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A note on scalar "generalized" invexity

Riccardo Cambini and Laura Carosi*

Abstract

The aim of this paper is to study the notion of invexity and of some "generalized" invexity properties for both smooth and nonsmooth scalar functions. In particular, it is studied how far "generalized" invex properties are different from invexity and it is established whether or not the use of more and more parameters and functionals in the definitions is really effective and helpful. As a conclusion, it is proved that several "generalized" invexity properties are actually equivalent to invexity. In other words, the introduction of parameters in defining "generalized" invexity properties does not yield "a priori" any kind of generalization.

Key words: Generalized convexity, generalized invexity.
AMS - 2010 Math. Subj. Class. 90C26, 90C30, 90C46.

1 Introduction

As it is very well known, invexity guarantees that critical points are global minimum points, Fritz-John conditions are sufficient optimality conditions and weak and strong duality results hold. In order to get larger and larger classes of functions which verify those nice properties, a huge amount of research papers provides many different generalizations of invexity. Even an hasty and superficial reading of the most recent contributions tells us that many different definitions of generalized invex functions are obtained by weakening the differentiable assumption and by introducing more and more parameters and/or functionals which are required to verify some nice conditions (see for example [8, 17] and reference therein). Unfortunately, the proposed definitions are not often so easy to be verified and in many cases it is impossible to find examples guaranteeing that these classes of functions are true generalizations of some other existing classes of generalized convex functions. Therefore two main questions arise: are all the introduced parameters really useful? Are these classes really different? Some other papers appeared in the literature sharing the same critical view, such as the ones by Craven and Glover [7], by Caprary [4] and by Mititelu [14], where equivalences among some generalized invexity properties are proved ([1]). Moreover, Zălinescu in [18] offers a critical review of some generalized invexity concepts and shows that the use of parameters in some generalized invexity definitions is not always correct from a mathematical point of view.

Our paper aims to strenghten the results by Caprari [4] and to go further in showing that various "generalized" invexity properties are not actually true generalizations of the class of scalar invex functions. This is proved both for smooth and nonsmooth functions. More precisely, we first describe several different conditions and we prove the equivalence among all of them (see Theorem 1 and Corollary 1). At a first sight, the introduced properties do not seem related with invexity and "generalized" invexity, but as soon as we properly specify them we are able to recognize many of the definition proposed in the literature of generalized invexity. For the sake of clearness, let us consider a function $f : X \to \Re$, $X \subseteq \Re^n$, and a set valued function $K : X \to \Re^n$ such that $K(x)$

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[1] See also Craven [6] for generalized invex vector function

is a nonempty convex and compact set for all $x \in X$. In Theorem 1 function f is said to verify condition *B1)* if there exists $\eta : (X \times X) \to \Re^n$ such that:

$$f(x_1) - f(x_2) \geq \xi^T \eta(x_1, x_2) \quad \forall \xi \in K(x_2),\ \forall x_1, x_2 \in X.$$

If we assume f to be differentiable and we take $K(x) = \{\nabla f(x)\}\ \forall x \in X$, condition *B1)* is nothing but invexity. In a similar way, starting from the other conditions presented in Theorem 1 and Corollary 1 we are able, for example, to recover the definition of pseudoinvexity, F-convexity, F-pseudoconvexity, strong pseudoinvexity, strong F-pseudoconvexity, up to the notion of $(\Im, b, \phi, \rho, \theta)$-univexity used by Zalmai in [19] for n-set functions.
Therefore, by proving the equivalence of all the properties given in Thereom 1 and Corollary 1 we show that all the above mentioned generalized invexity properties coincide with invexity. Thanks to their very general formulations, Theorem 1 and Corollary 1 allow to obtain the same equivalence results even in the nondifferentiable case. Moreover, Theorem 1 and Corollary 1 allow to determine other kinds of generalized invexity properties by means of suitable specifications of the set valued function K and of the other parameters.
Furthermore, condition *A)* in Theorem 1 does not use any kind of parameters or functionals and it is in turn equivalent to the other listed properties. As a direct consequence of Theorem 1 and Corollary 1 we come at the conclusion that the use of parameters and or functionals in defining "generalized" invexity properties does not yield "a priori" any kind of generalization.
The paper is organized as follows. In Section 2 several properties are introduced and their equivalence is proved. These equivalences results are used in Section 3 in order to obtain the equivalence among some "generalized" invexity properties for both smooth and non-smooth scalar functions.

2 Preliminary equivalences

Various general equivalences are proved in the following Theorem 1 and Corollary 1. These results will be the key tool for studying the relationships existing among the various generalized invexity-type properties proposed in the literature.

