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Local and global dynamics in an overlapping generations model with endogenous time discounting

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Abstract

The aim of the paper is to develop an overlapping generations model with endogenous time preference. We assume that individual's rate of time preference is socially determined by economy wide level of labour supply. In the case of constant impatience rate, there exists a unique steady state, saddle-point stable for the largest part of the parameters space. By constrast, in the case of socially determined impatience, there may exist multiple steady states and the equilibrium dynamics may exhibit (local or global) indeterminacy and complex behaviour.

1 Introduction

One of the main research fields in macroeconomics in recent years is to investigate how it is possible to produce endogenous fluctuations in microfounded models without introducing stochastic components or exogenous shocks. In particular, OLG models seems to provide a useful framework for such buildings. In his pioneeristic work, Reichlin (1986) shows that endogenous fluctuations are possible in perfectly competitive OLG economies with Leontief technology; Grandmont et al. (1998) generalize the result for economies with low values of the elasticity of substitution between capital and labor. Other sources for non linear dynamics may be, among others, increasing returns to scale, public spending, environmental factors, or consumption externalities (see respectively Cazzavillan et al., 1998; Cazzavillan, 1996; Antoci and Sodini, 2009; Antoci et al., 2010). The present paper aims at adding up another element for this analysis, by considering yet another factor that may be a source of fluctuations or more complex dynamics: namely, the social dimension in the determination of time preferences.

Nowadays, there exists a huge literature on "endogenous discounting factor" and several different frameworks were drawn, each one studying a possible way how economic (or social) dimension may influence time preferences: for example the role of addiction, income, mortality rate, uncertainty or parents' status (see Becker and Mulligan, 1997, for an exhaustive review). Many of these contributions underline that a non-constant discount rate may have strong repercussions in the dynamic properties of the models: Bhenabib and Day (1981) show the possibility of chaotic dynamics in a Diamond-like model in which discount rate of each generation is positively related to its lifetime wealth; Drugeon (1996) studies the possibility of endogenous growth trajectories and multiple equilibria in a continuous time model, in which increasing impatience with respect to present consumption is introduced; Bian and Meng (2004) study, in a continuos time exogenous growth model, the role endogenous discount rate for indeterminacy; Gomes (2008) develops a dicrete-time AK model where agents' degree of impatience positively depends on income, finding that the model is able to generate complex dynamics; Sarkar (2007) explores the decreasing marginal impatience hypothesis, showing that short-run damped oscillation are plausible near the steady states.

Following Reichlin (1986), in this paper we consider an overlapping generations model with endogenous labour supply, without first period consumption, in which we introduce an endogenous determination of time preferences, namely we assume that is it is influenced by economy wide level of time allocation: the more is the time devoted for future consumption at macro level, the less is the discounting factor for future consumption, in the single agent's problem.

The present paper has the following structure. Sections 2 and 3 define the set-up of the model and the associated dynamic system. Sections 4 and 5 deal with local stability analysis of fixed points and local indeterminacy. Section 6 shows that global indeterminacy and chaos can occur in our model. Section 7 contains the conclusions.

2 The model

We consider an overlapping generations economy with constant population. Time is discrete, $t = 1, 2, 3, \dots\dots\dots, \infty$; there exists a continuum of agents who live for two periods of time and two generations of individuals (*young* and the *old*) coexist in each period of time t. Individuals work when they are young and consume the private good when they are old[1]. The private good is produced by a continuum of perfectly competitive firms.

In each period t, the representative young individual has to allocate her time endowment fixed to 2 between leisure activity and labour supply l_t $(2 \geq l_t \geq 0)$ to the representative firm, remunerated at the wage rate W_t. The remuneration $l_t \cdot W_t$ is entirely invested in productive capital K_{t+1} (i.e. $K_{t+1} = l_t \cdot W_t$) that the individual will rent to the representative firm in time $t+1$ at the interest

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[1] This assumption is adopted in several overlapping generations models (see for example) and it was introduced by the seminal works on indeterminacy and non linear dynamics by Reichlin (1986) and Woodford (??). It simplifies our analysis by abstracting from the consumption-saving choices of agents.

factor R_{t+1}. The sum obtained, $W_t \cdot l_t \cdot R_{t+1}$, allows him to buy and consume the quantity $C_{t+1} = W_t \cdot l_t \cdot R_{t+1}$ of the good produced by the firm ($W_t \cdot l_t$ and $W_t \cdot l_t \cdot R_{t+1}$ are expressed in unities of the consumption good).

2.1 The utility function

We assume the following utility function:

$$U(2 - l_t, C_{t+1}, E_{t+1}) = \ln(2 - l_t) + S\frac{(C_{t+1}/B)^{1-\sigma}}{1-\sigma}$$

where $S \in (0, 1]$ indicates the discounting factor; B is a positive scale parameters that will be used to apply the "normalized steady state" technique, and σ is a positive parameter,with $\sigma \neq 1$, denoting the inverse of the intertemporal elasticity of substitution in consumption. It is worthwhile to notice that the utility function is jointly concave in $2 - l_t$ and C_{t+1}.

