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A note on semi-pseudoconvexity and "generalized" invexity

Riccardo Cambini and Laura Carosi*

Abstract

A definition of generalized convex functions is proposed and its relationships with the generalized convex properties commonly studied in the literature are analyzed. Furthermore, the proposed class is compared with the ones of invex and generalized invex functions. Unlike in the invexity and generalized invexity properties, the introduced concept, named semi-pseudoconvexity, does not require any use of parameter function. The new definition allows us to prove the equivalence among the classes of F-convex functions, F-pseudoconvex functions and strong pseudoinvex functions. Taking into account the equivalence results given by Craven and Clover and Caprary [4, 7], we obtain that several notions of generalized invexity coincide with the class of invex functions and hence they can not be seen as true generalizations of invexity. Moreover, the equivalence between invexity and semi-pseudoconvexity underlines that the use of parameters and/or functionals in defining invexity properties does not yield "a priori" a higher level of generalization. The semi-pseudoconvexity property is used to derive necessary and sufficient optimality conditions and to obtain duality results for vector optimization problems.

Key words: Generalized convexity, generalized invexity, duality conditions, vector optimization.
AMS - 2010 Math. Subj. Class. 90C29, 90C46, 90C26.

1 Introduction

Generalized convexity has been extensively studied over the last decades as it is witnessed by many texts and a huge amount of research papers. Different aspects of convex functions have been analyzed and studied in the literature. In particular, most of the attention has been devoted to the key role that convexity plays in deriving optimality conditions and to the relationship between convexity and duality results. Several generalizations of convex functions have been proposed with the attempt to extend the class of functions which maintains the nice properties of convexity. Among the different kinds of generalized convex functions, pseudoconvexity and invexity certainly occupy a prominent position. As it is very well known, both pseudconvexity and invexity guarantee that critical points are global minimum points, Fritz-John conditions are sufficient optimality conditions and weak and strong duality results hold. In order to get a larger and larger class of functions which verifies those nice properties, many different generalizations of pseudconvexity and invexity have been proposed in the recent literature. Even an hasty and superficial reading of the most recent papers on that topics provides many different definitions of generalized invex and pseudoconvex functions (just see for example [5, 9, 16]). Various classes of generalized convex functions needs of the existence of suitable parameters or functionals which are required to verify some nice conditions (just like invex functions, the so called F-convex functions, (F, ρ)-convex functions, and so on and so for). Unfortunately, the proposed definitions are not often so easy to be verified and in many cases it is impossible to find examples guaranteeing that these classes of functions are true

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generalizations of the existing class of generalized convex functions.
Due to this, following a sort of reverse process, we aim to propose a new definition of generalized convexity which is not based on any kind of parameter or suitable functional. The proposed property, named semi-pseudoconvexity, is simply related to the behavior of the gradient of the function. In Section 2 we underline that this concept coincides neither with the one of pseudoconvex functions nor the one of quasiconvex functions. Moreover we study the key role played by the semi-pseudoconvexity in optimization. In particular, it is shown that the condition "every critical point is a global minimum point" completely characterizes the semi-pseudoconvexity.
Taking into account this characterization of semi-pseudoconvexity, in Section 3 the results in [4, 7] are extended by proving that the class of semi-pseudoconvex functions (defined by means of neither parameters nor functionals) coincides with both invexity and some of the generalized invexity concepts proposed in the literature, that are F-convexity, F-pseudoconvexity, strong pseudoinvexity, strong F-pseudoconvexity. The aim of the provided results is twofold. First, it is shown that F-convexity, F-pseudoconvexity, strong pseudoinvexity and strong F-pseudoconvexity are not actually true generalizations of the class invex functions. Then, it is pointed out that the use of parameters and or functionals in defining "generalized" invexity properties does not yield "a priori" any kind of generalization.
Finally, in the last section of the paper semi-pseudoconvexity is used in obtaining duality results for a vector optimization problem where the primal objective function is ordered by an arbitrary convex, closed and pointed cone C, (not necessarily the Paretian cone) and the feasible region is defined by means of equality constraints and inequality ones. Two mixed type dual problems are given, a scalar and a vector one. Weak and strong duality results are provided assuming the semi-pseudoconcavity[1] of a suitable Lagrangean function. As in the tradition of the mixed type dual problems (see for all the pioneer paper by Xu [18] and the recent survey by Chinchuluun and Pardalos [5] and reference therein), specifying the value of the parameters in the dual problem, both Wolfe-type and Mond-Weir-type duality [15, 17] can be derived.