Theorem 1 *Let $f : X \to \Re$, $X \subseteq \Re^n$, be a scalar function and let $K : X \to \Re^n$ be a set valued function such that $K(x)$ is a nonempty convex and compact set for all $x \in X$. The following properties A), B1)-B6), C1)-C6), are all equivalent:*

A) it holds:

$$f(x_1) < f(x_2) \Longrightarrow \xi \neq 0 \quad \forall \xi \in K(x_2),\ \forall x_1, x_2 \in X$$

B1) there exists $\eta : (X \times X) \to \Re^n$ such that:

$$f(x_1) - f(x_2) \geq \xi^T \eta(x_1, x_2) \quad \forall \xi \in K(x_2),\ \forall x_1, x_2 \in X$$

B2) there exists $\eta : (X \times X) \to \Re^n$ such that:

$$f(x_1) < f(x_2) \Longrightarrow \xi^T \eta(x_1, x_2) < 0 \quad \forall \xi \in K(x_2),\ \forall x_1, x_2 \in X$$

B3) there exists $\eta : (X \times X) \to \Re^n$, there exists $\vartheta : (X \times X) \to \Re^n$, there exists $\rho \in \Re$, $\rho > 0$, such that:

$$f(x_1) - f(x_2) \geq \xi^T \eta(x_1, x_2) + \rho \|\vartheta(x_1, x_2)\|^2 \quad \forall \xi \in K(x_2),\ \forall x_1, x_2 \in X$$

B4) there exists $\eta : (X \times X) \to \Re^n$, there exists $\vartheta : (X \times X) \to \Re^n$, there exists $\rho \in \Re$, $\rho > 0$, such that:

$$f(x_1) < f(x_2) \Longrightarrow \xi^T \eta(x_1, x_2) < -\rho \|\vartheta(x_1, x_2)\|^2 \quad \forall \xi \in K(x_2),\ \forall x_1, x_2 \in X$$

B5) for every $\vartheta : (X \times X) \to \Re^n$ *there exists* $\eta : (X \times X) \to \Re^n$ *and there exists* $\rho \in \Re$, $\rho > 0$, *such that:*

$$f(x_1) - f(x_2) \geq \xi^T \eta(x_1, x_2) + \rho \|\vartheta(x_1, x_2)\|^2 \quad \forall \xi \in K(x_2),\ \forall x_1, x_2 \in X$$

B6) for every $\vartheta : (X \times X) \to \Re^n$ *there exists* $\eta : (X \times X) \to \Re^n$ *and there exists* $\rho \in \Re$, $\rho > 0$, *such that:*

$$f(x_1) < f(x_2) \Longrightarrow \xi^T \eta(x_1, x_2) < -\rho \|\vartheta(x_1, x_2)\|^2 \quad \forall \xi \in K(x_2),\ \forall x_1, x_2 \in X$$

C1) there exists $F : (X \times X \times \Re^n) \to \Re$, *with* $F(x_1, x_2, 0) \geq 0\ \forall x_1, x_2 \in X$, *there exists* $\psi : \Re^2 \to \Re$, *with* $\psi(y_1, y_2) < 0\ \forall y_1, y_2 \in \Re$ *such that* $y_1 < y_2$, *there exists* $b : (X \times X) \to \Re$, *with* $b(x_1, x_2) > 0\ \forall x_1, x_2 \in X$, *such that:*

$$\psi(f(x_1), f(x_2)) \geq F(x_1, x_2, b(x_1, x_2)\xi) \quad \forall \xi \in K(x_2),\ \forall x_1, x_2 \in X$$

C2) there exists $F : (X \times X \times \Re^n) \to \Re$, *with* $F(x_1, x_2, 0) \geq 0\ \forall x_1, x_2 \in X$, *there exists* $\psi : \Re^2 \to \Re$, *with* $\psi(y_1, y_2) < 0\ \forall y_1, y_2 \in \Re$ *such that* $y_1 < y_2$, *there exists* $b : (X \times X) \to \Re$, *with* $b(x_1, x_2) > 0\ \forall x_1, x_2 \in X$, *such that:*

$$\psi(f(x_1), f(x_2)) < 0 \Longrightarrow F(x_1, x_2, b(x_1, x_2)\xi) < 0 \quad \forall \xi \in K(x_2),\ \forall x_1, x_2 \in X$$

C3) there exists $F : (X \times X \times \Re^n) \to \Re$, *with* $F(x_1, x_2, 0) \geq 0\ \forall x_1, x_2 \in X$, *there exists* $\psi : \Re^2 \to \Re$, *with* $\psi(y_1, y_2) < 0\ \forall y_1, y_2 \in \Re$ *such that* $y_1 < y_2$, *there exists* $\vartheta : (X \times X) \to \Re^n$, *there exists* $\rho \in \Re$, $\rho > 0$, *there exists* $b : (X \times X) \to \Re$, *with* $b(x_1, x_2) > 0\ \forall x_1, x_2 \in X$, *such that:*

$$\psi(f(x_1), f(x_2)) \geq F(x_1, x_2, b(x_1, x_2)\xi) + \rho \|\vartheta(x_1, x_2)\|^2 \quad \forall \xi \in K(x_2),\ \forall x_1, x_2 \in X$$

C4) there exists $F : (X \times X \times \Re^n) \to \Re$, *with* $F(x_1, x_2, 0) \geq 0\ \forall x_1, x_2 \in X$, *there exists* $\psi : \Re^2 \to \Re$, *with* $\psi(y_1, y_2) < 0\ \forall y_1, y_2 \in \Re$ *such that* $y_1 < y_2$, *there exists* $\vartheta : (X \times X) \to \Re^n$, *there exists* $\rho \in \Re$, $\rho > 0$, *there exists* $b : (X \times X) \to \Re$, *with* $b(x_1, x_2) > 0\ \forall x_1, x_2 \in X$, *such that:*

$$\psi(f(x_1), f(x_2)) < 0 \Longrightarrow F(x_1, x_2, b(x_1, x_2)\xi) < -\rho \|\vartheta(x_1, x_2)\|^2 \quad \forall \xi \in K(x_2),\ \forall x_1, x_2 \in X$$

