogeneous δ for our results.

6 Asymmetric values for δ

Suppose that conformism is asymmetrically perceived when voting and when not voting. In other words, we consider a scenario in which δ_1 denotes the weight given to conformism when voting and $\delta_0 \neq \delta_1$ denotes the weight given to conformism when not voting. When conformism is population wide, a voter supporting candidate j, with private cost i, votes if

[7] By simulating the equilibria for a particular cost function in large electorate, we find that they do not change much with θ (Landi and Sodini, 2010).

and only if

$$P_j(\rho, \lambda, 1/2, n) \geq c_i + \delta_0 - (\delta_0 + \delta_1)\frac{\rho + \lambda}{2}$$

In this setup, symmetry of the cutoffs across groups still holds; and the interior equilibria are characterized by studying the equation

$$P_j(F(c), F(c), 1/2, n) = c + \delta_0 - (\delta_0 + \delta_1)F(c)$$

whereas corner equilibria are obtained from the conditions

$$P_R(0, \lambda, 1/2, n) \leq \delta_0 - (\delta_0 + \delta_1)\frac{\lambda}{2} \quad (13)$$

$$P_L(\rho, 0, 1/2, n) \leq \delta_0 - (\delta_0 + \delta_1)\frac{\rho}{2} \quad (14)$$

$$P_R(1, \lambda, 1/2, n) \geq 1 + \delta_0 - (\delta_0 + \delta_1)\frac{1 + \lambda}{2} \quad (15)$$

$$P_L(\rho, 1, 1/2, n) \geq 1 + \delta_0 - (\delta_0 + \delta_1)\frac{1 + \rho}{2} \quad (16)$$

The following results are a straightforward consequence of the analysis done in the base model with symmetric δ:

Proposition 11. *The corner equilibrium* $(0, 0)$ *exists if and only if* $\delta_0 \geq 1/2$*;*
The corner equilibrium $(1, 1)$ *exists if and only if* $\delta_1 \geq 1 - (2n\pi)^{-1/2}$.

Proposition 11 shows that corner equilibria do not depend on the relative distance between δ_0 and δ_1. In fact, since these equilibria are computed by comparing the marginal cost and marginal benefit of voting at the extremes (i.e. $c = 0$ and $c = 1$), it is only the conformism parameter governing the respective action (i.e. voting or not) that matters.
As a result, there can be parameter configurations for which none of the two exist, only one of the two exist, or both exist. These cases in turn affect the characterization of the interior equilibria. Specifically:

Proposition 12. *Suppose that* $\delta_0 \geq 1/2$ *and* $\delta_1 \geq 1 - (2n\pi)^{-1/2}$*. There are two corner equilibria,* $(0, 0)$ *and* $(1, 1)$ *and (generically) an odd number of interior equilibria.*
Suppose that $\delta_0 < 1/2$ *and* $\delta_1 \geq 1 - (2n\pi)^{-1/2}$*. There exist a corner equilibrium,* $(1, 1)$*, and (generically) an even, and possibly zero, number of interior equilibria.*
Suppose that $\delta_0 \geq 1/2$ *and* $\delta_1 < 1 - (2n\pi)^{-1/2}$*. There exist a corner equilibrium,* $(0, 0)$*, and (generically) an even, and possibly zero, number number of interior equilibria. In addition,*

there are no interior equilibria whenever, $F(c)$ is strictly concave or $f(c) < \infty$ for all $c \in [0,1]$ and

$$\delta_0 + \delta_1 < \min\left\{1, \frac{1}{f(M)}\right\}$$

Finally, suppose that $\delta_0 < 1/2$ and $\delta_1 < 1 - (2n\pi)^{-1/2}$. There are no corner equilibria and (generically) an odd number of interior equilibria. In addition, the interior equilibrium is unique whenever $f(c) < \infty$ for all $c \in [0,1]$, and

$$\delta_0 + \delta_1 < \min\left\{1, \frac{1}{f(M)}\right\}$$

Proposition 12 shows that multiplicity of equilibria remains an issue, because it follows from the externality of other citizens' actions into individual's utilities.

7 A smooth distribution of δ

So far we have assumed that society shares the same value for conformism. In this section we introduce incomplete information (i.e. heterogeneity) with respect to the parameter δ. One might consider heterogeneity of preference for conformism as a more realistic assumption, since it is quite unlikely that citizens agree on the level by which conformism matters. There are also some advantages of introducing a distribution for the parameter δ into the model: it is possible to verify the robustness of the multiplicity of equilibria with respect to the asymmetry of information in the underlying parameters of the model, just like in Palfrey and Rosenthal (1983, 1985). In addition, it allows to do comparative statics on the voting decisions with respect to the distribution of cohesiveness in societies.

Suppose that preference for conformism δ is distributed according to the continuous function $F_\delta(\delta)$ with support $[0, \hat{\delta}]$. Suppose that the private cost c is distributed according to the continuous distribution $F_c(c)$ with support $[o, \hat{c}]$. To make the problem interesting let $\min\{\hat{c}, \hat{\delta}\} > 1/2$. Last assume that both distributions are independent. Let $f_\delta(\delta)$ and $f_c(c)$ denote the respective densities. Now players are characterized by the triplet (j, k, h) indicating the alternative they prefer, their intensity of preference for conformism and their private cost of voting. A strategy is a function from this triple into the voting or not voting decision. Equilibria will be characterized by cutoffs (c_j, δ_j) such that voters below the cutoffs vote and voters above the cutoff abstain.

Since c and δ are drawn from the the same distributions for all players, and $\theta = 1/2$, the symmetric interior equilibrium requires cutoffs to be the same for supporters of both alternatives and can be obtained as the solution of the equation

$$P_R(\lambda, \lambda, 1/2, n) = c + \delta(1 - 2\lambda) \tag{17}$$

where λ is the share of voters, and it is obtained as the solution of the following fixed point problem

$$Prob_{c,\delta}\left[c + \delta(1 - 2\lambda) \leq P_R(\lambda, \lambda, 1/2, n)\right] = \lambda \tag{18}$$

To ease on notation we write $P(\lambda, n)$ to denote $P_R(\lambda, \lambda, 1/2, n)$. Let $l(\lambda)$ denote the left hand side of equation (18). Note that $l(\lambda)$ is a continuous function of λ, such that

$$l(0) = Prob_{c,\delta}(c + \delta \leq 1/2) > 0$$

and

$$l(1) = Prob_{c,\delta}(c - \delta \leq P(1, n)) \approx \int_0^{\bar{\delta}} F_c\left(c \leq \frac{1}{\sqrt{2n\pi}} + \delta\right) f_\delta(\delta) d\delta < 1$$

where the second equality follows from the approximations of the pivot probabilities. This suffices to establish existence of one fixed point. We have therefore that

Proposition 13. *Assume conformism is population wide, and citizens differ in their private cost of voting and intensity of preference for conformism. When c and δ are independently distributed, there exists at least one interior equilibrium.*

Note in particular that

$$l(1/2) = F_c\left(P(1/2, n)\right) < \frac{1}{2}$$

whenever the median cost, c_m, is larger than $P(1/2, n) = \frac{e^{-n/2}}{2}\left[I_0(n/2) + I_1(n/2)\right]$. Therefore we have that

Proposition 14. *For any distribution of the cost, $F_c(c)$ and of the intensity of preference for conformism, $F_\delta(\delta)$, we can find a sufficiently large n for which an interior equilibrium exists at low level of turnout, i.e. for which $\lambda < 1/2$.*

Note also that the condition on the population's size depends only on the specific functional form for the distribution of the cost.