2.2 The productive Sector

We assume constant returns to scale. In particular, the representative firm has the following Cobb-Douglas production function:

$$Y = A \cdot f(K_t, l_t) = A \cdot l_t^{1-\alpha} \cdot K_t^{\alpha} = A \cdot l_t \cdot k_t^{\alpha}$$

where $k_t := K_t/l_t$ and A is a positive parameter representing (exogenous) technological progress.

The economy is assumed perfectly competitive and so, in each period t, the representative firm maximizes the profit function:

$$A \cdot f(K_t, l_t) - W_t \cdot l_t - R_t \cdot K_t \tag{1}$$

taking the wage rate W_t and the interest factor R_t as exogenously given. As usual, this assumption gives rise to the following first order conditions:

$$W_t = A \cdot (1 - \alpha) \cdot k_t^{\alpha} \tag{2}$$

$$R_t = A \cdot \alpha \cdot k_t^{\alpha-1} \tag{3}$$

2.3 Variable intensity of discounting factor

Notice that in this kind of specification, with consumption only in old age, S may be also considered as the relative weight of the consumption goods of utility function. According to this last interpretation, in many papers centred on dynamic properties of such a specification (see Seegmuller 2003) this term is normalized to one and no explicit discounting of olds' consumption is introduced. The novelty of the present paper is due to the exploration of the case in which

S is not constant, but it is socially determinate: we assume that the more the average labour supply is high (or, considering its dual aspect, the less is the average allocation to leisure activity), the more the single agent tends to give an high value on S, i.e. $S'(L) > 0$.
The implication of this assumption is that a higher level of relative weight of leisure increases the relative weight of leisure to the agent's utility.
To avoid too strong effects of the dependence of S on L, we have assumed an S-shaped specification[2] of S, with $S(0) > 0$:

$$S(L_t) := \frac{\arctan(f\pi(L-1))}{\pi} + \frac{1}{2} \tag{4}$$

where $1/2$ represents the autonomous level of discounting factor, while the first addend is the variable part; $f > 0$ is a parameter measuring the strength of the social influence: the more f is high, the more the role of the comparison in discounting factor is high for intermediate value of f[3].

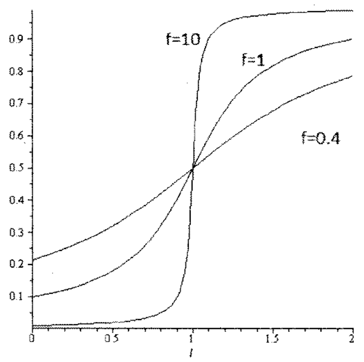

Figure 1: Different shape of S varying f: values showed in figure.

3 The Agent's problem

The representative agent maximizes his objective function:

$$\max U(l_t, C_{t+1})$$

under the constraints:

$$C_{t+1} = R_{t+1} \cdot W_t \cdot l_t \tag{5}$$

$$l_t \in [0, 2] \tag{6}$$

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[2] Similar specifications are used in several different contexts, for example to model investiments in a Kaldor-like business cycle model (Agliari et al. 2007) or to descibe the rule of adoption of particular strategies in financial market models (Dieci et al. 2006).

[3] In the next section we start our analysis form the equilibrium point for which L=1.

In our perfectly competitive economy, W_t and R_{t+1} are considered as exogenously given. Furthermore, S is taken as determined, in that the representative individual do not consider the effect of her choices on its value.

Under these assumptions, the first order condition for an interior solution of the representative individual's choice problem is:

$$\frac{1}{2-l_t} - S(L_t)\frac{[R_{t+1} \cdot W_t/B]^{1-\sigma}}{l_t^{\sigma}} = 0 \tag{7}$$

3.1 Equilibrium dynamics

We assume that each economic agent considers L_t as exogenously determined. However, being all economic agents identical, ex-post $l_t = L_t$. Thus, because the role of average labour input in S is not internalized, the choices of the representative individual are not (Pareto) optimal, however, the orbits followed by the economy are Nash equilibria, in that no single individual has interest to modify her choices if also the others don't revise theirs.