C5) for every $\vartheta : (X \times X) \to \Re^n$ *there exists* $F : (X \times X \times \Re^n) \to \Re$, *with* $F(x_1, x_2, 0) \geq 0$ $\forall x_1, x_2 \in X$, *there exists* $\psi : \Re^2 \to \Re$, *with* $\psi(y_1, y_2) < 0\ \forall y_1, y_2 \in \Re$ *such that* $y_1 < y_2$, *and there exists* $\rho \in \Re$, $\rho > 0$, *there exists* $b : (X \times X) \to \Re$, *with* $b(x_1, x_2) > 0\ \forall x_1, x_2 \in X$, *such that:*

$$\psi(f(x_1), f(x_2)) \geq F(x_1, x_2, b(x_1, x_2)\xi) + \rho \|\vartheta(x_1, x_2)\|^2 \quad \forall \xi \in K(x_2),\ \forall x_1, x_2 \in X$$

C6) for every $\vartheta : (X \times X) \to \Re^n$ *there exists* $F : (X \times X \times \Re^n) \to \Re$, *with* $F(x_1, x_2, 0) \geq 0$ $\forall x_1, x_2 \in X$, *there exists* $\psi : \Re^2 \to \Re$, *with* $\psi(y_1, y_2) < 0\ \forall y_1, y_2 \in \Re$ *such that* $y_1 < y_2$, *and there exists* $\rho \in \Re$, $\rho > 0$, *there exists* $b : (X \times X) \to \Re$, *with* $b(x_1, x_2) > 0\ \forall x_1, x_2 \in X$, *such that:*

$$\psi(f(x_1), f(x_2)) < 0 \Longrightarrow F(x_1, x_2, b(x_1, x_2)\xi) < -\rho \|\vartheta(x_1, x_2)\|^2 \quad \forall \xi \in K(x_2),\ \forall x_1, x_2 \in X$$

Proof It is first worth recalling that being $K(x)$ a nonempty convex and compact set then there exists an unique element $\bar{\xi}(x) \in K(x)$ such that $\|\bar{\xi}(x)\| \leq \|\xi\|\ \forall \xi \in K(x)$, and that it results $\bar{\xi}(x)^T \bar{\xi}(x) \leq \xi^T \bar{\xi}(x)\ \forall \xi \in K(x)$.

A)⇒B1). Let us define the following functional:

$$\eta(x_1,x_2) = \begin{cases} \frac{f(x_1)-f(x_2)}{\bar{\xi}(x)^T\bar{\xi}(x)}\,\bar{\xi}(x) & \text{if } 0 \notin K(x_2),\ f(x_1) < f(x_2) \\ 0 & \text{otherwise} \end{cases}$$

In the case $0 \notin K(x_2)$, $f(x_1) < f(x_2)$, for all $\xi \in K(x_2)$ it is:

$$\xi^T\eta(x_1,x_2) = (f(x_1) - f(x_2))\,\frac{\xi^T\bar{\xi}(x)}{\bar{\xi}(x)^T\bar{\xi}(x)} \leq f(x_1) - f(x_2)$$

since $(f(x_1) - f(x_2)) < 0$ and $\frac{\xi^T\bar{\xi}(x)}{\bar{\xi}(x)^T\bar{\xi}(x)} \geq 1$. In the case $0 \in K(x_2)$ property *A0)* implies $f(x_1) \geq f(x_2)$ and hence for all $\xi \in K(x_2)$ it is $\xi^T\eta(x_1,x_2) = 0 \leq f(x_1) - f(x_2)$; the same happens also in the case $0 \notin K(x_2)$, $f(x_1) \geq f(x_2)$. The whole result is then proved.

B1)⇒B2), B3)⇒B4), B5)⇒B6). Follows trivially just assuming $f(x_1) < f(x_2)$.

B1)⇒B3), B2)⇒B4). Just choose $\vartheta(x_1,x_2) = 0\ \forall x_1,x_2 \in X$.

A)⇒B5). Let us define the following functional where ρ is fixed to any positive value while ϑ is not fixed:

$$\eta(x_1,x_2) = \begin{cases} \frac{f(x_1)-f(x_2)-\rho\|\vartheta(x_1,x_2)\|^2}{\bar{\xi}(x)^T\bar{\xi}(x)}\,\bar{\xi}(x) & \text{if } 0 \notin K(x_2),\ f(x_1) < f(x_2) \\ \frac{-\rho\|\vartheta(x_1,x_2)\|^2}{\bar{\xi}(x)^T\bar{\xi}(x)}\,\bar{\xi}(x) & \text{otherwise} \end{cases}$$