2 Semi-pseudoconvexity: definitions and preliminary results

The aim of this section is to propose a new definition of generalized convexity. The introduced generalized convexity property is based on the behavior of the gradient of the function. No parameter function is used in the definition, for this very reason the considered concepts lie in the field of generalized convexity and not in the generalized invexity one.

Definition 1 *A differentiable scalar function $f : X \to \Re$, $X \subseteq \Re^n$ open convex set, is said to be :*

- *semi-pseudoconvex in X if for all $x_1, x_2 \in X$ it holds:*

$$f(x_1) < f(x_2) \quad \Rightarrow \quad \nabla f(x_2) \neq 0 \tag{1}$$

- *strictly semi-pseudoconvex in X if for all $x_1, x_2 \in X$, $x_1 \neq x_2$, it holds:*

$$f(x_1) \leq f(x_2) \quad \Rightarrow \quad \nabla f(x_2) \neq 0 \tag{2}$$

It is worth comparing the semi-pseudoconvexity concepts with the very well known classes of pesudoconvex and of quasiconvex functions, in order to point out that the introduced properties are not trivial. For the sake of completeness, let us recall that a differentiable scalar function $f : X \to \Re$, with $X \subseteq \Re^n$ open convex set, is said to be (see for all [1, 2]):

- *[strictly] pseudoconvex* if for all $x_1, x_2 \in X$ it holds:

$$f(x_1) < f(x_2) \ [\leq] \quad \Rightarrow \quad \nabla f(x_2)^T (x_1 - x_2) < 0 \tag{3}$$

[1]We recall that f is semi-pseudoconcave if $-f$ is semi-pseudoconvex

- *quasiconvex* if for all $x_1, x_2 \in X$ it holds:

$$f(x_1) \leq f(x_2) \quad \Rightarrow \quad \nabla f(x_2)^T (x_1 - x_2) \leq 0 \tag{4}$$

It's immediate to verify that [strictly] pseudoconvex functions are also [strictly] semi-pseudoconvex. Notice also that functions having no stationary points are strictly semi-pseudoconvex and hence also semi-pseudoconvex. Constant functions are both semi-pseudoconvex and pseudoconvex, but they are neither strictly semi-pseudoconvex nor strictly pseudoconvex. The following Example 1 provides a function which is both semi-pseudoconvex and strictly semi-pseudoconvex but neither pseudoconvex nor strictly pseudoconvex. Figure 1 summarizes the relationships between pseudoconvexity and semi-pseudoconvexity. Notice that all of the relationships are proper.

Example 1 *Function $f(x_1, x_2) = -x_1^2 - x_2$ is both semi-pseudoconvex and strictly semi-pseudoconvex in $\Re^2$ since $\nabla f(x_1, x_2)^T = (-2x_1, -1) \neq 0$ for all $(x_1, x_2) \in \Re^2$. Function f is not quasiconvex and hence neither pseudoconvex nor strictly pseudoconvex since $f(2, -1) < f(0, 0)$ while $\nabla f(0, 0)^T((2, -1) - (0, 0)) > 0$.*

pseudoconvex	$\Rightarrow$	semi-pseudoconvex
$\Uparrow$		$\Uparrow$
strictly pseudoconvex	$\Rightarrow$	strictly semi-pseudoconvex

Figure 1: Relationships between pseudoconvexity and semi-pseudoconvexity

On the other hand, there is no inclusion relationship between quasiconvexity and semi-pseudoconvexity. The previous Example 1 provides a function which is semi-pseudoconvex but not quasiconvex, while the following Example 2 is about a function which is quasiconvex but not semi-pseudoconvex (see Figure 2).

Example 2 *Function $f(x) = x^3$ is quasiconvex, but not semi-pseudoconvex in $\Re$.*

quasiconvex	$\not\Leftarrow$ $\not\Rightarrow$	semi-pseudoconvex

Figure 2: No relationship between quasiconvexity and semi-pseudoconvexity

Example 1 provides a function having no critical point which is semi-pseudoconvex but neither quasiconvex nor pseudoconvex. The following Example 3 underlines that such a kind of behavior is not due to the absence of critical points.