By plugging (1), (2) and (4) in (7) and taking into account that, by (2), it holds:

$$K_{t+1} = l_t \cdot k_{t+1} = l_t \cdot W_t = l_t \cdot A \cdot (1-\alpha) \cdot k_t^{\alpha}$$

the dynamic system representing the dynamics of the economy is:

$$-\frac{1}{2-L_t} + \left[\frac{\arctan\left(f\pi(L-1)\right)}{\pi} + \frac{1}{2}\right] \frac{[(\alpha \cdot (1-\alpha) \cdot A^2 \cdot k_t^{\alpha} \cdot k_{t+1}^{\alpha-1})/B]^{1-\sigma}}{L_t^{\sigma}} = 0 \tag{8}$$

$$k_{t+1} \cdot L_t = A \cdot (1-\alpha) \cdot k_t^{\alpha} \cdot L_t \tag{9}$$

4 Uniqueness and Multiplicity of Steady States

The system (8)-(9) defines k_{t+1} and L_{t+1} as functions of k_t and L_t. In this section, we study the stability of steady states of such discrete dynamic system. Since our model contains a large number of parameters, to make clear the study we apply the geometrical-graphical method developed by Grandmont et al. (1998) that allows us to characterize the stability properties of a specific steady state of a two-dimensional dynamic system. We impose some conditions on parameters under which a steady state $(k_{s.s}, L_{s.s}, S_{s.s})$ with $k_{s.s} = L_{s.s} = S_{s.s.} = 1$ exists. This allows us to analyze the effects on stability due to changes in parameters' values being sure that the steady state doesn't disappear. By requiring that $k_{s.s} = L_{s.s} = S_{s.s} = 1$ (by (8)-(9)) we obtain the following conditions on parameters' values:

$$A = A^* := \frac{1}{1-\alpha}, \; B := 2^{\frac{1}{\sigma-1}} \alpha A \tag{10}$$

Using conditions (10), the dynamic system (8)-(9) can be explicitly written as:

$$k_{t+1} = V(k_t, L_t) := \left[\frac{(2-L_t) k_t^{\alpha(1-\sigma)} (2 \arctan(L_t - 1) f\pi + \pi)}{\pi L_t^{\sigma}} \right]^{\frac{1}{(1-\alpha)(1-\sigma)}} \tag{11}$$

$$L_{t+1} = Z(k_t, L_t) := L_t \cdot k_t^{\alpha} \cdot \left[\frac{\pi L_t^{\sigma}}{(2-L_t) k_t^{\alpha(1-\sigma)} (2 \arctan(L_t - 1) f\pi + \pi)} \right]^{\frac{1}{(1-\alpha)(1-\sigma)}} \tag{12}$$

defined on the set $D := \left\{ (k_t, L_t) \in R^2 : k_t > 0, 0 < L_t < 2 \right\}$.
Notice that $k = 1$ always holds at the steady states of system (11)-(12), while the steady state values of L are given by the solutions of the equation

$$g(L) := \left[\frac{(2-L_t)(2 \arctan(L-1) f\pi + \pi)}{\pi L_t^{\sigma}} \right]^{\frac{1}{(1-\alpha)(1-\sigma)}} = 1 \tag{13}$$

Obviously, $L = 1$ is a solution of equation (13), that is the normalized steady state exists for every constellation of parameters. The following proposition holds.

Proposition 1 *There exist* f_1 *and* f_2, *with* $f_1 < f_2 \equiv \frac{1+\sigma}{2}$, *such that: for* $f < f_1$ *an unique steady state (1,1) exists; for* $f \in (f_1, f_2)$ *two other steady states exist, both with* $L < 1$. *For* $f > f_2$ *two non-normalized steady states exist, one with* $L < 1$ *and the other with* $L > 1$.

Proof. We present the proof only for the case $\sigma \in (0, 1)$, the other one follows in a similar way. Notice that $\lim_{L \to 0} g(L) = +\infty, \lim_{L \to 2} g(L) = 0, \forall f > 0, \forall \alpha \in (0, 1)$. Furthermore $g'(L)$ may change sign at most two times and $g(L)$ cuts the horizontal line 1 at the point L=1 with the slope $g'(1) = \frac{-1+2f-\sigma}{(1-\sigma)(1-\alpha)}$. Starting from f=0, if we let f increase, then $g'(1)$ is increasing and for $f = f_2 \equiv \frac{1+\sigma}{2}$ it vanishes. With straightforward computations it is easy to check that for $f = f_2$, we have $g''(1) = -\frac{2+\sigma+\sigma^2}{(1-\sigma)(1-\alpha)} < 0$, that is 1 is a maximum point. Summing up the previous properties, it follows that for $f = f_2$, an other intersection between the graph of g and the horizontal line still exists at the left of 1 and there exists a value $f_1 < f_2$ such that the graph of g is tangential the line (see Figure 2 on the left). At this point the result follows from simple geometric considerations. ■

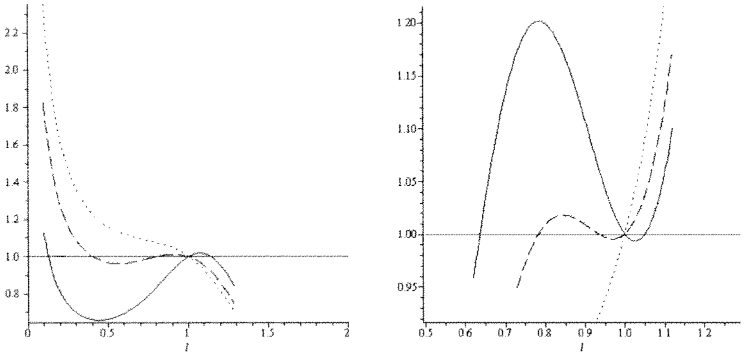

Figure 2: *Graphs of $g(L)$ in case $\sigma < 1$ (on the left) and in case $\sigma > 1$ (on the right)*

In the rest of the paper we denote the eventually two non-normalized steady states with $(1, \widehat{L},)$ and $(1, \widehat{\widehat{L}})$, with $\widehat{L} < \widehat{\widehat{L}}$.