In the case $0 \notin K(x_2)$, $f(x_1) < f(x_2)$, for all $\xi \in K(x_2)$ it is:

$$\begin{aligned} \xi^T\eta(x_1,x_2) &= \left(f(x_1) - f(x_2) - \rho\|\vartheta(x_1,x_2)\|^2\right)\frac{\xi^T\bar{\xi}(x)}{\bar{\xi}(x)^T\bar{\xi}(x)} \\ &\leq f(x_1) - f(x_2) - \rho\|\vartheta(x_1,x_2)\|^2 \end{aligned}$$

since $\left(f(x_1) - f(x_2) - \rho\|\vartheta(x_1,x_2)\|^2\right) \leq 0$ and $\frac{\xi^T\bar{\xi}(x)}{\bar{\xi}(x)^T\bar{\xi}(x)} \geq 1$. In the case $0 \in K(x_2)$ property *A0)* implies $f(x_1) \geq f(x_2)$ and hence for all $\xi \in K(x_2)$ it is

$$\xi^T\eta(x_1,x_2) + \rho\|\vartheta(x_1,x_2)\|^2 = \rho\|\vartheta(x_1,x_2)\|^2\left(1 - \frac{\xi^T\bar{\xi}(x)}{\bar{\xi}(x)^T\bar{\xi}(x)}\right) \leq 0 \leq f(x_1) - f(x_2)$$

since $\rho\|\vartheta(x_1,x_2)\|^2 \geq 0$ and $\frac{\xi^T\bar{\xi}(x)}{\bar{\xi}(x)^T\bar{\xi}(x)} \geq 1$; the same happens also in the case $0 \notin K(x_2)$, $f(x_1) \geq f(x_2)$. The whole result is then proved.

Bi)⇒Ci), $i \in \{1,\ldots,6\}$. Just choose $F(x_1,x_2,\xi) = \xi^T\eta(x_1,x_2)$, $\psi(y_1,y_2) = y_1 - y_2$, $b(x_1,x_2) = 1$.

C1)⇒C2), C3)⇒C4), C5)⇒C6). Follows trivially just assuming $\psi(f(x_1),f(x_2)) < 0$.

C1)⇒C3), C2)⇒C4). Just choose $\vartheta(x_1,x_2) = 0\ \forall x_1,x_2 \in X$.

C4)⇒A), C6)⇒A). Assuming $f(x_1) < f(x_2)$ it follows that $\psi(f(x_1),f(x_2)) < 0$ and hence for all $\xi \in K(x_2)$ it is:

$$F(x_1,x_2,b(x_1,x_2)\xi) < -\rho\|\vartheta(x_1,x_2)\|^2 \leq 0$$

Being $F(x_1,x_2,b(x_1,x_2)\xi) < 0$ it follows $b(x_1,x_2)\xi \neq 0$ so that $\xi \neq 0$ and the result is proved. □

In other words, the previous theorem shows that the use of parameter functionals in properties *B1)-B6)* and *C1)-C6)* is useless since they are equivalent to property *A)* which has no parameter functionals at all.

It is worth noticing that many other properties can be obtained from *C1)-C6)* by properly fixing some of the parameters. It is important to point out that the properties obtained in this way are not more restrictive than the original ones, they are actually equivalent to *A)*, *B1)-B6)* and *C1)-C6)*, as it is stated in the following corollary.

Corollary 1 *Let* $f : X \to \Re$, $X \subseteq \Re^n$, *be a scalar function and let* $K : X \to \Re^n$ *be a set valued function such that* $K(x)$ *is a nonempty convex and compact set for all* $x \in X$. *Let* P *be any property obtained from C1)-C6) in Theorem 1 by assuming one or more of the following conditions:*

- *i)* $\psi(f(x_1), f(x_2)) = f(x_1) - f(x_2)$,
- *ii)* $\psi(f(x_1), f(x_2)) = \varphi(f(x_1) - f(x_2))$ *for a suitable* $\varphi : \Re \to \Re$, *with* $\varphi(y) < 0 \ \forall y < 0$,
- *iii)* $F(x_1, x_2, \xi) = \xi^T \eta(x_1, x_2)$ *for a suitable* $\eta : (X \times X) \to \Re^n$,
- *iv)* $b(x_1, x_2) = 1$.

Then, P is equivalent to properties A), B1)-B6), C1)-C6), in Theorem 1.

Proof First note that conditions *i)-iv)* verify the assumptions of properties *C1)-C6)* in Theorem 1. Let P be obtained from property *Ci)*, with $i \in \{1, \ldots, 6\}$, by assuming one or more of conditions *i)-iv)*. The result then follows from Theorem 1 by noticing that property *Bi)* implies P and that property P itself implies *Ci)*. □

3 "Generalized" invexity for scalar functions

The equivalences discussed in Section 2 seem not necessarily related to invexity and optimality conditions. Neverthless, some "generalized" invexity properties for scalar functions can be recognized in Theorem 1 and Corollary 1 by properly specifying the set valued function K and the other parameters.