Other equilibria with high turnout can arise in this setting, as the following examples show. To start with, consider the case where both costs and preference for conformism are uniformly distributed. While costs are kept between 0 and 1, the preference for conformism ranges from 0 to Δ. Intuitively, the larger Δ the more likely it should be to have multiple equilibria. We are interested in the lower bound on Δ for which multiple interior equilibria arise.

Proposition 15. *Suppose that $F_c(c) = c$, $c \in [0, 1]$, and $F_\delta(\delta) = \delta/\Delta$, $\delta \in [0, \Delta]$. There are two interior equilibria with high turnout if and only if $\Delta > 4(1 - P(3/4, n))^2$.*

A similar result can be obtained when when costs are mainly small and preference for conformism is mainly large. We can see that by adopting a right skewed triangular distribution for the cost, and a (left skewed) power distribution, between 0 and 2, for the preference for conformism.

Proposition 16. *Suppose $F_c(c) = c(2 - c)$, $c \in [0, 1]$, and $F_\delta(\delta) = \delta^2/4$, with $\delta \in [0, 2]$. There are two interior high turnout equilibria.*

To wrap up, multiplicity of equilibria is not fragile to the introduction of uncertainty about the preference for conformism.

8 Discussion

In this paper we have characterized the equilibria of a voting model where agents care about others' choices. We have shown that for some parameter specifications we have multiplicity of equilibria, which in turn create a coordination problem. In a companion work (Landi and Sodini, 2010) we try to address such an issue from the point of view of evolutionary dynamics. However in this section we want to briefly discuss of our findings in light of the data on turnout.

Quite few countries adopt or adopted compulsory voting (henceforth CV) laws. By establishing a set of penalties for non compliance and/or by setting up procedures to ease the act of voting, these laws de facto reduce the individual cost of voting. According to cost benefit analysis, we should expect an increase in turnout rates when CV laws are introduced, and a decrease in turnout rates when these laws are abolished.

Indeed, there is evidence of positive correlations between turnout rates and introduction of CV laws and voters' turnout, and of negative correlation between abolishment of CV laws and voters' turnout (Jackman, 2001). However, the magnitude of these effects is quite big.

For example, after abolishing CV law, average turnout fell from 91.40% to 82.93% in Italy, from 94.70% to 81.34% in the Netherlands and from 86.37% to 44.82% in Venezuela. [8] Even more, turnout rates when CV laws are present dominate turnout rates when CV laws are abolished. The minimum turnout rates when CV laws were present are, respectively, 87.44%, 93.12%, and 60.00%, while maximum turnout rates after CV laws have been abolished are, respectively, 86.14%, 88.00%, and 56.55%. These figures are quite striking, in light of the fact that in almost all cases CV laws put no or purely symbolic penalties for non compliance, if they are mentioned at all. Therefore, to be able to match these numbers with a cost of voting model, we always need a big mass of marginal voters.

While this is definitely an interesting empirical question, we want to suggest a further explanation of this correlation: CV laws act as a coordinating device, a norm that allows citizens to coordinate towards the higher turnout equilibrium. Such laws, in fact, might be the expression of a majority that considers the act of voting as important and valuable and by putting this in black and white they generate the right coordinating device. As others (Jackman, 2001) have already noticed, a CV law may act as a social norm in some countries such as Italy, where "social embarrassment [is] an important sanction for non-compliance." A further motive to prefer this interpretation of the act of voting comes from the evidence that most of the penalties applied to non voters are smaller than a parking ticket violation. However we observe much more compliance with CV laws. This seems to corroborate the hypothesis that something else, besides the comparison at the margin of private cost and private benefit, is taken into account when deciding whether to vote. Something that can be framed within norm dictated behavior, and that we have studied in this paper, the preference for conformism.

9 Conclusion

This paper presents an alternative model of turnout where citizens have preference for conformism. If conformism plays a non marginal role in citizens' utilities, the voting game has multiple equilibria, and high turnout rates are observable. This result is robust with respect to the uncertainty underlying the cost of voting and the asymmetry and heterogeneity of the

[8] Authors' own calculations from turnout data available at `http://www.idea.int`.

parameter governing the preference for conformism.

In this model, turnout is interpreted as the solution of two types of coordination problems: one, which is standard, is due by the tendency of citizens who prefer the same alternative to free ride on others' efforts, which pushes turnout rates down. The other, with the population at large (but not necessarily so) whereby others' actions directly influence one's utility. This may reinforce the low willingness to vote if there is an expectation that the others will stay home, but may also counteract it if there is an expectation that others will turn out.

Different degrees of preferences for conformism will potentially induce different turnout rates. Yet, it is still possible to observe different turnout rates for the same level of preference for conformism, depending on how the equilibrium selection is solved. Multiplicity of equilibria can also be used to explain the high jumps in voter turnout that we observe after compulsory voting laws were abolished, as these laws can be interpreted as a coordinating device that help citizen to pick the right norm of behavior.

Our paper shows that for given intensity for conformism, high turnout equilibria are more likely to emerge when the reference group for conformism is more homogeneous. The next step on the research agenda is to deepen the role played by more structured network relations into the voting or not choice.

A Pivot probabilities

This is another way to write the pivot probabilities. It becomes useful when looking at corner equilibria:

$$P_R(\rho,\lambda,\theta,n) = \frac{1}{2}e^{-n(\rho\theta+\lambda(1-\theta))}\left[1+\lambda(1-\theta)n\right] + \\ + \frac{1}{2}\sum_{k=1}^{\infty}\frac{e^{-\rho\theta n}(\rho\theta n)^k}{k!}\frac{e^{-\lambda(1-\theta)n}(\lambda(1-\theta)n)^k}{k!}\left[1+\frac{\lambda(1-\theta)n}{k+1}\right] \quad \text{(A-1)}$$

and

$$P_L(\rho,\lambda,\theta,n) = \frac{1}{2}e^{-n(\rho\theta+\lambda(1-\theta))}\left[1+\rho\theta n\right] + \\ \frac{1}{2}\sum_{k=0}^{\infty}\frac{e^{-\rho\theta n}(\rho\theta n)^k}{k!}\frac{e^{-\lambda(1-\theta)n}(\lambda(1-\theta)n)^k}{k!}\left[1+\frac{\rho\theta n}{k+1}\right] \quad \text{(A-2)}$$

Based on equations (A-1) and (A-2) we can easily get

$$P_R(0,\lambda,\theta,n) = \frac{1}{2}e^{-n\lambda(1-\theta)}\left[1+\lambda(1-\theta)n\right] \quad \text{(A-3)}$$

$$P_L(0,\lambda,\theta,n) = \frac{1}{2}e^{-n\lambda(1-\theta)} \quad \text{(A-4)}$$

$$P_R(\rho,0,\theta,n) = \frac{1}{2}e^{-n\rho\theta} \quad \text{(A-5)}$$

$$P_L(\rho,0,\theta,n) = \frac{1}{2}e^{-n\lambda(1-\theta)}\left[1+\rho\theta n\right] \quad \text{(A-6)}$$

On the other hand, the pivot probabilities can be approximated to obtain (Abramowitz and Stegun, 1965; Myerson, 1998)

$$P_R(\rho,\lambda,\theta,n) \approx \frac{e^{-n(\sqrt{\theta\rho}-\sqrt{(1-\theta)\lambda})^2}}{4\sqrt{n\pi\sqrt{\theta(1-\theta)\lambda\rho}}}\frac{\sqrt{\theta\rho}+\sqrt{(1-\theta)\lambda}}{\sqrt{\theta\rho}} \quad \text{(A-7)}$$

$$P_L(\rho,\lambda,\theta,n) \approx \frac{e^{-n(\sqrt{\theta\rho}-\sqrt{(1-\theta)\lambda})^2}}{4\sqrt{n\pi\sqrt{\theta(1-\theta)\lambda\rho}}}\frac{\sqrt{\theta\rho}+\sqrt{(1-\theta)\lambda}}{\sqrt{(1-\theta)\lambda}} \quad \text{(A-8)}$$