5 Stability of the normalized steady state and local indeterminacy

In the present model, productive capital K_t is a state variable, so its initial value K_0 is given, while L_t is a jump variable in that it represents the representative individual's labor input, chosen taking into account of the wage, interest factor and discounting factor. It follows that individuals of the first generation have to choose the initial value L_0 (and consequently the initial value of $k_t = K_t/L_t$). Considering the (local) dynamic properties of stationary equilibrium, if the normalized steady state is a saddle and the initial condition of K is enough close to 1, then there exists an unique initial value of L_t, L_0, such that the orbit passing through (k_0, L_0) approaches the steady state. When the steady state is a sink, given the initial value K_0, then there exists a continuum of initial values L_0 such that the orbit passing through (k_0, L_0) approaches the steady state; consequently, the orbit the economy will follow is "locally indeterminate" in that it depends on the choice of the initial value L_0.
The Jacobian matrix of (11)-(12), evaluated at the normalized steady state, is:

$$J = \begin{pmatrix} \frac{\alpha}{1-\alpha} & \frac{-1+2f-\sigma}{(1-\sigma)(1-\alpha)} \\ \frac{-\alpha^2}{(1-\alpha)} & \frac{-\alpha+2+\sigma\alpha-2f}{(1-\sigma)(1-\alpha)} \end{pmatrix}$$

with

$$Tr(J) = \frac{2}{(1-\alpha)(1-\sigma)} - \frac{2f}{(1-\alpha)(1-\sigma)} \tag{14}$$

$$Det(J) = \frac{2\alpha}{(1-\alpha)(1-\sigma)} + \frac{2f\alpha}{(1-\alpha)(1-\sigma)} \tag{15}$$

Given the values of the other parameters, varying f, the point (see equations (15)-(14)):

$$(P_1, P_2) := \left(\frac{2}{(1-\alpha)(1-\sigma)} - \frac{2f}{(1-\alpha)(1-\sigma)}, \frac{2\alpha}{(1-\alpha)(1-\sigma)} + \frac{2f\alpha}{(1-\alpha)(1-\sigma)}\right)$$

draws in the plane $(Tr(J), Det(J))$ a half-line T_1 with slope α $(0 < \alpha < 1)$ (see Figures 3 and 4) starting from the point (obtained posing $f = 0$ in (15)-(14)):

$$(\overline{P}_1, \overline{P}_2) := \left(\frac{2}{(1-\alpha)(1-\sigma)}, \frac{2\alpha}{(1-\alpha)(1-\sigma)}\right) \tag{16}$$

The point $(\overline{P}_1, \overline{P}_2)$, varying α, draws in the plane $(Tr(J), Det(J))$ a half-line T_2 with slope 1 starting from the point $\left(\frac{2}{1-\sigma}, 0\right)$. If $\sigma \in (0,1)$, then $(\overline{P}_1, \overline{P}_2) \to (+\infty, +\infty)$ for $\alpha \to 1$ and $(P_1, P_2) \to (-\infty, -\infty)$ for $f \to +\infty$. If $\sigma > 1$, then $(\overline{P}_1, \overline{P}_2) \to (-\infty, -\infty)$ for $\alpha \to 1$ and and $(P_1, P_2) \to (+\infty, +\infty)$ for $f \to +\infty$. From these geometrical findings and from Proposition 1, we can deduce the following 2 propositions:

Proposition 2 *Assume that* $\sigma \in (0,1)$, *and let* $f_{fl} := \frac{3+\alpha-(1-\alpha)\sigma}{2(1+\alpha)}$, $f_{tr} := \frac{1+\sigma}{2}$, $f_{ns} := \frac{3\alpha-1}{2\alpha}$. *Then the following generically holds:*

1. If $\alpha \in (0, 1/2)$ (see Case 1 in Figure 3), the normalized steady state $(1,1)$ is saddle-point stable for $f < f_{tr}$; a transcritical bifurcation occurs for $f = f_{tr}$ and the normalized steady state becomes a sink for $f_{tr} < f < f_{fl}$. For $f = f_{fl}$ then a flip bifurcation occurs and the normalized steady state becomes saddle-stable again for $f > f_{fl}$.