Example 3 *Function $f(x_1, x_2) = -\frac{1}{8}(x^4 + 6x^3 - 7x^2 + 24x)e^{-y^2} + x^4$ is semi-pseudoconvex in $\Re^2$. It has one only critical point $(1, 0)$ which is the global minimum point. Function f is not quasiconvex, and hence neither pseudoconvex, since the restriction $f(-1, t)$ admits a maximum point at $t = 0$.*

The last example recalls that the class of functions, whose critical points are local and global minimum points, strictly contains the class of pseudoconvex functions. In this light, the following theorem proves that the semi-pseudoconvexity is both a necessary and sufficient condition guaranteeing a critical point to be a local and a global minimum point.

Theorem 1 *Let $f : X \to \Re$, $X \subseteq \Re^n$ open convex set, be a differentiable scalar function. The following properties are equivalent:*

i) f *is [strictly] semi-pseudoconvex in* X*;*

ii) for all $x_0 \in X$ *it holds:* $\nabla f(x_0) = 0$ *if and only if* x_0 *is a [the unique] global minimum point for* f *on* X*.*

Proof *i)*$\Rightarrow$*ii)* Let f be [strictly] semi-pseudoconvex in A and let $x_0 \in A$. If x_0 is a global minimum point then the openess of A implies that $\nabla f(x_0) = 0$. If $\nabla f(x_0) = 0$ then the semi-pseudoconvexity of f implies that $f(x) \geq f(x_0)$ $[f(x) > f(x_0)]$ for all $x \in A$ and hence x_0 is a [the unique] global minimum point.
ii)$\Rightarrow$*i)* Assume by contradiction that $\exists x_1, x_2 \in A$ such that $f(x_1) < f(x_2)$ $[f(x_1) \leq f(x_2)]$ and $\nabla f(x_2) = 0$. For *ii)* x_2 is a [the unique] global minimum point and this contradicts the inequality $f(x_1) < f(x_2)$ $[f(x_1) \leq f(x_2)]$. □

3 Semi-pseudoconvexity and generalized invexity

In this section we aim to compare the semi-pseudoconvexity with invexity and its generalizations. For the sake of completeness, let us first recall the definition of invex and pseudoinvex functions.

Definition 2 *Let* $f : X \to \Re$*,* $X \subseteq \Re^n$ *open and convex set, be a differentiable scalar function. Then,* f *is said to be:*

- *invex if the following property holds*

$$\exists \eta : (X \times X) \to \Re^n \text{ such that } f(x_1) - f(x_2) \geq \nabla f(x_2)^T \eta(x_1, x_2) \ \forall x_1, x_2 \in X \tag{5}$$

- *pseudoinvex if the following property holds*

$$\exists \eta : (X \times X) \to \Re^n \text{ such that } f(x_1) < f(x_2) \Longrightarrow \nabla f(x_2)^T \eta(x_1, x_2) < 0 \ \forall x_1, x_2 \in X \tag{6}$$

Nevertheless, taking into account the characterization of invex functions [7, 14] and the equivalence theorem between the class of invex and pseudo-invex functions (see for all [9]), the class of semi-pseudoconvex functions coincides with both the class of invex and pseudoinvex functions.

Starting from the concepts of invex and convex functions, further generalizations have been proposed in the last decades literature. New classes have been introduced in order to derive more general optimality conditions and to deepen the study of duality for both scalar and vector optimization problems (see for all [9, 10, 16] and reference therein). Here below we recall the following classes of generalized invex functions.

Definition 3 *Let* $f : X \to \Re$*,* $X \subseteq \Re^n$ *open and convex set, be a differentiable scalar function. Then,* f *is said to be:*

- F*-convex if there exists a functional* $F : (X \times X \times \Re^n) \to \Re$*, with* $F(x_1, x_2, 0) = 0 \ \forall x_1, x_2 \in X$*, such that*

$$f(x_1) \geq f(x_2) + F(x_1, x_2, \nabla f(x_2)) \quad \forall x_1, x_2 \in X \tag{7}$$

- F*-pseudoconvex if there exists a functional* $F : (X \times X \times \Re^n) \to \Re$*, with* $F(x_1, x_2, 0) = 0$ $\forall x_1, x_2 \in X$*, such that*

$$f(x_1) < f(x_2) \Longrightarrow F(x_1, x_2, \nabla f(x_2)) < 0 \quad \forall x_1, x_2 \in X \tag{8}$$