These approximations allow us to conclude that

$$P_R(\rho,\lambda,\theta,n) = P_L(\rho,\lambda,\theta,n)\sqrt{\frac{(1-\theta)\lambda}{\theta\rho}}$$

This relation is useful when considering the other type of corner equilibria, since

$$P_R(1,\lambda,\theta,n) = \sqrt{\frac{(1-\theta)\lambda}{\theta}}P_L(1,\lambda,\theta,n) \quad \text{(A-9)}$$

$$P_L(\rho,1,\theta,n) = \sqrt{\frac{\theta\rho}{1-\theta}}P_R(\rho,1,\theta,n) \quad \text{(A-10)}$$

B Bessel functions

The asymptotic results about pivot probabilities are computed by Myerson. Yet in the application at hand, they can be obtained by using modified Bessel functions. As those probabilities depend also on the tie breaking rule, which I assumed to be a flip of a fair coin, it is important to have an understanding of how these Bessel functions works, in case the tie breaking rule were to be changed, given the application at hand.

A modified Bessel function I of degree v is defined as

$$I_v(z) = \left(\frac{1}{2}z\right)^v \sum_{k=0}^{\infty} \frac{\left(\frac{1}{4}z^2\right)^k}{k!(k+v)!}$$

which, for fixed v z is sufficiently large (see Abramowitz and Stegun, 1965; Castanheira, Castanheira; Myerson, 2000) can be approximated by

$$I_v(z) \approx \frac{e^z}{\sqrt{2\pi z}}$$

This is the starting point for the computation of pivot probabilities. In fact, in a Poisson game, the probability that candidate 1 is ahead of exactly v votes is given by

$$Prob(v_1 = v_2 + v) = \sum_{k=0}^{\infty} \frac{e^{-r_1} r_1^{k+v}}{(k+v)!} \cdot \frac{e^{-r_2} r_2^k}{k!}$$

where r_i depends on the parameter of the Poisson distribution and the strategy of the players. After some algebra manipulation this becomes

$$Prob(v_1 = v_2 + v) = e^{-(r_1+r_2)} \left(\frac{r_1}{r_2}\right)^{v/2} (\sqrt{r_1 r_2})^v \sum_{k=0}^{\infty} \frac{\left(\sqrt{r_1 r_2}\right)^{2k}}{k!(k+v)!}$$

and the relation with a Bessel modified function becomes transparent, once we let $z = 2\sqrt{r_1 r_2}$. As a result the probability that candidate 1 is ahead of exactly v votes for large populations is approximated by

$$Prob(v_1 = v_2 + v) \approx \frac{e^{-(\sqrt{r_1}-\sqrt{r_2})^2}}{2\sqrt{\pi\sqrt{r_1 r_2}}} \left(\sqrt{\frac{r_1}{r_2}}\right)^v$$

By similar calculations we can see that the probability that candidate 2 is ahead of exactly w votes is approximately given by

$$Prob(v_2 = v_1 + w) \approx \frac{e^{-(\sqrt{r_1}-\sqrt{r_2})^2}}{2\sqrt{\pi\sqrt{r_1 r_2}}} \left(\sqrt{\frac{r_2}{r_1}}\right)^v$$

When ties are broken via a flip of a coin, candidate i's voter is pivotal any time her vote breaks a losing tie or forces a winning tie. This requires candidate i tieing or trailing by one vote.

Therefore, the probability that candidate 1's voter is pivotal, P_1, is the average of the probability of having a tie between the two candidates ($w = 0$) and the probability of candidate 2 being one vote ahead $w = 1$. Thus

$$P_1 \approx \frac{e^{-(\sqrt{r_1}+\sqrt{r_2})^2}}{4\sqrt{\pi\sqrt{r_1 r_2}}}\left(1+\sqrt{\frac{r_2}{r_1}}\right)$$

Similarly, the probability of candidate 2's voter being pivotal, P_2, is given by

$$P_2 \approx \frac{e^{-(\sqrt{r_1}+\sqrt{r_2})^2}}{4\sqrt{\pi\sqrt{r_1 r_2}}}\left(1+\sqrt{\frac{r_1}{r_2}}\right)$$

Remark that

$$\frac{P_1}{P_2} = \sqrt{\frac{r_2}{r_1}}$$

If ties were solved differently, these equations would change. Suppose for example that candidate 1 represents the status quo which can be overturn only by a majority of votes against, then a candidate 1's voter is pivotal any time candidate 1 is trailing by one vote ($w = 1$). Therefore the pivot probability is just

$$\tilde{P}_1 \approx \frac{e^{-(\sqrt{r_1}-\sqrt{r_2})^2}}{2\sqrt{\pi\sqrt{r_1 r_2}}}\sqrt{\frac{r_2}{r_1}}$$

On the other hand, a candidate 2's voter is pivotal any time there is a tie between the two candidates ($v = 0$). Therefore the pivot probability is just

$$\tilde{P}_2 \approx \frac{e^{-(\sqrt{r_1}-\sqrt{r_2})^2}}{2\sqrt{\pi\sqrt{r_1 r_2}}}$$

Note that, again, the ratio of these two probabilities is unchanged as

$$\frac{\tilde{P}_1}{\tilde{P}_2} = \sqrt{\frac{r_2}{r_1}}$$

Therefore the equilibrium relations between r_1 and r_2 should not depend on the particular tie breaking rule. This is the sense of the comment found in the literature about the fact that there is no loss in generality in assuming a random tie breaking rule.

The following are useful properties of Bessel functions of the first order which are reported or can be derived from (Abramowitz and Stegun, 1965, pages 375, 377):

1. $I_{-n}(x) = I_n(x)$
2. $\frac{dI_0(x)}{dx} = I_1(x)$
3. $\frac{dI_1(x)}{dx} = I_0(x) - \frac{I_1(x)}{x}$

4. $\frac{dx^n I_n(x)}{dx} = x^n I_{n-1}(x)$

Lemma 1. *The function*

$$l(c) = \frac{e^{-nF(c)}}{2}\left[I_0(nF(c)) + I_1(nF(c))\right]$$

is strictly decreasing. In addition it is strictly convex if and only if

$$f'(c) \le \frac{nf^2(c)}{I_1(nF(c))}\left[I_1(nF(c)) - I_0(nF(c))\right] + \frac{2f(c)^2}{F(c)}$$

Proof. By using the properties of the Bessel functions we have that

$$l'(c) = -\frac{e^{-nF(c)}}{2}\frac{f(c)I_1(nF(c))}{F(c)} < 0$$

and

$$\begin{aligned} l'' &= -\frac{e^{-nF(c)}}{2F(c)^2}\left[F(c)f'(c)I_1(nF(c)) - nF(c)f(c)^2 I_1(nF(c)) + nF(c)f^2(c)I_0(nF(c)) - 2f^2(c)I_1(nF(c))\right] \\ &= -e^{-nF(c)}\left\{\frac{f'(c)}{F(c)}I_1(nF(c)) + \frac{nf(c)^2}{F(c)}\left[I_0(nF(c)) - I_1(nF(c))\right] - 2\left[\frac{f(c)}{F(c)}\right]^2 I_1(nF(c))\right\} \end{aligned}$$

□

C Proofs of results

This section contains the proofs of our results.

Proof of Proposition 1. Consider a citizen j of group i with private cost c_{ij}. It is individually rational to vote if and only if the marginal benefit (expressed by the pivot probabilities) is no smaller than the marginal cost (expressed by the private cost net of the externality component coming from the behavior of the other citizens).