2. If $\alpha \in (1/2, 1)$ (see Case 2 in Figure 3), the normalized steady state $(1,1)$ is saddle-point stable for $f < f_{tr}$; a transcritical bifurcation occurs for $f = f_{tr}$ and the normalized steady state becomes a source. For $f = f_{ns}$ a Neimark-Sacker bifurcation occurs and the normalized steady is a sink for $f_{ns} < f < f_{fl}$. For $f = f_{fl}$ then a flip bifurcation occurs and the normalized steady state becomes saddle-stable again for $f > f_{fl}$.

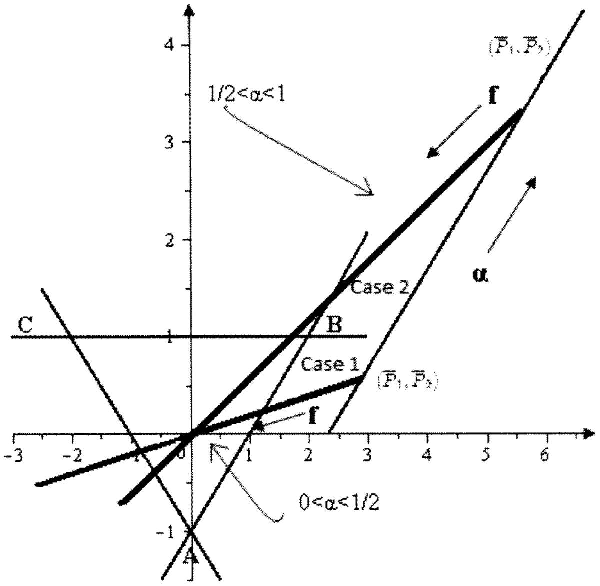

Figure 3: *Stability analysis of the normalized steady state in case $\sigma < 1$*

Proposition 3 *Let $f_{fl} := \frac{3+\alpha-(1-\alpha)\sigma}{2(1+\alpha)}$, $f_{tr} := \frac{1+\sigma}{2}$, $f_{ns} := \frac{3\alpha-1}{2\alpha}$.Then, the following generically holds:*

1. *If $\alpha \in (0, 1/2)$ and $\sigma > \frac{\alpha+3}{1-\alpha}$ (Case 1 in Figure 4), then the normalized steady state is a sink for $f <, f_{tr}$. For $f = f_{fl}$ then a transcritical bifurcation occurs and the normalized steady state becomes saddle-stable for $f > f_{tr}$.*

2. *If $\alpha \in (0, 1/2)$ and $\sigma \in (1, \frac{\alpha+3}{1-\alpha})$ (Case 2 in Figure 4), then the normalized steady state is saddle-point stable for $f < f_{fl}$. For $f = f_{fl}$ then a flip bifurcation occurs and the normalized steady state becomes a sink for $f \in (f_{fl}, f_{tr})$. For $f = f_{fl}$ then a transcritical bifurcation occurs and the normalized steady state becomes saddle-stable again for $f > f_{tr}$.*

3. *If $\alpha \in (1/2, 1)$ (Case 3 in Figure 4), then the normalized steady state is saddle-point stable for $f < f_{fl}$. For $f = f_{fl}$ then a flip bifurcation occurs and the normalized steady state becomes a sink for $f \in (f_{fl}, f_{ns})$. For $f = f_{ns}$ then a Niemark-Sacker bifurcation occurs and the normalized steady state becomes source. For $f = f_{fl}$ then a flip bifurcation occurs and the normalized steady state becomes saddle-stable again for $f > f_{fl}$*

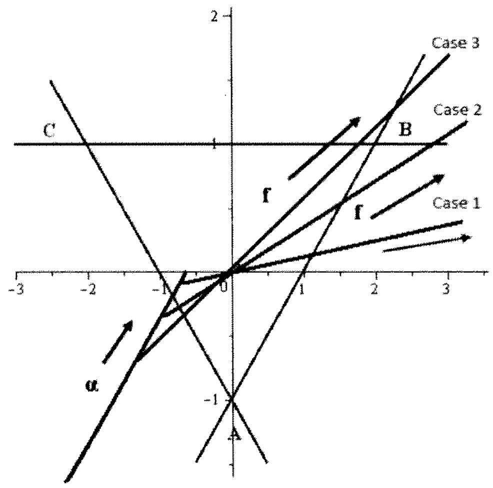

Figure 4: *Stability analysis of the normalized steady state in case* $\sigma > 1$

6 Global dynamics

Local stability analysis is not usually sufficient to fully characterize the model's behavior, especially if multiple steady states may exist. Furthermore the linearization techniques do not allow us to investigate the stability of invariant curves or limit cycles that arise when flip bifurcation and Niemark-Sacker bifurcation occurs.