- *strong pseudoinvex if for every* $\vartheta : (X \times X) \to \Re^n$ *there exists* $\eta : (X \times X) \to \Re^n$ *and there exists* $\rho \in \Re$*,* $\rho > 0$*, such that*

$$f(x_1) < f(x_2) \Longrightarrow \nabla f(x_2)^T \eta(x_1, x_2) < -\rho \|\vartheta(x_1, x_2)\|^2 \quad \forall x_1, x_2 \in X \tag{9}$$

- *strong F-pseudoconvex if for every* $\vartheta : (X \times X) \to \Re^n$ *there exists a functional* $F : (X \times X \times \Re^n) \to \Re$, *with* $F(x_1, x_2, 0) = 0\ \forall x_1, x_2 \in X$, *and there exists* $\rho \in \Re$, $\rho > 0$, *such that*

$$f(x_1) < f(x_2) \implies F(x_1, x_2, \nabla f(x_2)) < -\rho\|\vartheta(x_1, x_2)\|^2 \quad \forall x_1, x_2 \in X \tag{10}$$

The F-convex functions have been introduced by Hanson and Mond in [12] and for this very reason they are also called Hanson and Mond functions. Craven and Glover [7] prove that this class of function is nothing but the class of invex function. Furthermore, Caprari [4] proves the equivalence between the strong F-convexity and the strong pseudoinvexity.

The aim of the next Theorem 2 is to go further by proving that F-convexity coincides with both F-pseudoconvexity and strong pseudoinvexity. In other words, the results in [4, 7, 14] are deepened on by pointing out that the class of semi-pseudoconvex functions (not involving any kind of parameter functions) coincides with all the classes of functions presented in Definition 2 and in Definition 3 (based on parameter functions). Deeply speaking, some of the definitions presented in the recent literature with the aim to generalize the concept of invexity do not provide actually any kind of generalization (along this line see also [4, 9]). In addiction to this, the use of parameter functions aimed to look for invexity generalizations is useless, since those class of functions coincides with the one of semi-pseudoconvex functions which do not involve any kind of parameter functions being related only to the behavior of the gradient of the function f.

For the sake of completeness, in the next Theorem 2 all of the characterizations discussed so far are explicitly proved.

Theorem 2 *Let* $f : X \to \Re$ *be a differentiable scalar function on the open convex set* $X \subseteq \Re^n$. *The following conditions are equivalent:*

i) f *is semi-pseudoconvex;*

ii) f *is invex;*

iii) f *is pseudoinvex;*

iv) f *is F-convex;*

v) f *is F-pseudoconvex;*

vi) f *is strong pseudoinvex;*

vii) f *is strong F-pseudoconvex;*

Proof *i)⇒ii)* Let us define the following functional:

$$\eta(x_1, x_2) = \begin{cases} \frac{f(x_1) - f(x_2)}{\|\nabla f(x_2)\|^2} \nabla f(x_2) & \text{if } \nabla f(x_2) \neq 0 \\ 0 & \text{if } \nabla f(x_2) = 0 \end{cases}$$

In the case $\nabla f(x_2) = 0$ condition *i)* implies $f(x_1) \geq f(x_2)$ and hence $f(x_1) \geq f(x_2) + \nabla f(x_2)^T \eta(x_1, x_2)$. In the case $\nabla f(x_2) \neq 0$ it is $\nabla f(x_2)^T \eta(x_1, x_2) = f(x_1) - f(x_2)$. The whole result is then proved.

ii)⇒iii) Assuming $f(x_1) < f(x_2)$ condition *ii)* yields $\nabla f(x_2)^T \eta(x_1, x_2) \leq f(x_1) - f(x_2) < 0$.

iii)⇒v) Follows from *iii)* assuming $F(x_1, x_2, \nabla f(x_2)) = \nabla f(x_2)^T \eta(x_1, x_2)$.

ii)⇒iv) Follows from *ii)* assuming $F(x_1, x_2, \nabla f(x_2)) = \nabla f(x_2)^T \eta(x_1, x_2)$.

iv)⇒v) Assuming $f(x_1) < f(x_2)$ condition *iv)* yields $F(x_1, x_2, \nabla f(x_2)) \leq f(x_1) - f(x_2) < 0$.

v)⇒i) Assuming $f(x_1) < f(x_2)$ condition *iv)* implies $F(x_1, x_2, \nabla f(x_2)) < 0$ so that $\nabla f(x_2) \neq 0$ being $F(x_1, x_2, 0) = 0\ \forall x_1, x_2 \in X$.