Since voters use a cutoff rule, we have that $\lambda = F(\bar{c}_L)$ and $\rho = F(\bar{c}_R)$. Consider the following functions

$$\begin{aligned} E_R(c_R, c_L) &= P_R(F(c_R), F(c_L), \theta, n) - \delta[1 - 2((1-\theta)F(c_L) + \theta F(c_R))] \\ E_L(c_R, c_L) &= P_L(F(c_L), F(c_R), \theta, n) - \delta[1 - 2((1-\theta)F(c_L) + \theta F(c_R))] \end{aligned}$$

They represent the expected payoff from voting, net of the cost of voting. Use them to create the function $T : [0,1]^2 \to [0,1]^2$ such that

$$T(c_R, c_L) = \left(\max\left\{\min\left\{E_R(c_R, c_L), 1\right\}, 0\right\}, \max\left\{\min\left\{E_L(c_R, c_L), 1\right\}, 0\right\}\right)$$

Since this function is continuous we can apply the Brouwer fixed point theorem and conclude there exist a $(\bar{c}_R, \bar{c}_L)$ such that $T(\bar{c}_R, \bar{c}_L) = (\bar{c}_R, \bar{c}_L)$.
The next step is to show that such a pair corresponds to a cutoff equilibrium of the game. Whenever $\bar{c}_j \in (0,1)$, we have that group j citizens with lover private costs prefer to vote, and those with higher private costs prefer to abstain. Whenever $\bar{c}_j = 1$, it must be that $E_j(1, c_k) > 1$ which means that the marginal benefit of voting is larger than all the possible marginal costs. Last, whenever $\bar{c}_j = 0$, it must be that $E_j(0, c_k) < 0$ which means that the marginal benefit of voting is smaller than all the possible marginal costs. □

Proof of Proposition 2. The first part is a straightforward consequence of the conditions for corner equilibria.
The second part follows by the following two lemmata. □

Lemma 2. *There are no equilibria of the type* $(\bar{c}_R, 0)$, *and* $(0, \bar{c}_L)$.

Proof. Consider the candidate equilibrium $(\bar{c}_R, 0)$. We must have

$$\begin{aligned} P_R(F(\bar{c}_R), 0, \theta, n) &= \bar{c}_R + \delta(1 - 2\theta F(\bar{c}_R)) \\ P_L(F(\bar{c}_R), 0, \theta, n) &\leq \delta(1 - 2\theta F(\bar{c}_R)) \end{aligned}$$

Therefore we must have that

$$P_L(F(\bar{c}_R), 0, \theta, n) < P_R(F(\bar{c}_R), 0, \theta, n)$$

However

$$P_L(F(\bar{c}_R), 0, \theta, n) = \frac{e^{-F(\bar{c}_R)\theta n}}{2}(1 + F(\bar{c}_R)\theta n) > \frac{e^{-F(\bar{c}_R)\theta n}}{2} = P_R(F(\bar{c}_R), 0, \theta, n)$$

The same logic can be used to show that the other pair of cutoffs is not an equilibrium. □

Lemma 3. *A necessary condition for* $(\bar{c}_R, 1)$ *[resp.* $(1, \bar{c}_L)$*] to be an equilibrium is that* $\theta > 1/2$ *[resp.* $\theta < 1/2$*]. Therefore there are no equilibria of the type* $(\bar{c}_R, 1)$, *and* $(1, \bar{c}_L)$ *when* $\theta = 1/2$.

Proof. Consider the pair $(\bar{c}_R, 1)$. For it to be an equilibrium we must have

$$\begin{aligned} P_R(F(\bar{c}_R), 1, \theta, n) &= \bar{c}_R + \delta[1 - 2(\theta F(\bar{c}_R) + 1 - \theta)] \\ P_L(F(\bar{c}_R), 1, \theta, n) &\geq 1 + \delta[1 - 2(\theta F(\bar{c}_R) + 1 - \theta)] \end{aligned}$$

Therefore we must have that

$$P_L(F(\bar{c}_R), 1, \theta, n) > P_R(F(\bar{c}_R), 1, \theta, n) \tag{C-1}$$

Since we are looking at strictly positive voting probabilities, the pivot probabilities can be approximated by using (A-7) and (A-8)

$$P_R(F(c_R), 1, 1/2, n) \approx \frac{e^{-n/2(\sqrt{F(c_R)}-1)^2}}{2\sqrt{2n\pi\sqrt{F(c_R)}}} \frac{\sqrt{F(c_R)}+1}{\sqrt{F(c_R)}}$$

$$P_L(F(c_R), 1, 1/2, n) \approx \frac{e^{-n/2(\sqrt{F(c_R)}-1)^2}}{2\sqrt{2n\pi\sqrt{F(c_R)}}} [\sqrt{F(c_R)}+1]$$

and therefore inequality (C-1) becomes

$$\sqrt{\frac{F(\bar{c}_R)\theta}{1-\theta}} > \sqrt{\frac{1-\theta}{F(\bar{c}_R)\theta}}$$

This condition is possible, for $F(\bar{c}_R) \in (0,1)$, only if $\theta > 1/2$. In a similar fashion we can show that $\theta < 1/2$ is necessary for the other pair of cutoffs to be an equilibrium. □

Proof of Proposition 3. Based on Proposition 2, Lemma 2 and Lemma 3 it is enough to show that the interior equilibria, when they exist, are symmetric. When $F(c_j) > 0$ for all j, and n is large, we can approximate the pivot probabilities by using (A-7) and (A-8)

$$P_R(F(c_R), F(c_L), 1/2, n) \approx \frac{e^{-n/2(\sqrt{F(c_R)}-\sqrt{F(c_L)})^2}}{2\sqrt{2n\pi\sqrt{F(c_L)F(c_R)}}} \frac{\sqrt{F(c_R)}+\sqrt{F(c_L)}}{\sqrt{F(c_R)}} \tag{C-2}$$

$$P_L(F(c_R), F(c_L), 1/2, n) \approx \frac{e^{-n/2(\sqrt{F(c_R)}-\sqrt{F(c_L)})^2}}{2\sqrt{2n\pi\sqrt{F(c_L)F(c_R)}}} \frac{\sqrt{F(c_R)}+\sqrt{F(c_L)}}{\sqrt{F(c_L)}} \tag{C-3}$$

Notice that the conditions for interior equilibria are equivalent to

$$\sqrt{\frac{F(c_L)}{F(c_R)}} = \frac{c_R + \delta[1 - F(c_R) - F(c_L)]}{c_L + \delta[1 - F(c_R) - F(c_L)]} \tag{C-4}$$

$$P_L(F(c_R), F(c_L), n) = c_L + \delta[1 - F(c_R) - F(c_L)] \tag{C-5}$$

Let $(\bar{c}_L, \bar{c}_R)$ be an interior equilibrium. Substituting $\bar{c}_L$ in the previous equations, it is easy to verify that $c_R = \bar{c}_L$ is a solution. Now we prove that the solution $(\bar{c}_L, \bar{c}_L)$ is unique. Let $l(c_R)$ and $r(c_R)$ be respectively the left and the right sides of (C-4) when $c_L = \bar{c}_L$. We have that $r(c_R)$ is a decreasing function while with some more calculations we obtain that $sign\{r'(c_R)\} = sign\{\bar{c}_L + \delta[1 - F(c_R) - F(\bar{c}_L)] + \delta f(c_R)(c_R - \bar{c}_L)\}$. Note that by (C-5), we must have that $\bar{c}_L + \delta[1 - F(c_R) - F(\bar{c}_L)] \geq 0$. Therefore $sign\{r'(c_R)\} > 0$ for $c_R > \bar{c}_L$, and no equilibrium $(c_L, c_R) = (\bar{c}_L, \bar{c}_L + \varepsilon)$, $\varepsilon \neq 0$ exists. In a similar way, if we fix $c_R = \bar{c}_L$ we have that no equilibrium $(c_L, c_R) = (\bar{c}_L + \varepsilon, \bar{c}_L)$, $\varepsilon \neq 0$ exists. □