In this section we show that an increase in the parameter f, which measures the magnitude of the steep of S around the normalized steady state may be a source of global indeterminacy and chaos. We say that the model is global indeterminate if starting from the same initial value K_0 of the state variable K, the system may converge to different fixed points or ω-limit sets according to the initial choice L_0 of the control variable L.

Moreover, we will see that for different configurations of parameter, the model generates chaotic attractors and in this case because of sestitivity to the initial condition, we have a sort of indeterminacy in the long run behavior of the system even if the same attractor is reached.

For the next analysis it is useful to evidentiate that for some configurations of parameters, the map is not invertible and this aspect plays an important role to understand the dynamics of the model.

Firstly we define LC_{-1}, that is the locus of points at which the determinant of the Jacobian matrix vanishes.

Lemma 4 *The Jacobian of the map vanishes for* $f > f^* \simeq 0.896$ *along two horizontal lines* LC^a_{-1}, LC^b_{-1}.

Proof. With straightforward calculation we find that Jacobian vanishes for positive values of (L_t, k_t) if and only if $\frac{V'_{k_t}}{L_t} - \alpha\frac{V'_{L_t}}{k_t} = 0$, that is for

$$g(L_t) := (2 + L_t)\left(2\arctan(L_t - 1)f\pi + \pi\right) - \frac{2f\pi L_t(2 - L_t)}{(1 + (L_t - 1)^2 f^2\pi^2)} = 0$$

Notice that the expressions is independent from k_t, σ, α and $g'(L_t) = \frac{2f\pi L_t((f\pi)^2(1-L_t)+1)}{(1+(L_t-1)^2f^2\pi^2)}$, $g(0) > 0, g(2) > 0$. g admits a minimum in $L_t^* = \frac{(f\pi)^2-1}{(f\pi)^2}$ and, substituting, we find that $g(L_t^*) < 0 \iff f > f^*$, $f^* \simeq 0.896$. ■

Because of analytic specification it is not possible to explicit the inverse(s) of the map. Nonetheless it is possible to have some insight considering that the dynamic system is equivalent to

$$k_t^\alpha = k_{t+1}\left(\frac{\pi L_t}{(2 - L_t)(2\arctan(L_t - 1)f\pi + \pi)}\right)^{\frac{1}{(1-\sigma)(1-\alpha)}} \tag{17}$$

$$L_t = \frac{L_{t+1}k_{t+1}}{k_t^\alpha} \tag{18}$$

If we substitute (17) in (18) we have that the the components L_t of the inverse are given by the solutions (with respect to L_t) of the equation

$$h(L_t) := \left[\frac{(2 - L_t)(2\arctan((L_t - 1)f\pi) + \pi))}{L_t\pi}\right]^{\frac{1}{-1+\sigma}} = L_{t+1}k_{t+1}^\alpha$$

Notice that the right side varies from 0 to infinity, therefore, considering the shape of the graph of $h(L_t)$, we can determine the number of preimages. Specifically it is simple to verify that if $\sigma > 1$ (resp. $\sigma < 1$) than $\lim_{L_t \to 0} h(L_t) = +\infty$ and $\lim_{L_t \to 2^-} h(L_t) = 0$ (resp. $\lim_{L_t \to 0} h(L_t) = 0$ and $\lim_{L_t \to 2^-} h(L_t) = +\infty$); furthermore for low value of f, h is monotone and there exists a threshold value $\tilde{f}$ such that for $f > \tilde{f}$, h has a maximum $L_{\max}$ and a minimum $L_{\min}$. that is for every (L_{t+1}, k_{t+1}) there exist an unique inverse for $f < \tilde{f}$, while for $f > \tilde{f}$, using the notation used in Mira et al. (1996), it follows that the present map is a $Z_1 - Z_3 - Z_1$ map, that is there exist regions of plane whose points have one and three different rank-1 preimages, respectively. Such regions, or zones, are separated by the critical curves LC (i.e. the locus of points having two merging rank-1 preimages in points belonging to LC_{-1}) which in this case are made up of two branches of hyperbolas in the phase plane, say L_a and L_b, whose equations are $L = \frac{h(L_{\max})}{k^\alpha}$, $L = \frac{h(L_{\min})}{k^\alpha}$, respectively.

First scenario

We describe the typical scenario when $\sigma \in (0, 1)$ and[1] $1/2 < \alpha < \alpha^* < 1$. In the numerical simulations we fix $\alpha = .624$, $\sigma = .532$ and we let f vary. For $f < 0.682$,

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[1] We use α^* to indicate a threshold value of α changing the nature of the Neimark Sacker bifurcation involved in the successive analysis.

an unique (normalized) steady state exists and it is saddle (see Figure 3), that is an unique path converging to equilibrium exists. For $f \simeq 0.682$ a bifurcation occurs and 2 further steady states appears. At this stage, the 2 (external) saddle steady states coexist with an intermediate (unstable) focus. For $f = .765$ we have a first bifurcation that changes the position and the stability properties of $(1, \hat{\hat{L}})$ and the normalized steady state. When $f = f_{ns} \simeq 0.86$, the normalized steady state undergoes a subcritical Neimark-Sacker bifurcation and becomes stable, with its basin of attraction bounded by a repelling closed curve appeared at the bifurcation value.