i)⇒vi) Let us define the following functional where ρ is fixed to any positive value while ϑ is not fixed:

$$\eta(x_1, x_2) = \begin{cases} \frac{f(x_1) - f(x_2) - \rho\|\vartheta(x_1, x_2)\|^2}{\|\nabla f(x_2)\|^2} \nabla f(x_2) & \text{if } \nabla f(x_2) \neq 0 \\ 0 & \text{if } \nabla f(x_2) = 0 \end{cases}$$

Assuming $f(x_1) < f(x_2)$ condition *i)* implies $\nabla f(x_2) \neq 0$. Hence, it is:

$$\nabla f(x_2)^T \eta(x_1, x_2) = f(x_1) - f(x_2) - \rho\|\vartheta(x_1, x_2)\|^2 < -\rho\|\vartheta(x_1, x_2)\|^2$$

The result is then proved.

vi)⇒vii) Follows from *vi)* assuming $F(x_1, x_2, \nabla f(x_2)) = \nabla f(x_2)^T \eta(x_1, x_2)$.

vii)⇒i) Assuming $f(x_1) < f(x_2)$ condition *vii)* implies:

$$F(x_1, x_2, \nabla f(x_2)) < -\rho\|\vartheta(x_1, x_2)\|^2 \leq 0$$

so that $\nabla f(x_2) \neq 0$ being $F(x_1, x_2, 0) = 0 \ \forall x_1, x_2 \in X$. □

Remark 1 *It is worth noticing that in the recent literature different concepts of invexity have been introduced by means of a different use of the parameter function $\eta(x_1, x_2)$. Deeply speaking, concepts stronger than the ones recalled in Definition 2 have been proposed by requiring the parameter function $\eta(x_1, x_2)$ to be "a priori" fixed [13]:*

- *given a function $\eta : (X \times X) \to \Re^n$, f is said to be η-invex if the following property holds*

$$f(x_1) - f(x_2) \geq \nabla f(x_2)^T \eta(x_1, x_2) \ \forall x_1, x_2 \in X$$

- *given a function $\eta : (X \times X) \to \Re^n$, f is said to be η-pseudoinvex if the following property holds*

$$f(x_1) < f(x_2) \Longrightarrow \nabla f(x_2)^T \eta(x_1, x_2) < 0 \ \forall x_1, x_2 \in X$$

These properties hold for a specific parameter function $\eta(x_1, x_2)$ and for this very reason the classes of η-invex and of η-pseudoinvex functions are strictly contained in the ones of invex and of pseudoinvex functions, respectively. As a consequences, the concept of semi-pseudoconvexity (based on no parameter functions) generalizes both η-invexity and η-pseudoinvexity (based on the parameter function $\eta(x_1, x_2)$).

4 Semi-pseudoconvexity and duality results

The aim of this section is to point out the use of semi-pseudoconvexity in stating duality results. Actually, the maximization nature of the primal problems we are going to study needs of the use of [strictly] semi-pseudoconcave functions, that are nothing but functions f such that $-f$ is [strictly] semi-pseudoconvex. In particular, the following class of problems is considered:

$$P : \begin{cases} C^*\text{-}\max \ f(x) \\ x \in S_P \end{cases}$$

where $S_P = \{x \in X : g(x) \in V, \ h(x) = 0\}$, $X \subseteq \Re^n$ is a convex open set, $f : X \to \Re^s$, $g : X \to \Re^m$ and $h : X \to \Re^p$ are Fréchet differentiable vector valued functions with $s \geq 1$ and $m, p \geq 0$, $C \subset \Re^s$ and $V \subset \Re^m$ are closed convex pointed cones with nonempty interior (that is to say convex pointed solid cones), C^* is a cone such that $C^* = C$ or $Int(C) \subseteq C^* \subseteq C \setminus \{0\}$.

Given the cone C^*, a feasible point $x_0 \in S_P$ is said to be a *local C^*-efficient point* if there exists a suitable neighbourhood I_{x_0} of x_0 such that:

$$\nexists y \in I_{x_0} \cap S_P, \ y \neq x_0, \ \text{ such that } f(y) \in f(x_0) + C^*$$

or, in other words, such that:

$$f(y) \notin f(x_0) + C^* \quad \forall y \in I_{x_0} \cap S_P, \ y \neq x_0$$

In particular, $x_0 \in S_P$ is said to be a *local efficient point* in the case $C^* = C \setminus \{0\}$, it is said to be a *local weak efficient point* in the case $C^* = Int(C)$, it is said to be a *local strict efficient point* in the case $C^* = C$. Obviously, the previous definitions will be *global* instead of just local in the case the corresponding property holds for $I_{x_0} = X$. Denoting with C^+ and V^+ the positive polar cones of the cones C and V, respectively, the following scalar Lagrangean type function $\mathcal{L}_\mathcal{S} : (A \times C^+ \times V^+ \times \Re^p) \to \Re$ can be defined:

$$\mathcal{L}_\mathcal{S}(x, \alpha_f, \alpha_g, \alpha_h) = \alpha_f^T f(x) + \alpha_g^T g(x) + \alpha_h^T h(x)$$

so that its gradient with respect to variable x results to be:

$$\nabla \mathcal{L}_\mathcal{S}(x, \alpha_f, \alpha_g, \alpha_h)^T = \alpha_f^T J_f(x) + \alpha_g^T J_g(x) + \alpha_h^T J_h(x)$$

The classical Fritz John necessary optimality conditions hold for problem P in the case $x_0 \in S_P$ is a local efficient point:

- $\exists(\alpha_f, \alpha_g, \alpha_h) \in (C^+ \times V^+ \times \Re^p)$, $(\alpha_f, \alpha_g, \alpha_h) \neq 0$, such that:

$$\alpha_g^T g(x_0) = 0 \quad \text{and} \quad \nabla \mathcal{L}_\mathcal{S}(x_0, \alpha_f, \alpha_g, \alpha_h) = 0$$

that is nothing but the Karush-Kuhn-Tucker conditions in the case a constraint qualification condition holds (and $\alpha_f \neq 0$ is guaranteed).

In the following, two different mixed type duals will be considered. In this light, it is worth recalling that mixed duality has been proposed in the literature in order to manage in an unifying framework both the Mond-Weir and the Wolfe duals. The following notations are needed:

- $\delta \in \{0, 1\}$ is a $0-1$ parameter;
- $\mathcal{J} = \{J_1, J_2, J_3, J_4\}$ is a partition of $\mathcal{P} = \{1, \ldots, p\}$;
- $h(x) = [h_1(x), h_2(x), h_3(x), h_4(x)]$ and $\alpha_h = (\alpha_{h_1}, \alpha_{h_2}, \alpha_{h_3}, \alpha_{h_4})$ are partitioned accordingly to $\mathcal{J}$;
- $\Delta(x, \alpha_g, \alpha_{h_2}, \alpha_{h_3}, \alpha_{h_4}) = \delta \alpha_g^T g(x) + \alpha_{h_2}^T h_2(x) + \alpha_{h_3}^T h_3(x) + \alpha_{h_4}^T h_4(x)$;

The two following functions can then be defined (notice that $c \in Int(C)$):

$$\begin{aligned}
\mathcal{F}_\mathcal{S}(x, \alpha_f, \alpha_g, \alpha_h) &= \mathcal{L}_\mathcal{S}(x, \alpha_f, \alpha_g, \alpha_h) - \Delta(x, \alpha_g, \alpha_{h_2}, \alpha_{h_3}, \alpha_{h_4}) \\
&= \alpha_f^T f(x) + (1-\delta)\alpha_g^T g(x) + \alpha_{h_1}^T h_1(x) \\
\mathcal{F}_\mathcal{V}(x, \alpha_f, \alpha_g, \alpha_h) &= \mathcal{L}_\mathcal{V}(x, \alpha_f, \alpha_g, \alpha_h) - \frac{c}{\alpha_f^T c} \Delta(x, \alpha_g, \alpha_{h_2}, \alpha_{h_3}, \alpha_{h_4}) \\
&= f(x) + \frac{c}{\alpha_f^T c}\left[(1-\delta)\alpha_g^T g(x) + \alpha_{h_1}^T h_1(x)\right]
\end{aligned}$$

Notice that $\mathcal{F}_\mathcal{S}(x, \alpha_f, \alpha_g, \alpha_h)$ is a scalar function while $\mathcal{F}_\mathcal{V}(x, \alpha_f, \alpha_g, \alpha_h)$ is a vector one. Notice also that if $\alpha_f \neq 0$ then $\mathcal{F}_\mathcal{S}(x, \alpha_f, \alpha_g, \alpha_h) = \alpha_f^T \mathcal{F}_\mathcal{V}(x, \alpha_f, \alpha_g, \alpha_h)$.