3.1 The differentiable case

The aim of this subsection is to point out the use of Theorem 1 and Corollary 1 in studying "generalized" invexity properties for differentiable scalar functions. In particular, f is assumed to be differentiable and $K(x) = \{\nabla f(x)\}$, so that $\xi \in K(x_2)$ is replaced by $\nabla f(x_2)$. Under such assumptions, it is very easy to recognize in property *B1)* the very well known concept of invexity, in property *B2)* the concept of pseudoinvexity and in property *A)* the semi-pseudoconvexity concept considered in [3]. For the sake of completeness, let us explicitly recall that a differentiable scalar function $f : X \to \Re$, with $X \subseteq \Re^n$ open convex set, is said to be :

- *invex* in X if there exists a function $\eta : (X \times X) \to \Re^n$ such that for all $x_1, x_2 \in X$ it holds:
$$f(x_1) - f(x_2) \geq \nabla f(x_2)^T \eta(x_1, x_2)$$
- *pseudoinvex* in X if there exists a function $\eta : (X \times X) \to \Re^n$ such that for all $x_1, x_2 \in X$ it holds:
$$f(x_1) < f(x_2) \quad \Rightarrow \quad \nabla f(x_2)^T \eta(x_1, x_2) < 0$$
- *semi-pseudoconvex* in X if for all $x_1, x_2 \in X$ it holds:
$$f(x_1) < f(x_2) \quad \Rightarrow \quad \nabla f(x_2) \neq 0$$

Even if no kind of parameter is involved in the definition of semi-pseudoconvexity, Theorem 1 shows that this property is actually equivalent to both invexity and pseudoinvexity. For the sake of completeness, it is worth recalling that the equivalence between the semi-pseudoconvexity and the global optimality of critical points have been shown in [3], and that the equivalence between invexity, pseudoinvexity and the global optimality of critical points have been given by Craven and Glover in [7] (2).

Starting from the concepts of invex and convex functions, many generalizations have been proposed in the last decades literature. New classes have been introduced in order to derive more general optimality conditions and to deepen on the study of duality for both scalar and vector optimization problems (see for all [8, 9, 17] and reference therein). It is impossible to take into account of all the various proposed definitions, hence for the sake of convenience just the most used will be considered in the rest of this paper. These definitions share the same approach, that is the use of parameters and functionals aimed to weaken the invexity property. As an example, just take a look at the paper by Zalmai [19] who proposes for n-set functions the notions of $(F, b, \varphi, \rho, \vartheta)$-univexity and $(F, b, \varphi, \rho, \vartheta)$-pseudounivexity (3), where four functions and one real value are used as parameters. For the sake of completeness, here below we recall that a differentiable scalar function $f : X \rightarrow \Re$, with $X \subseteq \Re^n$ open convex set, is said to be :

- *F-convex* if there exists a functional $F : (X \times X \times \Re^n) \rightarrow \Re$, with $F(x_1, x_2, 0) = 0 \; \forall x_1, x_2 \in X$, such that
$$f(x_1) \geq f(x_2) + F(x_1, x_2, \nabla f(x_2)) \quad \forall x_1, x_2 \in X$$
- *F-pseudoconvex* if there exists a functional $F : (X \times X \times \Re^n) \rightarrow \Re$, with $F(x_1, x_2, 0) = 0$ $\forall x_1, x_2 \in X$, such that
$$f(x_1) < f(x_2) \implies F(x_1, x_2, \nabla f(x_2)) < 0 \quad \forall x_1, x_2 \in X$$
- *strong pseudoinvex* if for every $\vartheta : (X \times X) \rightarrow \Re^n$ there exists $\eta : (X \times X) \rightarrow \Re^n$ and there exists $\rho \in \Re$, $\rho > 0$, such that
$$f(x_1) < f(x_2) \implies \nabla f(x_2)^T \eta(x_1, x_2) < -\rho \|\vartheta(x_1, x_2)\|^2 \quad \forall x_1, x_2 \in X$$
- *strong F-pseudoconvex* if for every $\vartheta : (X \times X) \rightarrow \Re^n$ there exists a functional $F : (X \times X \times \Re^n) \rightarrow \Re$, with $F(x_1, x_2, 0) \geq 0 \; \forall x_1, x_2 \in X$, and there exists $\rho \in \Re$, $\rho > 0$, such that
$$f(x_1) < f(x_2) \implies F(x_1, x_2, \nabla f(x_2)) < -\rho \|\vartheta(x_1, x_2)\|^2 \quad \forall x_1, x_2 \in X$$
- *$(F, b, \varphi, \rho, \vartheta)$-univex* if there exists $F : (X \times X \times \Re^n) \rightarrow \Re$, with $F(x_1, x_2, 0) \geq 0 \; \forall x_1, x_2 \in X$, there exists $\varphi : \Re \rightarrow \Re$, with $\varphi(z) < 0 \; \forall z \in \Re$ such that $z < 0$, there exists $b : (X \times X) \rightarrow \Re$, with $b(x_1, x_2) > 0 \; \forall x_1, x_2 \in X$, such that:
$$\varphi(f(x_1) - f(x_2)) \geq F(x_1, x_2, b(x_1, x_2) \nabla f(x_2)) \quad \forall x_1, x_2 \in X$$
- *$(F, b, \varphi, \rho, \vartheta)$-pseudounivex* if there exists $F : (X \times X \times \Re^n) \rightarrow \Re$, with $F(x_1, x_2, 0) \geq 0$ $\forall x_1, x_2 \in X$, there exists $\varphi : \Re \rightarrow \Re$, with $\varphi(z) < 0 \; \forall z \in \Re$ such that $z < 0$, there exists $b : (X \times X) \rightarrow \Re$, with $b(x_1, x_2) > 0 \; \forall x_1, x_2 \in X$, such that:
$$\varphi(f(x_1) - f(x_2)) < 0 \implies F(x_1, x_2, b(x_1, x_2) \nabla f(x_2)) < 0 \quad \forall x_1, x_2 \in X$$
- *strong $(F, b, \varphi, \rho, \vartheta)$-univex* if there exists $F : (X \times X \times \Re^n) \rightarrow \Re$, with $F(x_1, x_2, 0) \geq 0$ $\forall x_1, x_2 \in X$, there exists $\varphi : \Re \rightarrow \Re$, with $\varphi(z) < 0 \; \forall z \in \Re$ such that $z < 0$, there exists $\vartheta : (X \times X) \rightarrow \Re^n$, there exists $\rho \in \Re$, $\rho > 0$, there exists $b : (X \times X) \rightarrow \Re$, with $b(x_1, x_2) > 0$ $\forall x_1, x_2 \in X$, such that:
$$\varphi(f(x_1) - f(x_2)) \geq F(x_1, x_2, b(x_1, x_2) \nabla f(x_2)) + \rho \|\vartheta(x_1, x_2)\|^2 \quad \forall x_1, x_2 \in X$$