Proof of Proposition 4. Write $H(c) = l(c) - r(c)$. The interior equilibria are therefore all the values of c for which $H(c) = 0$. The proof relies on the study of the sign of the following product: $H(0) \cdot H(1)$. Whenever $H(0) \cdot H(1) < 0$, $H(c) = 0$ can only have (generically) an odd number of solutions and will have at least one. Whenever $H(0) \cdot H(1) > 0$, $H(c) = 0$ can only have (generically) an even number of solutions and possibly none. Now remark that

$$H(0) = \frac{1}{2} - \delta$$
$$H(1) \approx (2n\pi)^{-1/2} - 1 + \delta$$

Therefore when $\delta < 1/2$ we have that $H(0) > 0$ and $H(1) < 0$, since $n > 1$. Likewise, when $\delta > 1 - (2n\pi)^{-1/2}$ we have that $H(0) < 0$ and $H(1) > 0$. □

Proof of Proposition 5. Remember that M is defined so that $f(M) \geq f(c)$ for all c. When $\delta < [2f(M)]^{-1}$, $r(c)$ is always increasing regardless of $F(c)$. In fact,

$$r'(c) = 1 - 2\delta f(c)$$

Since $l(c)$ is always decreasing, we conclude that $H(c)$ is decreasing. Since $[2f(M)]^{-1} < 1/2$, we have that $H(0) \cdot H(1) < 0$ and therefore the uniqueness result follows.
We have that $r(c_m) = c_m$ and

$$l(c_m) \approx (n\pi)^{-1/2}$$

and therefore $H(c_m) < 0$ if and only if $n\pi > c_m^{-2}$. This allows us to conclude that the solution to $H(c) = 0$ is to the left of c_m and therefore the equilibrium turnout rate is lower than 50%. We now need to show that as δ grows, the solution to $H(c) = 0$ decreases, thus generating lower equilibrium turnout. By the implicit function theorem we have that

$$\frac{dc}{d\delta} = -\frac{2F(c) - 1}{l'(c) - 1 + 2\delta f(c)}$$

The numerator is positive since the solution to $H(c) = 0$ is to the left of the median. The denominator is also negative since $l(c)$ is a decreasing function of c and $\delta < [2f(M)]^{-1}$. Thus the solution to $H(c) = 0$ is a decreasing function of δ. □

Proof of Proposition 6. Suppose $\delta > 1 - (2\pi n)^{-1/2}$. We have that $H(1) > 0$. In addition, since $n\pi > c_m^{-2}$, we know that $H(c_m) < 0$. Therefore there exists at least one interior equilibrium with high turnout.

We also know that

$$\frac{dc}{d\delta} = -\frac{2F(c) - 1}{l'(c) - 1 + 2\delta f(c)}$$

We now have that the numerator is positive since $c > c_m$. We also have that the denominator is positive. In fact, the denominator is just $H'(c)$ and at the smallest of the high turnout equilibria, $H(c)$ crosses the horizontal axis from below and therefore the function is increasing. □

C.1 Corner solutions with group specific conformism

This section proves Proposition 10. The analysis is carried in more detail, so that the results can be spelled out more precisely.

Proposition 17. *When conformism is group based the strategy profile* $(0, 1)$ *is an equilibrium for all* $\theta \in (0, 1)$ *if and only if* $\delta \geq 1 - \frac{e^{-n(1-\theta)}}{2}$.

In addition, let

$$\delta_0 = \frac{e^{-n(1-\theta)}}{2}[1 + n(1 - \theta)]$$

and

$$\delta_1 = 1 - \frac{e^{-n(1-\theta)}}{2}$$

There are no equilibria $(0, \bar{c}_L)$ *if* $\delta < \delta_0$. *When* $\delta \in (\delta_0, 1/2)$, *if an equilibrium of the above type exists, then we have that (generically) they are an odd number. When* $\delta \in (1/2, \delta_1]$, *there is (generically) an even number (and possibly zero) of equilibria* $(0, \bar{c}_L)$. *Finally, when* $\delta > \delta_1$ *there is (generically) an odd number of equilibria* $(0, \bar{c}_L)$.

Proof. Let x denote the share of one's group's citizens that chooses to vote. With group based conformism, given the strategy profile $(0, \bar{c}_L)$, we have that $x = 0$ for group R citizens and $x = F(\bar{c}_L)$ for group L citizens. Therefore, adapting from inequalities (3) and (6), the conditions for a corner equilibrium $(0, 1)$ boil down to

$$\delta \geq \max\left\{1 - \frac{e^{-n(1-\theta)}}{2}, [1 + n(1 - \theta)]\frac{e^{-n(1-\theta)}}{2}\right\} = 1 - \frac{e^{-n(1-\theta)}}{2}$$

where the equality follows because

$$e^{n(1-\theta)} > \frac{2+n(1-\theta)}{2}$$

for all $n > 0$ and $\theta \in (0,1)$.

Similarly, the conditions for the equilibrium $(0, \bar{c}_L)$ become

$$\frac{e^{-F(\bar{c}_L)(1-\theta)n}}{2} = \bar{c}_L + \delta(1 - 2F(\bar{c}_L)) \tag{C-6}$$

$$\delta > \frac{e^{-F(\bar{c}_L)(1-\theta)n}}{2}(1 + F(\bar{c}_L)(1-\theta)n) \tag{C-7}$$

Now observe that the study of equation (C-6) parallels Proposition 4. In addition,

$$\frac{e^{-F(\bar{c}_L)(1-\theta)n}}{2}(1 + F(\bar{c}_L)(1-\theta)n) \geq \frac{e^{-F(\bar{c}_L)(1-\theta)n}}{2}$$

for all $\bar{c}_L \in [0,1]$ with equality holding at $\bar{c}_L = 0$. Moreover, inequality (C-7) is always satisfied whenever $\delta \geq 1/2$ and $\bar{c}_L > 0$. Finally, when $\delta < \delta_0$, we have that inequality (C-7) is never met. □

Proposition 17 shows us that equilibria of the type $(0, barc_L)$ are guaranteed to exist whenever the preference for conformism is sufficiently strong. And replicating the analysis done for the interior equilibria of the base model, we can also conclude that such equilibria will be unique after δ meets a certain threshold.

With exact same arguments we can also prove the following result.

Proposition 18. *When conformism is group based, the strategy profile $(1,0)$ is an equilibrium for all $\theta \in (0,1)$ if and only if $\delta \geq 1 - \frac{e^{-n\theta}}{2}$.*

In addition, let

$$\delta_0 = \frac{e^{-n\theta}}{2}[1 + n\theta]$$

and

$$\delta_1 = 1 - \frac{e^{-n\theta}}{2}$$

There are no equilibria $(\bar{c}_R, 0)$ if $\delta < \delta_0$. When $\delta \in (\delta_0, 1/2)$, if an equilibrium of the above type exists, then we have that (generically) they are an odd number. When $\delta \in (1/2, \delta_1]$, there is (generically) an even number (and possibly zero) of equilibria $(\bar{c}_R, 0)$. Finally, when $\delta > \delta_1$ there is (generically) an odd number of equilibria $(\bar{c}_R, 0)$.