The following mixed type dual problems $D_\mathcal{S}$ and $D_\mathcal{V}$ will be referred to as the scalar and the vector mixed duals of P, respectively

$$D_\mathcal{S} : \left\{ \begin{array}{c} \min\ \mathcal{F}_\mathcal{S}(x, \alpha_f, \alpha_g, \alpha_h) \\ \nabla \mathcal{L}_\mathcal{S}(x, \alpha_f, \alpha_g, \alpha_h) = 0 \\ (x, \alpha_f, \alpha_g, \alpha_h) \in S_D \end{array} \right. \quad , \quad D_\mathcal{V} : \left\{ \begin{array}{c} C\_\min\ \mathcal{F}_\mathcal{V}(x, \alpha_f, \alpha_g, \alpha_h) \\ \nabla \mathcal{L}_\mathcal{S}(x, \alpha_f, \alpha_g, \alpha_h) = 0 \\ \alpha_f \neq 0 \\ (x, \alpha_f, \alpha_g, \alpha_h) \in S_D \end{array} \right.$$

where:

$$S_D : \left\{ \begin{array}{c} (x, \alpha_f, \alpha_g, \alpha_h) \text{ such that} \\ \delta\alpha_g^T g(x) + \alpha_{h_2}^T h_2(x) \leq 0 \\ \alpha_{h_3}^T h_3(x) = 0, \ \alpha_{h_4}^T h_4(x) \leq 0 \\ x \in A, \ \alpha_f \in C^+, \ \alpha_g \in V^+, \ \alpha_h \in \Re^p \end{array} \right.$$

Duality results concerning these two dual problems (a scalar and a vector one) based on the use of the semi-pseudoconcave properties have been deeply studied in [3] pointing out the provided extensions with respect to the current literature on generalized invexity. For the sake of completeness, the following results are recalled, suggesting the reader to refer to [3] for the proofs.

Theorem 3 *Let us consider the primal problem P and the dual problem $D_{\mathcal{S}}$. Assume also that function $\mathcal{L}_{\mathcal{S}}(x, \alpha_f, \alpha_g, \alpha_h)$ is semi-pseudoconcave with respect to variable x for all multipliers $(\alpha_f, \alpha_g, \alpha_h) \in (C^+ \times V^+ \times \Re^p)$. Then, the following results hold:*

i) [weak duality] $\forall x_1 \in S_P$ and $\forall (x_2, \alpha_f, \alpha_g, \alpha_h) \in S_D$ such that $\nabla \mathcal{L}_{\mathcal{S}}(x_2, \alpha_f, \alpha_g, \alpha_h) = 0$ it is $\alpha_f^T f(x_1) \leq \mathcal{F}_{\mathcal{S}}(x_2, \alpha_f, \alpha_g, \alpha_h)$

ii) If $x_0 \in S_P$ and $(x_0, \alpha_f, \alpha_g, \alpha_h)$ is feasible for $D_{\mathcal{S}}$, with $(1-\delta)\alpha_g^T g(x_0) = 0$, then $\alpha_f^T f(x_0) = \mathcal{F}_{\mathcal{S}}(x_0, \alpha_f, \alpha_g, \alpha_h)$, $(x_0, \alpha_f, \alpha_g, \alpha_h)$ is a global minimum for $D_{\mathcal{S}}$, x_0 is a global weak efficient point for P in the case $\alpha_f \neq 0$

iii) [strong duality] for all $x_0 \in S_P$ global efficient point for P $\exists \alpha_f \in C^+$, $\exists \alpha_g \in V^+$, $\exists \alpha_h \in \Re^p$, $(\alpha_f, \alpha_g, \alpha_h) \neq 0$, such that $\alpha_f^T f(x_0) = \mathcal{F}_{\mathcal{S}}(x_0, \alpha_f, \alpha_g, \alpha_h)$ and $(x_0, \alpha_f, \alpha_g, \alpha_h)$ is a global minimum point for $D_{\mathcal{S}}$

iv) for all $x_1 \in S_P$ global efficient point for P and for all $(x_2, \alpha_f, \alpha_g, \alpha_h)$ global minimum point for $D_{\mathcal{S}}$ $\exists \widehat{\alpha}_f \in C^+$ such that $\widehat{\alpha}_f^T f(x_1) = \mathcal{F}_{\mathcal{S}}(x_2, \alpha_f, \alpha_g, \alpha_h)$