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[2] See also the simpler proof given by Ben-Israel and Mond in [13] and for a wider discussion on this topic the book by Giorgi and Mishra [8].

[3] In the definition by Zalmai [19] the parameter ρ is not limited in sign.

- *strong* $(F, b, \varphi, \rho, \vartheta)$*-pseudounivex* if there exists $F : (X \times X \times \Re^n) \to \Re$, with $F(x_1, x_2, 0) \geq 0$ $\forall x_1, x_2 \in X$, there exists $\varphi : \Re \to \Re$, with $\varphi(z) < 0$ $\forall z \in \Re$ such that $z < 0$, there exists $\vartheta : (X \times X) \to \Re^n$, there exists $\rho \in \Re$, $\rho > 0$, there exists $b : (X \times X) \to \Re$, with $b(x_1, x_2) > 0$ $\forall x_1, x_2 \in X$, such that:

$$\varphi(f(x_1) - f(x_2)) < 0 \Longrightarrow F(x_1, x_2, b(x_1, x_2)\nabla f(x_2)) < -\rho\|\vartheta(x_1, x_2)\|^2 \quad \forall x_1, x_2 \in X$$

The F-convex functions have been introduced by Hanson and Mond in [11] and for this very reason they are also called Hanson and Mond functions; Craven and Glover in [7] prove that this class of function is nothing but the class of invex function. Furthermore, Caprari [4] proves the equivalence between the strong F-convexity and the strong pseudoinvexity.

Referring again to Theorem 1 and Corollary 1, strong pseudoinvex is nothing but a particular case of property *B6)* (let us recall to see $\xi \in K(x_2)$ as $\nabla f(x_2)$), while F-convex comes from *C1)* by replacing $\xi \in K(x_2)$ with $\nabla f(x_2)$ and by setting $\psi(f(x_1), f(x_2)) = f(x_1) - f(x_2)$ and $b(x_1, x_2) = 1$. Analogously, F-pseudoconvex and strong F-pseudoconvex are particular cases of properties *C2)* and *C6)*, respectively (let $\xi \in K(x_2)$ be $\nabla f(x_2)$, $\psi(f(x_1), f(x_2)) = f(x_1) - f(x_2)$ and $b(x_1, x_2) = 1$). Finally, $(F, b, \varphi, \rho, \vartheta)$-univexity, $(F, b, \varphi, \rho, \vartheta)$-pseudounivexity, strong $(F, b, \varphi, \rho, \vartheta)$-univexity and strong $(F, b, \varphi, \rho, \vartheta)$-pseudounivexity can be obtained from *C1)*, *C2)*, *C3)* and *C4)*, respectively, by setting $\psi(f(x_1), f(x_2)) = \varphi(f(x_1) - f(x_2))$.

Therefore, we are able to extend the analysis started first by Craven and Glover and later by Caprari. As a trivial consequence of Theorem 1 and Corollary 1 we show that F-convexity coincides with both F-pseudoconvexity, strong pseudoinvexity and $(F, b, \varphi, \rho, \vartheta)$-pseudounivexity. Hence, all the previously recalled classes coincide with the invex one.

Theorem 2 *Let $f : X \to \Re$ be a differentiable scalar function on the open convex set $X \subseteq \Re^n$. The following conditions are equivalent:*

i) f is semi-pseudoconvex;

ii) f is invex;

iii) f is pseudoinvex;

iv) f is F-convex;

v) f is F-pseudoconvex;

vi) f is strong pseudoinvex;

vii) f is strong F-pseudoconvex;

viii) f is $(F, b, \varphi, \rho, \vartheta)$-univex;

ix) f is $(F, b, \varphi, \rho, \vartheta)$-pseudounivex;

x) f is strong $(F, b, \varphi, \rho, \vartheta)$-univex;

xi) f is strong $(F, b, \varphi, \rho, \vartheta)$-pseudounivex.