Proof. Let x denote the share of one's group's citizens that chooses to vote. With group based conformism, given the strategy profile $(\rho, 0)$, we have that $x = 0$ for group L citizens and $x = \rho$ for group R citizens. Therefore, adapting from inequalities (5) and (4), the conditions for a corner equilibrium $(1, 0)$ boil down to

$$\delta \geq \max\left\{1 - \frac{e^{-n\theta}}{2}, [1 + n\theta]\frac{e^{-n\theta}}{2}\right\} = 1 - \frac{e^{-n\theta}}{2}$$

Similarly, the conditions for the equilibrium $(\bar{c}_R, 0)$ become

$$\frac{e^{-F(\bar{c}_R)n\theta}}{2} = \bar{c}_R + \delta(1 - 2F(\bar{c}_R))$$

$$\delta > \frac{e^{-F(\bar{c}_R)n\theta}}{2}(1 + F(\bar{c}_R)n\theta)$$

and the exact same logic as before applies. □

Last, consider the possibility of corner equilibria of the type $(1, \bar{c}_L)$ and $(\rho, 1)$. The conditions for the strategy profile $(1, \bar{c}_L)$ to be an equilibrium boil down to

$$P_R(1, F(\bar{c}_L), \theta, n) \geq 1 - \delta \tag{C-8}$$

$$P_L(1, F(\bar{c}_L), \theta, n) = \bar{c}_L + \delta(1 - 2F(\bar{c}_L)) \tag{C-9}$$

Now remark the following: $P_R(1, \lambda, \theta, n)$ has a peak at $\lambda = \frac{\theta}{1-\theta}$. In fact

$$P_R(1, \lambda, \theta, n) = \frac{e^{-n[(1-\theta)\lambda+\theta]}}{2}\left[I_0(2n\sqrt{\theta(1-\theta)\lambda}) + \sqrt{\frac{(1-\theta)\lambda}{\theta}}I_1(2n\sqrt{\theta(1-\theta)\lambda})\right]$$

By letting $x = 2n\sqrt{\theta(1-\theta)\lambda}$ one gets

$$P_R(1, x, \theta, n) = \frac{e^{-\frac{x^2}{4n\theta}-n\theta}}{2}\left[I_0(x) + \frac{x}{2n\theta}I_1(x)\right]$$

which has first derivative given by (see properties of Bessel functions)

$$\frac{dP_R(1, x, \theta, n)}{dx} = \frac{e^{-\frac{x^2}{4n\theta}-n\theta}}{2}\left[I_1(x) + \frac{x}{2n\theta}I_0(x) - \frac{x}{2n\theta}I_0(x) - \left(\frac{x}{2n\theta}\right)^2 I_1(x)\right]$$

which therefore is positive if and only if

$$1 \geq \frac{x}{2n\theta}$$

which holds if and only if (just plug back the definition of x)

$$\lambda \leq \frac{\theta}{1-\theta}$$

□

In addition, the maximum is interior if and only if $\theta < 1/2$. When the maximum is interior, the minimum is at either corner. In this case the value of the minimum is given by

$$\min\left\{\frac{1}{2}e^{-n\theta}, \frac{1}{2}e^{-n}\left[I_0\left(2n\sqrt{\theta(1-\theta)}\right) + \sqrt{\frac{1-\theta}{\theta}}I_1\left(2n\sqrt{\theta(1-\theta)}\right)\right]\right\}$$

Also, for any $\theta \geq 1/2$ the function is strictly increasing and has a maximum at $\lambda = 1$ and a minimum at $\lambda = 0$. In this case, the value of the minimum is given by

$$P_R(1, 0, \theta, n) = \frac{1}{2}e^{-n\theta}$$

Therefore, whenever $\delta > \min_\lambda\{P_R(1, \lambda, \theta, n)\}$ we have that inequality (C-8) is always satisfied and therefore it is enough to rely on the analysis done for the interior equilibria in the base model. Specifically we have the following sufficiency result.

Proposition 19. *Let* $\delta_0 = 1 - \min_\theta \min_\lambda P_R(1, \lambda, \theta, n)$*, and* $\delta_1 = 1 - P_L(1, 1, \theta, n)$*. Suppose* $\delta > \max\{\delta_0, \delta_1\}$*. There is, generically an odd number of equilibria of the type* $(1, F(\bar{c}_L))$*.*

Similarly, the equilibrium conditions for the strategy profile $(\bar{c}_R, 1)$ are given by

$$P_L(F(\bar{c}_R), 1, \theta, n) \geq 1 - \delta$$
$$P_R(F(\bar{c}_R), 1, \theta, n) = \bar{c}_R + \delta(1 - 2F(\bar{c}_R))$$

And a completely analogous reasoning holds. Namely:

Proposition 20. *Let* $\delta_0 = 1 - \min_\theta \min_\lambda P_L(\rho, 1, \theta, n)$*, and* $\delta_1 = 1 - P_R(1, 1, \theta, n)$*. Suppose* $\delta > \max\{\delta_0, \delta_1\}$*. There is, generically an odd number of equilibria of the type* $(F(\bar{c}_R), 1)$*.*

C.2 Multiple interior equilibria with heterogenous δ

Proof of Proposition 15. Rewrite equation (17) as

$$c = \delta(2\lambda - 1) + P(\lambda, n)$$

We focus on the case of $\lambda \geq 1/2$ since we are interested in the possibility of multiple interior equilibria with high turnout.

By the convolution formula, the probability that $c \leq \delta(2\lambda - 1) + P(\lambda, n)$ is given by

$$Prob_{c,\delta} = \frac{1}{\Delta}\int_0^\Delta F_c(\delta(2\lambda - 1) + P(\lambda, n))d\delta$$

which can be further computed as

$$Prob_{c,\delta} = \begin{cases} \frac{1}{\Delta}\int_0^{\Delta}[\delta(2\lambda-1)+P(\lambda,n)]d\delta & \text{if } \Delta(2\lambda-1)+P(\lambda,n)\leq 1 \\ \frac{1}{\Delta}\int_0^{B(\lambda)}[\delta(2\lambda-1)+P(\lambda,n)]d\delta + \frac{1}{\Delta}\int_{B(\lambda)}^{\Delta} d\delta & \text{if } \Delta(2\lambda-1)+P(\lambda,n)>1 \end{cases}$$

where $B(\lambda) = \frac{1-P(\lambda,n)}{2\lambda-1}$. After some lines of algebra we get

$$Prob_{c,\delta} = \begin{cases} \frac{(2\lambda-1)\Delta}{2} + P(\lambda,n) & \text{if } \Delta(2\lambda-1)+P(\lambda,n)\leq 1 \\ 1-\frac{(1-P(\lambda,n))^2}{2\Delta(2\lambda-1)} & \text{if } \Delta(2\lambda-1)+P(\lambda,n)>1 \end{cases}$$

Note that $P(\lambda,n)$ can be approximated by $(2n\pi\lambda)^{-1/2}$ because we are working under the assumption of $\lambda > 1/2$. Therefore one can see that there exists a unique $\lambda \in (1/2,1)$ for which $P(\lambda,n) = 1-\Delta(2\lambda-1)$.