If we let f increase further, for $f \simeq 0.885$, a first qualitative change of the basin of attraction occurs: the stable manifold of $(1, \hat{L})$ collides with the invariant closed curve, "merging" together (see Figure 6). A subsequent bifurcation of the structure of the basins of attraction, leading a simply connected basin to a connected basin but with holes (multiply connected area) is due to the creation of a bay H_0 in the region Z_3 after a contact bifurcation between L_b and the basin boundary (arising at $f \simeq 0.94$). If f exceeds the value $f_{fl} \simeq 1.054$ the normalized steady state loses its stability through a flip bifurcation, crating a periodic attractor with increasing period and then chaotic. For $f = 1.095$ the chaotic attractor occupies an area quite close to the frontier of its basin. A contact between the two transforms the chaotic attractor into a chaotic repellor at $f \simeq 1.1$

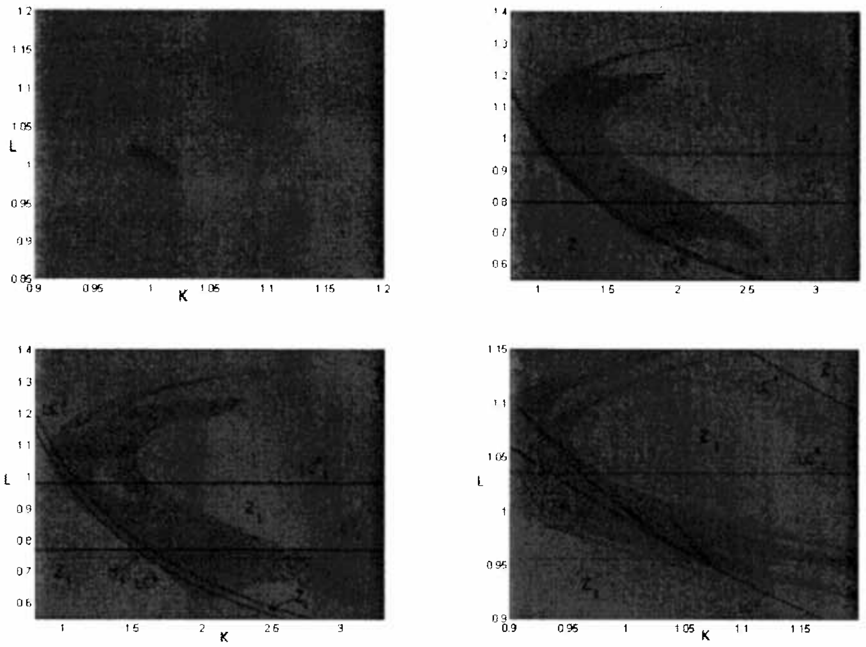

Figure 5: *Evolution of basin of attraction: (a) for $f = 0.86$ the map is invert-*

ible and the normalized steady state is surrounded by an invariant closed curve that delimitates its basin of attraction; (b) simply connected basin of attraction for $f = 0.9$ (the map has just lost the invertiblity); (c) multiply connected (or connected with holes) basin of attraction for f=0.96; (d) chaotic attractor just before its death due to contact to the border of the basin of attraction f=1.095.

Second scenario

Using numerical simulation we verify that, for a given value of $\sigma < 1$, there exists a threshold value $\alpha^* > 1/2$ reversing the nature of the Neimark Sacker. In this case, for $f = f_{ns.}$, the normalized steady state is repelling and surrounded by an attractive closed invariant curve. In the numerical simulation we fix $\alpha = 0.765$, $\sigma = 0.52$. If we consider lower values of f, the dynamics become less regular because the invariant curve has a contact to the critical curve LC_{-1} and this collision induces a wavy shape of Γ. If we let f decrease further, we obtain the effect of destroying the attractor and almost all orbits diverge to infinity[5].Notice that analogously to the previous case, several holes are present in the basin of attraction.