Theorem 4 *Let us consider the primal problem P and the dual problem $D_{\mathcal{V}}$, and let C^* be a cone such that $Int(C) \subseteq C^* \subseteq C \setminus \{0\}$. Assume also that at least one of the following conditions (C_1) and (C_2) is verified for all multipliers $(\alpha_f, \alpha_g, \alpha_h) \in (C^+ \times V^+ \times \Re^p)$, $\alpha_f \neq 0$:*

(C_1) function $\mathcal{L}_{\mathcal{S}}(x, \alpha_f, \alpha_g, \alpha_h)$ is strictly semi-pseudoconcave with respect to x;

(C_2) $C^* = Int(C)$ and $\mathcal{L}_{\mathcal{S}}(x, \alpha_f, \alpha_g, \alpha_h)$ is semi-pseudoconcave with respect to x.

Then, the following results hold:

i) [weak duality] $\forall x_1 \in S_P$ and $\forall (x_2, \alpha_f, \alpha_g, \alpha_h) \in S_D$ such that $\nabla \mathcal{L}_{\mathcal{S}}(x_2, \alpha_f, \alpha_g, \alpha_h) = 0$, $\alpha_f \neq 0$, it is $f(x_1) \notin \mathcal{F}_{\mathcal{V}}(x_2, \alpha_f, \alpha_g, \alpha_h) + C^*$

ii) If $x_0 \in S_P$ and $(x_0, \alpha_f, \alpha_g, \alpha_h)$ is feasible for $D_{\mathcal{V}}$, with $(1-\delta)\alpha_g^T g(x_0) = 0$, then $f(x_0) = \mathcal{F}_{\mathcal{V}}(x_0, \alpha_f, \alpha_g, \alpha_h)$, $(x_0, \alpha_f, \alpha_g, \alpha_h)$ is a global C^*-minimum point for $D_{\mathcal{V}}$, x_0 is a global C^*-efficient point for P

Theorem 5 *Let us consider the primal problem P and the dual problem $D_{\mathcal{V}}$, and let C^* be a cone such that $Int(C) \subseteq C^* \subseteq C \setminus \{0\}$. Assume also that function $\mathcal{L}_{\mathcal{S}}(x, \alpha_f, \alpha_g, \alpha_h)$ is strictly semi-pseudoconcave with respect to x for all multipliers $(\alpha_f, \alpha_g, \alpha_h) \in (C^+ \times V^+ \times \Re^p)$, $\alpha_f \neq 0$, and that a constraint qualification condition holds for problem P. Then, the following results hold:*

i) [strong duality] for all $x_0 \in S_P$ global efficient point for P $\exists \alpha_f \in C^+ \setminus \{0\}$, $\exists \alpha_g \in V^+$, $\exists \alpha_h \in \Re^p$, such that $f(x_0) = \mathcal{F}_{\mathcal{V}}(x_0, \alpha_f, \alpha_g, \alpha_h)$ and $(x_0, \alpha_f, \alpha_g, \alpha_h)$ is a global $(C \setminus \{0\})$-minimum point for $D_{\mathcal{V}}$

ii) for all $x_1 \in S_P$ global efficient point for P and for all $(x_2, \alpha_f, \alpha_g, \alpha_h)$ global $(C \setminus \{0\})$-minimum point for $D_{\mathcal{V}}$ it results $f(x_1) - \mathcal{F}_{\mathcal{V}}(x_2, \alpha_f, \alpha_g, \alpha_h) \notin (C \cup -C) \setminus \{0\}$

5 Conclusions

In this paper the class of semi-pseudoconvex functions is defined and studied, pointing out the relationships existing with the classes of quasiconvex and pseudoconvex functions. This generalized convexity concept is based on no parameter function and can be usefully applied in the study of optimality conditions. It is also proved that semi-pseudoconvexity is equivalent to some of the generalized invexity concepts introduced in the recent literature with the aim of generalizing invexity. In particular, it is pointed out that the use of parameter functions aimed to determine convexity generalizations may be useless and that some of the "generalized" invex properties proposed in the recent literature actually provide no generalization at all. This research topic can be furthermore deepened on by extending the given results to the multicriteria case or to the nondifferentiable case (that is using Lipschitzian functions).

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Anno: **2000**

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Anno: **2001**

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Anno: **2002**

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Anno: **2003**

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Anno: **2004**

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Anno: **2005**

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Anno: **2006**

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Anno: **2007**

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Anno: **2008**

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Anno: **2009**

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principali definizioni, proprietà
e applicazioni

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Anno: **2010**

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Anno: **2011**

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