Deeply speaking, some of the definitions presented in the recent literature with the aim to generalize the concept of invexity do not provide actually any kind of generalization (along this line see also [4, 8, 18]). Moreover, the use of parameter functions aimed to look for invexity generalizations is useless, since those class of functions coincides with the one of semi-pseudoconvex functions which is related only to the behavior of the gradient of function f and does not involve any kind of parameter functions.

Remark 1 *It is worth noticing that in the recent literature different concepts of invexity have been introduced by means of a different use of the parameter function* $\eta(x_1, x_2)$. *Concepts stronger than the ones previously recalled have been proposed by requiring the parameter function* $\eta(x_1, x_2)$ *to be "a priori" fixed [12]:*

- *given a function* $\eta : (X \times X) \to \Re^n$, f *is said to be* η*-invex if the following property holds*
$$f(x_1) - f(x_2) \geq \nabla f(x_2)^T \eta(x_1, x_2) \ \forall x_1, x_2 \in X$$
- *given a function* $\eta : (X \times X) \to \Re^n$, f *is said to be* η*-pseudoinvex if the following property holds*
$$f(x_1) < f(x_2) \Longrightarrow \nabla f(x_2)^T \eta(x_1, x_2) < 0 \ \forall x_1, x_2 \in X$$

These properties hold for a specific parameter function $\eta(x_1, x_2)$ *and for this very reason the classes of* η*-invex and of* η*-pseudoinvex functions are strictly contained in the ones of invex and of pseudoinvex functions, respectively. As a consequences, the concept of semi-pseudoconvexity (based on no parameter functions) generalizes both* η*-invexity and* η*-pseudoinvexity (based on the parameter function* $\eta(x_1, x_2)$*). For an exhaustive discussion about the use of parameter* $\eta(x_1, x_2)$ *see [8].*

3.2 The nondifferentiable case

Invexity properties have been extended in the non differentiable case following various approaches, such as using Dini derivatives [9, 15] or Clarke's subdifferential [4, 7, 8]. The latter one has been the most used and will be analyzed in this subsection. Specifically speaking, invexity properties can be extended to the nondifferentiable case by assuming function f to be locally Lipschitz and by using the Clarke's subdifferential of f at x, denoted with $\partial^c f(x)$, which results to be nonempty, convex and compact due to the local Lipschitzianity of f. In this light, some of the "generalized" invexity properties discussed so far can be rewritten as follows:

- *invexity* : there exists a function $\eta : (X \times X) \to \Re^n$ such that
$$f(x_1) - f(x_2) \geq \xi^T \eta(x_1, x_2) \quad \forall \xi \in \partial^c f(x_2), \ \forall x_1, x_2 \in X$$
- *pseudoinvexity* : there exists a function $\eta : (X \times X) \to \Re^n$ such that
$$f(x_1) < f(x_2) \quad \Rightarrow \quad \xi^T \eta(x_1, x_2) < 0 \quad \forall \xi \in \partial^c f(x_2), \ \forall x_1, x_2 \in X$$
- *semi-pseudoconvexity* : it holds:
$$f(x_1) < f(x_2) \quad \Rightarrow \quad \xi \neq 0 \quad \forall \xi \in \partial^c f(x_2), \ \forall x_1, x_2 \in X$$
- *F-convexity* : there exists a functional $F : (X \times X \times \Re^n) \to \Re$, with $F(x_1, x_2, 0) = 0$ $\forall x_1, x_2 \in X$, such that
$$f(x_1) \geq f(x_2) + F(x_1, x_2, \xi) \quad \forall \xi \in \partial^c f(x_2), \ \forall x_1, x_2 \in X$$
- *F-pseudoconvexity* : there exists a functional $F : (X \times X \times \Re^n) \to \Re$, with $F(x_1, x_2, 0) = 0$ $\forall x_1, x_2 \in X$, such that
$$f(x_1) < f(x_2) \Longrightarrow F(x_1, x_2, \xi) < 0 \quad \forall \xi \in \partial^c f(x_2), \ \forall x_1, x_2 \in X$$
- *strong pseudoinvexity* : for every $\vartheta : (X \times X) \to \Re^n$ there exists $\eta : (X \times X) \to \Re^n$ and there exists $\rho \in \Re$, $\rho > 0$, such that
$$f(x_1) < f(x_2) \Longrightarrow \xi^T \eta(x_1, x_2) < -\rho \|\vartheta(x_1, x_2)\|^2 \quad \forall \xi \in \partial^c f(x_2), \ \forall x_1, x_2 \in X$$

- *strong F-pseudoconvexity* : for every $\vartheta : (X \times X) \to \Re^n$ there exists a functional $F : (X \times X \times \Re^n) \to \Re$, with $F(x_1, x_2, 0) \geq 0 \ \forall x_1, x_2 \in X$, and there exists $\rho \in \Re$, $\rho > 0$, such that

$$f(x_1) < f(x_2) \implies F(x_1, x_2, \xi) < -\rho\|\vartheta(x_1, x_2)\|^2 \quad \forall \xi \in \partial^c f(x_2), \ \forall x_1, x_2 \in X$$