Let $\mathcal{I}^- = \{\lambda \in [1/2,1] | 2\lambda\Delta + P(\lambda,n) \leq 1+\Delta\}$. Let $\mathcal{I}^+ = \{\lambda \in [1/2,1] | 2\lambda\Delta + P(\lambda,n) > 1+\Delta\}$. First note that there is no fixed point for $\lambda \in \mathcal{I}^-$. Suppose there is one, then it must satisfy

$$2\lambda(\Delta-1)+2P(\lambda,n)-\Delta=0$$

Solving for $P(\lambda,n)$

$$P(\lambda,n) = \frac{\Delta}{2} - \lambda(\Delta-1)$$

Since $P(1/2,n) < \Delta/2$, both sides of the above equation are decreasing function of λ, and the right hand side is also linear, the existence of a fixed point requites $P(1) > 1-\Delta/2$. In other words, necessary condition for such a fixed point to exist is that

$$\Delta > 2(1-P(1))$$

Now plug $P(\lambda,n) = \frac{\Delta}{2} - \lambda(\Delta-1)$ into the inequality constraint contained in $\mathcal{I}^-$, which generates the condition

$$\lambda \leq \frac{2+\Delta}{2(1+\Delta)}$$

In other words, if such a point exists, λ needs to be bounded above by $\frac{2+\Delta}{2(1+\Delta)}$. This condition puts also a bound on $P(\lambda,n)$, which needs to satisfy the following inequality

$$P(\lambda,n) \geq 1+\frac{\Delta}{2}$$

Nevertheless, since we also need $\Delta > 2(1-P(1))$, it must be the case that

$$P(\lambda,n) > 1-P(1)$$

which generates the desired contradiction. □

Therefore the fixed point can only be found in the set $\mathcal{I}^+$. The fixed point problem can be rewritten as

$$(1 - P(\lambda, n))^2 = 2\Delta(2\lambda - 1)(1 - \lambda)$$

The left hand side increases from $1/4$ to $(1 - P(1))^2$, a number very close to 1. The right hand side is positive in the range $1/2$ and 1, is zero at the extremes, and has maximum value equal to $\Delta/4$ at $\lambda = 3/4$.

Therefore there are two fixed points if and only if

$$\frac{\Delta}{4} > (1 - P(3/4))^2$$

Last, focus on the inequality constraint contained in $\mathcal{I}^+$. It requires $P(\lambda) > 1 - \Delta(2\lambda - 1)$. By using the lower bound on Δ we see that this inequality implies that

$$1 - \Delta(2\lambda - 1) < 1 - 4(1 - P(3/4))^2 < 0$$

if and only if $P(3/4) < 1/2$, which is true for all n. In fact

$$P(3/4, n) \approx \sqrt{\frac{2}{3n\pi}} < \frac{1}{2}$$

if and only $n\pi > 8/3$, which holds for all $n > 1$. This allows to conclude that the two fixed points are within the range of values for λ and concludes the proof.

□

Proof of Proposition 16. Given our assumptions, $F_c(c) = c(2 - c)$, with $c \in [0, 1]$ and $F_\delta(\delta) = \delta^2/4$ with $\delta \in [0, 2]$. Using the convolution formula, we have that

$$Prob_{c,\delta}\left(c + \delta(1 - 2\lambda) \le P(\lambda, n)\right) = \int_0^2 F_c\left(P(\lambda, n) + \delta(2\lambda - 1)\right) \frac{\delta}{2} d\delta$$

We want to show that there are two interior equilibria with $\lambda \ge 1/2$. The above probability can be written as

$$Prob_{c,\delta} = \begin{cases} \int_0^2 [\delta(2\lambda - 1) + P(\lambda, n)][2 - P(\lambda, n) - \delta(2\lambda - 1)]\frac{\delta}{2} d\delta & \text{if } 4\lambda + P(\lambda, n) \le 3 \\ \int_0^{B(\lambda)} [\delta(2\lambda - 1) + P(\lambda, n)][2 - P(\lambda, n) - \delta(2\lambda - 1)]\frac{\delta}{2} d\delta + \int_{B(\lambda)}^2 \frac{\delta}{2} d\delta & \text{if } 4\lambda + P(\lambda, n) > 3 \end{cases}$$

with $B(\lambda) = \frac{1 - P(\lambda, n)}{2\lambda - 1}$. After some algebra one gets

$$Prob_{c,\delta} = \begin{cases} -8\lambda^2 - P(\lambda, n)^2 + \frac{8}{3}\lambda[5 - 2P(\lambda, n)] - \frac{14}{3}[1 - P(\lambda, n)] & \text{if } 4\lambda + P(\lambda, n) \le 3 \\ \frac{23 - 96\lambda(1 - \lambda) + 4P(\lambda, n)(1 + P(\lambda, n)^2) - P(\lambda, n)^2(6 + P(\lambda, n))}{24(2\lambda - 1)^2} & \text{if } 4\lambda + P(\lambda, n) > 3 \end{cases}$$

As $\lambda > 1/2$, we can approximate $P(\lambda, n)$ with $(2n\pi\lambda)^{-1/2}$. The analytical analysis of this fixed point problem is a little cumbersome. However numerical computations allows us to prove the result holds. A graphical analysis of the problem is provided in figure 6.

[Figure 6 about here.]

□

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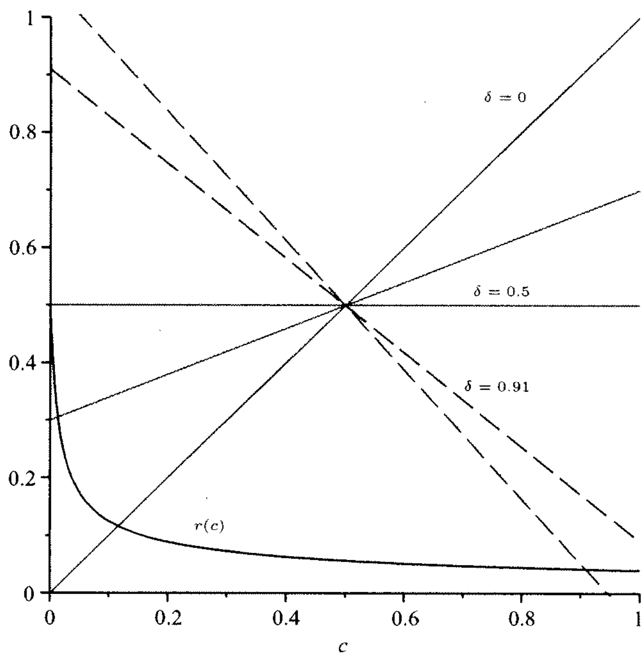

Figure 1: Equilibrium analysis when costs are uniformly distributed, population size $n = 100$, and $\theta = 1/2$. The curve $l(c)$ is drawn for different values of δ.

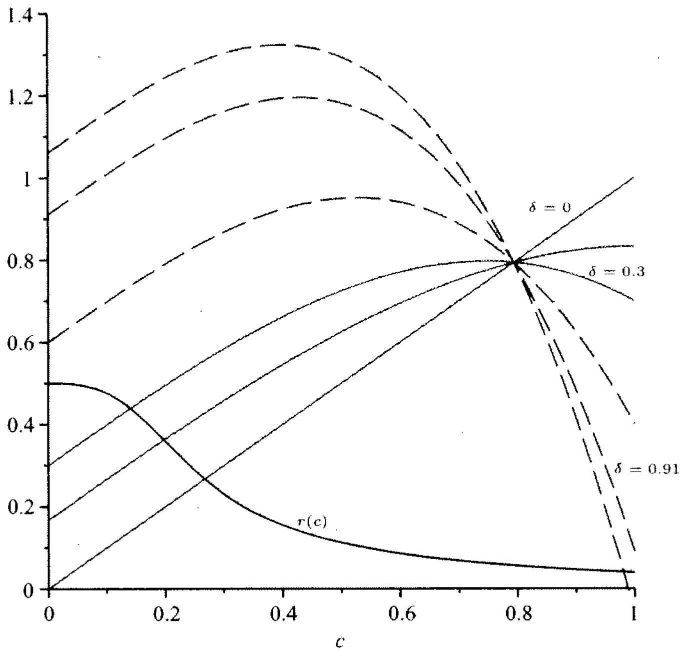

Figure 2: Equilibrium analysis for the distribution $F(c) = c^3$, population size $n = 100$ and $\theta = 1/2$. The curve $l(c)$ is drawn for different values of δ.