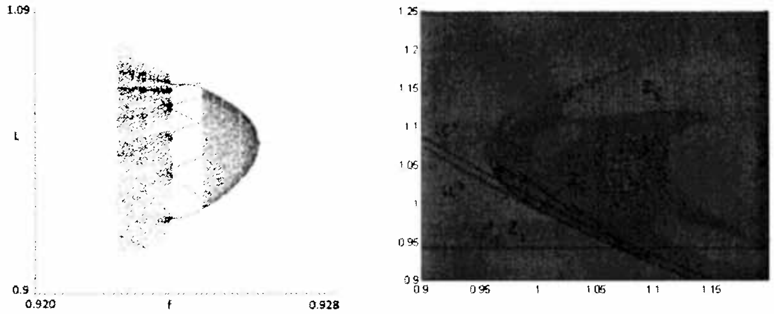

Figure 7: *(a) Bifurcation diagram: Neimark-Sacker bifurcation; (b) Basin of attraction of* Γ *after the contact with* LC^b

Third scenario

In the previous scenarios, the system is characterized by the existence of an unique attractor. This case deals with the possible coexistence and evolution of two different attractors. We consider $\alpha = 0.2$, $\sigma = 1.4$. For $f < 1.18$ an unique attractor exists; for $f = f^* \simeq 1.18$ we have the appearance of an other locally stable steady state through a saddle node bifurcation and the basins of attraction of the two attractors are separated by the stable manifold of $(1, \widehat{\widehat{L}})$, $\widehat{\widehat{L}} < 1$. With the increase of f, firstly, for $f = f_{tr}$, we have a change of position and of stability between the normalized steady state and $(1, \widehat{\widehat{L}})$, then two sequences of not synchronized flip bifurcations arise, leading to the coexistence of two chaotic attractors. At this stage an increasement of f makes attractors wider and closer

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[5] Considering $\sigma > 1$, we verify that at most a unique attracting fixed point (the normalized steady state) exists and it is destabilized by a supercritical Neimark-Sacker and a supercritical flip bifurcation for high and low values of f, respectively. Considering the basin of attraction, similar phenomena to what described in the Scenario 2 arise, hence, for the sake of briefness we don't deepen this part.

to borders of their basins of attraction. The asymmetry of the map implies that the contacts do not occur at the same time. At $f = \widehat{f} \simeq 1.6$, a first collision between the attractor below and the border of its basin of attraction causes the death of the attractor and almost all trajectories in this area are captured by the attractor above. At $f = \hat{\hat{f}} \simeq 1.627$, a new contact of the attractor above with the stable manifold of the saddle $(1,1)$ generates a "fusion" of the survived attractor and the repellor, creating a new attractor that occupies a large area in the phase space. Hence, this new stage is characterized by large oscillations in the time evolution of the variables (see Tramontana et al. 2009 for a similar behavior).

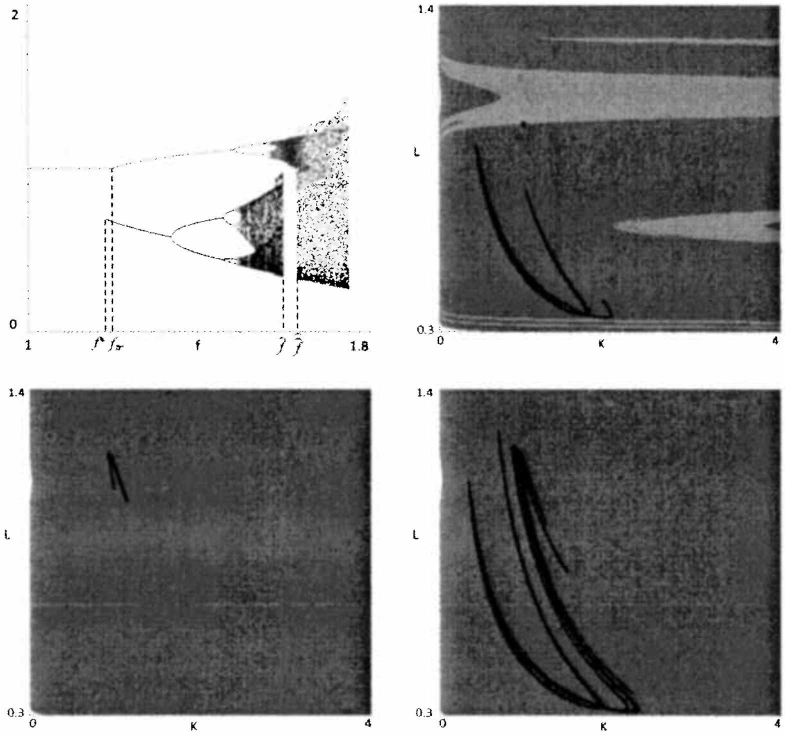

Figure 7: *(a) Bifurcation diagram: the black part correspond to initial condition enough close to* $(1,\hat{\hat{L}})$ *when two attractors exist; (b) coexistence of two attractors (f=1.58) separated by the stable manifold of (1,1); (c) a unique attractor, located in the top part of D, captures almost all converging initial conditions; (d) enlargement of the attractor just after* $\hat{\hat{f}}$.

7 Conclusion

In this work we have explored an overlapping generations model with an endogenous determination of discount rate. In this setting we have analyzed the properties of the dynamic two-dimensional system generated by the model. The model generates a wide range of possible dynamic patterns, depending on parameter configurations: coexistence of attractors, local and global indeterminacy occur for a wide range of specifications of parameters. Furthermore, chaotic attractors and global bifurcation may occur.

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