- $(F, b, \varphi, \rho, \vartheta)$*-univexity* : there exists $F : (X \times X \times \Re^n) \to \Re$, with $F(x_1, x_2, 0) \geq 0 \ \forall x_1, x_2 \in X$, there exists $\varphi : \Re \to \Re$, with $\varphi(z) < 0 \ \forall z \in \Re$ such that $z < 0$, there exists $b : (X \times X) \to \Re$, with $b(x_1, x_2) > 0 \ \forall x_1, x_2 \in X$, such that:

$$\varphi(f(x_1) - f(x_2)) \geq F(x_1, x_2, b(x_1, x_2)\xi) \quad \forall \xi \in \partial^c f(x_2), \ \forall x_1, x_2 \in X$$

- $(F, b, \varphi, \rho, \vartheta)$*-pseudounivexity* : there exists $F : (X \times X \times \Re^n) \to \Re$, with $F(x_1, x_2, 0) \geq 0 \ \forall x_1, x_2 \in X$, there exists $\varphi : \Re \to \Re$, with $\varphi(z) < 0 \ \forall z \in \Re$ such that $z < 0$, there exists $b : (X \times X) \to \Re$, with $b(x_1, x_2) > 0 \ \forall x_1, x_2 \in X$, such that:

$$\varphi(f(x_1) - f(x_2)) < 0 \implies F(x_1, x_2, b(x_1, x_2)\xi) < 0 \quad \forall \xi \in \partial^c f(x_2), \ \forall x_1, x_2 \in X$$

- *strong* $(F, b, \varphi, \rho, \vartheta)$*-univexity* : there exists $F : (X \times X \times \Re^n) \to \Re$, with $F(x_1, x_2, 0) \geq 0 \ \forall x_1, x_2 \in X$, there exists $\varphi : \Re \to \Re$, with $\varphi(z) < 0 \ \forall z \in \Re$ such that $z < 0$, there exists $\vartheta : (X \times X) \to \Re^n$, there exists $\rho \in \Re$, $\rho > 0$, there exists $b : (X \times X) \to \Re$, with $b(x_1, x_2) > 0 \ \forall x_1, x_2 \in X$, such that:

$$\varphi(f(x_1) - f(x_2)) \geq F(x_1, x_2, b(x_1, x_2)\xi) + \rho\|\vartheta(x_1, x_2)\|^2 \quad \forall \xi \in \partial^c f(x_2), \ \forall x_1, x_2 \in X$$

- *strong* $(F, b, \varphi, \rho, \vartheta)$*-pseudounivexity* : there exists $F : (X \times X \times \Re^n) \to \Re$, with $F(x_1, x_2, 0) \geq 0 \ \forall x_1, x_2 \in X$, there exists $\varphi : \Re \to \Re$, with $\varphi(z) < 0 \ \forall z \in \Re$ such that $z < 0$, there exists $\vartheta : (X \times X) \to \Re^n$, there exists $\rho \in \Re$, $\rho > 0$, there exists $b : (X \times X) \to \Re$, with $b(x_1, x_2) > 0 \ \forall x_1, x_2 \in X$, such that:

$$\varphi(f(x_1) - f(x_2)) < 0 \implies F(x_1, x_2, b(x_1, x_2)\xi) < -\rho\|\vartheta(x_1, x_2)\|^2 \quad \forall \xi \in \partial^c f(x_2), \ \forall x_1, x_2 \in X$$

By assuming $K(x) = \partial^c f(x)$ in Theorem 1 and in Corollary 1 it is then trivial to prove that all the equivalences given in Theorem 2 hold in the nondifferentiable case too.

Theorem 3 *Let $f : X \to \Re$, with $X \subseteq \Re^n$ open and convex set, be locally Lipschitz and let $\partial^c f(x)$ be the Clarke's subdifferential of f at x. Then, conditions i)-xi) given in Theorem 2 are equivalent.*

4 Conclusions

In this paper we prove the equivalence of various classes of "generalized" invex functions pointing out that the use of parameters and functionals does not yield "a priori" any kind of generalization. This has been shown in both the differentiable case and the nondifferentiable one. In this latter case locally Lipschitz functions are considered as well as their Clarke's subdifferential.

Looking at further classes of generalized invex functions, Mititelu [14] shows preinvexity and prepseudoinvexity are equivalent to invexity and pseudoinvexity respectively. Therefore, taking into account our equivalence results we get that even prepseudoinvexity coincides with invexity. It is worth noticing that we just consider differentiable functions on an open set, while Mititelu [14] and Mititelu and Postoloache [15] deal with non-smooth functions on arbitrary set by using the upper Dini derivatives. A possible extension of our analysis is to consider invexity properties based on the use of upper Dini derivatives.

It is worth noticing that the general formulation of Theorem 1 and Corollary 1 could suggest further equivalence results among other classes of "generalized" invex functions the interested reader can find in the huge literature on this topic.

Regarding vector valued functions, in [6] Craven presents various inclusions among different classes of generalized invexity. Following his lines and taking into account our equivalence results it could be interesting analyzing how they can be extended to the vector case.

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