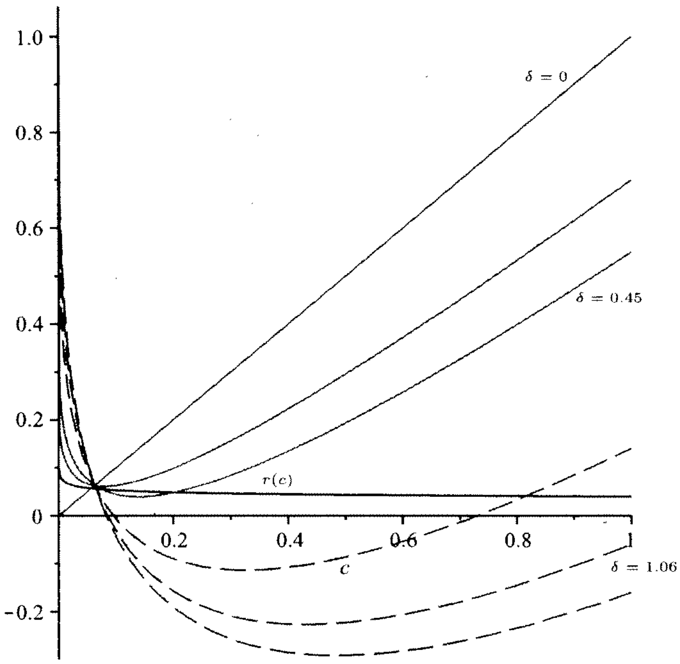

Figure 3: Equilibrium analysis for a power distribution with parameters 1 and 1/4 and population size $n = 100$ and $\theta = 1/2$. The curve $l(c)$ is drawn for different values of δ.

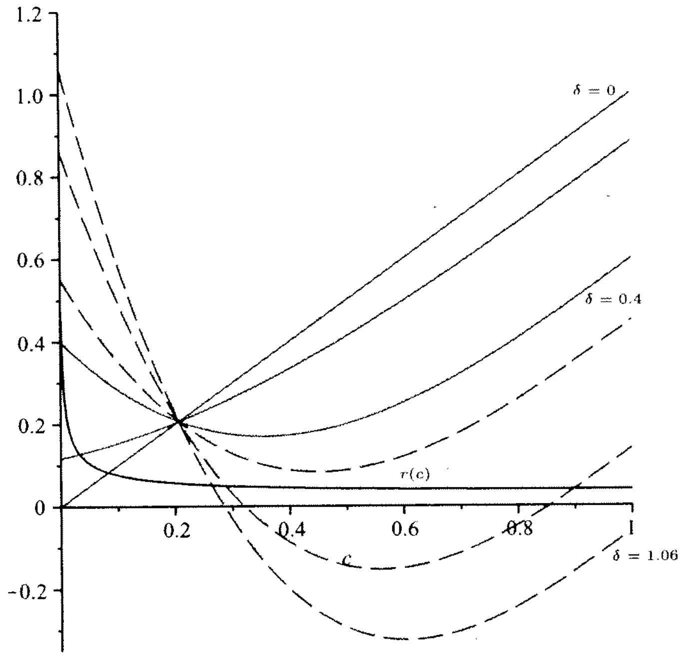

Figure 4: Equilibrium analysis for the distribution $F(c) = 1 - (1 - c)^3$ and population size $n = 100$ and $\theta = 1/2$. The curve $l(c)$ is drawn for different values of δ.

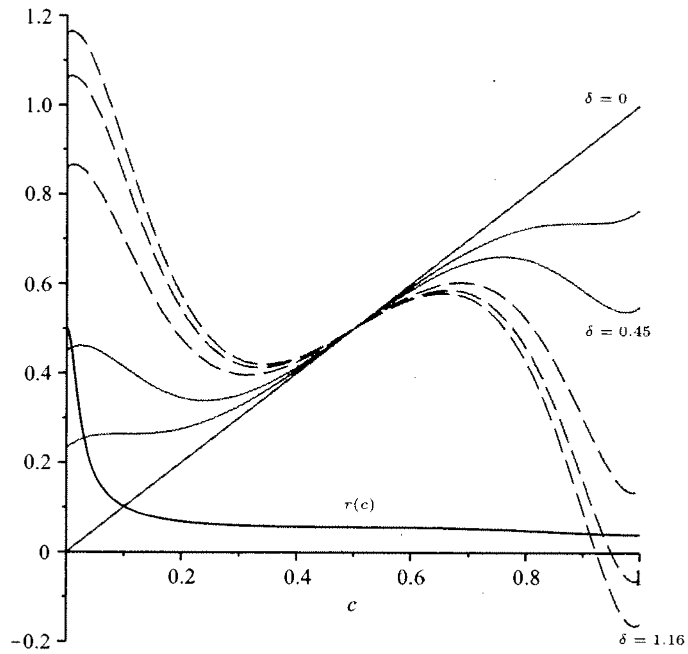

Figure 5: Equilibrium analysis for a distribution with modes at $M_1 = 0.1$ and $M_2 = 0.9$, and population size $n = 100$ and population size $n = 100$ and $\theta = 1/2$. The curve $l(c)$ is drawn for different values of δ.

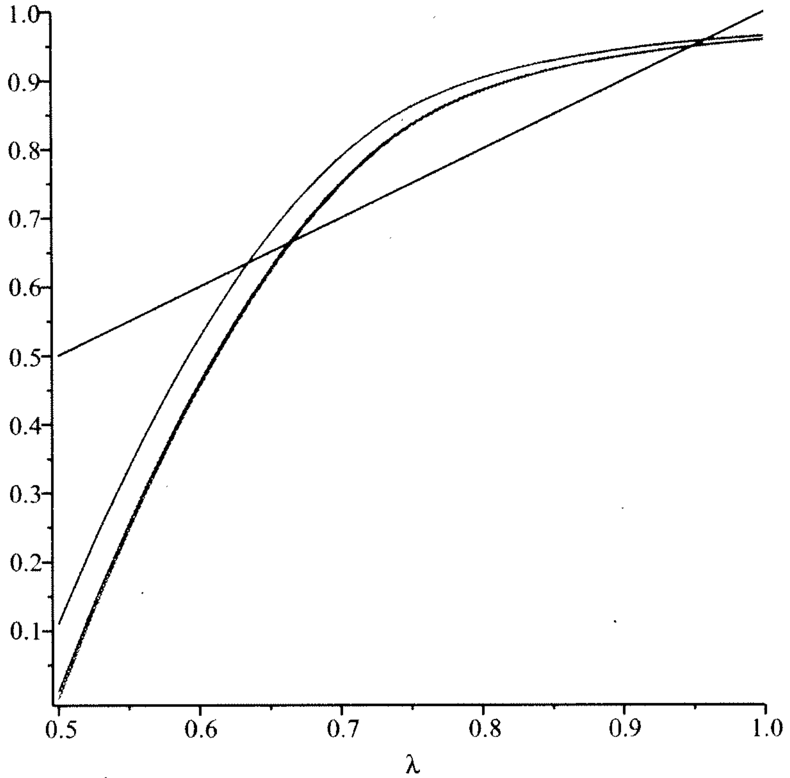

Figure 6: High turnout equilibria with heterogenous δ, for $n = 100$ (blue), $n = 10,000$ (red), and $n = 10,000,000$ (green).

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